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REUSE OF DRAINAGE WATER PROJECT

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REUSE OF DRAINAGE WATER MODEL

Calculation method of drainage water and watertable depth

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REUSE OF DRAINAGE WATER MODEL

Calculation method of drainage water and watertable depth

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C O N T E N T S

	blz.
1. INTRODUCTION	1
2. SOIL WATER BALANCE	2
2.1. Ponded water case	4
2.2. Non-ponded water case	5
3. DRAINAGE OF RICE FIELDS	7
4. DESCRIPTION OF INPUT AND OUTPUT	11
4.1. Input and output	12
4.2. Programme structure	12
4.2.1. Initialization of constants	14
4.2.2. Drainage of rice fields	14
4.2.3. Drainage of non-rice fields	16
LITERATURE	18

1. INTRODUCTION

The Reuse of Drainage Water Project is a research project financed by the Ministry of Irrigation in Egypt and the Ministry of Foreign Affairs in the Netherlands. The responsability for the implementation of this research has been delegated to the Drainage Research Institute (DRI) in Egypt and the Institute for Land and Water Management Research (ICW) in the Netherlands. The main objective of the project is to assist the Ministry of Irrigation in Egypt in the planning of future water-management strategies incorporating reuse of drainage water practices. In order to achieve this main objective a comprehensive measurement programme has been initiated and a mathematical model is being formulated for the prediction of future effects of different watermanagement strategies.

In the model approach the agricultural crop and its reaction to different watermanagement practices is of prime importance. A separate model has been formulated to calculate the irrigation water distribution between the subarea's distinguished in the Nile Delta (RIJTEMA and BOELS, 1985). On subarea level a model has been formulated simulating the farmers behaviour with respect to the (unofficial) reuse of drainage water if the irrigation water supply is insufficient.

For each identified subregion and for each crop present in this subregion the calculation of crop water use, drainage rate and soil salinity forms the core of the Reuse of Drainage Water Model. This part of the model can be subdivided into four related submodels: the calculation of the irrigation efficiency, the calculation of the actual evapotranspiration, the calculation of drainage rates and the calculation of soil and drainage water salinity. The irrigation efficiency submodel (BOELS, 1986) and the actual evapotranspiration submodel (ABDEL KHALIK, ROEST and RIJTEMA, 1986) have been formulated and programmed. On the calculation of salinity a separate report will be issued. In the present report the calculation of drainage rates will be treated.

Recently RIJTEMA and ABDEL KHALIK (1985) formulated an analytical solution of the watertable behaviour and drainage rates under seepage and leakage conditions acounting for the differences in phreatic water level between two parallel drains. This solution, however accurate, involves a summation of terms rendering it impractical for application on a regional scale due to the computer time required. In the present approach the average phreatic waterlevel will be considered and the resistance against drainage flow will be assumed near the drains. Under seepage conditions capillary rise promoted by the wateruptake of plants may affect the watertable and thus the drainage rate.

For rice fields two waterbalances are involved: the waterbalance for the standing water layer and the waterbalance for the phreatic water. These balances are solved taking into account the possibilities of entrapped air between the phreatic water level and the saturated soil surface associated with rice field conditions.

2. SOIL WATER BALANCE

In the reuse of drainage water model two soil water storages are distinguished: the unsaturated storage reservoir and the saturated storage reservoir. The unsaturated storage reservoir and its interactions with the saturated reservoir (capillary rise) and the atmosphere (evapotranspiration) has been treated by ABDEL KHALIK et al (1986). The present report deals with the saturated storage reservoir, the contents of which is also referred to as 'drainable water', and its interactions with drainage to and from the aquifer and the drainage system.

During irrigation of the soil both reservoirs are filled to a certain degree. The distribution of the irrigation water over both reservoirs depends on the soil moisture deficit before irrigation, the crackvolume of the soil and the quantity of irrigation water applied (BOELS, 1986). The quantity of drainable water has been defined with respect to drainage depth:

$$M_{dr} = \mu\, h(t) \qquad (1)$$

where: M_{dr} = quantity of drainable water in m
μ = drainable porosity ($= \theta_s - \theta_{fe}$)
θ_s = moisture fraction at saturation
θ_{fc} = moisture fraction at field capacity
$h(t)$ = height of phreatic level in m above drain depth at time t

For the mathematical description of the drain discharge the head losses due to the restricted permeability of the soil are considered concentrated at the location of the drain. Neglecting the contribution of the saturated soil system above drainage level, and assuming equidistant parallel drains, this drainage resistance can be given (ERNST, 1962):

$$C_d = wL + \frac{L^2}{8kD} \qquad (2)$$

where: C_d = drainage resistance in days
w = entrance resistance of the drain in $d.m^{-1}$
L = drain distance in m
k = saturated permeability in $m.d^{-1}$
D = depth of the layer participating in the horizontal drainage flux in m

For the parameter D the distance between drainage depth and a resistance layer above the aquifer is used. If such a resistance layer is found at a depth below $\frac{1}{4}L$ this value can be taken as $\frac{1}{4}L$ and equation (2) reduces to:

$$C_d = wL + \frac{L}{4k} \quad \text{if} \quad D \geq \tfrac{1}{4}L \qquad (3)$$

Four fluxes from the saturated groundwater system have to be considered in the soil water balance:

f_d - drainage flux in $m.d^{-1}$
f_l - leakage/seepage flux in $m.d^{-1}$
f_c - capillary rise flux in $m.d^{-1}$
E - evapotranspiration flux in $m.d^{-1}$

The drainage flux is positive when the groundwater table is above drainage depth and zero when the phreatic level is below drainage depth. The leakage/seepage flux is positive in the case of leakage (phreatic level above aquifer piezometric head) and negative in the case of seepage (aquifer pressure above phreatic water level). The capillary flux is assumed to occur under seepage conditions only (aquifer piezometric head above drain depth) when the moisture fraction in the rootzone is below field capacity (ABDEL KHALIK et al, 1986). Evapotranspiration directly from the saturated groundwater reservoir is assumed only under ponded conditions (water on the land surface). The model formulation for ponded conditions is significantly different from non-ponded conditions. The occurrence of surface drainage is treated in the irrigation efficiency submodel (BOELS, 1986).

2.1. Ponded water case

For the ponded water case the mass balance equation for the drainable water can be formulated:

$$\frac{d\,h(t)}{dt} = - f_d - f_l - E \qquad h(t) \geq d_d \tag{4}$$

where d_d = drainage depth in m.

This equation if valid as long as the water level is above field surface. The drainage flux is directly related to the phreatic level:

$$f_d = \frac{h(t)}{C_d} \qquad h(t) \geq 0 \tag{5}$$

The leakage/seepage flux is related to the water level and the aquifer piezometric head:

$$f_l = \frac{h(t) - h_{aq}}{C_{aq}} \tag{6}$$

where: h_{aq} = aquifer piezometric pressure in m above drain level
C_{aq} = vertical resistance in days for the leakage/seepage flow

During the ponding period evapotranspiration is assumed to extract water from the ponded water. If a crop is present extraction will actually take place by the plant roots from the effective rootzone, but the extracted water will be imediately replaced by the ponded water. The actual evapotranspiration rate has been calculated in the evapotranspiration submodel (ABDEL KHALIK et al, 1986) and is input for the present model.

2.2. Non-ponded water case

For phreatic levels below soil surface drainage is assumed to take place from the drainable water reservoir defined by the drainable porosity:

$$\mu \frac{d\ h(t)}{dt} = - f_d - f_l - f_c \qquad (7)$$

For the drainage flux equation (5) is valid as long as the phreatic waterlevel is above drain depth (is positive). For waterlevels below drain depth the drains fall dry and the drainage flux becomes zero. For the leakage/seepage flux equation (6) is used and the capillary rise flux has been calculated in the evapotranspiration sub-model and is input in the present model.

2.3. Soil water balance algorithm

By substitution of the valid relations for the drainage fluxes three solutions of the soil water balance are found. All three can be reduced to the general form:

$$\frac{d\ h(t)}{dt} = B - A\ h(t) \qquad (8)$$

where: A = parameter with dimension d^{-1}
B = parameter with dimension $m.d^{-1}$

The value for the parameters A and B can be found by substitution of equations (5) and (6) in equation (4) and (7) respectively and are given in table 1.

Table 1. Parameter values drainage model

Condition	A	B	h_b
$h(t) \geq d_d$	$1/C_{aq} + 1/C_d$	$h_{aq}/C_{aq} - E$	d_d
$0 \leq h(t) < d_d$	$1/\mu C_{aq} + 1/\mu C_d$	$h_{aq}/\mu C_{aq} - f_c/\mu$	0
$h(t) < 0$	$1/\mu C_{aq}$	$h_{aq}/\mu C_{aq}$	-

In table (1) the boundary value for h(t) restricting the validity of the relation has been included. The general solution of equation (8) for the boundary condition $h = h(t_o)$ for $t = t_o$ gives:

$$h(t) = \frac{B}{A} + \{h(t_o) - \frac{B}{A}\} e^{-At} \qquad (9)$$

In principle equation (9) can be applied only on the condition that parameter A does not equal zero. The only possibility for A to become zero is for the condition of a phreatic level below drain depth and an aquifer resistance approaching infinity. Since these two conditions are contradicting A will never become zero.

The validity of equation (9) is restricted by the boundary value of the validity of the A and B parameters (table 1). The corresponding time period can be calculated by solving equation (9) for t taking the boundary value equal to h(t):

$$T = \frac{1}{A} \ln \left\{ \frac{A\, h(t_o) - B}{Ah_b - B} \right\} \qquad (10)$$

where: T = period of validity of parameters A and B in days

h_b = boundary value of h(t) in m

The cumulative drain discharge during the period T can be calculated by integrating equation (9) with respect to time and dividing the result by the drain resistance:

$$F_d = \frac{1}{C_d} \left\{ \frac{B}{A} T + \frac{1}{A} \left[h(t_o) - \frac{B}{A} \right] (1 - e^{-AT}) \right\} \qquad (11)$$

where: F_d = cumulative draindischarge in m during the period T

The cumulative leakage/seepage flux can be calculated as the balance of initial and final waterlevel and drainage, capillary and evapotranspiration flux respectively.

3. DRAINAGE OF RICE FIELDS

Land preparation for the rice fields by the Egyptian farmers starts with a dry tillage, crusting the large soil elements into finer ones. This dry crusting procedure is followed by dry landleveling. By this procedure the fine soil elements are transported into the cracks that are at this time in the season, following the wheat crop, maximally developed. During the succeeding pre-irrigation the dried soil starts swelling and, because the cracks have been filled, the soil becomes more compacted than prior to land preparation. Mathematically the reduction in permeability associated with a higher soil compaction can be described by a resistance layer in the first few centimeters of the soil.

During pre-irrigation the larger pores and the cracks in the soil will be filled with irrigation water in first instance. Assuming that the swelling of the clay soils is a fast process, the soil will become saturated at the surface and air entrapped in the larger soil elements below soil surface cannot escape. Due to the soil matrix potential the infiltrated water will be redistributed in time.

Based on these assumptions the volume of water that infitrates into the soil during the first few minutes after irrigation starts can be formulated:

$$V_i = V_c + \mu \{ d_d - h(t_o) \} \qquad (12)$$

where: V_i = volume of water in m that infiltrates into the soil during the first few minutes after irrigation

V_c = crack volume of the soil in m

The crack volume depends on the initial waterlevel, the initial moisture content and the soil type (BOELS, 1986). The volume of entrapped air also depends on the initial moisture content:

$$V_a = M_o - M(t_o) - V_c \qquad (13)$$

where: V_a = volume of entrapped air in m in the soil
M_o = available moisture for evapotranspiration at field capacity in m
$M(t_o)$= initial available moisture in m

Assuming a redistribution of moisture and air in such a way that the soil reaches field capacity the air volume must be stored in the drainable pore space. The waterlevel immediately after irrigation follows then:

$$h(t) = d_d - \frac{V_a}{\mu} \qquad (14)$$

Assuming further that the entrapped air is not compressible and cannot escape as long as the topsoil is saturated (a standing water layer is present) and assuming that the moisture content in the unsaturated zone remains at field capacity the drain discharge can be described by the piezometric pressure at drain level, exerted by the standing water layer:

$$f_d = \frac{h_p(t)}{C_d} \qquad h_p(t) \geq 0 \qquad (15)$$

where: $h_p(t)$ = piezometric pressure in m at drain depth.

For the leakage/seepage flux the formulation is similar:

$$f_l = \frac{h_p(t) - h_{aq}}{C_{aq}} \qquad (16)$$

Due to the standing waterlayer the entrapped air will be under pressure. This pressure will influence the infiltration flux from the standing waterlayer into the soil:

$$f_i = \frac{h^*(t) - h_a(t)}{C_p} \qquad h^*(t) \geq 0 \qquad (17)$$

where: f_i = infiltration flux at the soil surface in $m.d^{-1}$
$h^*(t)$ = standing waterlayer in m
$h_a(t)$ = air pressure expressed in m water column with respect to atmospheric pressure

Due to the assumption that the soil moisture storage does not change the infiltration flux must be equal to the drainage and leakage/seepage flux as long as the piezometric pressure at drain level is possitive:

$$f_i = f_d + f_l \qquad h_p(t) \geq 0 \qquad (18)$$

Introduction of the relation between air pressure and piezometric pressure at drain level:

$$h_a(t) = h_p(t) - h_o \qquad (19)$$

where: h_o = (constant) waterlevel above field drains in m

and combining with equations (15-18) gives the following expression for the piezometric pressure:

$$h_p(t) = \frac{\frac{h_{aq}}{C_{aq}} + \frac{h^*(t) + h_o}{C_p}}{\frac{1}{C_{aq}} + \frac{1}{C_d} + \frac{1}{C_p}} \qquad h_p(t) \geq 0 \qquad (20)$$

The validity of equation (20) is given by a positive piezometric pressure at drain level. This condition can be translated into a boundary value for the standing waterlayer by taking equation (20) equal to zero; and solving for $h^*(t)$:

$$h_b^* = - h_{aq} \frac{C_p}{C_{aq}} - h_o \qquad (21)$$

where: h_b^* = boundary value for the standing water layer in m at which the drain discharge becomes zero

For values of the standing water layer below the boundary value the piezometric pressure is negative and the infiltration flux at soil surface equals the leakage/seepage flux:

$$f_i = f_l \qquad h^*(t) < h_b^* \qquad (18a)$$

Solution of this equation for the piezometric pressure gives:

$$h_p(t) = \frac{\frac{h_{aq}}{C_{aq}} + \frac{h^*(t) + h_o}{C_p}}{\frac{1}{C_{aq}} + \frac{1}{C_p}} \qquad h^*(t) < h_b^* \qquad (20a)$$

Assuming that water used for transpiration by the plant canopy is extracted by the plant roots from the first few centimeters of saturated soil above the puddled layer the moisture balance of the standing water layer can be drafted:

$$\frac{d\, h^*(t)}{dt} = - E - f_d - f_l \qquad (22)$$

The general form of this equation is identical to equation (8):

$$\frac{d\, h^*(t)}{dt} = B - A\, h^*(t) \qquad (23)$$

where: B = parameter with dimension $m.d^{-1}$
A = constant with dimension d^{-1}

By substitution of equation (15) and (16) into equation (22) and inserting the expression for the piezometric pressure at draindepth, equation (20) and solving equation (23) for A and B gives for the different boundary conditions:

$$A = \frac{1}{C_{aq} + C_p} \qquad 0 \leq h^*(t) \leq h_b^* \tag{24a}$$

$$A = \frac{C_{aq} + C_d}{C_p C_{aq} + C_d C_{aq} + C_d C_p} \qquad h^*(t) > h_b^* \quad \text{and} \quad h^*(t) > 0 \tag{24b}$$

$$B = \frac{h_{aq}}{C_{aq}} - E - \frac{h_{aq} \frac{C_p}{C_{aq}}}{C_p + C_{aq}} \qquad 0 \leq h^*(t) \leq h_b^* \tag{25a}$$

$$B = \frac{h_{aq}}{C_{aq}} - E - \frac{h_{aq} C_p \left[1 + \frac{C_d}{C_{aq}}\right] + h_o (C_{aq} + C_d)}{C_p C_{aq} + C_d C_{aq} + C_p C_d}$$

$$h^*(t) > h_b^* \quad \text{and} \quad h^*(t) > 0 \tag{25b}$$

The general solution of equation (23) has been given by equation (9). The validity of this equation can be calculated using equation (10) with the proper boundary conditions ($h^* = h_b^*$ or $h^* = 0$).

The cumulative drain discharge during the period of validity T of equation (23) can be found by integrating equation (15) after substitution of equation (20):

$$F_d = \frac{(h_{aq} C_p + h_o C_{aq})\, T + C_{aq} \left\{\frac{B}{A} T + \frac{1}{A}\left[h^*(t_o) - \frac{B}{A}\right]\left[1 - e^{-AT}\right]\right\}}{C_p C_{aq} + C_d C_{aq} + C_d C_p} \tag{26}$$

The cumulative leakage/seepage flux can be found as the balance of initial and final standing waterlayer and the drainage and evapotranspiration fluxes.

4. DESCRIPTION OF INPUT AND OUTPUT

The drainage submodel is one of the four submodels calculating the water and salt balance of an area with one crop for one irrigation interval. These submodels are: the irrigation efficiency submodel (BOELS, 1986); the evapotranspiration submodel (ABDEL KHALIK et al, 1986); the drainage water generation submodel, and the salt distribution submodel.

4.1. Input and output

The following input is required for the drainage submodel:

$h(t_o)$ - initial height of the watertable (m)
$h^*(t_o)$ - initial height of the standing waterlayer (m)
E - evapotranspiration rate ($m.d^{-1}$)
f_c - capillary flux ($m.d^{-1}$)
μ - drainable porosity
C_p - resistance puddled layer (days)
C_d - drainage resistance (days)
C_{aq} - seepage/leakage resistance (days)
t - length of the irrigation interval (days)
d_d - drainage depth (m)
h_{aq} - aquifer pressure above draindepth (m)
rice code - in case of rice crop this code should equal 1

The following output is produced by the drainage subroutine:

h(t) - final height of the watertable (m)
h*(t) - final height of standing waterlayer (m)
f_d - average drainage flux ($m.d^{-1}$)
f_l - average leakage (positive) /seepage (negative) flux ($m.d^{-1}$)

4.2. Programme structure

In the first part of the programme the constants required in the calculations are determined (fig. 1).

If the crop is a rice crop and initially a standing water layer is present on the field the standing water balance has to be used to calculate the drainage. If the rice fields fall dry during the time step the remaining period will be calculated according to the phreatic water balance algorithm.

The cumulative drainage water quantity is calculated during each distinguished part of the time step and in the final part of the programme the leakage/seepage is determined from the balance of initial and final waterlevel and standing waterlayer, the cumulative drainage and the actual evapotranspiration during ponding time and the capillary flux calculated in the 'EVA' submodel.

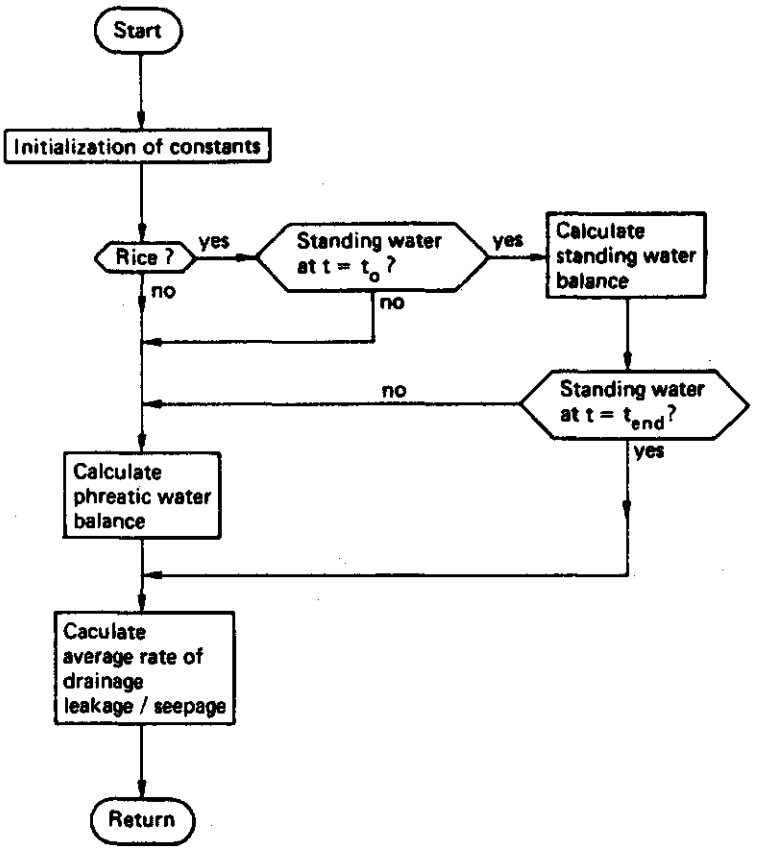

Fig. 1. General structure of subroutine 'DRAGE'

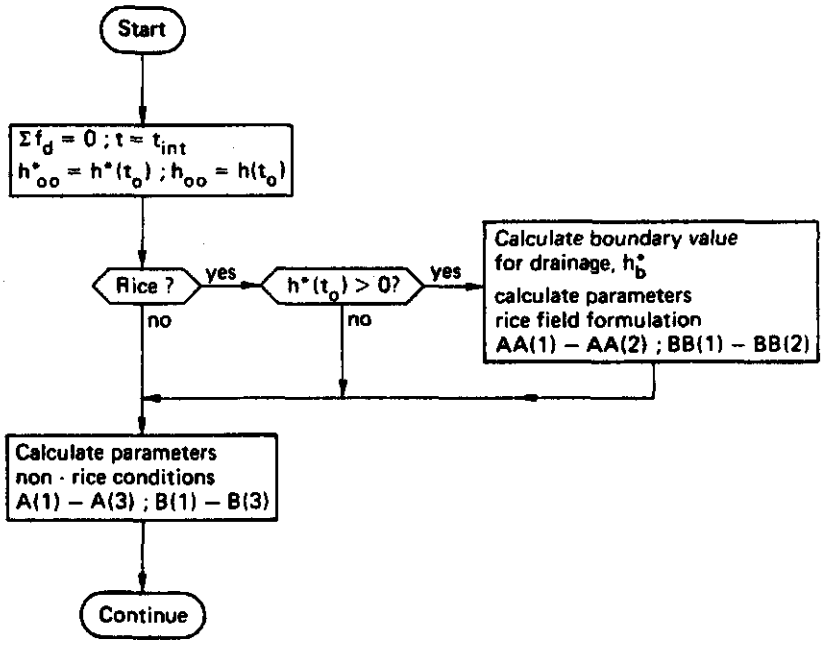

Fig. 2. Flow programme 'Initialization of constants' in subroutine 'DRAGE'

4.2.1. Initialization of constants

In this section of the programme (fig. 2) the sum used for the cumulative drainage calculation is put at zero. The initial values of the irrigation interval, the initial waterlevel and the initial standing waterlayer have to be saved, because during the calculation process their values may be changed.

For rice field conditions (rice code equals 1) and in the presence of an initial standing waterlayer the parameters A and B (equation 24 and 25) are calculated. The boundary value for the standing water layer at which the drainage system stops to function due to the piezometric pressure (equation 21) is calculated.

Because at this moment during programme execution it is not known whether the rice fields will fall dry in all cases (rice fields and non-rice fields) the A and B parameters for non-rice conditions (table 1) are calculated.

4.2.2. Drainage of rice fields

The drainage of rice fields is treated separately due to the presence of a resistance layer in the topsoil and the possibility of entrapped air in the soil profile. If the rice code equals 1 and initially a standing waterlayer is on the field the final value for the waterlevel is equal to the initial value (no changes in water storage below the soil surface). If the boundary value for the standing waterlayer is positive and the initial standing waterlayer is above this boundary value the time to reach this boundary value is calculated with function 'TT' where equation (10) has been programmed. If this boundary value is not reached within the time step the conditions will not change and the final standing waterlayer is calculated with the function 'DIF' where equation (9) has been programmed and the drainage water quantity with equation (26) where the time-integrated value of the standing waterlayer is calculated with the function 'HINT'. In this case during the complete time-step the soil has been ponded and the evapotranspiration has been assumed from the standing waterlayer ($T_{rice} = t_{oo}$) (fig. 3).

If the boundary value is reached within the time step the drainage is calculated during this part of the time step only. The initial value for the standing waterlayer for the remaining part of the time step equals the boundary value in this case.

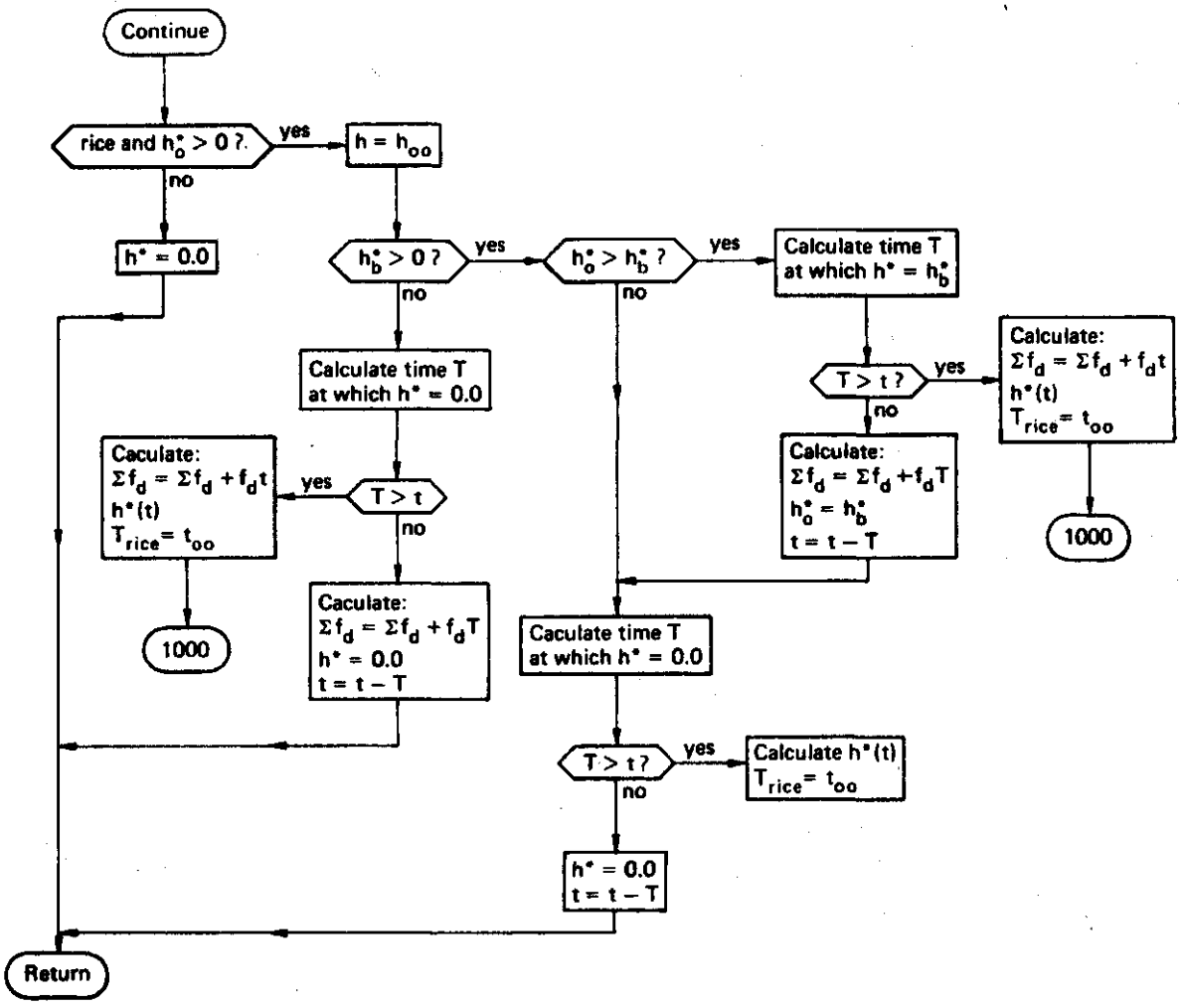

Fig. 3. Flow diagramme 'standing water balance' in subroutine 'DRAGE'

During the remaining part of the time step, untill the rice field falls dry no drainage will take place, in this case (standing water-layer below boundary value). The procedure of calculation is similar to the case of the standing waterlayer above the boundary value.

If the boundary value is less than zero this parameter will not affect the calculations and drainage will take place untill the rice fields fall dry. Again, the calculation procedure (calculate the time at which the field falls dry; check whether this happens within the irrigation interval; calculate final standing waterlayer and drainage quantity) is similar to the previous cases.

If the boundary values (h_b^* and/or 0.0) have not been reached execution has been transferred to statement 1000 where the average drainage and leakage/seepage rate are calculated. In all other cases during the remaining part of the time step drainage will be calculated as for the non-rice fields.

4.2.3. Drainage of non-rice fields

For the non-rice fileds the calculations are based on the 'drainable water' balance. First, the ponded case is treated by testing whether the initial waterlevel is above soil surface. If this is the case the time required for the waterlevel to reach the soil surface is calculated using the function 'TT'. If the waterlevel does not reach the soil surface the final waterlevel is calculated with equation (11) where the time-integrated value of the waterlevel is calculated in function 'HINT'. If the field falls dry during the time step the initial waterlevel for the remaining part of the time step equals drainage depth. For the calculation of the drainage quantity only the ponding time is used in function 'HINT' in this case.

The total ponding time (including the rice field conditions) is calculated as the difference between the irrigation interval and the remaining part of the time step with non-ponded conditions (fig. 4).

If a capillary flux has been calculated in the 'EVA' subroutine this flux will apply only to the non-rice, non-ponded conditions. Because the output of subroutine 'EVA' is given as average fluxes during the irrigation interval and because capillary fluxes by definition (ABDEL KHALIK et al, 1986) do not occur under rice field conditions and when the moisture fraction in the soil root zone is at or above field capacity a correction is made for the B coefficient for this case, if T_{rice} and f_c are greater than zero (table 1):

$$B = \frac{h_{aq}}{\mu\, C_{aq}} - \frac{f_c}{\mu} - f_c \frac{t_i - T_{rice}}{\mu\, t} \qquad (27)$$

where: t_i = irrigation interval length (days)
T_{rice} = ponded period in days
t = remaining part of the time step in days with non-ponded conditions

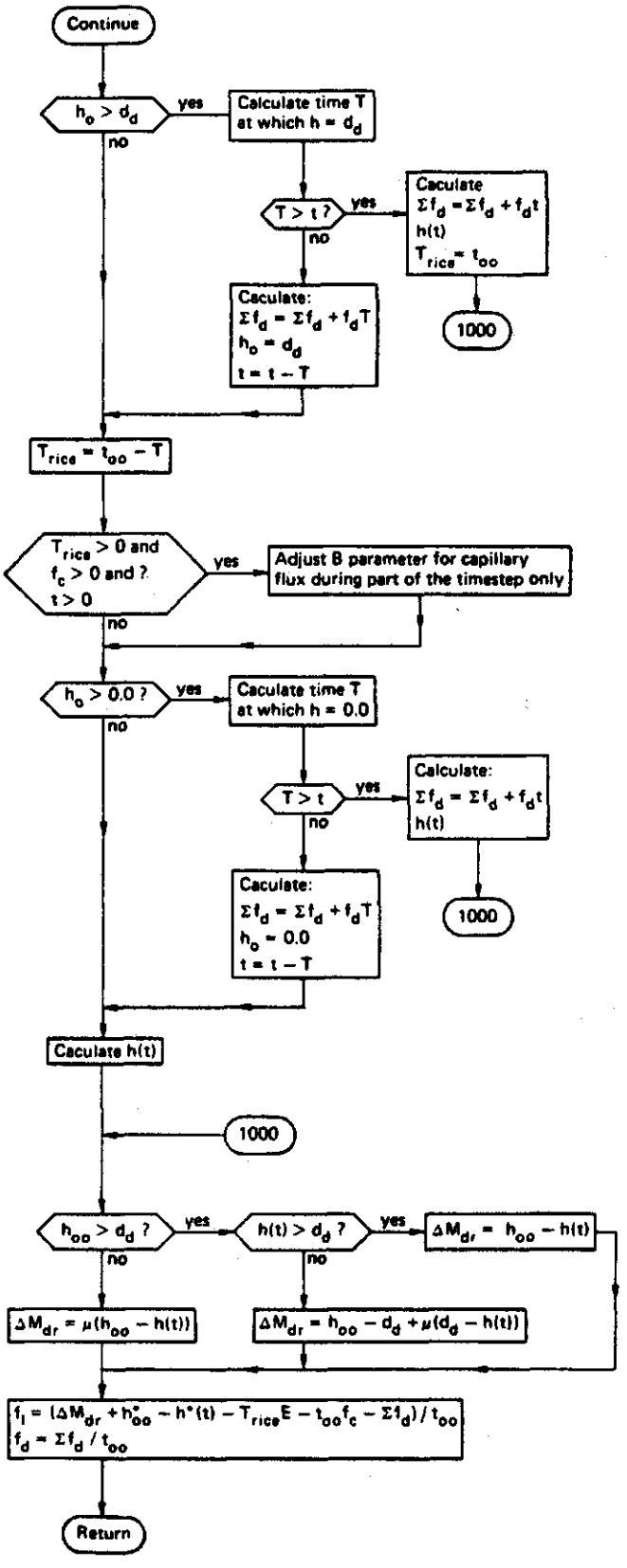

Fig. 4. Flow diagramme 'Phreatic water balance' in subroutine 'DRAGE'

If the initial waterlevel is greater than zero the time required for the waterlevel to reach drainlevel is calculated and the ensuing calculations proceed similar to the ponded water case.

If the initial waterlevel is below the field drains or is during part of the time step the waterlevel is below drainage depth the final waterlevel is calculated using the function 'DIF' and the appropriate A and B values.

In the final part of the programme the leakage/seepage rate is calculated as the balance of initial and final waterlevel and standing waterlayer, the evapotranspiration during ponding time T_{rice}, the capillary flux (input from 'EVA') and the calculated cumulative drainage. Both the drainage flux and seepage/leakage flux are calculated as average during the irrigation interval.

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