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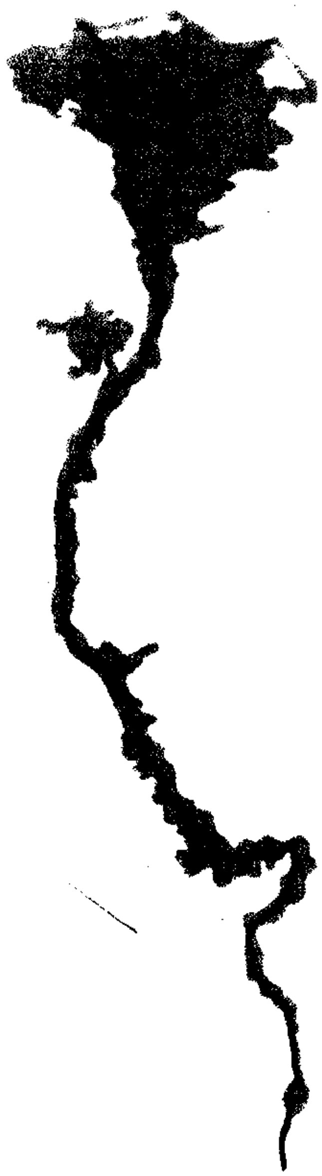
REUSE OF DRAINAGE WATER PROJECT



Formulation for the irrigation water distribution model in the Nile Delta

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C O N T E N T S

1. INTRODUCTION

The irrigation canal system in the Nile Delta can be divided into two components on basis of their main functions:

- The main canal system transports the irrigation water from the Delta Barrage to the irrigation command areas. This main canal system has a regional transport function and is under complete control of the Ministry of Irrigation.
- The transport and distribution of irrigation water below the level of the command canal inlet is not completely controlled by the Ministry of Irrigation but it also depends on farmer's activities in irrigation. At this level also the rotation in the irrigation schedule has to be introduced.

The areas served at distributary level varies from about 500 to 7000 feddans. The water delivery and the cropping pattern in these areas will be considered as being diffuse. Each unit will have its own properties with respect to crops, soil type, soil moisture deficit, salinity and so on. The spatial distribution and numbering of units, served by a distributary, will be indicated with respect to the inlet point of the distributary in the transport canal system. Farmer's behaviour and workmanship with respect to irrigation determine largely the water availability and the water distribution in the different unit areas.

2. THE MAIN CANAL SYSTEM

The water transport model (Susy-model) has to describe quantitatively and qualitatively the regional transport of the irrigation water from Delta Barrage into the Delta, and the mixing with drainage water in the main canal system. This part of the path way is under complete control of the Ministry of Irrigation. The water transport at this level can be considered as being proportional to the ratio of the water requirement of the area served over the total requirement of the Delta and the sum of delivery at the Delta Barrage and the quantity of drain water officially recharged to the irrigation canal system either by Reuse Pumping Stations, or by gravity from drain canals. This situation can be described by eq. (1):

$$W(n, t) = \left(R(n, t) / \sum_{n=1}^{n_{max}} R(n, t)\right) \cdot \left(W_{db}(t) + \sum_{m=1}^{m_{max}} D(m, t)\right) \qquad (1)$$

where: $W(n, t)$ = quantity of water delivered in irrigation command canal n at time t in $m^3.s^{-1}$

$R(n, t)$ = water requirement in command canal area n at time t in $m^3.s^{-1}$

$W_{db}(t)$ = water delivery at Delta Barrage at time t in $m^3.s^{-1}$

$D(m, t)$ = drain water recharge at location m at time t in $m^3.s^{-1}$

$R(n, t)$ is determined by the area covered by a certain crop rotation at time t and the water requirement of that rotation. So $R(n, t)$ can be given as:

$$R(n, t) = \sum_{c=1}^{c_{max}} A(c, n, t) \times r(c, n, t)$$

where: $A(c, n, t)$ = area occupied by crop rotation c at time t in feddans

$r(c, n, t)$ = water requirement of crop rotation c at time t in $m^3.feddan^{-1}.s^{-1}$

Though a large number of crops are grown in the Delta only the major crop rotations will be considered. The present cropping pattern is based on the growth of a summer and a winter crop. In the Southern Delta no cultivation of rice is present, but part of this area is used for vegetable production. In the Northern Delta rice becomes the predominant crop in summer, while occasionally barley is grown instead of wheat on saline soils.

The values of $r(c, n, t)$ can be derived at this level from the idealized water duties given by the Water Master Plan, or can be defined for new crop rotations. For the present situation the following crop rotations will be considered:

Rotation 1: berseem - maize - berseem - cotton - wheat - vegetables
Rotation 2: berseem - maize - berseem - cotton - wheat - rice
Rotation 3: berseem - cotton - wheat - rice
Rotation 4: berseem - rice - berseem - cotton - wheat - rice

With a further intensification through the introduction of short growing varieties and new technologies additional crop rotation schemes can be defined.
The farmers will get irrigation water available in the distributaries by a scheme of 5 days water and 10 days closed in winter. In areas without rice cultivation the summer scheme is 7 days water, 7 days closed, whereas in the rice areas a scheme of 4 days water, 4 days closed is followed. In the early stages of rice cultivation the standing water layer will be refreshed when the water temperature becomes above 35°C.
The irrigation system will be closed for maintenance between January 15th and February 15th.
Based on the idealized schedule for irrigation gifts derived from the Water Master Plan, taking into account 10% for conveyance losses and considering a field irrigation efficiency of 80% gives the calculated water duties for the different crop rotations as presented in Table 1.

The value of $\sum_{m=1}^{m_{max}} D(m, t)$ in eq. (1) is subject to restrictions depending on the difference between the water requirement in the Delta and the water delivery at the Delta Barrage, as well as on the availability of drainage water at the drain water recharge points. The minimum conditions can be described by the expression:

$$\sum_{m=1}^{m_{max}} D(m, t) = \mathrm{MIN}\left(\sum_{n=1}^{n_{max}} (R(n, t) - W_{db}(t)) \; ; \; \sum_{m=1}^{m_{max}} D_{av}(m, t) \right) \qquad (3)$$

where: $D_{av}(m, t)$ = expected maximum quantity of drain water available for recharge at point m at time t in $m^3.s^{-1}$

The expected drain water quantity $D_{av}(m, t)$ can be derived from the available quantities in previous years, during the period under consideration and the supply policy followed.

Table 1. Water duties in m^3 per feddan for 4 different crop rotations for half month periods

Period		Crop rotation number			
		1	2	3	4
January	I	385	385	385	385
January	II	-	-	-	-
February	I	-	-	-	-
February	II	585	585	585	585
March	I	330	330	285	330
March	II	350	350	395	350
April	I	340	340	220	340
April	II	340	340	220	340
May	I	770	575	220	330
May	II	575	430	430	310
June	I	375	575	910	575
June	II	465	735	910	735
July	I	530	725	880	970
July	II	550	730	880	970
August	I	550	725	880	970
August	II	550	775	880	970
September	I	395	595	595	770
September	II	375	575	575	770
October	I	310	310	310	595
October	II	285	285	285	575
November	I	440	440	440	465
November	II	440	440	440	420
December	I	245	245	245	245
December	II	240	240	240	240

3. IRRIGATION CANAL DIMENSIONS

Irrigation command canals serve regions of different sizes, and it is necessary to adapt the canal dimensions to the area served. For modelling purposes use will be made of available design data. Table 2 gives the design data for irrigation canals serving regions with different surface areas.

Table 2. Design data of irrigation canals

Area served feddans	Q $m^3.s^{-1}$		Water height in m		Bottom width (m)	Bottom slope x 10^{-5}	Max. velocity $m.s^{-1}$
	max.	min.	max.	min.			
< 750	0.435	0.315	0.86	0.74	1.0	13	.284
1,000	0.576	0.420	0.88	0.74	1.5	12	.284
2,000	1.175	0.850	1.14	0.97	2.0	11	.322
3,500	2.030	1.470	1.35	1.12	3.0	9	.346
5,000	2.958	2.142	1.57	1.28	3.5	9	.375
8,100	4.698	3.403	1.78	1.45	5.0	8	.402
9,100	5.278	3.822	1.80	1.42	5.5	8	.404
10,500	6.090	4.410	1.91	1.55	6.0	8	.419
12,500	7.250	5.520	2.05	1.64	6.5	7	.436
14,100	8.178	5.922	2.07	1.66	7.0	7	.438
17,500	10.150	7.300	2.08	1.68	8.5	7	.440
20,500	11.890	8.610	2.24	1.84	9.0	7	.461
22,500	13.050	9.450	2.30	1.82	9.5	7	.496
25,500	14.790	10.710	2.41	1.96	10.0	7	.484
27,500	15.950	11.550	2.46	1.95	10.5	7	.490
30,100	17.458	12.642	2.51	2.07	11.0	7	.496
32,500	18.840	13.650	2.50	2.07	12.0	7	.498

From the data given in Table 2 the following relations can be derived, expressing the different variables as functions of the area served in feddans (A_s). The following relations hold:

- Maximum command in $m^3.s^{-1}$:

$$Q_{max} = 5.8 * 10^{-4} * A_s \qquad r^2 = 0.999 \qquad (4)$$

- Minimum command in $m^3.s^{-1}$:

$$Q_{min} = 4.2 * 10^{-4} * A_s \qquad r^2 = 0.999 \qquad (5)$$

- Maximum water height in m:

$$h_{max} = 8.54 * 10^{-4}(0.0124\ A_s^{-0.403} + 1.5 10^{-4})^{-1} \qquad r^2 = 0.996 \qquad (6)$$

- Minimum water height in m:

$$h_{min} = 6.94 * 10^{-4}(0.0124\ A_s^{-0.403} + 1.5 10^{-4})^{-1} \qquad r^2 = 0.992 \qquad (7)$$

- Bottom slope:

$$s = (69.377\ A_s^{-0.314} + 4.0)\ 10^{-5} \qquad r^2 = 0.953 \qquad (8)$$

- Maximum velocity in $m.s^{-1}$:

$$V_{max} = 0.097\ A_s^{0.158} \qquad r^2 = 0.991 \qquad (9)$$

- Bottom width in m:

$$W = 8.68 10^{-2}\ A_s^{0.439} + 10.5 10^{-4}\ A_s^{0.842} - \frac{8.54 10^{-4}}{0.0124\ A_s^{-0.403} + 1.5 10^{-4}}$$

$$r = 0.990 \qquad (10)$$

The flow of water in a command canal can for steady state conditions be described with the Chezy formula:

$$Q = C_{ch} A \cdot \sqrt{Rs} \qquad (11)$$

where: Q = flow of water in $m^3.s^{-1}$

C_{ch} = Chezy coefficient in $m^{\frac{1}{2}}.s^{-1}$

A = area of wetted cross section in m^2

R = hydraulic radius in m

s = bottom slope in $m.m^{-1}$

For canals with only a slight variation in the water height the Chezy formula can be rewritten as:

$$Q = C_{ch} \cdot \bar{w} \cdot h \sqrt{Rs} = c^* h \qquad (11a)$$

Substituting the combination of equations (4) and (6), respectively (5) and (7) in (11a) yields:

$$c^*_{max} = 0.679\, A_s(0.0124\, A_s^{-0.403} + 1.5.10^{-4}) \qquad (11b)$$

and

$$c^*_{min} = 0.605\, A_s(0.0124\, A_s^{-0.403} + 1.5.10^{-4}) \qquad (11c)$$

with as average value:

$$c^* = 0.642\, A_s(0.0124\, A_s^{-0.403} + 1.5.10^{-4}) \qquad (11d)$$

Extrapolation of the derived relations to bigger canals gives results which look very reasonable. Some data are given in Table 3.

Table 3. Dimensions of big canals derived from extrapolation with the relations (4) through (10)

Area served feddans	$Q\ m^3.s^{-1}$ max.	$Q\ m^3.s^{-1}$ min.	Water height max.	Water height min.	Bottom width m	Bottom slope $\times 10^{-5}$	Max. velocity $m.s^{-1}$
100,000	58.0	42.0	3.17	2.57	27.46	5.9	0.598
200,000	116.0	84.0	3.55	2.88	48.96	5.5	0.667

4. DISCHARGE REGULATING STRUCTURES

The water distribution to irrigation canals of lower order regions is regulated by discharge controlling structures as weirs with rectangular control sections and either freely discharging or submerged orifices. The delivery to branch canals can also be regulated by pumping stations. Due to the slope of the land, it might be necessary to use water level control structures in the canal system to prevent too high velocities in the system. All these structures have different relations between water height and discharge. In the Nile Delta generally two different main types of weirs are present and will be used in the formulation of the transport model.

4.1. Weirs with rectangular control sections

This type of weirs will not be used very often for the regulation of the water distribution to branch canals. They might be present as water level regulating structures, but they will generally be used as spillways. Two main types will be introduced in the model.

4.1.1. Sharp crested weirs

The discharge of sharp crested weirs, with rectangular control sections can be given by the equation:

$$Q(t) = C_e(2/3\sqrt{2}g)G \,.\, W_{str}\,.\, (h(t)-h_o)^{1.5} \qquad (12)$$

where: $Q(t)$ = discharge in $m^3.s^{-1}$

C_e = efficiency coefficient

W_{str} = crest width in m

$h(t)$ = upstreams water height in m

h_o = crest height in m

g = acceleration due to gravity in $m.s^{-2}$

The efficiency factor $C_e = 0.595 + 0.03 \left(\frac{h-h_o}{h}\right)$ and $W_{str} = 0.8$ bottom width of the approach canal.

Taking $C_e = 0.63$ gives as general expression:

$$Q(t) = 1.88\ W_{str}(h(t) - h_o)^{1.5} \tag{12a}$$

4.1.2. Broad crested weirs

The discharge relation for broad crested weirs is given by the expression:

$$Q(t) = C_d \cdot C_v\ 2/3(2/3g)^{0.5}\ W_{str}(h(t) - h_o)^{1.5} \tag{12b}$$

where: C_d = discharge efficiency coefficient
C_v = velocity correction coefficient

The other symbols have the same meaning as in eq. 21.

$C_d = 0.848$ if $0.08 < \frac{h - h_o}{L} \leq 0.33$, with L equal to the crest length, and $\frac{h - h_o}{h} \leq 0.35$.

C_v varies from 1.0 to 1.10.

Spillways can be considered as broad crested weirs that fullfill the above mentioned conditions. The crest height of the spillway can be given as the maximum water height, as defined in eq. (6). Taking as average value for C_v equal to 1.05 gives as general equation for the spillways:

$$Q_t = 1.53\ W_{str}(h(t) - h_o)^{1.5} \tag{12c}$$

When the conditions for C_d are not realized a multiplication factor must be applied that is always more than unity. This factor can vary from 1.0 to 1.25 and the coefficient in equation (12b) approaches the value for the sharp crested weir.

4.1.3. Linearization of the discharge relation

For modelling purposes it might be convenient to linearize the given relations. Expressing the discharge relation in a dimensionless form yields:

$$\frac{Q(t)}{Q_{max}} = \left\{\frac{h(t) - h'_o}{h_{max} - h_o}\right\}^{1.5} \tag{12d}$$

where: h_o = minimum crest height in m

h'_o = variable crest height in m

Q_{max} and h_{max} are expected extreme maximum values exceeding the maximum design values by 10 or 20%.

Eq. (12d) can be approximated by 3 linear relations valid for the following set of conditions:

$$\frac{h(t) - h'_o}{h_{max} - h_o} \leq 0 \qquad Q(t)/Q_{max} = 0 \tag{13a}$$

$$0 \leq \frac{h(t) - h'_o}{h_{max} - h_o} \leq 0.131 \quad Q(t)/Q_{max} = 0.3175\left(\frac{h(t)-h'_o}{h_{max}-h_o}\right) \tag{13b}$$

$$0.131 \leq \frac{h_t - h'_o}{h_{max} - h_o} \leq 0.467 \quad Q(t)/Q_{max} = -0.0641+0.8084\left(\frac{h(t)-h'_o}{h_{max}-h_o}\right) \tag{13c}$$

$$0.467 \leq \frac{h_t - h'_o}{h_{max} - h_o} \leq 1.00 \quad Q(t)/Q_{max} = -0.269 +1.2471\left(\frac{h(t)-h'_o}{h_{max}-h_o}\right) \tag{13d}$$

The results of the linear relations in comparison with the original expression are given in Table 4.

Table 4. Relation between the relative water height and relative discharge as calculated with eqs. (12d), (13b), (13c) and (13d)

$\frac{h(t) - h'_o}{h_{max} - h_o}$	Q_t/Q_{max} (eq. 12d)	Q_t/Q_{max} (eq. 13b)	Q_t/Q_{max} (eq. 13c)	Q_t/Q_{max} (eq. 13d)
1.0	1.000			0.978
0.9	0.854			0.853
0.8	0.716			0.729
0.7	0.586			0.604
0.6	0.465			0.479
0.5	0.354		0.340	0.355
0.467	0.319		0.313	0.313
0.4	0.253		0.259	0.230
0.3	0.164		0.178	
0.2	0.0894	0.0635	0.0976	
0.131	0.0474	0.0416	0.0418	
0.1	0.0316	0.0318	0.0167	
0.07	0.0185	0.0222		
0.05	0.0112	0.0159		
0.02	0.0028	0.0063		
0.0	0.0	0.0		

4.2. Orifices

An orifice can be classified as a well defined opening in a plate or a bulkhead, the top of which is placed well below the upstream water level. Orifices are the older devices used for regulating and measuring water. If the upstream water level drops below the top of the opening, it no longer performs as an orifice but as a weir.

4.2.1. Freely discharging orifices

The discharge for freely discharging orifices is given by the expression:

$$Q(t) = C_d \cdot C_v\sqrt{2g} \cdot A(h(t) - h_o)^{0.5} \qquad (14)$$

where: h_o = height of centre of the opening at the upstream side in m
A = cross section of the opening

The other symbols have the same meaning as in the previous paragraphs. For freely discharging orifices $C_d \approx 0.85$ and C_v varies from 1.0-1.07. So as general expression to be used in the model formulation holds:

$$Q(t) = 3.913\ A(h(t) - h_o)^{0.5} \qquad (14a)$$

4.2.2. Submerged orifices

The discharge of submerged orifices is also dependent on the downstream water height. The discharge can be given by the expression:

$$Q(t) = C_d \cdot C_v\sqrt{2g} \cdot A(h_u(t) - h_d(t) + \Delta b)^{0.5} \qquad (15)$$

where: $h_u(t)$ = water height upstreams in m
$h_d(t)$ = water height downstreams in m
Δb = difference in bottom height between the upstream and downstream canal in m

For submerged orifices $C_d = 0.61$, so as general expression in the model formulation holds:

$$Q(t) = 2.817\ A(h_u(t) - h_d(t) + \Delta b)^{0.5} \qquad (15a)$$

4.2.3. Linearization of the discharge relation

Expressing the discharge relations in a dimensionless form yields:

$$Q(t)/Q_{max} = \frac{h(t) - h'_o}{h_{max} - h_o}^{0.5} = \left(\frac{H(t)}{H_{max}}\right)^{0.5} \tag{14b}$$

and for the submerged case:

$$Q(t)/Q_{max} = \left(\frac{\Delta h(t) + \Delta b}{\Delta h_{max} + \Delta_b}\right)^{0.5} = \left(\frac{H(t)}{h_{max}}\right)^{0.5} \tag{15b}$$

where Q_{max} and H_{max} are expected maximum values for discharge and pressure head difference respectively. This equation can be approximated by linear expressions for the following set of conditions:

$$0 \qquad \frac{H(t)}{H_{max}} \leqq 0.094 \quad \frac{Q(t)}{Q_{max}} = 0.045+2.939\left(\frac{H(t)}{H_{max}}\right) \tag{16}$$

$$0.094 \quad \frac{H(t)}{H_{max}} \leqq 0.404 \quad \frac{Q(t)}{Q_{max}} = 0.224+1.049\left(\frac{H(t)}{H_{max}}\right) \tag{16a}$$

$$0.404 \quad \frac{H(t)}{H_{max}} \leqq 1.000 \quad \frac{Q(t)}{Q_{max}} = 0.401+0.610\left(\frac{H(t)}{H_{max}}\right) \tag{16b}$$

Table 5 gives a comparison between the values calculated with eq. (14b) and (15b) and those calculated with eq. (16), (16a) and (16b).

Table 5. Relation between relative water height and relative discharge as calculated with eq. (14b) and eq. (16), (16a) and (16b)

$H(t)/H_{max}$	$Q(t)/Q_{max}$ (eq. 14b)	Q_t/Q_{max} (eq. 16)	Q_t/Q_{max} (eq. 16a)	Q_t/Q_{max} (eq. 16b)
1.0	1.000			1.011
0.9	0.949			0.950
0.8	0.894			0.889
0.7	0.837			0.828
0.6	0.775			0.767
0.5	0.707		0.749	0.706
0.404	0.636		0.648	0.647
0.4	0.632		0.644	0.645
0.3	0.548		0.539	0.589
0.2	0.447	0.633	0.434	
0.1	0.316	0.339	0.329	
0.094	0.307	0.321	0.323	
0.05	0.224	0.192	0.276	
0.02	0.141	0.104		
0.0	0.00	0.045		

4.3. Irrigation pumping stations

The capacity of the irrigation pumping stations depends on the lifting head. So a minimum value must be defined at minimum water height in de delivery canal. For the purpose of the present study it will be assumed that the capacity can be given as a linear function of the water height in the delivery canal so:

$$Q(t) = a(h(t) - h_{min}) \tag{17}$$

4.4. Dimensions of structures

Structures used for regulation of the water distribution generally have variable crest heights and gate openings. To approximate the dimensions of structures, it is assumed that at maximum design capacities of the canals combined with maximum design water heights, the controlling structures also reach their maximum design capacity. This means under these conditions for weirs a minimum crest height and for orifices a maximum opening. With these assumptions is design Q_{max} of the structure equal to the maximum design capacity of the receiving branch canal, and also directly dependent on the area served as given in eq. (4). It is assumed that the structure width (W_{str}) equals 0.8 times the bottom width of the water receiving branch canal. The upstreams water height at maximum capacity is determined by the area served by the upstream canal. Based on the preceding discussion the following design relations will be used:

Sharp crested weirs:

$$h_o - h_{max} = \left(\frac{Q_{max}}{1.38\ W_{str}}\right)^{\frac{1}{1.5}} \tag{18}$$

where: h_{max} = maximum design water height of the delivery canal to be calculated with eq. (6), using the area served by the upstreams canal

Q_{max} = maximum design capacity of the receiving branch canal to be calculated with eq. (4)

W_{str} = 0.8 W of the receiving branch canal to be calculated with eq. (10)

h_o = minimum crest height

Broad crested weirs:

$$h_o = h_{max} - \left(\frac{Q_{max}}{1.53\ W_{str}}\right)^{\frac{1}{1.5}} \tag{19}$$

The symbols have the same meaning as for the sharp crested weirs.
It is assumed that the fixed crest height of spillways is equal to the maximum design water height. The structure width of the spillways is 0.80 W_u where W_u in that case is the upstreams bottom width. Distributaries might have spillways which are circular shaped. In that case the structure width is equal to $2\pi r$, with $r = 0.4\ W_u$.

Freely discharging orifices:
For freely discharging orifices an additional assumption must be made, as two unknown factors are present, namely the maximum opening of the structure and the height of the bottom construction. It is assumed that at maximum design capacity the centre of the orifice opening is at 0.5 h_{max} of the delivering canal section. In that case the following relation holds:

$$h_{max}^{str} = \frac{Q_{max}}{\sqrt{0.5\ h_{max}}} * \frac{1}{3.913 * W_{str}} \qquad (20)$$

where: h_{max}^{str} = the maximum height of the orifice opening in m
Q_{max} = maximum design capacity downstreams canal
h_{max} = maximum design water height of upstreams canal
W_{str} = width of structure opening = 0.8 W of downstreams canal

The solid bottom height of the structure can be calculated as:

$$h_{bot} = 0.5(h_{max} - h_{max}^{str}) \qquad (20a)$$

The variables h_o and h'_o in eq. (14b) can be calculated as:

$$h_o = h_{bot} + \frac{1}{2} h_{max}^{str} \qquad (21)$$

and

$$h'_o = h_{bot} + \frac{1}{2} h_{str} \qquad (21a)$$

Submerged orifices:

Submerged orifices will reach their maximum design capacity with maximum design water heights for the upstream and downstream side. The width of the structure equals 0.8 W of the receiving canal. The difference in bottom height between the upstream and downstream canals is dependent on the local situation. To get an impression of maximum gate openings a range of calculations have been performed based on the given assumptions. Table 6 gives the results of these calculations as a function of the area served. Q_{max} and h_{max} are the maximum design capacity and maximum water height at the downstreams side of the structure. The width of the opening is calculated as 0.8 W from the downstreams bottom width. The height of the structure opening has been calculated for different values of the difference of the upstreams and downstreams water head. In particular for small differences in water head the variation in required maximum height of the structure opening becomes considerable.

Table 6. Some design characteristics of submerged orifices serving distributaries

Area served feddans	Q_{max} $m^3.s^{-1}$	h_{max} m	W_{str} m	Δh in m				
				0.2	0.4	0.6	0.8	1.0
				h^{str}_{max} in m				
750	0.435	0.85	0.80	0.43	0.31	0.25	0.22	0.19
1,000	0.580	0.93	0.98	0.47	0.33	0.27	0.23	0.21
2,000	1.160	1.17	1.52	0.61	0.43	0.35	0.30	0.27
3,000	1.740	1.33	1.98	0.70	0.49	0.40	0.35	0.31
4,000	2.320	1.45	2.39	0.77	0.54	0.44	0.39	0.34
5,000	2.900	1.55	2.77	0.83	0.59	0.48	0.42	0.37
6,000	3.480	1.64	3.13	0.88	0.62	0.51	0.44	0.39
7,000	4.060	1.71	3.47	0.93	0.66	0.54	0.46	0.42
8,000	4.640	1.77	3.80	0.97	0.69	0.56	0.48	0.43
9,000	5.220	1.83	4.11	1.01	0.71	0.58	0.50	0.45
10,000	5.800	1.89	4.41	1.04	0.74	0.60	0.52	0.47

4.5. General remarks

It follows from the canal design data, that the design ratio Q_{max}/Q_{min} = 0.72. If it is assumed that the Q_{max} as given in the linearized equations exceeds the design maximum with 25%, the ratio with the design maximum Q will be 0.8 and with the design minimum 0.58. Both figures are in the range of a single linear relationship. The corresponding values of the water height values can be calculated from the structure design data.

5. THE IRRIGATION COMMAND CANAL

An irrigation command canal serves generally 50,000-200,000 feddans. Such an area can be divided in a number of subregions, each served by a main distributary, that takes it water from the command canal. The water balance of the command canal can be given by the expression:

$$\frac{dV_c(t)}{dt} = Q_{ic}(t) - \sum_{k=1}^{k_{max}} Q_{sr}(k, t) - Q_c(t) - Q_{sp}(t) \qquad (22)$$

where: $V_c(t)$ = volume of water in the command canal in m^3
$Q_{ic}(t)$ = water intake from higher order canal in $m^3.s^{-1}$
$Q_{sr}(k, t)$ = water intake in subregion number k in $m^3.s^{-1}$
$Q_c(t)$ = conveyance losses in command canal in $m^3.s^{-1}$
$Q_{sp}(t)$ = spillway losses from the command canal in $m^3.s^{-1}$

Generally $Q_{ic}(t)$ is the sum of the quantities of water supplied at the command canal inlet and the quantities supplied by recharge points from the drainage canal system either as free discharge by gravity flow or by pumping of reuse pumping stations.

Equation (22) cannot be solved analytically for the command canal as a whole, so a numerical approach will be used by dividing the canal in k_{max} sections.

In each command canal the following specifications will be used:

k = number of canal section, numbered from the main inlet point in downstreams direction
$V(k, t)$ = volume of water in section k in m^3
$\Delta L(k)$ = length of canal section k in m
$\bar{W}(k)$ = mean width of the canal in section k in m
$h(k, t)$ = mean water height in section k in m
$s(h)$ = mean bottom slope in section k in $m.m^{-1}$

For zero flow conditions the relation between the mean water heights in two successive sections can be given as:

$$h(k + 1, t) = h(k, t) + 0.5 \sum_{k=k}^{k+1} s(h) \,.\, \Delta L(h) \qquad (23)$$

Considering the flux of water from one section to another as being linearly dependent on the difference in hydraulic head between the two sections, gives the following expression:

$$Q(k, k + 1, t) = a(k)\left\{h(k, t) - h(k + 1, t) + 0.5 \sum_{k=k}^{k+1} s(k)\ \Delta L(k)\right\} \qquad (24)$$

For steady state conditions the situation can be given by expression (11a):

$$Q(k,\ k + 1) = C^*(k)\ h(k,\ t) = C^*(k + 1)\ h(k + 1,\ t) \tag{25}$$

From this it follows that:

$$h(k + 1,\ t_o) = \frac{C^*(h)}{C^*(k + 1)} \cdot h(k,\ t_o) \tag{26}$$

So for steady-state conditions eq. (24) can be rewritten as:

$$Q(k,\ k + 1) = C^*(k)\ h(k,\ t_o) = a\left[h(k,\ t_o)\left(1 - \frac{C^*(k)}{C^*(k + 1)}\right) + \right.$$

$$\left. + 0.5 \sum_{k=k}^{k+1} (s(k)\quad L(k)\right] \tag{24a}$$

Solving for a yields:

$$a(k,\ t) = \frac{C^*(k)\ h(k,\ t_o)}{h(k,\ t_o)\left(1 - \frac{C^*(k)}{C^*(k + 1)}\right) + 0.5 \sum\limits_{k=k}^{k+1} s(h)\ \Delta L(k)} \tag{27}$$

$a(k,\ t)$ can be considered as a conductance for transport from section k to $k + 1$, and is expressed in $m^2.s^{-1}$.

For each section in the command canal the following water balance equation can be given:

$$\frac{dV(k,\ t)}{dt} = Q(k - 1,\ k,\ t) + Q_s(k,\ t) - Q_{sr}(k,\ t) - Q(k,\ k + 1,\ t) -$$

$$+ Q_c(k,\ t) \tag{28}$$

where $V(k,\ t)$ = volume of water in section k in m^3

$Q(k - 1,\ k,\ t)$ = flow from section k - 1 to k in $m^3.s^{-1}$

$Q(k,\ k + 1,\ t)$ = flow from section k to k + 1 in $m^3.s^{-1}$

$Q_s(k,\ t)$ = inflow from other sources in section k in $m^3.s^{-1}$

$Q_{sr}(k,\ t)$ = flow to lower order subregion in $m^3.s^{-1}$

$Q_c(k,\ t)$ = conveyance losses in section k in $m^3.s^{-1}$

Branching of the main canal, and the delivery of water to lower order subregions will always be considered in connection with discharge regulating structures. The ratio of delivery to branch canals is determined by the water duties for the areas served by these branches and the return flow of drainage water to them.

A special case exists when water level control structures are present in the irrigation canal for velocity regulation. These structures will be considered as permanently operating spillways. The relation between discharge by structures and water height will be given by the linear relationships discussed in Chapter 4. This relation will be given by the general form:

$$Q_{str} = A_{str} + B_{str} \times h(k, t)$$

It must be realized that in case of weirs A_{str} depends on the actual crest height and this constant changes by manupulations with the crest height. In the case of orifices both A_{str} and B_{str} depend on the relative gate opening.

Generally conveyance losses will be taken as a fraction of the water quantity transported. Since in the present formulation this quantity is related to the water height in the successive sections, it will be assumed that the conveyance losses in a section are proportional with height.

Linearizations of eq. (28) with respect to time gives:

$$\frac{V(k, t) - V(k, t_o)}{t - t_o} = \frac{1}{2}[Q(k - 1, k, t) + Q(k - 1, k, t_o) +$$

$$- Q_s(k, t_o) + Q_s(k, t_o) - Q_{sr}(k, t) - Q_{sr}(k, t_o) - Q(k, k + 1, t)$$

$$+ Q(k, k + 1, t_o) - Q_c(k, t) - Q_c(k, t_o)] \qquad (28a)$$

Rearranging of terms gives:

$$V(k, t) - \frac{1}{2}(t - t_o)\{Q(k - 1. k, t) + Q_s(k, t) - Q_{sr}(k, t) - Q(k, k + 1, t) -$$

$$+ Q_c(k, t)\} = V(k, t_o) + \frac{1}{2}(t - t_o)\{Q(k - 1, h, t_o) + Q_s(k, t_o) -$$

$$+ Q_{sr}(k, t_o) - Q(k, k + 1, t_o) - Q_c(k, t_o)\} \qquad (28b)$$

Equation (28b) can be rewritten in the general form, expressing all variables in water heights as:

$$- A(k)\ h(k - 1, t) + (1 + B(k))\ h(k, t) - C(k) + 1, t) =$$

$$A(k)\ h(k - 1, t_o) + (1 - B(k))\ h(k, t_o) + C(k + 1, t_o) + 2D(k)$$

(28c)

with.

$$A(k) = a(k-1)\left[\frac{2}{t-t_o} W(k)\ \Delta L(k)\right]^{-1}$$

$$B(k) = \left[a(k-1) + B_{sr}(k) + a(k) + b(k)\right]\left[\frac{2}{t-t_o} W(k)\ \Delta L(k)\right]^{-1}$$

$$C(k) = a(k)\left[\frac{2}{t-t_o} W(k)\ \Delta L(k)\right]^{-1}$$

$$D(k) = \left[0.5\ a(k-1) \sum_{k=k-1}^{k} (s(k)\ \Delta L(k)) + Q_s(k) - A_{sr}(k) + \right.$$

$$\left. - 0.5\ a(k) \sum_{k=k}^{k+1} s(k)\ \Delta L(k)\right] * \left[\frac{2}{t-t_o} W(k)\ \Delta L(k)\ ^{-1}\right]$$

For some specific values of k the following Q-values must be specified:

$k = 1\ Q(k-1, k, t) = Q_i(t)$

For each section k with a water level control structure:

$k = k_c\ Q(k, k+1, t) = A_{strc} + B_{strc} h(k, t)$

and for

$k = k_{max} = Q(k, h+1, t) = MAX\{0, A_{strc}(k) + B_{strc}\ h(k, t)\}$

For these values of k some of the constants A(k), B(k), C(k) and D(k) will change:

$k = 1 \quad A(1) = 0$

$$B(1) = (B_{sr}(1) + a(1) + b(1)\left[\frac{2}{t-t_o} W(1)\ \Delta L(1)\right]^{-1}$$

$$D(1) = \left[Q_i + Q_s(1) - A_{sr}(1) - 0.5\ a(k) \sum_{k=1}^{2} s(k)\ \Delta L(k)\right] *$$

$$* \left[\frac{2}{t-t_o}(W(1) * \Delta L(1))\right]^{-1}$$

$$k = k_{strc} \quad B(k_{strc}) = (a(k-1) + B_{sr}(k) + B_{strc}(k) + b(k)) \left[\frac{2}{t - t_o} W(k)\ \Delta L(k)\right]^{-1}$$

$$C(k_{strc}) = 0$$

$$D(k_{strc}) = \left[0.5\ a(k-1) \sum_{k=k-1}^{k} s(k)\ \Delta L(k) - A_{sr}(k) - A_{strc}(k)\right] *$$

$$* \left[\frac{2}{t - t_o}(W(k)\ .\ \Delta L(k)\right]^{-1}$$

For spillways two situations can be present. For discharging spillways at $k = k_{max}$ the constants follow the formulation as for $k = k_{strc}$. With the water height below the crest height of the spillway and zero discharge gives:

$$k = k_{max} \quad B(k_{max}) = (a(k-1) + B_{sr}(k) + b(k)) \left[\frac{2}{t - t_o} W(k)\ \Delta L(k)\right]^{-1}$$

$$C(k_{max}) = 0$$

$$D(k_{max}) = \left[0.5\ a(k-1) \sum_{k=k-1}^{k} s(k)\ .\ \Delta L(k) + Q_s(k) - A_{sr}(k)\right] *$$

$$* \left[\frac{2}{t - t_o} W(k)\ .\ L(k)\right]^{-1}$$

If more subregions are served from the same canal section the relation $Q_{sr}(k, t)$ can be given as:

$$Q_{sr}(k, t) = \sum_{\ell=1}^{\ell_{max}} (A_{str}(k, \ell) + B_{str}(k, \ell)\ h(k, t)$$

Equation (28c) can be rewritten as:

$$A^*(k)\ h(k-1, t) + B^*(k)\ h(k, t) + C^*(k)\ h(k+1, t) = K(k) \qquad (28d)$$

The whole canal is now expressed in a series of linear equations, giving the folling general matrix:

$$
\begin{array}{cccccccc}
B_1 & C_1 & \cdot & \cdot & \cdot & \cdot & \cdot & K_1 \\
A_2 & B_2 & C_2 & \cdot & \cdot & \cdot & \cdot & K_2 \\
\cdot & A_3 & B_3 & C_3 & \cdot & \cdot & \cdot & K_3 \\
 & & & & & & & \\
\cdot & \cdot & \cdot & \cdot & A(k_{max}-1) & B(k_{max}-1) & C(k_{max}-1) & K_{max}-1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & A(k_{max}) & B(k_{max}) & K_{max}
\end{array}
$$

Depending on the order of the command canal the subregional outflow $Q_{sr}(k, t)$ can be a rotational one. In that case attention must be given in the formulation to the number of lower order canals taking water from the command canal. Even with one crop rotation in the command area, it is necessary to have at least 6 distributaries taking water from the command canal, due to the different irrigation schemes during summer and winter.

6. THE IRRIGATION WATER DISTRIBUTARY

The distributary canal serves 500-7000 feddans, with an average area of 5000 feddans. It is assumed that the water uptake from the distributary system is diffuse. The water balance equation for this system can be written as:

$$\frac{dV(\ell, t)}{dt} = Q_{sr}(k, t) - Q_{irr}(\ell, t) - Q_s(\ell, t) \tag{29}$$

where: $V(\ell, t)$ = volume of water in the distributary system in open connection with the uptake point in m^3

$Q_{sr}(k, t)$ = intake through an orifice from canal (k), generally by submerged orifices in $m^3.s^{-1}$

$Q_{irr}(k, t)$ = extraction from the distributary by the farmers = $n\bar{q}$ $m^3.s^{-1}$

n = number of irrigation units working

$\bar{q}$ = mean capacity of the units in $m^3.s^{-1}$

$Q_s(\ell, t)$ = spillway losses in $m^3.s^{-1}$

$k = k_{strc}$ $\quad B(k_{strc}) = (a(k-1) + B_{sr}(k) + B_{strc}(k) + b(k))$

$$\left[\frac{2}{t - t_o} W(k)\ \Delta L(k)\right]^{-1}$$

$$C(k_{strc}) = 0$$

$$D(k_{strc}) = \left[0.5\ a(k-1) \sum_{k=k-1}^{k} s(k)\ \Delta L(k) - A_{sr}(k) - A_{strc}(k)\right] *$$

$$* \left[\frac{2}{t - t_o}(W(k)\ .\ \Delta L(k)\right]^{-1}$$

For spillways two situations can be present. For discharging spillways at $k = k_{max}$ the constants follow the formulation as for $k = k_{strc}$. With the water height below the crest height of the spillway and zero discharge gives:

$k = k_{max}$ $\quad B(k_{max}) = (a(k-1) + B_{sr}(k) + b(k))\left[\frac{2}{t - t_o} W(k)\ \Delta L(k)\right]^{-1}$

$$C(k_{max}) = 0$$

$$D(k_{max}) = \left[0.5\ a(k-1) \sum_{k=k-1}^{k} s(k)\ .\ \Delta L(k) + Q_s(k) - A_{sr}(k)\right] *$$

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If more subregions are served from the same canal section the relation $Q_{sr}(k, t)$ can be given as:

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A_2 & B_2 & C_2 & \cdot & \cdot & \cdot & \cdot & K_2 \\
\cdot & A_3 & B_3 & C_3 & \cdot & \cdot & \cdot & K_3 \\
 & & & & & & & \\
\cdot & \cdot & \cdot & \cdot & A(k_{max}-1) & B(k_{max}-1) & C(k_{max}-1) & K_{max}-1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & A(k_{max}) & B(k_{max}) & K_{max}
\end{array}
$$

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n = number of irrigation units working

$\bar{q}$ = mean capacity of the units in $m^3.s^{-1}$

$Q_s(\ell, t)$ = spillway losses in $m^3.s^{-1}$

The main difference with the main canal system is due to the fact, that the extraction rate by the farmers is not a continuous one, but it reaches a maximum during day time and a minimum during the night. Secondly, the extraction by pumping may exceed the supply considerably during short periods near sunrise, whereas the reverse might be true near sunset.

Spillway losses, as a consequence, will be highly variable with time, with minimum losses during day time and maximum losses during night.

The water intake $Q_{sr}(k, t)$ can be independent of the water height during the time step under consideration, as it is with freely discharging orifices. In case of submerged orifices the water intake is not only dependent on the water height in the upstreams canal, but also on the water height in the distributary itself. Both systems can be present in the Delta, although it must be expected, due to the small differences in height, that in most cases submerged ones will be present.

6.1. Volume of water present in the distributary area

Within the area served by a distributary water delivery and cropping pattern will be considered diffuse. Nevertheless it will be necessary to describe the irrigation network with some detail. The irrigation network intensity can be schematically described with the following characteristic parameters:

A_c - area served by the command canal;
L_c - length of the command canal;
n_d - number of distributaries branching off from command canal;
n_m - number of meskaa's branching off from distributary;
n_s - number of sakkia's along the meskaa;
u_f - number of fields irrigated by a sakkia.

Using these parameters the following 'average' characteristics can be calculated:

- average length distributaries : $\bar{L}_d = A_c/2L_c$ (30)

- average area served by distributary: $\bar{A}_d = A_c/n_d$ (31)

- average length of meskaa's : $\bar{L}_m = \bar{A}_d/2\bar{L}_d = L_c/n_d$ (32)

- Average area served by meskaa's : $\bar{A}_m = \bar{A}_d/n_m = A_c/(n_m * n_d)$ (33)
- average length area irrigated by sakkia : $\bar{L}_s = \bar{A}_m/2\bar{L}_m = A_c/(2n_m * L_c)$ (34)
- average area served by sakkia : $\bar{A}_s = A_c/(n_d * n_m * n_s)$ (35)
- average length of a field plot : $L_F = \bar{A}_s/2L_s = L_c/(n_d * n_s)$ (36)

A sakkia serves normally 10 to 20 feddans, with an average of 16 feddans (A_s). When it is assumed, that the field plot has an area of 1 feddan (4200 m^2), with a length width ratio of 2, than $L_f = 91.65$ m. Substitution of L_F in eq. (36) gives for n_s:

$$n_s = L_c/(n_d * 91.65) \tag{37}$$

Substitution of n_s and A_s in eq. (35) gives:

$$n_m = 5.73\ A_c/L_c \tag{38}$$

Substitution of this expression in eq. (33) yields:

$$\bar{A}_m = L_c/(5.73 * n_d) \tag{39}$$

The average length of the Meskaa, was given in eq. (32). Dividing the length by the area served (eq. (39)), shows that the meskaa length per feddan is independent of the size of the area served and equals 5.73 m/feddan. The distributary length per feddan can be obtained from eq. (30) and (31) and can be given as $n_d/(2 * L_c)$ m/feddan. With the equations given in the preceding discussion and known values of L_c, n_d and A_d, the expected intensity of the canal network in a distributary area can be calculated, using the relations given in Table 7.

Using the eq. (4) through (11), the required dimensions of the network can be calculated, keeping in mind that for areas less than 750 feddans, the design data are constant.

Table 7. Parameters to be used for the calculation of network intensity at distributary level, with given values of the length of the command canal (L_c), number of distributaries branching off from this canal (N_d) and the area served by the distributary (A_d). Branching at both sides of the command canal

Canal type	Number present	Area served feddan	Unit intensity m/feddan	Mean length canal
distributary	1	A_d	$2100\ n_d/2L_c$	$2100\ (n_d\ A_d)/L_c$
Meskaa	$n_m=2.73.10^{-3}\ L_d$	$\bar{A}_m=A_d/n_m$	5.73	$5.73\ A_d/n_m$

6.2. Irrigation intensity

Irrigation starts when a minimum water height is present in the distributary system. The full rate of irrigation will be reached at the moment that the water height reaches its maximum design capacity. As the capacity of a sakkia is on the average 25 $\ell.s^{-1}$ and of a diesel engine about 100 $\ell.s^{-1}$, with an average supply area of 16 feddans shows that the maximum capacity of all irrigation units together exceeds the inlet capacity considerably. Generally the operation time for an irrigation unit will be less than the water inlet time at the inlet structure. Generally farmers at the beginning of the distributary will finish their required irrigation sooner than farmers at the end of the system. Farmers will have a preference for irrigation during day time, but farmers at the end of the system can be forced to irrigate partly at night, because of lack of water during day time. So it is necessary to distinguish between day and night irrigation. The system can be expressed by the expression:

$$Q_{irr}(t) = n(t)\ \bar{q} = a^*(\ell)(h(k,\ t)) \tag{40}$$

where the factor a* depends on the number of units in operation.

The minimum required working time (MRWT) depends on the water requirement of the area served. This water requirement can be calculated from the moisture deficits in the fields covered by different crops:

$$r(t) = \sum_{c=1}^{c_{max}} a(c, t) * M_d(c, t) * (1 + LR(c, t))/e_{fa}(c, t) \quad (41)$$

where: $r(t)$ = the water requirement in the area served at time t

$a(c, t)$ = area covered by crop c at time t in feddan

$M_d(c, t)$ = moisture deficit at the beginning of the irrigation interval in $m^3.feddan^{-1}$

$LR(c, t)$ = leaching requirement

e_{fa} = field application efficiency

Generally the water requirement will be calculated from the evapotranspiration model.

The mean minimum required working time can be calculated, as MRWT = $r(t)/n\bar{q}$, which can be used as an indicator for the number of units in operation, as function of the time.

6.3. Spillway losses

From the distributary dimensions also the bottom height at the end of the distributary can be calculated. The difference in bottom height between the inlet point and the spillway can be considerable, and it seems useless to state that the crest height is at maximum design level of the last distributary section. It will be assumed that the free board at the end of the distributary permits a crest height that coincides with the minimum design water height at the intake point.

The spill discharge will be calculated as:

$$Q_{sp}(\ell, t) = \mathrm{Max}\{0; B_s(\ell) * h(\ell, t) - A_s(\ell)\} \quad (42)$$

6.4. The water balance of the distributary

Distributaries are generally served by submerged orifices. This means that the intake by the distributary is affected by the variation in water height of the command canal. The water balance equation for this system is given as:

$$S_w(\ell) \frac{dh(\ell, t)}{dt} = Q_{sr}(k, t) - Q_{irr}(\ell, t) - Q_s(\ell, t) \quad (29a)$$

where: $S_w(\ell)$ = surface area of the waters in open connection with the intake point in m^2

$h(\ell, t)$ = water height at the intake point in m

$Q_{sr}(k, t)$ = intake from command canal section k in $m^3.s^{-1}$

$Q_{irr}(\ell, t)$ = total water uptake from the distributary system in $m^3.s^{-1}$

$Q_s(k, t)$ = spillway losses in $m^3.s^{-1}$

The discharge of the submerged orifice can be given as:

$$Q_{sr}(k, t) = B_{str}(k)\ h(k, t) - B_{str}(k)\ h(\ell, t) + A_{str}(k) \tag{43}$$

where: h(k, t) is the water height in canal command section k in m:

$$Q_{irr}(k, t) = a^*h(\ell, t) \tag{44}$$

$$Q_s(\ell, t) = B_s(\ell)\ h(\ell, t) + A_s(\ell) \text{ or } 0 \tag{45}$$

Substitution of these relations in eq. (29a) gives:

$$S_w(\ell) \frac{dh(\ell, t)}{dt} = B_{str}(k)\ h(k, t) - B_{str}(k)\ h(\ell, t) + A_{str}(k) + \\ - a^*(\ell)\ h(\ell, t) - B_s(\ell)\ h(\ell, t) - A_s(\ell) \tag{29b}$$

Generally, the variation in water height in command canal k with time will be small compared with the changes in water height in the distributary. The variation in water height in the command canal can be given as a linear function of the time, during a time step so:

$$h(k, t) = h(k, o) + \beta * t$$

Substitution of this expression in eq. (29b) gives:

$$\frac{dh(\ell, t)}{dt} = \frac{B_{str}(k)}{S_w(\ell)} * h(k, o) + \frac{B_{str}(k)}{S_w(\ell)} * \beta * t - \\ + \frac{(B_{str}(k) + a^*(\ell) + B_s(\ell))}{S_w(\ell)} h(\ell, t) + \frac{A_{str}(k) - A_s(\ell)}{S_w(\ell)} \tag{29c}$$

or:

$$\frac{dh(\ell, t)}{dt} + \{(B_{str}(k) + a^*(\ell) + B_s(\ell))/S_w(\ell)\}\ h(\ell, t) =$$

$$\frac{B_{str}(k)}{S_w(\ell)} * (h(k, o) + \beta * t) + (A_{str}(k) - A_s(\ell))/S_w(\ell)$$

Integration yields:

$$h(\ell, t) = \left\{\frac{B_{str}(k)(h(k, o) + \beta(k) * t)}{B_{str}(k) + a^*(\ell) + B_s(\ell)}\right\} - \frac{S_w(\ell) * B_{str}(k) * \beta(k)}{(B_{str}(k) + a^*(\ell) + B_s(\ell))^2}$$

$$+ \frac{A_{str}(k) - A_s(\ell)}{B_{str}(k) + a^*(\ell) + B_s(\ell)} + K\ e^{-((B_{str}(k) + a^*(\ell) + B_s(\ell))/S_w) * t} \tag{29d}$$

For t = o and h(ℓ, t) = h(ℓ, o) gives:

$$K = h(\ell, o) - \frac{B_{str}(k) * h(k, o)}{B_{str}(k) + a^*(\ell) + B_s(\ell)} + \frac{S_w(\ell) * B_{str}(k) * \beta(k)}{(B_{str}(k) + a^*(\ell) + B_s(\ell))^2}$$

$$- \frac{(A_{str}(k) - A_s(\ell))}{B_{str}(k) + a^*(\ell) + B_s(\ell)}$$

Substitution of $h(k, o) + \beta * t = h(k, t)$ and $\beta = \frac{h(k, t) - h(k, o)}{t}$ with t now equalling the length of the time step and rearranging of terms yields:

$$h(\ell, t) = \frac{B_{str}(k)}{B_{str}(k) + a^*(\ell) + B_s(\ell)} * \left\{1 - \frac{S_w(\ell)}{(B_{str}(k) + a^*(\ell) + B_s(\ell)) * t}\right.$$

$$\left. * \left(1 - e^{-(B_{str}(k) + a^*(\ell) + B_s(\ell)/S_w(\ell)) \quad t}\right)\right\} * (h(k, t) - h(k, o) -$$

$$+ \frac{B_{str}(k)\ h(k, o) + A_{str}(k) - A_s(\ell)}{B_{str}(k) + a^*(\ell) + B_s(\ell)} *$$

$$* \left\{1 - e^{-(B_{str}(k) + a^*(\ell) + B_s(\ell)/S_w(\ell)) * t}\right\} +$$

$$+ h(\ell, o)\ e^{-(B_{str}(k) + a^*(\ell) + B_s(\ell)/S_w(\ell)) * t} \tag{29e}$$

Setting $t = t - t_o$ and substitution of eq. (29e) in eq. (43) gives:

$$Q_{sr}(k, t) = B_{str}(k) \Bigg[1 - \frac{B_{str}(k)}{B_{str}(k) + a^*(\ell) + B_s(\ell)} *$$

$$* \left\{ 1 - \frac{S_w(\ell)}{B_{str}(k) + a^*(\ell) + B_s(\ell)} * \frac{1}{t - t_o} * \right.$$

$$\left. * \left(1 - e^{-(B_{str}(k) + a^*(\ell) + B_s(\ell)/S_w(\ell))(t - t_o)} \right) \right\} \Bigg] h(k, t) +$$

$$- B_{str}(k) \Bigg[B_{str}(k) + a^*(\ell) + B_s(\ell) * \left\{ 1 - \frac{S_w(\ell)}{B_{str}(k) + a^*(\ell) + B_s(\ell)} \quad \frac{1}{t - t_o} * \right.$$

$$\left. * \left(1 - e^{-(B_{str}(k) + a^*(\ell) + B_s(\ell)/S_w(\ell))(t - t_o)} \right) \right\} * h(k, t_o) +$$

$$- \frac{B_{str}(k) * h(k, t_o) + A_{str}(k) - A_s(\ell)}{B_{str}(k) + a^*(\ell) + B_s(\ell)} *$$

$$* \left(1 - e^{-(B_{str}(h) + a\ (\ell) + B_s(\ell)/S_w(\ell))(t - t_o)} \right) +$$

$$+ h(\ell, t_o)\ e^{-(B_{str}(k) + a\ (\ell) + B_s(\ell)/S_w(\ell))(t - t_o)} \Bigg] + A_{str}(k)$$

This equation has the same shape as those for the freely discharging control structures, but the constants A'_{str} and B'_{str} now also depend on the activities in the distributary area and the length of the time step. It must be remembered, that as long as $h(\ell, t) < h_s(\ell)$ the spillway constants equal zero. So it appears that with submerged orifices, the same calculation for the command canal can be executed.

6.5. Integrated irrigation water uptake by farmers

Setting:

$$K_1(\ell) = \frac{B_{str}(k) * h(k, t_o)}{B_{str}(k) + a^*(\ell) + B_s(\ell)}$$

$$K_2(\ell) = \frac{B_{str}(k) * \beta(k)}{B_{str}(k) + Q^*(\ell) + B_s(\ell)}$$

$$K_3(\ell) = \frac{S_w(\ell) * B_{str}(k) * \beta(k)}{(B_{str}(\ell) + a^*(\ell) + B_s(\ell))^2} - \frac{A_{str}(k) - A_s(\ell)}{B_{str}(k) + a^*(\ell) + B_s(\ell)}$$

$$K_4(\ell) = h(\ell, t_o) - K_1 + K_3$$

$$\alpha(\ell) = (B_{str}(k) + a^*(\ell) + B_s(\ell))/S_w(\ell)$$

$$\beta(k) = \frac{h(k, t) - h(k, t_o)}{(t - t_o)}$$

reduces eq. (29d) to:

$$h(\ell, t) = K_1(\ell) + K_2(\ell) \cdot t - K_3(\ell) + K_4(\ell)\, e^{-\alpha(\ell)} \cdot t$$

The time integrated water uptake by the farmers in the area of the distributary equals:

$$\int_{t=o}^{t} Q_{irr}\, dt = \int_{t=o}^{t} a^*(\ell)\, h(\ell, t)\, dt$$

or:

$$\int_{t=o}^{t} Q_{irr}\, dt = \int_{t=o}^{t} a^*(\ell)\Big\{\{K_1(\ell) - K_3(\ell)\} + K_2(\ell) \cdot t + K_4(\ell)\, e^{-\alpha(\ell)} \cdot t\Big\}\, dt \tag{44a}$$

$$\int_{t=o}^{t} Q_{irr}\, dt = a^*(\ell)\Big\{(K_1(\ell) - K_3(\ell))\, t + \frac{1}{2} K_2(\ell)\, t^2 + \frac{K_4(\ell)}{\alpha(\ell)}\Big(1 - e^{-\alpha(\ell) * t}\Big)\Big\}$$

Setting $t = t - t_o$ yields:

$$\int_{t=o}^{t-t_o} Q_{irr}\, dt = a^*(\ell)\left\{(K_1(\ell) - K_3(\ell))(t - t_o) + \frac{1}{2} K_2(\ell)(t - t_o)^2 + \right.$$

$$\left. - \frac{K_4(\ell)}{\alpha(\ell)}\left(1 - e^{-\alpha(\ell)(t - t_o)}\right)\right\}$$

6.6. Integrated spillway losses from distributaries

Spillways of distributaries are not discharging continuously, so the integrated losses are given by the expression:

$$\int_{t=o}^{t-t_o} Q_s(\ell, t)\ .\ dt = \mathrm{Max}\left(0, \int_{t=o}^{t-t_o} (B_s(\ell) * h(\ell, t) + A_s(\ell))dt\right) \qquad (45a)$$

The integral of the second part of the right hand side of eq. (45a) equals:

$$\int_{t=o}^{t-t_o} Q_s(\ell, t)\, dt = B_s(\ell) * \left\{(K_1(\ell) - K_3(\ell))(t - t_o) + \frac{1}{2} K_2(\ell)(t - t_o)^2 + \right.$$

$$\left. - \frac{K_4(\ell)}{\alpha(\ell)}\left(1 - e^{-\alpha(\ell)(t - t_o)}\right)\right\} + A_3(\ell)(t - t_o)$$

The solution for the command canal gives a series of spillway losses for all the distributaries branching off from these canals. The discharges of the spillways do not start at the same time. It means that the time steps should be choosen in such a way, that at the end of a time step a next distributary starts to spill. With a series of for instance 15 distributaries it means that the model has to choose perhaps 15 different small time steps to start all the spills. To prevent this, it will be assumed that spillways not operating at the beginning of a time step, are not operating during that time step. In this way it will be possible to use a limited number of short time step, that have been preset. The underestimation of spillway losses during the critical time step will be partly compensated during the next time step. This situation plays a dominant role at the end of the day, when farmers stop with irrigation. Moreover it will be assumed, that in the morning when farmers start to irrigate, the spillway losses become immediately zero.

7. DETERMINATION MAXIMUM TIME STEP

The duration of the time step to be applied strongly depends on the flux conditions in the different canal sections. Due to the linearization used in the solution of the flux equations, the model can become instable, when the time step is too large. There is a direct relation between the length of the time step and the deviation from steady state conditions in the canal section under consideration. An approximation of maximum acceptable time step can be obtained, using eq. (28):

$$w(k) * \Delta L(k) \frac{dh(k, t)}{dt} = A^*(k)\ h(k-1, t) - B^*(k)\ h(k, t) +$$

$$+ C^*(k)\ h(k+1, t) + D^*(k)$$

Now taking $h(k-1, t) \simeq h(k-1, t_o) * h(k, t)/h(k, t_o)$ and

$$h(k+1, t) \simeq h(k+1, t_o) * h(k, t)/h(k, t_o) \text{ and}$$

$$D^*(k) \simeq D^*(k)\ h(k, t)/h(k, t_o)$$

gives as expression:

$$\frac{dh(k, t)}{dt} \simeq \{A^{**}(k) - B^{**}(k) + C^{**}(k) + D^{**}(k)\}\ h(k, t)$$

where

$$A^{**}(k) = a(k-1) * (h(k-1, t_o)/h(k, t_o)) * \{w(k) * \Delta L(k)\}^{-1}$$

$$B^{**}(k) = \{a(k-1) + B_{sr}(k) + a(k) + b(k)\}\{w(k) * \Delta L(k)\}^{-1}$$

$$C^{**}(k) = a(k) * (h(k+1, t_o)/h(k, t_o)) * \{w(k) * \Delta L(k)\}^{-1}$$

$$D^{**}(k) = \left\{0.5 * a(k-1) \sum_{k=k-1}^{k} (s(k) * \Delta L(k)) + Q_s(k) - A_{sr}(k) - \right.$$

$$\left. + 0.5\ a(k) \sum_{k=k}^{k+1} s(k)\ .\ \Delta L(k)\right\} * (1/h(k,\ t_o)) * \{w/k) * \Delta L(k)\}^{-1}$$

For some specific values of k the constants will change:

$k = 1 \quad A^{**}(1) = 0$

$$B^{**}(1) = (B_{sr}(1) + a(1) + b(1)) * \{w(1) * \Delta L(1)\}^{-1}$$

$$C^{**}(1) = a(1)\{h(2,\ t_o)/h(1,\ t_o)\} * \{w(1) * \Delta L(1)\}^{-1}$$

$$D^{**}(1) = \left\{\bar{Q}_i + Q_s(1) - A_{sr}(1) - 0.5\ a(1) * \sum_{k=1}^{2} s(k) * \Delta L(k)\right\} *$$

$$* (1/h(1,\ t_o)) * \{w(1) * \Delta L(1)\}^{-1}$$

For $k = k_{str}$ or $k = k_{max}$ with discharging spillway:

$$A^{**}(k) = a(k-1) * (h(k-1,\ t_o)/h(k,\ t_o)) * \{w(k) * \Delta L(k)\}^{-1}$$

$$B^{**}(k) = [a(k-1) + B_{sr}(k) + B_{str}(k) + b(k)] * \{w(k) * \Delta L(k)\}^{-1}$$

$$C^{**}(k) = 0$$

$$D^{**}(k) = \left[0.5\ a(k-1) \sum_{k=k-1}^{k} s(k) * \Delta L(k) + Q_s(k) - A_{sr}(k) - A_{str}(k)\right] *$$

$$* \{w(k) * \Delta L(k)\}^{-1}$$

For $k = k_{max}$ and zero spillway discharge:

$$A^{**}(k) = a(k-1) * (h(k-1,\ t_o)/h(k,\ t_o)) * \{w(k) * \Delta L(k)\}^{-1}$$

$$B^{**}(k) = [a(k-1) + B_{sr}(k) + b(k)]\{w(k) * \Delta L(k)\}^{-1}$$

$$C^{**}(k) = 0$$

$$D^{**}(k) = \left[0.5 * a(k-1) \sum_{k=k-1}^{k} s(k) * \Delta L(k) + Q_s(k) - A_{sr}(k) \right] * \{1/h(k, t_o\} *$$

$$* \{w(k) \ . \ \Delta L(k)\}^{-1}$$

The model remains stable when according to Euler holds:

$$\Delta t \leqq \left| \frac{2}{A^{**}(k) - B^{**}(k) + C^{**}(k) + D^{**}(k)} \right|$$

So the maximum allowed time step can be calculated for each canal section, and this can be compared with a given preset value of Δt. The model can choose the most suitable time step.