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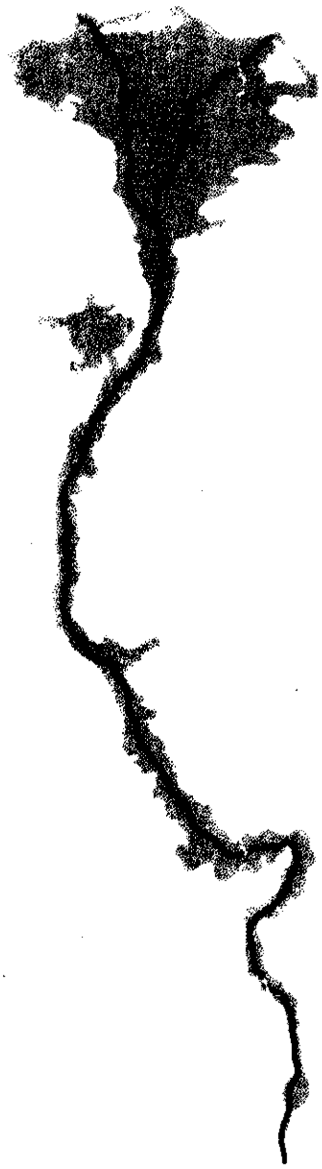
REUSE OF DRAINAGE WATER PROJECT



Calculation of drainage water quantity and depth of water tables in Delta Areas with seepage and leakage

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C O N T E N T S

1. INTRODUCTION

In many river valleys and coastal plains semi-confined aquifers are present. Depending on the piezometric pressure in the aquifer natural leakage in the higher parts of the area, or upward seepage from the aquifer in the lower parts is a common phenomenon. For the design of drainage systems under these conditions it is often assumed that the leakage and seepage flow may be considered as a constant flow, evenly distributed over the whole area.

In the case of parallel drains, however, leakage and seepage will not be evenly distributed over the surface. Leakage will be greater in the middle between two drains, whereas seepage will be greater near the drains as long as the transmissivity of the underlying aquifer is high enough to maintain a constant piezometric head of the groundwater there-in

The problem of vertical water movement through a clay cap overlying an aquifer is important in groundwater management studies. It is of special interest when agricultural drainage is concerned. Land drainage is an essential practice in irrigated agriculture for preventing salinization. While natural drainage (leakage) is considered as a favourable condition, upward vertical seepage could be a restricting factor for crop production. Saline groundwater moving upward will evaporate resulting in salt accumulation in the root zone.

Particularly in studies on the reuse of drainage water for irrigation purposes the occurrence of seepage and leakage is essential for the quantity and quality of available drainage water.

The drainage of steady rainfall and seepage of artesian water in stratified soils consisting of two layers, overlying an aquifer was considered by PANKOW and RIJTEMA (1970).

Recently, a solution of a rising water table in an artesian area subject to upward seepage was developed by WESSELING and WESSELING (1984).

They considered humid conditions, where a water table above drains is established by a piezometric pressure from an underlying aquifer, then a steady rate of precipitation starts falling and lasts for relatively long time. Their transient solution permits the determination of the rate and amount of water table rise and the maximum water table height above drains due to simultaneous steady rates of rainfall and seepage.

A good survey for the calculation of drain spacing for falling water tables in a clay overlying an artesian aquifer has been given by ABDEL-DAYEM (1984).

In areas with subsurface drains, however, the piezometric head of the aquifer water may be at any height relative to the drain level. This height and the position of the phreatic surface will determine together, whether seepage, leakage or some intermediate condition will prevail during the water table fluctuation. The increase in water table height, immediately after irrigation, may cause temporarily leakage in situations with the piezometric head of the aquifer groundwater above the height of the drains in the toplayer.

The main objective of the present study is to develop an analytical solution to describe the rate of drainage water generation, the leakage and seepage rates and the rate at which the phreatic water table moves from a maximum height above the drains immediately afer irrigation to a minimum one just before the next irrigation cycle starts.

2. THEORY

In most cases the thickness of the covering layer below the drain level will be large as compared to het height of the phreatic level above the drain, so the transmissivity of the top layer is assumed to be constant.

When the thickness of the covering layer is not too large as compared to the drain spacing, the Dupuit-Forchheimer assumptions for horizontal flow may be considered to be valid for flow to drains. In that case is for a falling water table after irrigation the following differential equation valid:

$$\mu \frac{\partial h}{\partial t} = kD \frac{\partial^2 h}{\partial x^2} - \frac{1}{c}(h-h_a) \qquad (1)$$

where: μ = drainable pore space of the covering layer in $m^3.m^{-3}$

- h = height of the phreatic level above drain level in m
- h_a = piezometric height of the aquifer groundwater in m
- k = horizontal permeability of the covering layer in $m.day^{-1}$
- D = thickness of the covering layer below drain level in m
- kD = transmissivity of the covering layer in $m^2.day^{-1}$
- c = vertical resistance of the covering layer in days
- t = time in days
- x = distance in m
- L = drain distance in m

Suppose that at $t = 0$ instantaneous irrigation takes place that causes an increase in the amount of water in the saturated system of the covering layer, resulting in a uniform height of the phreatic level. For this situation eq. (1) holds with the following initial and boundary conditions:

$$h(x, 0) = h_o \quad (2)$$

$$h(0, t) = 0 \quad (3)$$

$$h(L, t) = 0 \quad (4)$$

For leakage conditions the following boundary conditions hold:

$$h(x, \infty) = 0 \qquad h_a = 0 \quad (5)$$

$$h(x, t_o) = 0 \qquad h_a < 0 \qquad \mu \frac{dh}{dt} = -\frac{1}{c}(h-h_a) \quad (6)$$

For seepage conditions the following boundary conditions must be applied:

$$t = \infty \qquad h_a > 0 \qquad kD \frac{d^2h}{dx^2} = \frac{1}{c}(h-h_a) \quad (7)$$

The general solution of eq. (1) subject to the given boundary conditions can be obtained assuming:

$$h(x, t) = X.T \quad (8)$$

Differentiation after t and separation of variables yields:

$$\frac{\mu}{kD}\frac{T''}{T'} = \frac{X'' - \frac{1}{kDc}X}{X} = -\kappa \tag{8a}$$

Integration gives the following general solution:

$$h(x,\ t) = \left\{C_1 + C_2\ e^{-\kappa/\alpha^2 t}\right\}\left\{C_3\ \sin\ x\sqrt{\beta} + C_4\ \cos\ x\sqrt{\beta}\right\} \tag{9}$$

with $\alpha^2 = \frac{\mu}{kD}$ and $\kappa = \beta + \frac{1}{kDc}$

From the boundary condition $h(0,\ t) = 0$ follows $C_4 = 0$

From the boundary condition $h(L,\ t) = 0$ follows $\beta = \frac{n^2\pi^2}{L^2}$

2.1. Leakage to the aquifer

For conditions with leakage to the aquifer the boundary condition $h(x,\ t_c) = 0$ must be ful-filled, with $h_a < 0$. A special case is present when $h_a = 0$, since then the condition $h(x,\ \infty) = 0$ must be followed. Both conditions are valid if for $t = t_c$ holds:

$$T = C_1 + C_2\ e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\cdot t_c} = 0$$

or:

$$C_1 = -\ C_2\ e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\cdot t_c}$$

for $t_c = \infty$ follows: $C_1 = 0$

Substitution in eq. (9) yields:

$$h(x,\ t) = \sum_{n=o}^{\infty} B_n \left\{ e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\cdot t} - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\cdot t_c} \right\} \sin\frac{n\pi x}{L} \tag{9a}$$

From the initial condition $h(x, o) = h_o$ follows:

$$h(x,\ t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4h_o}{n\pi} \frac{e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\cdot t} - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\cdot t_c}}{1 - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\cdot t_c}} \sin\frac{n\pi x}{L} \tag{9b}$$

For the special case that $h_a = 0$ and $t_c = \infty$ eq. (9b) reduces to:

$$h(x,\ t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4h_o}{n\pi}\, e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\cdot t} \sin\frac{n\pi x}{L} \tag{9c}$$

The main problem under leakage conditions is to find a relation between the value of t_c and the hydrological parameters. The total change in water storage in the soil due to discharge equals for $t \leqq t_c$ and $0 \leqq x \leqq \frac{1}{2} L$:

$$\int_o^{\frac{1}{2}L} \mu \frac{\partial h}{\partial t} \cdot dx = \sum_{n=1,3,5}^{\infty} -\mu \frac{4h_o L}{n^2\pi^2}\left[\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right]\frac{kD}{\mu} \frac{e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\cdot t}}{1-e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\cdot t_c}} \tag{10}$$

For $t \geqq t_c$ the change in storage equals the leakage flux so:

$$\int_o^{\frac{1}{2}L} \mu \frac{\partial h}{\partial t}\, dx = -\frac{L}{2c}(h(x,\ t) - h_a) \tag{11}$$

For $t = t_c$ both equations are valid with $h(x, t) = 0$. Setting both equations equal yields:

$$h_a = \frac{-8kDch_o}{\pi^2} \sum_{n=1,3,5}^{\infty} \frac{1}{n^2}\left[\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right] \frac{e^{-\left(\frac{u^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\,.\,t_c}}{1-e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\,.\,t_c}}$$

This equation must be solved for t_c by trial and error. However, it appears that neglecting the higher order terms of n for $t = t_c$ yields a good approximation for the value of t_c, using the expliciet expression:

$$t_c = \frac{1}{\left(\frac{\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}} \,.\, \ln\left[1 - \frac{8kDc}{\mu^2}\left(\frac{h_o}{h_a}\right)\left(\frac{\pi^2}{L^2} + \frac{1}{kDc}\right)\right] \tag{13}$$

The discharge to the drains can for $t < t_c$ be given as:

$$q_d = - kD\left(\frac{\partial h(x,\ t)}{\partial x}\right)_{x=o} =$$

$$= - \frac{4kD}{L}\,h_o \sum_{n=1,3,5}^{\infty} \frac{e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\,.\,t} - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\,.\,t_c}}{1-e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\,.\,t_c}} \tag{14}$$

The mean drainage rate expressed in $m.d^{-1}$ for $t < t_c$ is given by the expression:

$$\bar{f}_d(t) = \frac{8kDh_o}{L^2} \sum_{n=1,3,5}^{\infty} \frac{e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\,.\,t} - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\,.\,t_c}}{1-e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\,.\,t_c}} \tag{15}$$

The total drainage water production between t_o and t equals:

$$\int_{t=t_o}^{t} \bar{f}_d(t).dt = \frac{8kDh_o}{L^2} \sum_{n=1,3,5}^{\infty} \left[\left(1-e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \cdot t_c} \right)^{-1} * \right.$$

$$* \left\{ \left(\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \right)^{-1} * \left(e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \cdot t_o} - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\pi} \cdot t} \right) + \right.$$

$$\left. \left. - (t-t_o)\, e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \cdot t_c} \right\} \right] \qquad (15a)$$

The mean leakage rate to the aquifer follows for $t < t_c$ from:

$$\bar{f}_\ell(t) = \frac{2}{L} \int_{o}^{\frac{1}{2}L} \frac{1}{c}\left[h(x,\ t) - h_a\right] dx$$

or

$$\bar{f}_\ell(t) = \frac{1}{c}\left[\frac{8h_o}{2} \left\{ \sum_{n=1,3,5}^{\infty} \frac{1}{n^2}\ \frac{e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \cdot t} - \left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \cdot t_c}{1-e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \cdot t_c}} \right\} - h_a \right] \qquad (16)$$

The total quantity of water lost by leakage between t_o and t equals:

$$\int_{t=t_o}^{t} \bar{f}_\ell(t) \cdot dt = \frac{1}{c}\left[\frac{8h_o}{\pi^2} \sum_{n=1,3,5}^{\infty} \frac{1}{n^2} \left(1-e^{-\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\ \frac{kD}{\pi} \cdot t_c} \right)^{-1} * \right.$$

$$\left\{ * \left\{ \left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \right\}^{-1} \left(e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \cdot t_o} - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \cdot t} \right) + \right.$$

$$\left. \left. + (t-t_o)\, e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\pi} \cdot t_c} \right\} - h_a(t-t_o) \right] \qquad (16a)$$

For the situation that $t \geqq t_c$ the following relation has to be used for the depth of the groundwater table:

$$h(x,\ t) = h_a\left(1-e^{-\frac{1}{\mu c}(t-t_c)}\right) \tag{17}$$

The drain discharge rate $\bar{f}_d$ equals in that case 0, while the mean leakage rate to the aquifer is given by the expression:

$$\bar{f}_\ell(t) = -\frac{h_a}{c}\, e^{-\frac{1}{\mu c}(t-t_c)} \tag{18}$$

The quantity of water lost by leakage for the conditions that $t_o \geqq t_c$ is given as:

$$\int_{t=t_o}^{t} \bar{f}_\ell(t) = -\mu h_a\left(e^{-(t_o-t_c)/\mu c} - e^{-(t-t_c)/\mu c}\right) \tag{19}$$

2.2. Seepage from the aquifer

In case of seepage from the aquifer, the drawdown of the groundwater table is smaller than when leakage is present. In connection with the salinization of the topsoil by saline seepage water a capillary upward flux must be taken into account. Taking a constant upward flux to the rootzone eq. (1) can be rewritten as:

$$\mu \frac{\partial h}{\partial t} = kD \frac{\partial^2 h}{\partial x^2} - \frac{1}{c}(h - h_a + c * f_c) \tag{16}$$

where f_c is the capillary flux, with a maximum value equalling the seepage flux present when the groundwater table equals drain depth. Now taking h_a equal to $h_a + c * f_c$ results in a differential equation equal to eq. (1).

The solution of equation (1) subject to the boundary conditions (2), (3), (4) and (7) for seepage conditions can be obtained by differentiating twice eq. (9) after x and by substituting the results in eq. (7) for t equalling ∞. The result gives:

$$- kD \sum_{n=1}^{\infty} \frac{n^2\pi^2}{L^2}(C_1 \sin \frac{n\pi x}{L}) = \frac{1}{c}\left[\sum_{n=o}^{\infty} (C_1 \sin \frac{n\pi x}{L}) - h_a\right] \tag{20}$$

or:

$$\sum_{n=o}^{\infty}\left(- kDc \frac{n^2\pi^2}{L^2} - 1\right) C_1 \sin \frac{n\pi x}{L} = - h_a \tag{20a}$$

or:

$$C_1 = \frac{4h_a}{n\pi} kDc\left\{\frac{n^2\pi^2}{L^2} + 1\right\}^{-1} \tag{20b}$$

Substitution in eq. (9) yields:

$$h(x, t) = \sum_{n=o}^{\infty}\left[\frac{4h_a}{n\pi}\left\{kDc \frac{n^2\pi^2}{L^2} + 1\right\}^{-1} + C_2 e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \cdot t}\right] \sin \frac{n\pi x}{L} \tag{9d}$$

The initial condition $h(x, o) = h_o$ gives:

$$h(x, o) = h_o = \sum_{n=o}^{\infty}\left[\frac{4h_a}{n\pi}\left\{kDc \frac{n^2\pi^2}{L^2} + 1\right\}^{-1} + C_2\right] \sin \frac{n\pi x}{L} \tag{21}$$

or:

$$C_2 = \frac{4h_o}{n\pi} - \frac{4h_a}{n\pi}\left\{kDc \frac{n^2\pi^2}{L^2} + 1\right\}^{-1} \tag{22}$$

Substitution in eq. (9d) yields as general solution for seepage conditions:

$$h(x, t) = \sum_{n=1,3,5...}^{\infty}\left[\left\{\frac{4h_o}{n\pi} e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \cdot t} + \frac{4h_a}{n\pi}\left\{kDc \frac{n^2\pi^2}{L^2} + 1\right\}^{-1}\left(1-e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right) \frac{kD}{\mu} \cdot t}\right)\right\} \sin \frac{n\pi x}{L}\right] \tag{9e}$$

The discharge to the drain can be given as:

$$q_d = kD\left(\frac{\partial h(x,\ t)}{\partial x}\right)_{x=o} =$$

$$\frac{-4kD}{L} \sum_{n=1,3,5}^{\infty} \left[h_o\ e^{-\left(\frac{n^2\,{}^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\ .\ t} + h_a\left\{kDc\ \frac{n^2\pi^2}{L^2} + 1\right\}^{-1} x \right.$$

$$\left. x\left(1 - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\ .\ t}\right)\right] \qquad (23)$$

The mean drainage rate expressed in $m.d^{-1}$ is given by the expression:

$$\bar{f}_d(t) = \frac{8kD}{L^2} \sum_{n=1,3,5}^{\infty} \left[h_o\ e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\pi}\ .\ t} + h_a\left\{kDC\ \frac{n^2\pi^2}{L^2} + 1\right\}^{-1} x \right.$$

$$\left. x\left(1 - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\ .\ t}\right)\right] \qquad (24)$$

The total drainage water production between t_o and t equals under these conditions:

$$\int_{t=t_o}^{t} f_d(t)\ .\ dt = \frac{8kD}{L^2} \sum_{n=1,3,5}^{\infty} \left[h_o\left\{\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\right\}^{-1} \left\{ e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\ .\ t_o} + \right.\right.$$

$$\left. - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\ .\ t} \right\} + \frac{h_a}{\mu c}\left\{\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\right\}^{-1}(t-t_o) +$$

$$\left. - \frac{h_a}{\mu c}\left\{\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\right\}^{-2} \left\{ e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\ .\ t_o} - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\ .\ t} \right\} \right]$$

(24a)

Depending on the values of h_o and h_a the net flux from the aquifer will change as a function of t from a net leakage to the aquifer for small t values to seepage for large values of t. So immediately after irrigation leakage might be present, changing gradually into seepage with increasing value of t. The net aquifer flux can be given by the expression:

$$\bar{f}_s = \frac{2}{L}\int_o^{\frac{1}{2}L} \frac{1}{c}[h(x, t) - h_d]\, dx =$$

$$\frac{1}{c}\left[\left[\sum_{n=1,3,5}^{\infty} \frac{8h_o}{n^2\pi^2} e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu} \cdot t} + \frac{8h_a}{n^2\pi^2}\left\{kDc\,\frac{n^2\pi^2}{L^2} + 1\right\}^{-1} x\right.\right.$$

$$\left.\left. x\left(1-e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu} \cdot t}\right)\right] - h_a\right] \qquad (25)$$

For positive values of $\bar{f}_s$ net leakage inflow to the aquifer is present, whereas for negative values of $\bar{f}_s$ net seepage from the aquifer is present.

The total seepage contribution to the drainage water production between t_o and t is calculated from the expression:

$$\int_{t=t_o}^{t} \bar{f}_s \cdot dt = \frac{1}{c}\left[\sum_{n=1,3,5}^{\infty}\left[\frac{8h_o}{n^2\pi^2}\left\{\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\right\}^{-1}\left\{e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu} \cdot t_o} +\right.\right.\right.$$

$$\left. - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu} \cdot t}\right\} + \frac{8h_a}{n^2\pi^2\mu c}\left\{\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\right\}^{-1}(t-t_o) +$$

$$- \frac{8h_a}{n^2\pi^2\mu c}\left\{\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\right\}^{-2}\left\{e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu} \cdot t_o} +\right.$$

$$\left.\left.\left. - e^{-\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu} \cdot t}\right\}\right] - h_a(t-t_o)\right]$$

Now the terms:

$$\frac{4h_a}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n}\left\{kDc\,\frac{n^2\pi^2}{L^2} + 1\right\}^{-1} \qquad \text{in eq. 9a,}$$

$$\frac{8kDh_a}{L^2} \sum_{n=1,3,5}^{\infty} \left\{kDc\,\frac{n^2\pi^2}{1^2} + 1\right\}^{-1} \qquad \text{in eq. (24), eq (24a) and}$$

$$\frac{8h_a}{\pi^2\mu c^2} \sum_{n=1,3,5}^{\infty} \frac{1}{n^2}\left\{\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\right\}^{-1} \qquad \text{in eq. (25a)}$$

do not converge very rapidly. However, the convergence of the series solutions can be improved considerably by replacing the expressions by:

$$\frac{4h_a}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n}\left\{kDc\,\frac{n^2\pi^2}{L^2} + 1\right\}^{-1} = \frac{4h_aL^2}{\pi^3 kDc} * \left\{\frac{\pi^3}{29.479261} + \sum_{n=1,3,5}^{\infty}\left(\frac{1}{n\left(n^2 + \frac{L^2}{\pi^2 kDc}\right)} - \frac{1}{n^3}\right)\right\}$$

$$\frac{8kDh_a}{L^2} \sum_{n=1,3,5}^{\infty} \left\{kDc\,\frac{n^2\pi^2}{L^2} + 1\right\}^{-1} = \frac{8h_a}{\pi^2 c} * \left\{\frac{\pi^2}{8} + \sum_{n=1,3,5}^{\infty}\left(\frac{1}{n^2 + \frac{L^2}{\pi^2 kDc}} - \frac{1}{n^2}\right)\right\}$$

and

$$\frac{8h_a}{\pi^2\mu c^2} \sum_{n=1,3,5}^{\infty} \frac{1}{n^2}\left\{\left(\frac{n^2\pi^2}{L^2} + \frac{1}{kDc}\right)\frac{kD}{\mu}\right\}^{-1} =$$

$$= \frac{8h_aL^2}{\pi^4 kDc^2} * \left\{\frac{\pi^4}{96} + \sum_{n=1,3,5}^{\infty}\left(\frac{1}{n^2\left(n^2 + \frac{L^2}{\pi^2 kDc}\right)} - \frac{1}{n^4}\right)\right\}$$

It must be kept in mind that only the positive values of this equation give the part of seepage water in the drainage water production. The negative values voor small t values give the leakage to the deep aquifer following immediately after irrigation.

3. DRAINAGE WATER GENERATION MODEL

On basis of the formulation given in the previous chapters a computer programme has been constructed. The programme consists of a main programme with 4 subroutines and some data files. Some test runs have been made for plot 5 of the Mashtul Pilot Area near Zagazig. The input data used were, $k = 0.09\ m.day^{-1}$, $D = 3$ m, $\mu = 0.065$, $C = 10{,}000$ days, $L = 30$ m and $h_d = -3.0$ m below drain level. The results of the calculations are shown in Fig. 1, where the fulldrawn lines represent the simulated results for phreatic water table height, drainage rate and leakage rate. The dots give the observed water table heights over a period of 90 days.

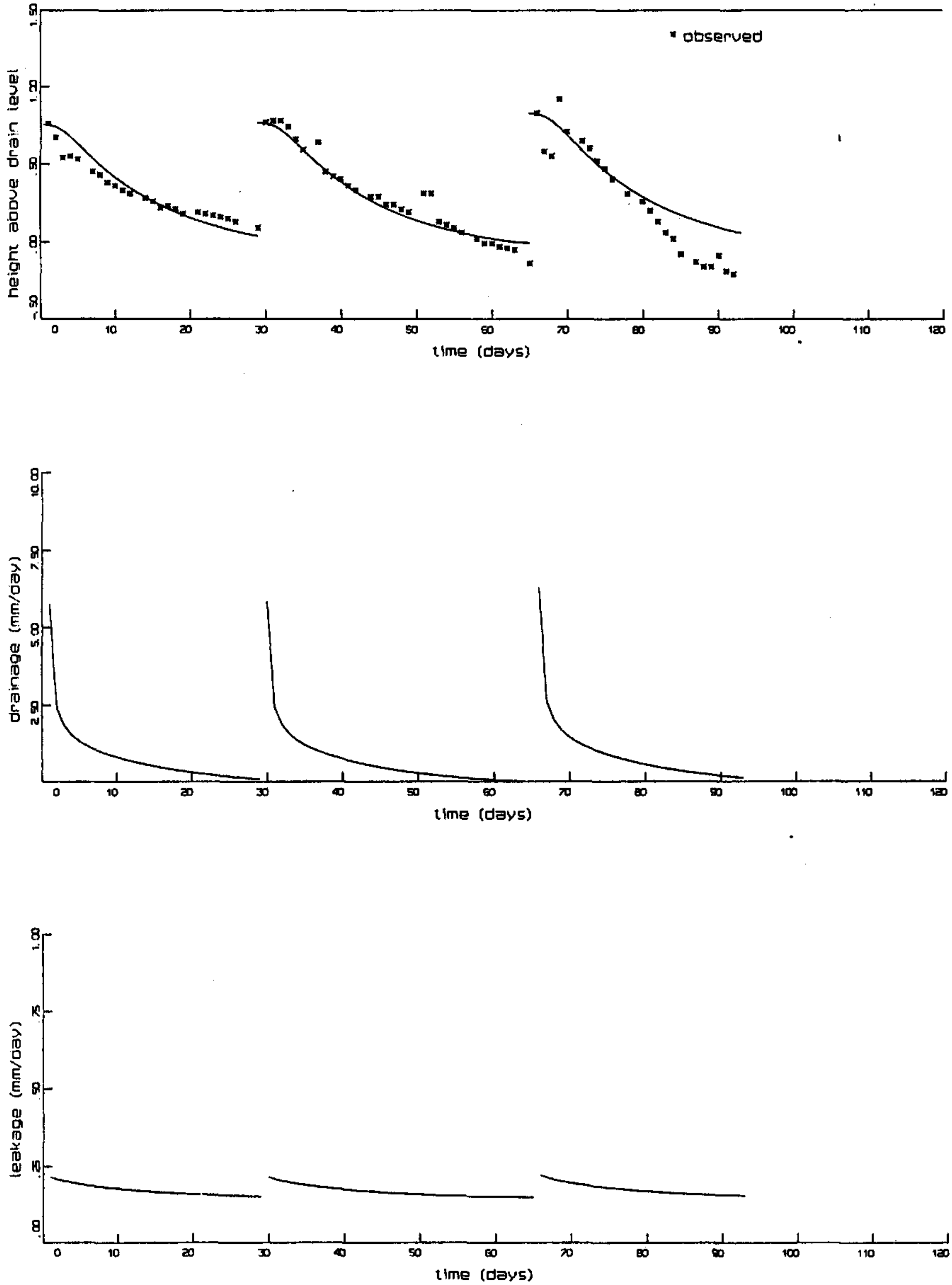

Fig. 1. Results of some test runs of the model simulation for Plot 5 Mashtul Pilot Area

LITERATURE

ABDEL DAYEM, M.S.Y., 1984. Drain spacing for falling water table in a clay cap over an artesian aquifer. Thesis Cours University 1984: 229 pp.

PANKOW, J. and P.E. RIJTEMA, 1970. De resultaten van het waterbalans onderzoek in 1968 voor de objecten met een constant slootpeil in Hoenkoop. ICW-nota 567.

WESSELING, J.G. and J. WESSELING, 1984. The influence of seepage on the depth of water tables in drainage. J. Hydrol. 73: 289-297 ICW TB No. 34.