REUSE OF DRAINAGE WATER PROJECT

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CALCULATION OF ON-FARM IRRIGATION EFFICIENCY

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CONTENTS

1. INTRODUCTION

The model of the water management in the Nile Delta of Egypt comprises the calculation of the irrigation water distribution. As this model also generates the drainage rates, it requires input items to distinguish the different components, producing the ultimate drainage water quantity. Once delivered into the field irrigation channels, the water can follow different pathways(Fig. 1):

1. infiltration from the channels into the soil, feeding the shallow groundwater reservoir;
2. direct evaporation;
3. infiltration from furrows or basins into the soil;
4. through the shallow groundwater reservoir into the (open) drainage system;
5. via surface drainage into the drainage system;
6. evapotranspiration by crops and evaporation from the bare soil surface.

The items 1 and 2 are the on farm conveyance losses.
Determining the on farm irrigation efficiency means determining the quantity of each item mentioned per unit area, given the total quantity irrigation water delivered to the farm systems.

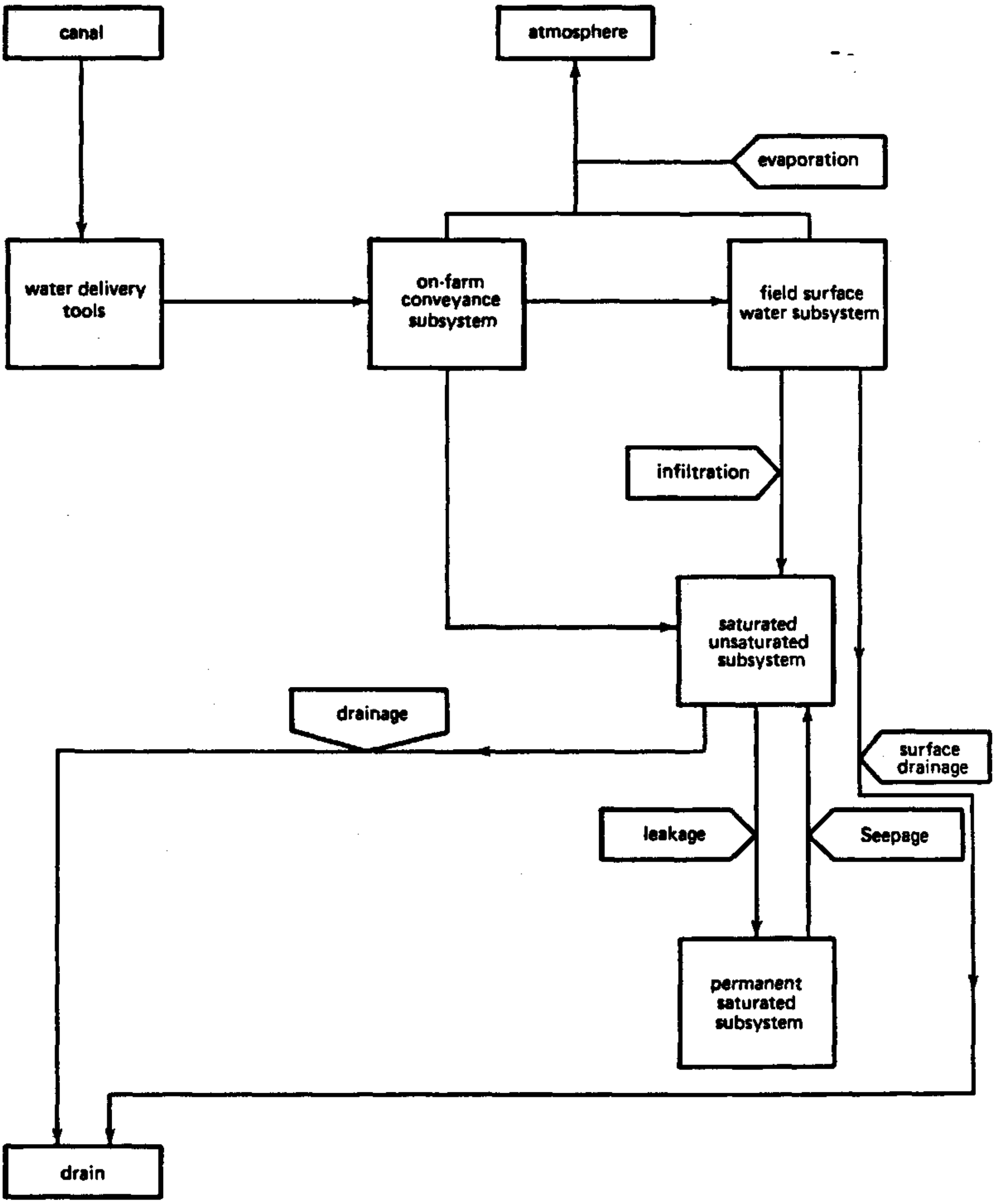


Fig. 1. Flow component interaction between various subsystems (adapted from UTWILLER et al, 1984)

2. DEFINITIONS

ε_{fi} – Accordingly to EWUP (1984), the on-farm irrigation efficiency is defined as the ratio of the water stored in the field during irrigation to the water entering the farm from the delivery system.

ε_{fc} – The on-farm conveyance efficiency is defined as the ratio of the water delivered by an on-farm conveyance channel to the field during irrigation to the water entering the farm from the delivery system.

ε_{ga} – The gross application efficiency will be defined as the ratio of the water infiltrating into the soil during irrigation to the water applied to the field.

ε_{na} – The net application efficiency is the ratio of the water stored in the soil during irrigation to the water infiltrated into the soil.

B – Width of a field (m); L – Length of a field (m).

A_i – Capacity of irrigation tool ($m^3.sec^{-1}$).

M_o – Total moisture deficit of the soil, preceeding irrigation (m).

M_f – Moisture deficit at field capacity (m).

V_c – Volume of cracks, preceeding irrigation ($m^3.m^{-2}$).

ρ_o – Minimum dry-bulk denstity ($10^{-3}.kg.m^3$).

θ_o – Moisture content at which ρ_{min} occurs ($m^3.m^{-3}$).

θ_a – Air content at moisture content θ_o.

I – Infiltration rate ($m.sec^{-1}$).

S_o – Slope of energy line

S – Slope of plot.

n – Manning coefficient

cnl – Conveyance losses ($m^3.sec^{-1}$).

T_o – Operation time of irrigation tool (sec).

T_s – Operation time to realize full soil saturation (sec)

T_p – Maximum duration of ponding a field (sec)

T_e – Time at which waterfront reaches the tail of a field or where movement stops (sec).

T_{ma} – Maximum application time (sec) to realize full saturation.

V_{t_x} – Volume of water infiltrated into the soil during period $t_x < T_e$ (m^2).

From the water balance during some period Δt, it can be shown that (see RIJTEMA and ROEST, 1984):

$$\frac{dx}{dt} = \frac{q}{y} \qquad (3)$$

and as

$$y = \left(\frac{q.n}{\sqrt{S_o}}\right)^{3/5} \qquad (3b)$$

the above equation reads:

$$\frac{dx}{dt} = \left(\frac{\sqrt{S_o}}{n}\right)^{3/5} . \; q^{2/5} = \left(\frac{\sqrt{S_o}}{n}\right)^{3/5} . \; \left\{q_o - I.X\right\}^{2/5} \qquad (4)$$

The boundary conditions are: $x = 0$ when $t = 0$.
Integrations yields:

$$t_x = \frac{5}{3} . \frac{1}{I} . \left(\frac{n}{\sqrt{S_o}}\right)^{3/5} \left\{q_o^{3/5} - (q_o - I.X)^{3/5}\right\} \qquad (5)$$

The totale volume infiltrated per unit width of the field during time t_x is:

$$V_{t_x} = \int_o^X I(t_x - t_x)\; dx \qquad (6)$$

or after integration: $(0 \leqq X \leqq \frac{q_o}{I})$:

$$V_{t_x} = \frac{5}{3} . \left(\frac{n}{\sqrt{S_o}}\right)^{3/5} \left[\frac{5}{8} . \frac{1}{I} . q_o^{8/5} - (q_o - I.X)^{3/5} \left\{\frac{5}{8} . \frac{1}{I}(q_o - I.X) + X\right\}\right] \qquad (7)$$

3.2. Cracking soils

Due to shrinkage a volume of cracks per unit area (eq. V_c) are developed. During irrigation these cracks are instantaneously filled. Eq. 3 is now written like:

$$\frac{dx}{dt} = \frac{q}{Y + V_c} \tag{8}$$

Substituting eq. 3b and eq. 1, yields:

$$\int \left\{ \left(\frac{n}{\sqrt{S_o}}\right)^{3/5} (q_o - I.x)^{-2/5} + \frac{V_c}{(q_o - I.x)} \right\} dx = \int dt \tag{9}$$

The boundary conditions are: x = 0 when t = 0 and x = X when t = T_x, so:

$$t_x = \frac{5}{3} \cdot \frac{1}{I} \cdot \left(\frac{n}{\sqrt{S_o}}\right)^{3/5} \left\{ q_o^{3/5} - (q_o - I.X)^{3/5} \right\} + \frac{V_c}{I} \cdot \ln \frac{q_o}{q_o - I.X} \tag{10}$$

The total amount of water entering the soil since irrigation starts untill time T_X

$$V_{T_X} = \int_o^X (T_X - t_x) \cdot I\, dx + V_c \cdot X \tag{11}$$

or:

$$V_{T_X} = \frac{5}{3}\left(\frac{n}{\sqrt{S_o}}\right)^{3/5} \left[\frac{5}{8} \cdot \frac{1}{I} \cdot q_o^{8/5} - (q_o - I.X)^{3/5} \left\{ \frac{5}{8} \frac{1}{I}(q_o - I.X) + X \right\} \right]$$

$$+ V_c \cdot \frac{q_o}{I} \cdot \ln \frac{q_o}{q_o - IX} \quad \text{, provided } IX < q_o \tag{12}$$

4. VOLUME OF WATER DELIVERED TO A FIELD PLOT

The maximum total net amount available for the field plot per unit width is the gross amount minus the conveyance losses:

$$V'_I = V_I\left(1 - \frac{CNL}{Q_i}\right) \qquad (13)$$

The actual amount delivered to a field plot under normal irrigation practice is restricted to the maximum net amount available, or to the amount required for a complete soil saturation, or to the amount that infiltrates during the admissable ponding period.

The admissable ponding period, however, should never be exceeded.

Amounts exceeding the actual required ones are assumed to be released to the open drainage canals.

The net stream size per unit plotwidth available, equals for basin irrigation ($\alpha = 1$):

$$q_o = Q_i - \frac{CNL}{\alpha B} \qquad (14)$$

For furrow irrigation the same approach will be used. Then, however only a part of the plot is wetted. In that case $\alpha < 1$ and reflects the relative wetted width.
The infiltration used is now:

$$I = \alpha I^* \qquad (15)$$

The streamsize permits, if no other restriction are present, to irrigate a plot length:

$$X_e = \frac{q_o}{I} \qquad (16)$$

If X_e exceeds the real plot length L, it is set to L. When $X_e < L$, t_x (eq. 10) becomes infinite large as $q_o - IX_e = 0$. To prevent calculation errors X_e has to be reduced by some small value.
To saturate the soil completely, the total duration of the infiltration in $x = X_e$ should be:

$$T_s = T_e + \frac{M_o - V_c}{I} \tag{17}$$

The total volume infiltrated is then:

$$V_s = V_e + (T_s - T_e) \cdot I \tag{18}$$

The volume of water infiltrated into the soil during the admissible ponding period is:

a. when $T_p \geq T_e$:

$$V_p = V_e + (T_p - T_e) \cdot I \tag{19}$$

b. when $T_p < T_e$:

In this situation the distance that the waterfront travelled is calculated from eq. 10, for $t_x = T_p$, following an iterative method. This distance is denoted by X_p.
Now V_p is calculated for $x = X_p$ with eq. 12.

The actual irrigated amount, V_a is now determined:

$$V_a = V_s \text{ when } V_s < V_p \text{ and } V_s < V_I' \tag{20a}$$

$$V_a = V_p \text{ when } V_s > V_p \text{ and } V_p < V_I' \tag{20b}$$

$$V_a = V_I' \text{ when } V_s \geq V_I' \text{ and } V_I' \leq V_p \tag{20c}$$

The required operation time of the irrigation tool is:

$$T_o = \frac{V_a}{q_o} \tag{21}$$

The conveyance losses are now:

$$V_c = T_o \cdot CNL$$

and the excess of irrigation water released to the drainage system is:

$$V_{sd} = V_I - V_a - V_c \tag{22a}$$

N.B.: All items are expressed in m^2 per unit plotwidth.

Following the definitions for the irrigation efficiency parameters given in Chapter 2 we define:

$$\varepsilon_{fi} = \frac{V_a - V_d}{V_i} \tag{22b}$$

$$\varepsilon_{fc} = \frac{V_I - V_c}{V_I} = \frac{V_{sd} + V_a}{V_I} \tag{22c}$$

$$\varepsilon_{ga} = \frac{V_a}{V_I - V_c} \tag{22d}$$

$$\varepsilon_{na} = \frac{V_a - V_d}{V_a} \tag{22e}$$

So:

$$\varepsilon_{fi} = \varepsilon_{fi} \cdot \varepsilon_{ga} \cdot \varepsilon_{na} \tag{22f}$$

5. ESTIMATION OF DRAINAGE VOLUME

The drainage volume can be estimated by asuming that the amount is drained that infiltrates above the quantity needed to refill the soil to field capacity.

Two general cases can be distinguished. One where the waterfront did not reach the tail end of the field, and one where it actually did.

5.1. $X_e < L$

This condition is met with eq. 17 and when $V_a = V_p$ while $T_p < T_e$. In the latter case X_e should be set to X_p.

When the irrigation tool stops operation, a watertable is present at the soil surface. The water depth at each location is described by eq. 3b for $q = q_o - I.X$:

$$Y_x = \left(\frac{n}{\sqrt{S_o}}\right)^{3/5} (q_o - IX)^{3/5} \tag{23}$$

It is assumed that after T_o the watertable recedes, starting in x=0. A horizontal watertable is assumed to be formed, starting in X = 0 untill it reaches at the original watertable downstream.

The volume of water infiltrated when the horizontal watertable just joins the original watertable in X is calculated with (see Fig. 2):

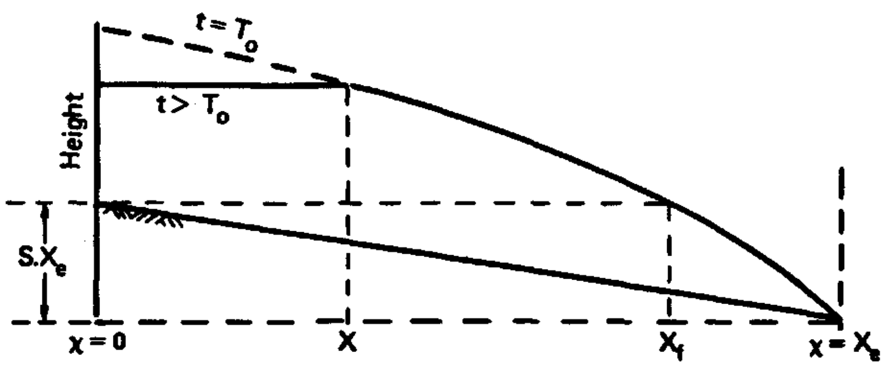


Fig. 2. Assumed watertable after irrigation stops

$$\Delta V = V_{so} - X\{Y_x - \frac{1}{2} XS\} - \frac{5}{8}\frac{1}{I}\left(\frac{n}{\sqrt{S_o}}\right)^{3/5}\{(q_o - I.x)^{8/5} - (q_o - I.X_e)^{8/5}\} \quad (24)$$

where V_{so}, the water quantity on the soil surface at time T_o:

$$V_{so} = \int_0^{X_e} Y_x dx = \frac{5}{8}\frac{1}{I}\left(\frac{n}{\sqrt{S_o}}\right)^{3/5}\{q_o^{8/5} - (q_o - I.X_e)^{8/5}\} \quad (25)$$

The required time is estimated with:

$$\Delta T = \frac{\Delta V}{I.X_e} \quad (26)$$

The watertable in x = 0 just falls on the soil surface when:

$$Y_x - X.S = 0 \quad (27)$$

This occurs for $x = X_f$, at the time moment: $T_o + \Delta T^*$. X_f is solved from eq. 27. The infiltrated volume at that moment is denoted by ΔV^*.
The remaining volume on the soil surface at time $T_o + \Delta T^*$ is assumed to infiltrate in a time period ΔT_r where

$$\Delta T_r = \frac{V_{so} - \Delta V^*}{\frac{1}{2} I.X_e} \quad (28)$$

The time moment where infiltration stops in location x is:

$$T_{x,e} = T_o + \Delta T^* + \Delta T_r \cdot \frac{x}{X_e} \quad (29)$$

The time moment at which infiltration starts is calculated with eq. 10 and denoted by $T_{x,s}$. The infiltrated amount in location x is now calculated with:

$$V_{i,x} = (T_{x,e} - T_{x,s}) \cdot I \quad (30)$$

Different conditions can be distinguished for drainage:

Case 5.1a. $(T_o+\Delta T^*) \cdot I < M_o - M_f - V_c$ and

$$g(x) = -T_{x,s} + \frac{x}{X_e} \cdot \Delta T_r < 0 \quad \text{for} \quad 0 \leq x \leq X_e \tag{31}$$

The latter condition is also met when $g'(x) < 0$ in $x = 0$ (see Fig. 3), or:

$$g'(0) = \frac{\Delta T_r}{X_e} - \left(\frac{n}{\sqrt{S_o}}\right)^{3/5} q_o^{-2/5} - \frac{V_c}{q_o} < 0 \tag{32}$$

Under these conditions no drainage occurs.

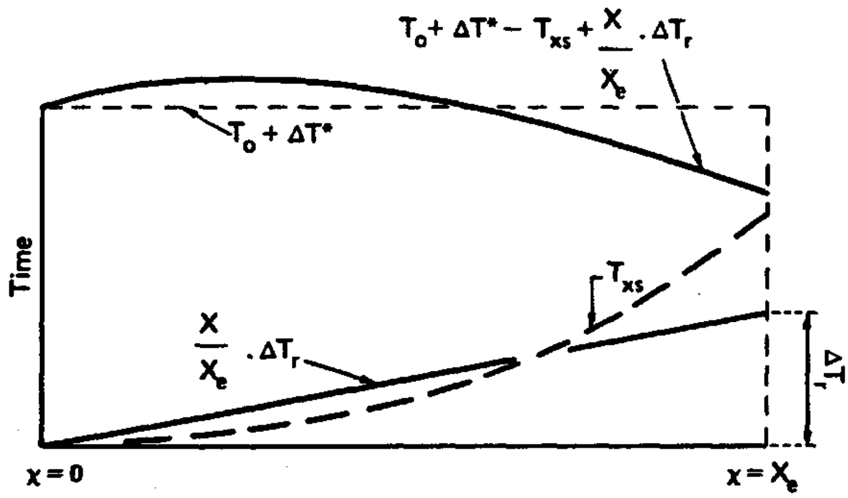


Fig. 3. Schematic representation $(T_{x,e}-T_{x,s})$ in eq. 30, 31

Case 5.1b. $(T_o+\Delta T^*) \cdot I < (M_o-M_f-V_c)$ and $g'(0) > 0$

Now the maximum value of $g(x)$ in some location X_m is determined X_m is found for $g'(x) = 0$ and derived from:

$$\left(\frac{n}{\sqrt{S_o}}\right)^{3/5} (q_o-I.X_m)^{-2/5} - V_c(q_o-IX_m)^{-1} = \frac{\Delta T_r}{X_e} \tag{33}$$

An iterative procedure is applied, while the starting value for X_m is set to $\frac{1}{2} X_e$.

No drainage occurs when:

$$(T_o+\Delta T^*)\ I + g(X_m)\ .\ I \leq M_o - M_f - V_c \tag{34}$$

When, however, the condition in eq. 34 is not met, drainage occurs. The location is defined as $x_{min} \leq x \leq x_{max}$.
The values x_{min} and x_{max} are solved from:

$$\frac{x}{X_e}\ .\ \Delta T_r - T_{x.s} = \frac{M_o - M_f - V_c}{I} - (T_o+\Delta T^*) \tag{35}$$

The function $T_{x,s}$ is identical to eq. 10. The solution for X_{min} is found on $0 < X_{min} \leq X_m$ and for x_{max} on $X_m < x_{max}$. Using a convenient iterative procedure X_{min} is solved with a starting value of $\frac{1}{2} X_m$, X_{max} with a starting value of $\frac{1}{2}(X_m+X_e)$. If $x_{max} > X_e$ then $x_{max} = X_e$. The total drainage is now:

$$V_d = \int_{x_{min}}^{x_{max}} V_{i,x}dx - (x_{max}-X_{min})(M_o-M_f-V_c)$$

$$= V(X_{min}, X_{max}) + \frac{X^2_{max} - X^2_{min}}{2X_e}\ .\ I \tag{36}$$

The function $V(X_{min}, X_{max})$ is defined as:

$$V(x_{min}, x_{max}) = T_o+\Delta T^*)(x_{max}-x_{min})\ .\ I$$

$$- \frac{5}{3}\left(\frac{n}{\sqrt{S_o}}\right)^{3/5} \{(x_{max}-x_{min})\,q_o^{3/5} - \frac{5}{8}.\frac{1}{I}(q_o-I.X_{max})^{8/5} - (q_o-IX_{min})^{8/5}\}$$

$$- V_c\{(x_{max}-x_{min})\ \ln q_o + \frac{1}{I}((q_o-I.X_{max})\ \ln(q_o-I.x_{max}) -$$

$$- (q_o-I.X_{min})\ \ln(q_o-I.X_{min}) - (X_{max}-x_{min}))\}$$

$$- (x_{max}-X_{min})(M_o-M_f-V_c) \tag{37}$$

Case 5.1c. $(T_o+\Delta T^*)\ .\ I \geq (M_o-M_f-V_c)$ and $g'(0) < 0$

Drainage occurs in $0 \leq x < X_{max}$. Where X_{max} is derived from eq. 35.

When solving this equation by some iterative procedure the starting value for x is set to $\frac{1}{2} X_e$. For $x_{min} = 0$ and $x_{max} = X_{max}$ the drained volume is calculated with eq. 36.

Case 5.1d. $(T_o+\Delta T^*)\ I \geq (M_o-M_f-V_c)$ and $g'(0) > 0$

Now drainage occurs on $0 < x < X_e$ the drainage volume is now:

$$V_d = T_o \cdot q_o - X_e(M_o-M_f) \tag{38}$$

5.2. $X_e = L$

This situation occurs when the water front reaches at the tail of the field plot and T_o exceeds T_e. After T_e, a part of the stream size, $(q-I.X_e)$, is stored on the field above the amount that was already present on T_e.

For simplicity it is assumed that the storage causes a horizontal watertable starting in $x = X_e$ (see Fig. 4). When irrigation is stopped at T_o, the horizontal watertable extends to X_b. Beyond time T_o the watertable recedes, starting in $x = 0$ and creating a horizontal watertable.

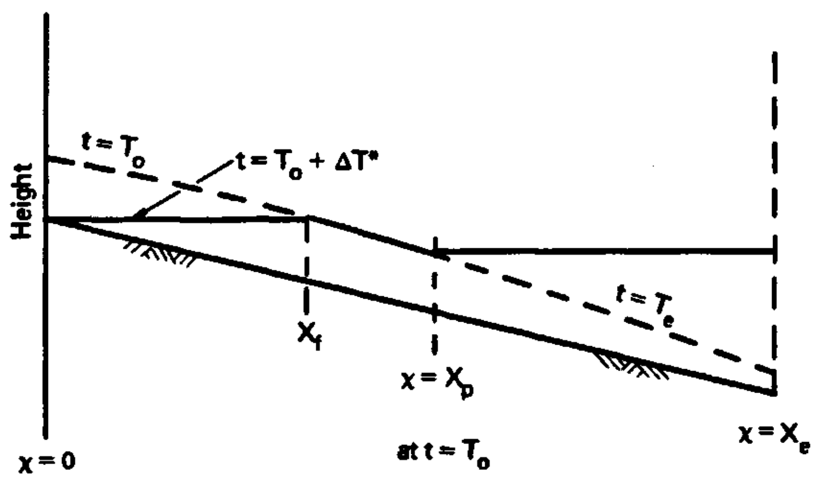


Fig. 4. Assumed watertables when $X_e = L$ at T_e, T_o and $T_o+\Delta T^*$

The quantity X_b is solved from:

$$V_{ad} = (T_o - T_e)(q_o - I.X_e)$$

$$= \left\{Y_{x_b} + \frac{1}{2}(X_e - X_b)\ S\right\}(X_e - X_b)$$

$$- \frac{5}{8} \cdot \frac{1}{I}\left(\frac{n}{\sqrt{S_o}}\right)^{3/5}\left\{(q_o - I.X_b)^{8/5} - (q_o - I.X_e)^{8/5}\right\} \qquad (39)$$

Again a convenient iterative procedure is applied to solve X_b, with an initial solution for $x = \frac{1}{2} X_e$.

When the watertable would recede till the soil surface in $x = 0$, the horizontal watertable would be extended to X_f. The latter value is obtained from eq. 27.

Now two main cases can be distinguished: $X_f \leq X_b$ and $X_f > X_b$. Each will be separately treated.

5.2.1. $X_f \leq X_b$

At time $T_o + \Delta T^*$ the water depth in $x = 0$ is just 0. ΔT^* is calculated with eq. 24 and 26 for $x = X_f$.

The quantity stored on the surface at time $T_o + \Delta T^*$ is:

$$\Delta V_r = \frac{5}{8}\,\frac{1}{I}\left(\frac{n}{\sqrt{S_o}}\right)^{3/5}\left\{q_o^{8/5} - (q_o - I.X_e)^{8/5}\right\} + V_{ad} - \Delta T^*.q_o \qquad (40)$$

This quantity is assumed to be redistributed instantaneously, forming a horizontal watertable, starting in $x = X_s$, where:

$$X_s = X_e - \frac{2\Delta V_r}{S} \qquad (41)$$

The total infiltration in location x is calculated from:

$$V_{i,x} = \begin{cases} (T_o + \Delta T^* - T_{x,s}) \cdot I + V_c & ;\ 0 \leq x \leq X_s \qquad (42a) \\ (T_o + \Delta T^* - T_{x,s}) \cdot I + (x - X_s) \cdot S + V_c & ;\ x > X_s \qquad (42b) \end{cases}$$

While the duration of the infiltration at location x is calculated from (see also Fig. 5):

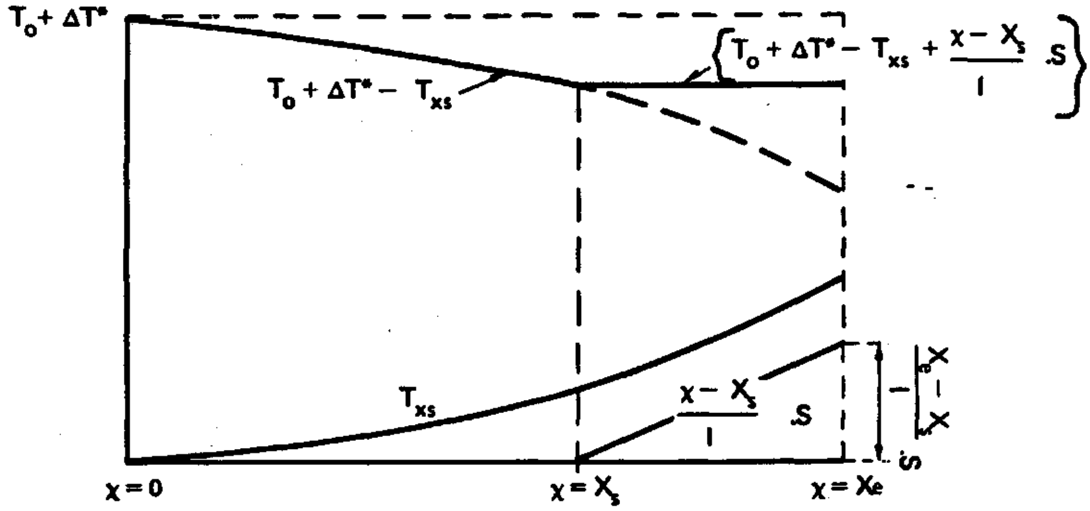


Fig. 5. Schematic depicture of eq. 43a and b

$$T_{i,x} = \begin{cases} T_o+\Delta T^*-T_{x,s} & ;\ 0 \leq x \leq X_s \quad (43a) \\ T_o+\Delta T^*-T_{x,s} + \dfrac{(x-X_s)\ .\ S}{I} & ;\ x > X_s \quad (43b) \end{cases}$$

Now a number of different conditions will be distinguished.

Case 5.2.1a. $T_{i,X_s} > T_i$, than drainage on $0 \leq x \leq X_s$

The variable T_i is calculated from:

$$T_i = \frac{M_o - M_f - V_c}{I} \tag{44}$$

Now we define $x_{min} = 0$ and $x_{max} = X_s$ and the drainage quantity on $0 \leq x \leq X_s$ is denoted by $V_{d,1}$:

$$V_{d,1} = V(X_{min}, X_{max}) \tag{45}$$

while the function $V(X_{min}, X_{max})$ is defined in eq. 37.

Case 5.2.1b. $T_{i,X_s} < T_i$ and $T_{i,o} > T_i$, than drainage on $0 \leq x \leq X_s - \Delta X$

The location $X_{max} = X_s - \Delta X$ is derived from eq. 43a, setting $T_{i,x}$

to T_i and solving X_{max} from this equation. Again $X_{min} = 0$ and $V_{d,1}$ is calculated with eq. 45.

Case 5.2.1c. $T_{i,o} < T_i$, than $V_{d,1} = 0$

Case 5.2.1d. $T_{i,X_s} > T_i$ and $T_{i,X_e} \geq T_i$ than $x_{min} = X_s$ and $X_{max} = X_e$

The drainage quantity on $X_s < x \leq X_e$ is denoted by $V_{d,2}$ and is:

$$V_{d,2} = V(x_{min}, X_{max}) + \frac{1}{2} \cdot S(X_e - X_s)^2 \tag{46}$$

Case 5.2.1e. $T_{i,X_s} > T_i$ and $T_{i,X_e} < T_i$

Drainage occurs on $X_s < x < X_e - \Delta X$. The location $X_{max} = X_e - \Delta X$ is solved from eq. 43b setting $T_{i,x}$ to T_i. As $x_{min} = X_s$, the drainage quantity is:

$$V_{d,2} = V(X_{min}, X_{max}) + \frac{1}{2} \cdot S(X_{max} - X_s)^2 \tag{47}$$

Case 5.2.1f. $T_{i,X_s} < T_i$ and $T_{i,X_e} > T_i$

Drainage occurs on $X_s + \Delta X \leq x < X_e$ and $X_{min} = X_s + \Delta X$ is solved from eq. 43b, accordingly to the procedure for Case 5.2.1e.
Now $X_{max} = X_e$ and:

$$V_{d,2} = V(X_{min}, X_{max}) + S(X_e - X_{min})\left(\frac{X_e + X_{min}}{2} - X_s\right) \tag{48}$$

Case 5.2.1g. $T_{i,X_s} < T_i$ and $T_{i,X_e} < T_i$, than $V_{d,2} = 0$

The total drainage quantity is:

$$V_d = V_{d,1} + V_{d,2} \tag{49}$$

5.2.2. $X_f > X_b$

From eq. 39 V_{ad} is calculated and X_b is solved. Inserting X_b in eq. 23 yields the water depth Y_{x_b}.

The quantity that has to infiltrate into the soil, until a horizontal watertable all over the field plot is obtained, is calculated with eq. 24 and 25 and is denoted by $\Delta V'$. The required time is estimated by eq. 26 inserting $\Delta V'$, and denoted by $\Delta T'$. The water depth in $X = 0$ at that moment is:

$$Y_o = Y_{x_b} - S.X_b \tag{50}$$

The total infiltration in location x is calculated from:

$$V_{i,x} = (T_o + \Delta T' - T_{x,s}) \cdot I + Y_o + X.S + V_c \tag{51}$$

while the duration of the infiltration is:

$$T_{i,x} = T_o + \Delta T' - T_{x,s} + (Y_o + X.S) \cdot \frac{1}{I} \tag{52}$$

A number of cases can be distinguished:

Case 5.2.2a. $T_{i,o} > T_i$ and $T_{i,X_e} > T_i$

Here the quantities $T_{i,o}$ and T_{i,X_e} are calculated from eq. 52 for for $x = 0$ and $x = X_e$ respectively. T_i is calculated from eq. 44:

$$V_d = T_o \cdot q_o - X_e(M_o - M_f) \tag{53}$$

Case 5.2.2b. $T_{i,o} > T_i$ and $T_{i,X_e} < T_i$

Drainage occurs on $0 \leq x \leq X_{max}$. The location $X_{max} < X_e$ is solved from eq. 52, setting $T_{i,x}$ to T_i. As $x_{min} = 0$, the drainage quantity is calculated from:

$$V_d = V(X_{min}, X_{max}) + (Y_o + \frac{1}{2} X_{max} \cdot S) \cdot X_{max} \tag{54}$$

Case 5.2.2c. $T_{i,o} \leq t_i$ and $T_{i,X_e} > T_i$

Drainage occurs on $X_{min} \leq x < X_e$. The location X_{min} is solved from eq. 53, setting $T_{i,x}$ to T_i. As now $x_{max} = X_e$, the drainage quantity is:

$$V_d = V(X_{min}, X_{max}) + (Y_o + \frac{X_{min} + X_e}{2} \cdot S)(X_e - X_{min}) \qquad (55)$$

Case 5.2.2d. $T_{i,o} < T_i$ and $T_{i,X_e} \leq T_i$

In this case no drainage occurs and

$$V_d = 0 \qquad (56)$$

6. CALCULATION OF CRACK VOLUME

Clay soils show swelling and shrinkage when within some limits the moisture content changes. Under natural circumstances this process results in the formation of cracks during drying and a subsidence of the soil surface. The crack volume in the context of this report is that volume of macro pores, that is present after drying, caused by shrinkage. Although subsidence is caused by shrinkage, the volume change due to this process is not included in the crack volume.

During drying, four fases are distinguished (see BRONSWIJK, 1985):

a. structural shrinkage, when water leaves macro pores without causing significant shrinkage. The water quantity that leaves will be denoted by θ_a;
b. normal shrinkage where the volume of water leaving the soil system almost results into an almost equal reduction of the soil mass volume;
c. residual shrinkage, where the reduction in soil mass volume is less than the volume of water leaving the soil system;
d. no shrinkage.

The swelling and shrinkage process can be characterized by a relationship between the actual dry bulk density (ρ) and moisture content (θ). Once known this relationship, the crack volume can be calculated.

A cube,which dimensions are the units of length, is considered. The minimum dry bulk density when the maximum porosity occurs, is denoted by ρ_o and the related moisture volume by θ_o.

In old soils as prevail in the Nile Delta, the swelling processes and shrinkage processes, follow the same pathway while swelling and shrinkage is uniformly in all directions (BRONSWIJK, 1985).

Due to shrinkage the dimensions of the considered cube change to $1 - \varepsilon$. The resulting dry bulk density is then:

$$\rho = \frac{\rho_o}{(1-\varepsilon)^3} \qquad (57)$$

and the effective crack volume:

$$V'_c = \{1 - (1-\varepsilon)^2\}(1-\varepsilon) \qquad (58a)$$

The subsidence of the top of the cube is considered not to contribute to the crack volume.

From eq. 57 (1−ε) is solved and substituted into eq. 58 which yields:

$$V_c' = 1 - \left(\frac{\rho_o}{\rho}\right)^{2/3} \left(\frac{\rho_o}{\rho}\right)^{1/3} \tag{58b}$$

The crack volume of a profile is:

$$V_c = \int_o^D \left\{ 1 - \left(\frac{\rho_o}{\rho}\right)^{2/3} \right\} dz \tag{59}$$

where D, the depth below soil surface, where cracks are formed, measured at the moment when cracks actually are present.

Now ρ will be expressed as a linear function of θ, with $\frac{d\rho}{d\theta}$ constant:

$$\rho = \rho_o + (\theta - \theta_o) \frac{d\rho}{d\theta} \tag{60}$$

When the air content at θ_o is θ_a the moisture deficit is calculated from:

$$M_o = \int_0^D \left\{ (\theta_o + \theta_a) \cdot \left(\frac{\rho_z}{\rho_o}\right)^{1/3} - \theta_z \right\} dz + \int_D^G (\theta_o + \theta_a - \theta_z)\, dz \tag{61}$$

where G is the depth of groundwater below soil surface.

A linear distribution of θ_z with respect to the depth is assumed with θ_m in z = 0, θ_o in z = D and $\theta_G = \theta_o + \theta_a$ in z = G. So:

$$\theta_z = \theta_m + (\theta_g - \theta_m) \cdot \frac{z}{G} \tag{62}$$

Calling:

$$A = 1 - \frac{\theta_o - \theta_m}{\rho_o} \cdot \frac{d\rho}{d\theta} \tag{63a}$$

$$B = \frac{\theta_g - \theta_m}{\rho_o} \cdot \frac{1}{G} \cdot \frac{d\rho}{d\theta} \tag{63b}$$

$$C = \frac{\theta_g - \theta_m}{G} \tag{63c}$$

$$D = \frac{\theta_o - \theta_m}{\theta_g - \theta_m} \cdot G \tag{63d}$$

Integration of eq. 61 yields:

$$M_o = \frac{3}{4B} \cdot \theta_G\{(A+B.D)^{4/3} - A^{4/3}\} - \frac{1}{2C}\{(\theta_m+C.D)^2 - \theta_m^2\}$$

$$+ (\theta_G - \frac{1}{2}\,\theta_a)(G-D) \tag{64}$$

From the subroutine that calculates the actual evapotranspiration, the moisture deficit M_o is known. From the drainage water generation model, the groundwater table is known. The quantities θ_o and θ_a and $\frac{d\rho}{d\theta}$ relate to soil types, and are known.
So the quantity θ_m has to be solved from eq. 64 and the crack volume is calculated from (integrating eq. 59):

$$V_c = D - \frac{3}{B}\{A^{1/3} - (A+B.D)^{1/3}\} \tag{65}$$

7. SOME SOIL PHYSICAL DATA

7.1. Infiltration rate

Infiltration rates are determined by actual soil physical properties and initial moisture distribution. On cracking soils infiltration is more complicate while cracks will be filled with water causing a horizontal infiltration into soil peds.

For simplicity the cumulative infiltrated depth, Y_t, will be described by a linear equation:

$$Y_t = V_c + I.t \qquad (66)$$

Where I in considered as a long term infiltration rate. Under this assumption the crack volume should include the moisture volume, θ_a, that can be stored in the macro pores.

From a series of 21 infiltration tests on a Vertisol near Kafr El-Sheikh, covered with wheat, an average long term infiltration rate of 0.127 $m.d^{-1}$ is obtained (LITWILLER et al., 1984). The average crack volume applying eq. 67 was 0.0574 $m^3.m^{-2}$, provided no significant drainage occurred during the infiltration tests.

It is assumed that the long term infiltration rate equals the unsaturated hydraulic conductivity at almost saturation. For clay soils, with a clay content more than 34% the unsaturated hydraulic conductivity is about 0.115 $m.d^{-1}$ at a moisture suction of 0.001 atm (SHAWKY and WAHDAN, 1977). For Vertisols an infiltration rate of about 0.10 $m.d^{-1}$ seems reasonable.

7.2. Swelling shrinkage behaviour

The swelling and shrinkage in the Nile Delta can be expected on soils classified as Vertisols and Entisols accordingly to the American soil classification (SOIL SURVEY STAFF, 1975).

In Egypt the Vertisols have a clay content of 30-60%, the Entisols of 20-30%.

Calculation of the crack volume accordingly to Chapter 6, requires a relationship between the moisture content and the dry bulk density of the soil matrix itself. Two sources are available from which this relationship can be derived. One provides dry bulk densities and moisture contents from soil samples (EL KITTAB, 1983). Other results stem from an unpublished thesis given ratios of changes in void ratio over moist ratios ($\frac{de}{d\nu}$).

Defining: volume of solids $= \frac{\rho}{m}$

volume of voids $= (1 - \frac{\rho}{m})$

$$e = \text{void ratio} = \frac{\text{volume of voids}}{\text{volume of solids}} = (\frac{m}{\rho} - 1)$$

$$\nu = \text{moist ratio} = \frac{\text{volume of moist}}{\text{volume of solids}} = m \cdot \frac{\theta}{\rho}$$

Where θ is the moist content in volume per unit soil volume, and m is the specific weight of solids ($2.65\ 10^3$ kg m^{-3}).

A relationship can be derived between $\frac{d\rho}{d\theta}$ and $\frac{de}{d\nu}$ like:

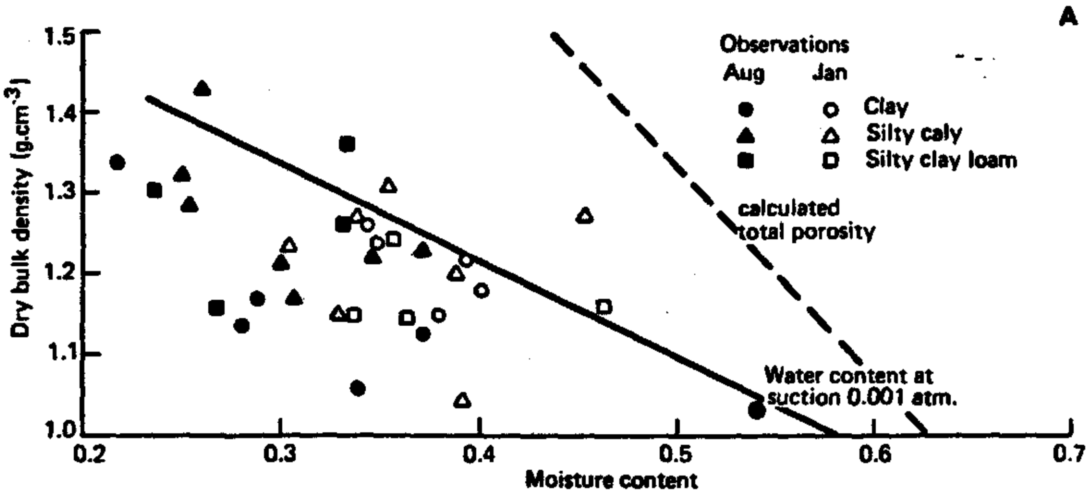
$$\frac{d\rho}{d\theta} = \rho \cdot \left\{\theta - \frac{1}{\frac{de}{d\nu}}\right\}^{-1} \tag{67}$$

Typical values for $\frac{de}{d\nu}$ in the Kafr El Sheikh area are 0.6-0.8, varying with depth and moisture content. For the Giza area these are 0.50-0.70.

Deriving the ratio $\frac{d\rho}{d\theta}$ from data given by El Kittab, showed that due to the sampling methods, at random cracks were incorporated. Similar experience is reported by BERNDT and COUGHLAN (1977) as is referred by BRONSWIJK (1985).

For a good estimation, the highest dry bulk densities are taken. They most probably reflect the actual soil matrix density (see Fig. 6A and B). For the Vertisols (Fig. 6A) an estimated ratio $\frac{d\rho}{d\theta}$ is -1,2. Inserting this value in eq. 67 yields for $\frac{de}{d\nu}$ a value of 0.7 which equals almost the measured quantity in the Kafr El Sheikh area. Following the same procedure for the Entisols yield values for $\frac{d\rho}{d\theta} = -1.85$ and $\frac{de}{d\nu} \simeq 0.99$. These values could be too high. It might be possible that soils with the lowest clay content have a some what higher dry bulk density than soils with higher clay content at the same moisture content. For a Fayoum soil the highest $\frac{de}{d\nu}$ value of 0.94 is reported, but at high moisture content.

From swelling and shrinkage experiments, performed by DU BOIS (1976) on a heavy Dutch clay soil a value for $\frac{d\rho}{d\theta} = -1.94$ could be derived,



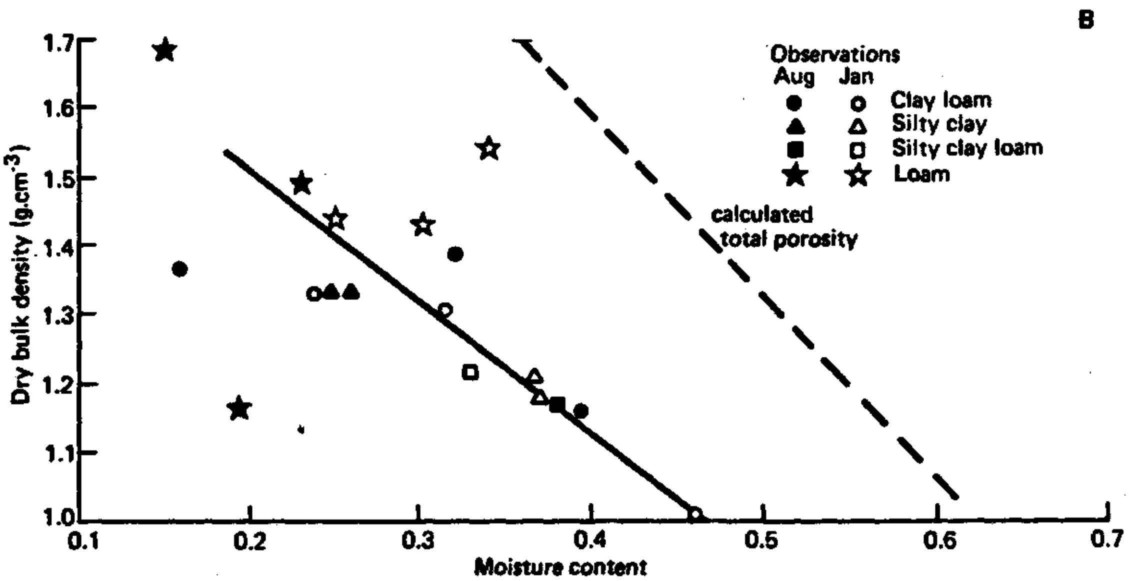


Fig. 6A and B. Relationship between moisture content and dry bulk density (after EL-KITTAB (1983)). A. Vertisols in the Nile Delta; B. Entisols in the Nile Delta

while $\frac{de}{d\nu} = 0.63$. BRONSWIJK (1985) doing similar experiments on a Dutch clay soil with 65% clay found $\frac{d\rho}{d\theta} = -2.56$ and $\frac{de}{d\nu} \simeq 0.49$.
Most probably the difference in the types of clay minerals are due to cause the differences in shrinkage behaviour of Dutch soils and Egyptian soils. For the Entisols a ratio of $\frac{de}{d\nu} = 0.85$ will be applied as hold for some Fayoum soils. The related: $\frac{d\rho}{d\theta} = -1.49$. For both, Vertisols and Entisols the minimum dry bulk density will be kept to $1.0\ 10^3$ kg m^{-3}. The air content when shrinkage starts, θ_a, is 0.045 and 0.09 while the moisture contents then are 0.58 and 0.47 respectively for Vertisols and Entisols.

8. SOME DATA ON STREAMSIZE, FIELD SIZE AND RETARDANCE COEFFICIENTS (VALUES OF MANNING'S n) CONVEYANCE LOSSES

As an average field sizes vary from 0.25-1.0 feddan (EWUP, 1984). Farmers typically irrigate basin crops in large flat basins, bounded by bunds. The largest basins are used for rice. The widths vary from 15 m to 40 m and the length from 40 to 220 m. The convential irrigation method for irrigating row crops is furrows in small basins which are approximately 15-20 m square.

As in most areas rice is a major crop, the field plots are dead level. No actual data on the Manning's coefficient are available. Recommended values for design purposes are given in Table 1.
VEN TE CHOW (1959) gives some average Manning's n-values: Corn 0.06; pasture 0.05; meadow 0.1; small grains 0.1 and brush and waste 0.12. For furrows a n-value of 0.04 will be used and for border irrigation 0.15 seems reasonable.

The streamsize available depends on the type of irrigation tool: sakia or engine driven pump, the lifting head and conveyance losses. The HAFR EL SHEIKH TEAM (1983) reports sakia discharges on the order of 30-35 l sec^{-1} at dynamic lifting heads less than 1 m. For lifting head of 1 m, Table 2 gives some typical sakia discharges. The relationship between sakia discharge and dynamic lifting head will be treated in Chapter 8.1.

Table 1. Manning's n-values for design purposes (KAFR EL SHEIKH TEAM, 1983)

Surface conditions	n-value
Smoot, bare soil surfaces (furrows)	0.04
Small grain, drill rows parallel to borderstrip	0.10
Alfalfa, mint, broadcast small grain and similar crops	0.15
Dense sod crops, drill row of small grain across borderstrip	0.25

Table 2. Some typical data of Sakia's (radius 1.5 m) at 3.3 revolutions per minute and 1 m dynamic lifting head

Area served (fed)	Discharge (m^3 sec^{-1})	Source	Reference
13	0.0158	Manoufia University	WAHBY et al (1980)
17	0.0234	EWUP	WAHBY et al (1980)
14	0.0089	Mastoul, Pilot Area	BRUINSMA, E. (1985)

The motor pumps give a discharge depending on its horse power and lifting head.
WAHBY et al (1980) give a typical value of 47 l sec^{-1} for a 9 HP diesel pump at a dynamic lifting head of 3.5 m. The area served is typically 28 feddan. For a 12 HP diesel pump the figures are 83 l sec^{-1} and 50 feddan.

8.1. Relationship between dynamic lifting head and sakia discharge

Accordingly to an empirical relationship the typical sakia discharge can be calculated from:

$$Q = K\, n\left(\frac{r-h}{r}\right)^{Z} \qquad (68)$$

where: Q = discharge (m^3 sec^{-1})
K = constant = 0.01408
n = revolutions per minute
r = radius of sakia (m)
h = lift head in (m)
Z = empirical constant = 0.6252 (WAHBY et al, 1980)

The delivery height, which can be assumed as about 0.3 m above soil surface, will be denoted by H_1 and the level in the canal H, so $h = H_1 - H$. Inserting the latter equality in eq. 68 yields a relationship between sakia discharge and canal level. When the area served by the sakia equals A_s, then the specific sakia discharge is:

$$q = \frac{K \cdot n}{A_s}\left\{\frac{r-H_1+H}{r}\right\}^Z \qquad (69)$$

In the water distribution model a linear relation is required. So:

$$q_1 = C \cdot \frac{K \cdot n}{A_s}\left\{\frac{r-H_1+H}{r}\right\} \qquad (70)$$

The regression coefficient is solved from:

$$\frac{d}{dC} \int_{H_1}^{H_2} \frac{K \cdot n}{A_s}\left(\left(\frac{r-H_1+H}{r}\right)^{Z+2} - C^2\left(\frac{r-H_1+H}{r}\right)^2\right) dH \quad = 0 \qquad (71)$$

and

$$C = \frac{3}{Z+2} \frac{\left(\frac{r-H_1+H_2}{r}\right)^{Z+2} - \left(\frac{r-H_1+H_1}{r}\right)^{Z+2}}{\left(\frac{r-H_1+H_2}{r}\right)^3 - \left(\frac{r-H_1+H_1}{r}\right)^3} \qquad (72)$$

The value for K, accordingly to WAHBY et al (1980) and other typical data: $r = 1.5$ m, $H_1 - H_2 = 0$ and $H_1 - H_1 = 1.5$, then $C = 1.143$.

For average conditions $(H_1-H) \simeq 1.0$ m and the number of revolutions is 3.3, giving an average sakia discharge of 84 m^3h^{-1} (0.0234 m^3 sec^{-1}).

BRUINSMA (1985) and DE LOUW (1984) report linear relationships between sakia discharge and dynamic lifting heads. As an average out of calibration curves for nine sakia's the relationship:

$$q_1 = \frac{n}{A_s}\{0.0139-0.0112(H_1-H)\} \qquad (73)$$

is derived.
This relationship holds for a sakia with diameter 3 m.

The eq. 70 and 73 can be combined to some average relationship for sakia with a radius of 1.5 m. This results in:

$$\bar{q}_1 = 0.00236\{1.371-H_1+H\} \quad (m^3.sec^{-1}\ fed^{-1}) \tag{74}$$

8.3. Conveyance losses

Conveyance losses from on farm channels can be calculated when applying Ernst's (1956) equation:

$$Q_C = \pi \cdot K_r \ \ln^{-1}\left(\frac{a \cdot D_r}{u}\right). \ \Delta h$$

where: K_r = radial hydraulic conductivity, $m.d^{-1}$

u = wet perimeter

D_r = elevation waterlevel in channel above impervious layer

a = factor depending on flow condition ≃ 1 for homogeneous soil

Δh = difference in water pressure at the boundaries of radial flow area and at wet perimeter

The dimensions of on-farm channels are roughly width ± 0.8 m, water depth ± 0.3 m and wet perimeter ≃ 1.10 m D_r can be estimated by 2 m.

Measurements in Abu Raya, Kafr El Sheikh show marwa losses of about 0.018 $m^3.sec^{-1}$ (KAFR EL SHEIKH TEAM, 1983). The length of the marwa (= on-farm conveyance channel) is not mentioned but could be 400 m. So on a daily base the losses per m length are 3.9 $m^3.d^{-1}$. This indicates, when Δh ≃ 0.5 m that the radial conductivity is about 1.5 $m.d^{-1}$. This can occur only in heavily cracked soil.

Due to evaporation at the free water surface, also water is lost. This quantity depends on the location and season. For the central Delta evaporation varies roughly from 1.4 in January till 7.3 $mm.d^{-1}$ in June (ABOUKHALED et al, 1975).

Keeping the radial hydraulic conductivity at 0.1 $m.d^{-1}$, which resembles the infiltration rate on Vertisols, the water pressure differences at 0.5 m and a length of the conveyance channel at 150 m, the conveyance losses are roughly 40 $m^3.d^{-1}$ exclusive evaporation. The latter quantity ranges from 0.15 $m^3.d^{-1}$ - 0.82 $m^3.d^{-1}$, which can be neglected.

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