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Method for allocating surface water
supply in the Mendoza region (Arg.)

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PREFACE

The work on this paper and the accompanying computer programmes was done within the project "Analysis of regional water resources and their management by means of numerical models and satellites in Mendoza (Argentina)" (ICW project number 100.35).

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1. Introduction

In the Mendoza region irrigation is practised on a large scale. Both groundwater and surface water are used. Surface water is preferred to groundwater because of the lower costs that are involved. However, surface water is limited in supply. Since there is a limited supply of surface water, there clearly is a need for a management strategy concerning its allocation. Strategies considered here are

- striving for equity of water distribution, i.e. that each irrigation unit receives an amount of water that is proportional to the irrigated area;
- striving for minimum total costs;
- a combination of these two.

The first listed strategy is the current policy. The second is just an example of an alternative. The assumption is that the total amount of irrigation remains the same, independent of the surface water supply. Supplied surface water simply substitutes groundwater as the source of water.

A third alternative can consist of a combination of the two mentioned ones, e.g. the striving for minimum total costs under additional constraints in the form of upper bounds on the deviations of the water supply from the 'equity' value.

Ideally, a procedure for determining the best water allocation for achieving a certain management strategy should take into account the limitations of a supply system all the way down to the points at which farm delivery takes place. However, such an approach has the serious drawback that the resulting mathematical formulation of the posed problem becomes extremely complex and the solution procedure unwieldy, if available at all. A more practical approach is to decompose the allocation problem into the following two subproblems:

- the 'planning subproblem' of determining the target volumes that each irrigation unit should receive;
- the 'scheduling subproblem' of devising a rotation scheme that approaches as much as possible the water distribution determined by solving the first subproblem.

In solving the first subproblem the limitations of the supply network are only explicitly taken into account down to the level at which there still is continuous delivery of water. If the solving of the second subproblem turns out to be infeasible, the formulation of the first subproblem has to be revised. Thus it may be necessary to iterate between the solving of the two subproblems.

Here the focus is on the planning subproblem. The way of tackling the scheduling subproblem and how the two solution procedures are combined will be described in a subsequent publication.

2. Formulation of the planning subproblem

The planning subproblem is formulated in terms of a mathematical programming problem of the form:

> maximize (or minimize) the 'objective function' F,
> subject to a number of constraints.

F is a function of the so-called decision variables, being the variables to which the optimization algorithm must attach a value. The constraints define the 'feasible region', indicating the set of combinations of values of decision variables that the algorithm can choose from. Preferably, all the involved mathematical relations should have a linear form, i.e. involve only linear expressions of the decision variables of the form

$$c1*x1 + c2*x2 + \ldots. ,$$

where c1, c2, ... are the coefficients of the decision variables x1, x2, ... If all the expressions are linear then the relatively straightforward method of Linear Programming (LP) can be used for the optimization.

The choice of objective function depends on the strategy that the authorities responsible for the water distribution want to follow. The giving of mathematical expressions for the corresponding objective functions is of course not possible without first having defined the meaning of the involved decision variables. Some of these decision variables are only used in the construction of a specific objective function.

In the following, first a description is given of the manner in which the supply system is mathematically represented. Then follow the constraints reflecting the physical characteristics of the supply system. Finally, the descriptions are given of the two objective functions that correspond with the two alternative strategies mentioned above. These descriptions also cover the decision variables and the constraints that are specially introduced for being able to construct a certain objective function.

2.1 Schematization of the supply network

The supply network is schematized into a network of nodes and arcs. The supply to a secondary sector is assumed to take place on a continuous basis. In the initial formulation of the planning subproblem, the assumption is made that there are no difficulties concerning the distribution of water within the secondary sectors (involving rotation and possibly even subrotation). Thus in the schematization of the network used in the formulation of the planning subproblem (Fig. 1), the tertiary sectors belonging to a certain secondary sector are all connected to the same inlet point.

The following indices are used for the numbering of nodes, arcs and tertiary sectors:
- k for nodes;
- j for arcs;
- i for tertiary sectors.

The subdivision of the tertiary sectors into farms is not shown in Fig. 1 and not included in the mathematical formulations of the

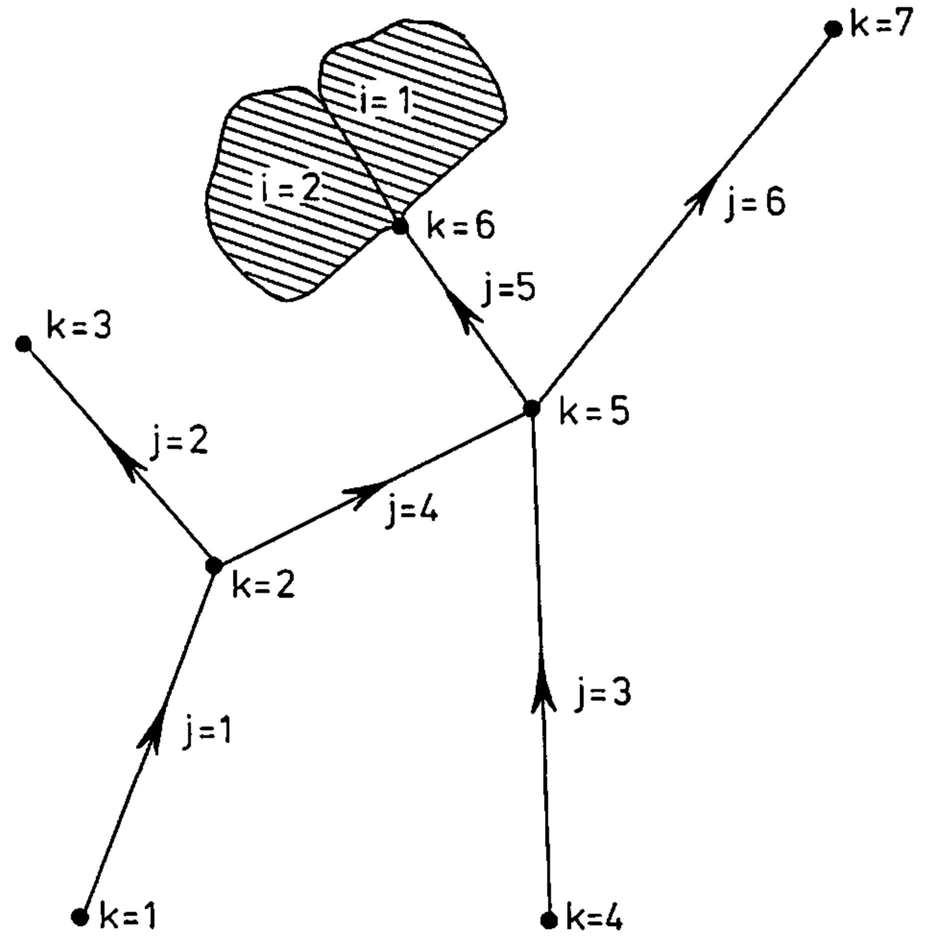

Fig. 1. Numbering within a schematized supply network:
- k for nodes
- j for arcs
- i for tertiary sectors

planning and the scheduling subproblems: this subdivision is assumed to be at a level of detail that can be neglected in the allocation procedure.

As can be seen from Fig. 1, the used schematization can involve alternative routings within the supply system. Even multiple supply inlets are allowed for. For decribing the geometry of the network the following symbols are further introduced:

- nk : total number of nodes;
- nj : total number of arcs;
- ni : total number of tertiary sectors;
- nd(k) : number of downstream arcs starting from node k;
- nu(k) : number of upstream arcs ending in node k;
- jd(k,lj) : j-value of downstream arc number lj (lj=1,...,nd(k)) starting from node k (lj is used for the local numbering of arcs, whereas j is used for the numbering on the scale of the whole network);
- ju(k,lj) : j-value of upstream arc lj (lj=1,...,nd(k)) ending in node k;
- ki(i) : k-value of node i from which the supply of a tertiary sector is thought to take place (the tertiary sectors that belong to the same secondary sector have the same k-value).

Each arc is attributed a certain conveyance capacity denoted by qc(j), deriving from the hydraulic properties of the relevant canal section in combination with the available water level gradient. Only steady state values are thus considered; backwater effects from one section to the other can only be taken into account by reducing the values of qc(j) in an approximate manner.

2.2 Flow through the network

The flow through the network is described in terms of yearly values, as occuring in a 'representative year'. Thus no time discretization is included in the mathematical formulations. For describing the flow through the network the following symbols are introduced:

- qi(j) : inflow rate of an arc (m3/yr);
- qo(j) : outflow rate of an arc (m3/yr);
- qs(k) : external supply of water to a node k (only relevant for upstream terminal nodes) (m3/yr);
- qx(i) : supply of water to a tertiary sector i, from node ki(i) (m3/yr).

Depending on whether there is infiltration or drainage in a certain canal section represented by an arc, either the inflow or the outflow rate is limited by the conveyance capacity. Both situations are accounted for by introducing the bounds

qi(j) =< qc(j), (1a)
qo(j) =< qc(j), (1b)

for all j, where qi(j), qo(j) and qc(j) are measured in m3/yr.

The relation between the inflow and the outflow rate is simply modeled with the following linear relation :

qo(j) = ce(j) * qi(j) + cd(j), (2)

for all j, where
ce(j) - conveyance efficiency (-)
cd(j) - intercept term (m3/yr)

In the nodes of the network, the outflow(s) must be balanced by the inflow(s). The following water balance terms can play a role:
a. supply of water (only in the upstream terminal nodes);
b. use of water by a tertiary sector (only in the downstream terminal nodes);
c. inflow from upstream arcs;
d. outflow to downstream arcs.

A definition sketch indicating all the flows that can play a role is given in Fig. 2. (In a specific node at most three of the four terms can play a role at the same time.)

The term a. is represented by the decision variable qs(k). This variable is subject to the upper bound qsmax(k) (measured in m3/yr) indicating the available supply rate (which in all the nodes except the upstream terminal nodes will be equal to zero):

qs(k) =< qsmax(k), for all k. (3)

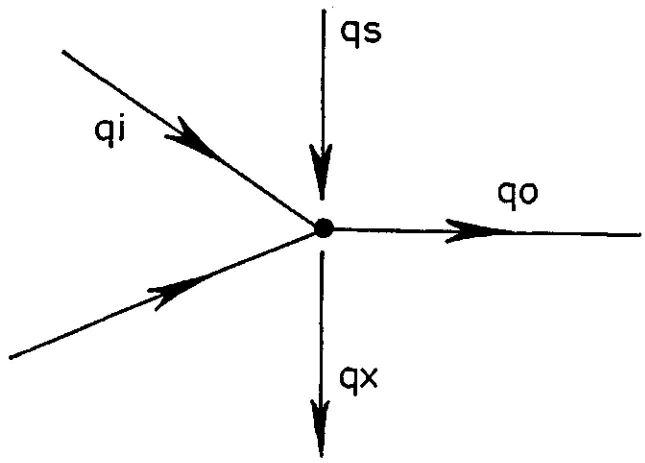

Fig. 2. Water balance terms that can play a role at a node k:

- qi : inflow from an upstream arc
- qo : outflow to a downstream arc
- qs : external supply of water
- qx : use of water by a tertiary sector

Term b. is represented by the sum of the variables qx(i) of which ki(i) is equal to the k-value of the considered node:

$$\sum_{i=1}^{ni} qx(i) \Big|_{ki(i)=k}$$

Term c. is represented by

$$\sum_{lj=1}^{nu(k)} qo(ju(k,lj)),$$

and term d. by

$$\sum_{lj=1}^{nd(k)} qi(jd(k,lj)) .$$

Combination of the four terms to a water balance equation for a certain node gives:

$$qs(k) - \sum_{i=1}^{ni} qx(i)\Big|_{ki(i)=k} =$$

$$\sum_{lj=1}^{nu(k)} qo(ju(k,lj)) - \sum_{lj=1}^{nd(k)} qi(jd(k,lj)), \qquad (4)$$

for all k.

2.3 Objective function

As stated above, the choice of water management strategy determines the appropriate objective function used in the optimization of the water allocation. So far three options have been anticipated upon:

- striving for equity;
- striving for minimum total costs.
- striving for minimum total costs, under constraints with respect to inequity of the water distribution;

2.3.1 Striving for equity

For the purpose of constructing the mathematical expression for the objective function reflecting the overall deviation from equity, the following equality constraints are introduced:

$$wp(i) - wm(i) = qx(i)/a(i) - wt \qquad (5)$$

for all i, where

- wt - mean water supply to the tertiary sectors (m/yr)
- wp(i) - 'positive deviation' of supply to a certain tertiary sector i from the mean value wt (m/yr)
- wm(i) - 'negative deviation' of supply to a certain tertiary sector i from the mean value wt (m/yr)
- a(i) - irrigated area within a tertiary sector i (m2)

Both wp(i) and wm(i) are non-negative decision variables. Either of the two variables should be zero, unless the water supply to a tertiary sector exactly equals zero, in which case both variables become equal to zero. It is, however, not necessary to include a constraint that forces at least one of wp(i) and wm(i) to zero (for each i, e.g. by setting wp(i)*wm(i) to zero). This is because the objective function is of the following form:

$$F1 = \sum_{i=1}^{ni} a(i) * (wp(i) + wm(i)) \qquad (6)$$

where F1 is to be minimized. F1 is the weighted sum of the deviations from the mean supply, with the areas of tertiary sectors as the weights. Minimization of this function leads automatically to either wm(i) or wp(i) becoming zero: if both are positive, then the set of values attached to the decision-variables is not optimal because there exists another solution in which both wm(i) and wp(i) have been reduced by the minimum value of the two. For instance, a solution in which wp(i)=20.0 and wm(i)=1.0 is discarded by the optimization algorithm and instead the solution in which wp(i)=19.0 and wm(i)=0.0 is chosen.

The mean water supply to the tertiary sectors has to be specified as an input parameter. That is because if it would be determined as part of the optimization, the result would be trivial: the mean water supply and the deviations would all be simply set to zero. The constraint that forces the mean to the specified value is:

$$wt = \sum_{i=1}^{ni} qx(i) \;/\; \sum_{i=1}^{ni} a(i) \tag{7}$$

2.3.2 Striving for minimum total costs

Supply of surface water to a tertiary sector causes a reduction of costs, as was explained in Section 1. In a certain tertiary sector there can be several wells in operation for the pumping of groundwater. Clearly, supply of water will lead to the closing down of the wells in the order that is most cost saving, i.e. the most costly wells will be closed down first. Thus the costs reduction as a function of surface water supply is characterised by a decreasing marginal reduction rate.

In order to be able to incorporate this aspect in a linear formulation of the resulting optimization problem (so that linear propgramming, LP, can be used), the costs reduction function is approximated by means of piece-wise linearization, as indicated by the straight segments in Fig. 3. For implementing this piece-wise linearization a number of auxiliary decision variables are introduced. These variables are given upper bounds that are equal to the lengths of the intervals of the piece-wise linearization:

$$xw(i,p) =< xwmax(i,p), \text{ for } i=1,...,ni \text{ and } p=1,..np(i), \tag{8}$$

where np(i) is the number of straight segments of the linearized function for a tertiary sector i. The total water supply to a certain tertiary sector is simply the sum of the respective auxiliary variables:

$$qx(i) = \sum_{p=1}^{np(i)} xw(i,p), \text{ for all } i. \tag{9}$$

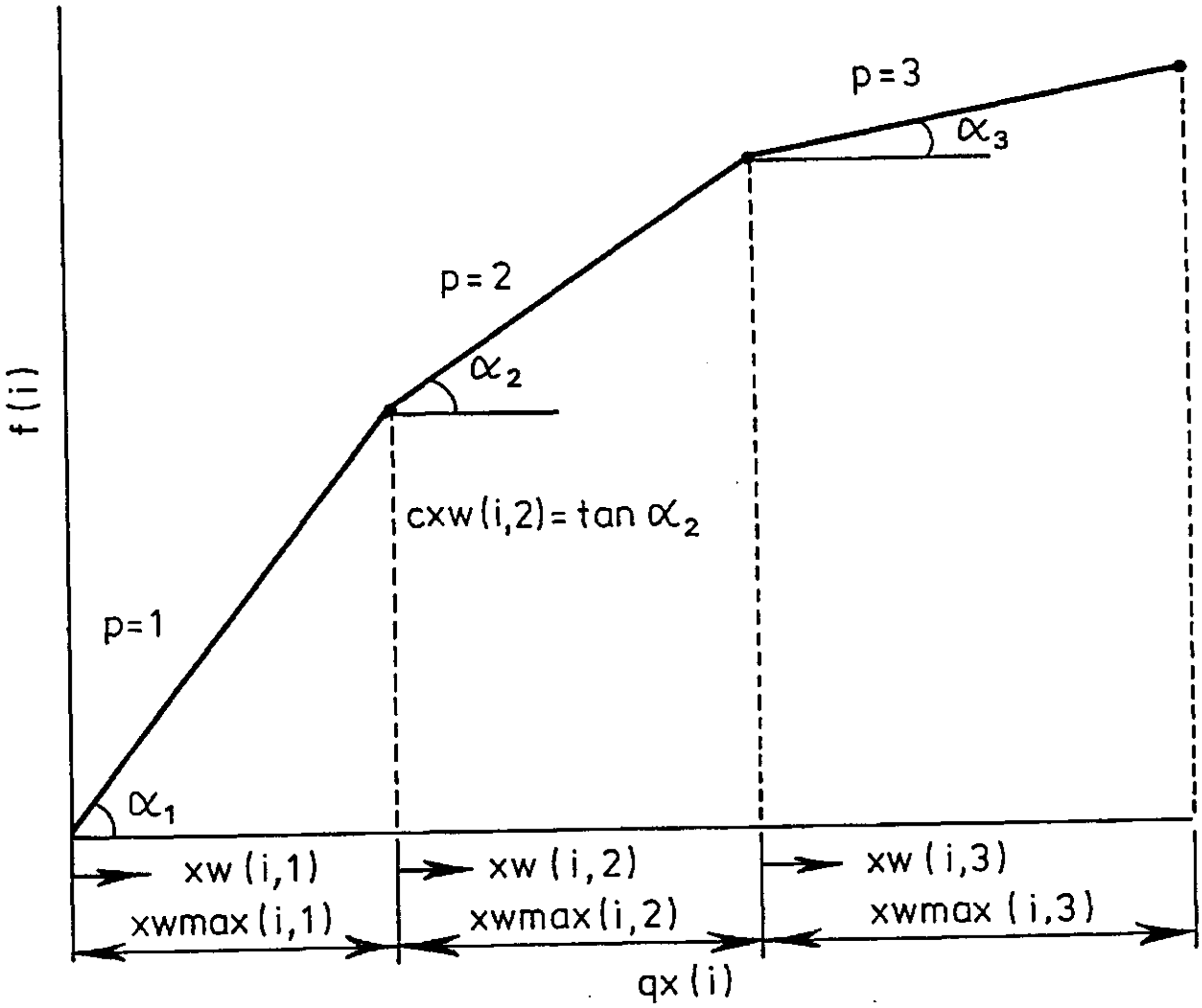

Fig. 3. Piece-wise linearization of the costs-reduction function f(i) for a tertiary sector i:

- p : index for segments
- np(i) : total number of segments
- xw(i,p) : auxiliary variable for qx(i) within interval p
- xwmax(i,p): length of interval p
- cxw(i,p) : costs reduction per unit of qx(i) within interval p

The objective function equalling the total costs reduction of water supply can now be written as:

$$F2 = \sum_{i=1}^{ni} \sum_{p=1}^{np(i)} cxw(i,p)*xw(i,p) \qquad (10)$$

Due to the convexity of the costs reduction function in combination with the maximization of the function F2, the optimization algorithm will automatically only select a certain auxiliary variable to become non-zero after the auxiliary variables with lower p-values have been 'filled up' to their upper bounds.

2.3.3 Striving for minimum total costs, under constraints with respect to the inequity of the water distribution

For implementing the strategy of minimizing costs under constraints with respect to the inequity of the water distribution, the constraints listed in Sections 2.3.1 and 2.3.2 are simply combined and some extra ones are added. The variables wt and F1 are treated differently, however, than when the first strategy is followed. The value of wt now does not have to be specified as an input parameter, because due to the maximization of F2 there is no danger that the trivial result of a zero wt-value will be obtained. And F1 is now clearly not taken as the objective function. Instead, an upper bound can be specified, which puts a limit to the overall inequity of the water distribution:

$$F1 \le F1max \qquad (11)$$

And for the deviations of the supplies to the tertiary sectors there can be constraints of the form:

$$wm(i) \le wmmax(i), \text{ and} \qquad (12a)$$

$$wp(i) \le wpmax(i), \qquad (12b)$$

for all i.

3. User manual

3.1 Overview of programmes and files

The names of input and output files are respectively INALL.DAT and SOLUT.DAT. The commands for running the program have been collected into the file BAT.COM. The commands contained in this file are:

- RUN GENER
- RENAME MAT.DAT MPS.DAT
- RUN MINOS

Execution is simply by giving the command @BAT (VMS) or whatever is appropriate under the operating system that the used computer is being run. The programme GENER has been created using the pre-processor GEMINI (Lebedev, 1984). The actual optimisation can for instance be done by the programme MINOS (Murtagh & Saunders, 1983). The file MPS.DAT is a standard way for describing the input data of programmes for LP. Thus many other software packages can be used instead of MINOS.

3.2 Input file

3.2.1 Specifications

The input of data is done in the so-called list-directed way, which means that the data do not have to be in a certain 'format'. They do of course have to be in a certain order; and the distinction between 'integers' (indicated by I) and 'reals' (R) should be respected. Care should also be taken to follow the instructions concerning the starting of a new record in the input file. In the specifications given below the requirement to start a new record is indicated by an asterisk in the column 'new rec'. Parameters that for a certain value of the option parameter IO are not required should be contained as 'dummies' (e.g. value 0.0).

new rec	symbol	description	type
*	IO	option parameter for objective func-	I
		tion : IO=1 for F1, IO=2 for F2	
*	NI	number of tertiary sectors	I
	NJ	number of arcs in the supply network	I
	NK	number of nodes in the supply network	I
*	KI(I),	number of the node from which a terti-	I
	I=1,NI	ary sector gets its water	
*	NP(I),	number of segments of piece-wise	I
	I=1,NP(I)	linear costs reduction function	
For K = 1,NK :			
*	NU(K)	number of upstream arcs of node K	I
	JU(K,LJ),	J-values of upstream arcs	I
	LJ=1,NU(K)		
	ND(K)	number of downstream arcs of node K	I
	JD(K,LJ),	J-values of downstream arcs	I
	ND(K)		
*	WTF	specified value of mean water supply	R
		to tertiary sectors (m/yr)	
		(only relevant for IO=1, else a dummy)	
*	F1MAX	maximum allowed value of F1-function	R
		(m/yr) (only relevant for IO=2)	
For J = 1,NJ :			
*	QC(J)	capacity of arc J (m3/yr)	R
	CE(J)	conveyance efficiency of arc J (-)	R
	CD(J)	intercept term of conveyance function	R
		(m3/yr)	
For K = 1,NK :			
*	QSMAX(K)	maximum external supply capacity at	R
		node K (m3/yr)	

```
------------------------------------------------------------|
For I = 1,NI :                                        |     |
   *    | AA(I)     | irrigated area in tertiary sector I  |  R  |
        |           | (m2)                                |     |
  FOR P = 1,NP(I) - - - - - - - - - - - - - - - - - - - - - - |
        |XWMAX(I,P)| upper bound of auxiliary variable     |  R  |
        |           | XW(I,P) of costs reduction function   |     |
        |           | (m3/yr)                               |     |
        | CXW(I,P)  | marginal costs reduction  (f/m3/yr)   |  R  |
  - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - |
------------------------------------------------------------|

------------------------------------------------------------|
For I = 1,NI :                                        |     |
   *    | WMMAX(I)  | maximum 'negative deviation' of water |  R  |
        |           | supply to a tertiary sector (m3/yr)   |     |
        | WPMAX(I)  | maximum 'positive deviation' of water |  R  |
        |           | supply to a tertiary sector (m3/yr)   |     |
------------------------------------------------------------|
```

3.2.2 Example

An example of the file INALL.DAT is:

```
 1
 4  10  10
 4   6  10  8
 2   2   2  2
 0            1  1
 1  1         2  2  7
 1  2         3  3  4  6
 1  3         0
 1  4         1  5
 1  5         0
 1  7         2  8  9
 1  8         0
 2  6  9      1  10
 1  10        0
 0.1
 10000.
 100.0  0.8  0.0
 100.0  0.8  0.0
 100.0  0.8  0.0
 100.0  0.8  0.0
 100.0  0.8  0.0
 100.0  0.8  0.0
 100.0  0.8  0.0
 100.0  0.8  0.0
 100.0  0.8  0.0
 100.0  0.8  0.0
 200. 0. 0. 0. 0. 0. 0. 0. 0. 0.
 30. 45.  0.2  45  0.1
 30. 45.  0.2  45  0.1
 30. 45.  0.2  45  0.1
 30. 45.  0.2  45  0.1
 10000. 10000.
 10000. 10000.
 10000. 10000.
 10000. 10000.
```

3.3 Example of an output file

The output file SOLUT.DAT that MINOS makes for the given example of an input file is given below. This file can be further processed for producing specific tables, graphics, etc.

```
PROBLEM NAME    GEMINI              OBJECTIVE VALUE   1.1999928000E+01

STATUS          OPTIMAL SOLN        ITERATION   13    SUPERBASICS   0

OBJECTIVE       CRITER   (MIN)
RHS             RHSIDE
RANGES
BOUNDS          BOUND1
```

SECTION 1 - ROWS

NUMBER	...ROW..	STATE	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY	..I
50	WJ001	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	1
51	WJ002	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	2
52	WJ003	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-1.999980E+00	3
53	WJ004	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-1.999980E+00	4
54	WJ005	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-1.999980E+00	5
55	WJ006	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-1.599984E+00	6
56	WJ007	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	7
57	WJ008	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	4.174815E-17	8
58	WJ009	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	9
59	WJ010	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	2.076475E-17	10
60	WK001	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	11
61	WK002	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	12
62	WK003	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	1.599984E+00	13
63	WK004	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	1.999980E+00	14
64	WK005	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	1.599984E+00	15
65	WK006	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	1.999980E+00	16
66	WK007	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	17
67	WK008	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	18
68	WK009	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	19
69	WK010	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	20
70	DW1	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-1.199993E+02	21
71	DV0001	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-3.000000E+01	22
72	DV0002	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-3.000000E+01	23
73	DV0003	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	3.000000E+01	24
74	DV0004	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	3.000000E+01	25
75	FF1	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+00	26
76	FF2	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	27
77	CRITER	BS	1.199993E+01	-1.199993E+01	-1.000000E+20	1.000000E+20	-1.000000E+00	28

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	STATE	...ACTIVITY...	.OBJ GRADIENT.	..LOWER LIMIT.	..UPPER LIMIT.	REDUCED GRADNT	M+J
1	GI001	BS	2.783213E+01	0.000000E+00	0.000000E+00	1.000000E+02	0.000000E+00	29
2	GI002	D BS	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+02	0.000000E+00	30
3	GI003	D BS	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+02	0.000000E+00	31
4	GI004	D BS	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+02	0.000000E+00	32
5	GI005	D BS	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+02	3.853410E-17	33
6	GI006	LL	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+02	3.199968E-01	34
7	GI007	BS	2.226571E+01	0.000000E+00	0.000000E+00	1.000000E+02	0.000000E+00	35
8	GI008	BS	3.750038E+00	0.000000E+00	0.000000E+00	1.000000E+02	3.339852E-17	36
9	GI009	BS	1.406253E+01	0.000000E+00	0.000000E+00	1.000000E+02	0.000000E+00	37
10	GI010	BS	1.125002E+01	0.000000E+00	0.000000E+00	1.000000E+02	1.661180E-17	38
11	GO001	BS	2.226571E+01	0.000000E+00	0.000000E+00	1.000000E+02	0.000000E+00	39
12	GO002	A LL	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+02	0.000000E+00	40
13	GO003	D BS	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+02	-1.363223E-16	41
14	GO004	LL	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+02	3.999960E-01	42
15	GO005	D BS	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+02	0.000000E+00	43
16	GO006	D BS	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+02	0.000000E+00	44
17	GO007	BS	1.781257E+01	0.000000E+00	0.000000E+00	1.000000E+02	0.000000E+00	45
18	GO008	BS	3.000030E+00	0.000000E+00	0.000000E+00	1.000000E+02	-4.174815E-17	46
19	GO009	BS	1.125002E+01	0.000000E+00	0.000000E+00	1.000000E+02	0.000000E+00	47
20	GO010	BS	9.000018E+00	0.000000E+00	0.000000E+00	1.000000E+02	-2.076475E-17	48
21	GS001	BS	2.783213E+01	0.000000E+00	0.000000E+00	2.000000E+02	0.000000E+00	49
22	GS002	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	50
23	GS003	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-1.599984E+00	51
24	GS004	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-1.999980E+00	52
25	GS005	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-1.599984E+00	53
26	GS006	EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-1.999980E+00	54
27	GS007	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	55
28	GS008	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	56
29	GS009	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	57
30	GS010	A EQ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	58
31	XW00011	D BS	1.027581E-16	0.000000E+00	0.000000E+00	4.500000E+01	0.000000E+00	59
32	XW00012	A LL	0.000000E+00	0.000000E+00	0.000000E+00	4.500000E+01	-4.502750E-17	60
33	XW00021	D BS	-5.137906E-17	0.000000E+00	0.000000E+00	4.500000E+01	0.000000E+00	61
34	XW00022	A LL	0.000000E+00	0.000000E+00	0.000000E+00	4.500000E+01	-4.502750E-17	62
35	XW00031	BS	9.000018E+00	0.000000E+00	0.000000E+00	4.500000E+01	0.000000E+00	63
36	XW00032	A LL	0.000000E+00	0.000000E+00	0.000000E+00	4.500000E+01	-4.502762E-17	64
37	XW00041	BS	3.000030E+00	0.000000E+00	0.000000E+00	4.500000E+01	2.998778E-17	65
38	XW00042	A LL	0.000000E+00	0.000000E+00	0.000000E+00	4.500000E+01	2.252433E-17	66
39	WP0001	LL	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+04	6.000000E+01	67
40	WP0002	LL	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+04	6.000000E+01	68
41	WP0003	BS	1.999976E-01	0.000000E+00	0.000000E+00	1.000000E+04	-6.660754E-16	69
42	WP0004	A LL	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+04	-1.332151E-15	70
43	WM0001	BS	1.000000E-01	0.000000E+00	0.000000E+00	1.000000E+04	0.000000E+00	71
44	WM0002	BS	1.000000E-01	0.000000E+00	0.000000E+00	1.000000E+04	0.000000E+00	72
45	WM0003	LL	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+04	6.000000E+01	73
46	WM0004	LL	0.000000E+00	0.000000E+00	0.000000E+00	1.000000E+04	6.000000E+01	74
47	WT1	EQ	1.000000E-01	0.000000E+00	1.000000E-01	1.000000E-01	1.199993E+02	75
48	FO1	BS	1.199993E+01	1.000000E+00	0.000000E+00	1.000000E+20	0.000000E+00	76
49	FO2	BS	2.400010E+00	0.000000E+00	0.000000E+00	1.000000E+20	0.000000E+00	77

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