REVIEW OF TECHNICAL CONSIDERATIONS IN THE DESIGN AND ANALYSIS OF EXPERIMENTS FOR ESTIMATING INPUT/OUTPUT RELATIONSHIPS

By Th. J. Ferrari,
Institute for Soil Fertility,
Groningen, the Netherlands.

Introduction

It is the task of agronomic-economic research to help the farmer choose among alternative technical possibilities he has to choose in a certain situation. Answering such questions can be very hard indeed ; the agronomist is often in a difficult position because he has both to take into account a great number of factors and to include some which are difficult to measure. These difficulties apply particularly for the agriculturist studying the economics of milk production. He must know which alternative possibilities of a mixed farming-system of Western Europe should be undertaking. What are the financial consequences, for example, of an increase of the grass yield through an increase in the nitrogen dressing via the cycle : soil, crop and cattle ? To a lifestock farmer, words of mathematician BROSS' book "Design for Decision" apply particularly : "It is much more difficult to be a good farmer than a good mathematician because the farmer must deal with so many vague and complex problems". It is the task of research to give solutions to these problems.

Clearly it is necessary, in making a justified choice between alternatives, to have at one's disposal a quantitative description (as complete as possible) of the technical possibilities and their consequences. Whether one has to advise the farmer with the help of programming models, or to analyse a certain farm result, this knowledge is always necessary. In the first case, the technical relationships are taken for granted ; in the second case, the accent lies on the explanation of certain phenomena. The procedures correspond since one is making a representation of the relationship or phenomenon by means of models in both cases. In the first case, the model is assumed to be known ; in the second case, the model is constructed as an hypothesis whose reality is tested by an experiment Consequently research concerns itself with the model mainly as an hypothesis. In this paper, we deal only with problems involving the investigation into the explanation of relationships. We know, however that the results of this research can be used for all sorts of programming purposes.

Use of models in agricultural research

What do we understand by models and what are their functions in research ? Models are simplifying abstractions of reality, in which only those elements already familiar to us and which are of reality in the science concerned, are considered. The abstraction is expressed in some language, that may be in words or in diagrams, mathematically or materially. Within the given limits we try to describe the reality as completely as possible.

It is of great importance in research that these models should have the attribute that the conclusions drawn from them hold true for reality. In other words, the reality of an assumed model is closely connected with that of the conclusions. It also appears that the hypotheses, so important for progress in sciences, are very suitable for being expressed in models. In this way we obtain the connection between models and research as follows. As in all empirical sciences the systematic increase of our knowledge in agricultural research is acquired by the formulation of hypotheses, which are tested against reality by means of observations and predictions. Absence of agreement between observations and predictions rejects the hypothesis ; presence makes the hypothesis more acceptable. In connection with the complex and practical character of agriculture, it appears fruitful to build models in the form of mathematical equations. In view of the particular character of the object, e.g. plant or milk production, some difficulties are met in the testing and quantifying of the parameters of the models. These difficulties are also present in other sciences such as sociology, economy and astronomy.

It is clear that the ultimate criterion in agriculture is that production must be expressed in some economic terms. In the model, production (e.g. the milk yield in kg. per ha., will be brought in causal relation with a number of factors. In a simple case the function has the following form : milk yield depends on the amounts of roughage and concentrates. This is a very simple model, which applies perhaps for cow stall feeding under certain conditions. It is much more difficult, however, to relate farm economic results with the amount of dressings of nitrogen. No direct relation between these two factors exists and all kinds of factors may interfere. It is clear, however, that the hypothetical model of these relationships becomes more complex and that testing and quantifying will encounter some difficulties. The following groups can be distinguished among those factors which influence the yield or economic results of an operation :

a) controllable factors which can be varied, such as dressings of nitrogen and amount of concentrates ;

b) pre-determined or non-changeable factors, which can be measured previously or predicted : soil profile and ground-water level, size of holdings, number of cows per ha., etc. ;

c) uncontrollable or non-changeable factors, which cannot be predicted such as weather, diseases and pests, economic situation, etc.

The complex character of production, especially under farm conditions, and the peculiar attributes of the above mentioned factor groups have certain consequences in research for the construction and testing of models.

It is well-known that the testing of a hypothesis in natural sciences takes place mainly by means of an artificial variation, ceteris-paribus, according to the idea that a change of a factor, assumed to be a cause, must also result in a corresponding change of the effect. In this, the ceteris-paribus assumption is very important. The introduction of a variation is difficult or impossible when we are dealing with factors of the second or third group, for they are not changeable. Astronomy shows that it is possible to obtain important results in science without artificial change. Furthermore, it is dubious whether the ceteris-paribus principle, under an artificial change, can be maintained in many cases. Changes in groundwater table or in dressings of nitrogen cause a chain of changes of other factors which can, in turn, affect production. The result is that conclusions about a factor causing a phenomenon cannot be drawn. Also, it is difficult to investigate effects of certain changes under farm conditions. Restrictions under farm conditions, costs,etc., prevent the introduction of experiments with artificial changes in farm economic research. There are perhaps possibilities in certain production branches of artificial changes which can be measured apart from uncontrolled changes. Examples might include feeding which takes place in the cowstall and with purchased feeding stuffs only.

A second difficulty is the great number of factors which influence the production and their interdependence. In practice, consequently, the research worker always has to investigate many factors together.

The normal experiment, with which the influence of one or two factors is investigated, is not well fitted to solve practical questions. It is a well-known fact that an increase in the number of factors investigated soon becomes impossible ; an increase in the number of factors increases the size of the experiment, by which the residual variance (inevitable with field experiments) becomes the main factor. Although statisticians have tried to eliminate this drawback by introducing the principle of confounding, a satisfying solution has not yet been obtained.

The limitation remains, that the results of these experiments only hold for the special conditions of soil, climate, treatment, etc. under which the experiment was conducted. The experience is, therefore, that the results of the different investigations can diverge strongly. The research worker can try to solve this difficulty by carrying out a large number of experiments to obtain a good sample of production circumstances. However, only an average result is obtained. Neither does a sub-division, according to geographical units, usual in sociology, satisfy. Without a more profound analysis of the factors causing the differences, an extrapolation from the average result to future individual cases remains dangerous.

Such an analysis is possible, however, since the test hypothesis need not be restricted to empirical data obtained by an artificial change only. Under the influence of natural sciences, many research workers are of the opinion that the so-called experiment with an artificial change [*controlled experiment* (1)] is the only correct method. However, it is quite feasibly possible to test a hypothesis by means of data from an experiment without this artificial change [*uncontrolled experiment* (1)], in which the variation of nature is used. As far as the logic of experimentation is concerned, this distinction is of no account at all. The testing of the hypothesis by

---

(1) In the Dutch language respectively "*proef met ingreep*" and "*proef zonder ingreep*", in the German language "*Experiment mit Eingriff*" and "*Experiment ohne Eingriff*".

means of deduction with given predictions is decisive. The word "experiment", derived from the latin verb "experiri", i.e. to test expresses this already. However, by the methods and results of the physical and chemical sciences, the word "experiment" has come to have quite a different sense, viz. artificial change, and the original sense is often forgotten. Of course, it must be said that an uncontrolled experiment also involves certain difficulties, particularly in obtaining a sufficient separation between the possible causal factors. The difficulties of a controlled experiment should not be understated either. We have mentioned already the unreal assumption of the ceteris-paribus situation. In a previous paper we compared the advantages and the disadvantages of both methods. It was evident that an experiment without artificial change makes it possible to test and to quantify models in which factors of the second and third group are taken up ; factors therefore which are not or are hardly changed, and by which differences between the results of the experiments can be explained.

Two-variable models ; single equation

Which models and which functions are generally used in agricultural research ? In the following discussion we have tried to illustrate the statements with examples derived from investigations into milk production, but it is very difficult to find appropriate examples. The general complaint in the literature available is that very little is known about the nature of the function representing the relationship between feeding-stuff and milk production. By force of circumstances we have taken examples derived out of our own soil fertility studies. We restrict ourselves, for the present, to models which can be described in a single equation with one or more factors.

The most simple model is the hypothesis that the yield differences can be explained by one or more factors without a further description of the function form. This is the point of view of the analysis of variance. The drawback, of course, is that a possibility to interpolate and to extrapolate is difficult because of the absence of a function. Economic interpretations are difficult in that case.

Another possibility is the model represented by a linear equation : a unit increase of the independent variable increases the effect by a constant amount, regardless of the value of the first variable. We know that this assumption is, in many cases, not real, especially in milk production investigations : the linearity can be useful in a limited region of the production. However, according to experiences in agricultural research it would serve a more useful purpose to use non-linear functions which allow reaching a maximum. There are advantages in choosing the most simple function in this case. In the literature many equations have been proposed. The most well-known equation is the Mitscherlich equation (followed by a depression). Some other functions are included below :

$$y = A(1 - 10^{-cx}) \qquad \text{Mitscherlich}$$

$$y = ax^{b} \qquad \text{Cobb-Douglas}$$

$$y = A.10^{-z\left(\log \frac{x+i}{a+i}\right)^{n}} \qquad \text{von Boguslawski-Schneider}$$

$$y = bx - cx^{2}$$

$$y = b\sqrt{x} - cx$$

We now abandon discussion of the general and particular properties of these functions. For this, we refer to the publication of HEADY and DILLON and to the paper of HOFFMANN and DÖRFEL.

These equations have one thing in common. They are developed mainly heuristic ; their theoretical base is very small. By this we mean to say that there is no preference for some equation from a physiological or a biochemical standpoint. The only theoretical derivation we know is the one for the Mitscherlich equation by LINSER and KAINDL in the field of plant nutrition. It is striking indeed that so little basic research on production functions has been undertaken. It appears that we are urgently in need of more biologically derived equations, especially in view of the great possibilities which computers give in the solution of these equations.

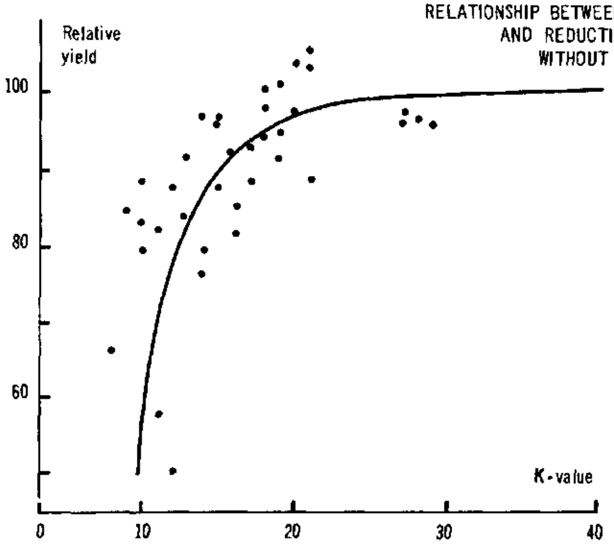


Figure 1

RELATIONSHIP BETWEEN POTASH-STATUS OF THE SOIL AND REDUCTION IN POTATO YIELD, WITHOUT POTASH-DRESSING

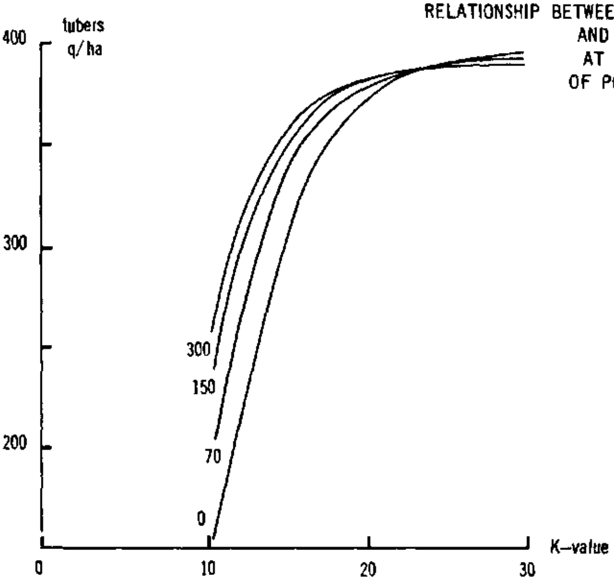


Figure 2

RELATIONSHIP BETWEEN POTASH-STATUS OF THE SOIL AND POTATO YIELD, AT FOUR LEVELS OF POTASH-DRESSING

Personal preference determines which equation is chosen ultimately. The choice is often made by the suggestion of the observational data provided. A study on milk production by HEADY, SCHNITTKER, JACOBSON and BLOMM, leaves the choice between three functions, viz. the logarithmic, the quadratic or the square root equations. It is clear that the function ultimately chosen should be used again as an hypothesis in the next investigation ; experience shows that this has often not been done. The uncertainty about the function to be used and the impossibility to compute at that time (there were no calculating machines) are reasons why over the last 30 years the graphic method in our soil fertility research has developed, always employing suggestions provided by observational data. The same method has been employed in economic research in the United States of America. As an example we show in Figure 1 the results of an investigation on the relationship between potash-status of the soil and the loss in potato yield without potash-dressing expressed in percentages of the maximal yield. Each point represents the result of one field experiment, the differences in potash-status are acquired without artificial change by taking natural situations. We may expect that the differences between the graphic and numerical method will be small.

Multi-variable models ; single equation

We know that two-variable models mostly do not meet the needs of a complete or satisfying description of production processes. With a view to this description, functions with more factors have been developed such as :

$$y = b_1x_1 + b_2x_2 + \ldots\ldots\ldots$$

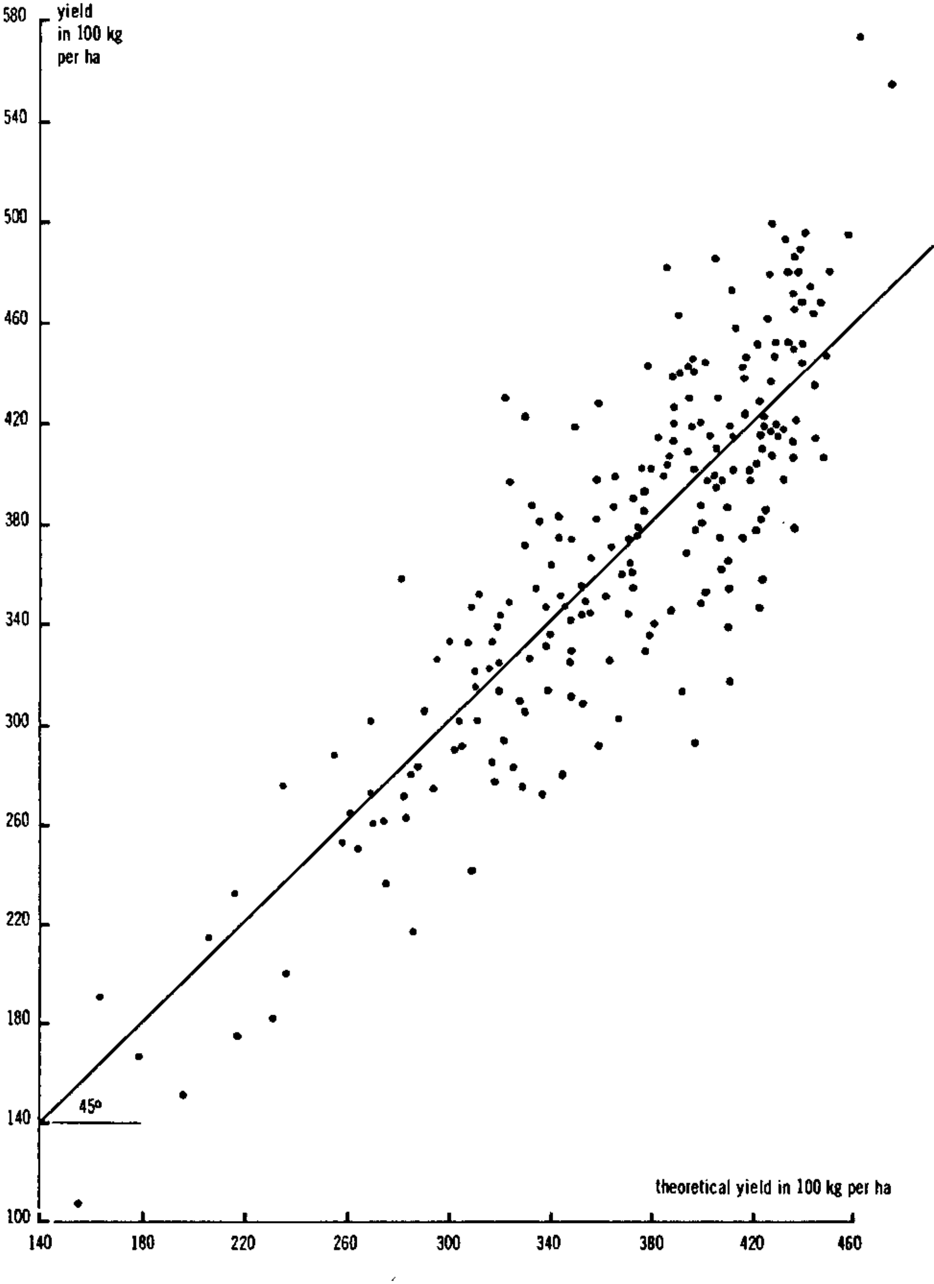
$$y = A(1-10^{-c_1x_1})(1-10^{-c_2x_2})\ldots\ldots \qquad \text{Mitscherlich}$$

$$y = ax_1^{b_1}\, x_2^{b_2} \ldots\ldots\ldots\ldots \qquad \text{Cobb-Douglas}$$

$$y = b_1x_1 + b_2x_2 + b_{12}x_1x_2$$

Figure 3

CORRELATION BETWEEN ACTUAL AND CALCULATED POTATO YIELDS



The properties of these equations will not be discussed either, although they are important in connection with such terms as isoclines, substitution rates, etc. We only point to the possibility of including equation terms for interactions ; in the last equations, the product term represents the interaction. Although the interaction, in our opinion, is mostly none other than a word to mark our lack of knowledge. Figure 2 shows a tested model with interaction in which the effect of potash-dressing depends on the potash-status of the soil.

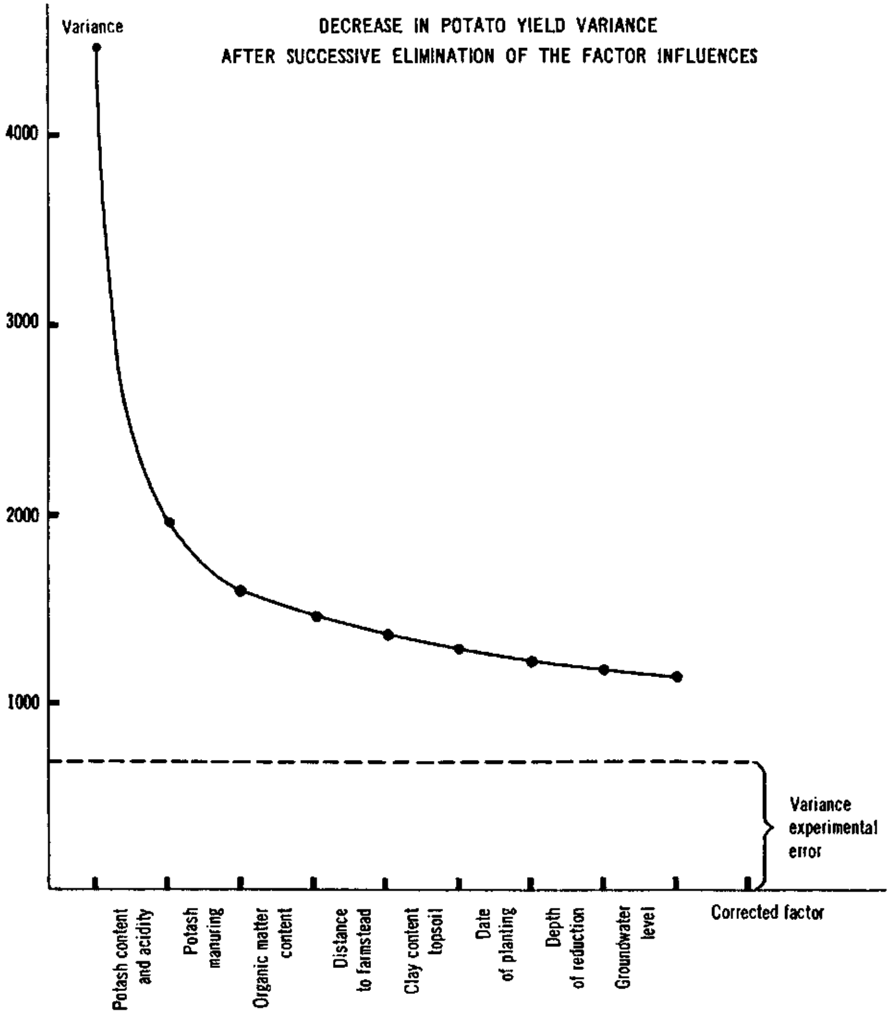
The extreme consequences of the possibilities of an experiment without artificial change and of multi-variate models are the investigations in which the research worker tries to find, in a graphic or numerical way, an explanation for the differences in yield or economic farm results by means of single plots or farms, respectively. A model has been drawn up which aims to give an explanation of the total variance present in nature. The upbuilding of the model with many factors goes rather far. In contrast to the design of the analysis of variance, to make the rest variance as small as possible, these multi-factorial investigations are interesting especially in a large starting variance. Figure 3 shows the possibility of such an analysis by means of the correlation between actual and estimated yields. This analysis was based on a model with thirteen factors, of which nine had a statistically significant influence. Figure 4 shows the decrease of the yield variance by successive elimination of the factor influences. The diagram also shows a phenomenon which is probably more general : while many factors have a small influence, only a few factors have a relatively great influence. We will return later to this subject.

Multi-equation models, chain processes

The equations of the models discussed above are essentially normal regression equations. The regression model is characterised by the hypothesis that a causal relationship exists between the so-called independent or causal factors and the dependent factor or

Figure 4

DECREASE IN POTATO YIELD VARIANCE AFTER SUCCESSIVE ELIMINATION OF THE FACTOR INFLUENCES



effect. It is also assumed that a change in an independent factor affects the dependent variable only and does not affect the other independent factors. The same assumption must also be made in the experiment with artificial change according to the ceteris-paribus principle. We find, however, that these assumptions are not often in agreement with the facts, both in the experiment with artificial change and in the experiment without this change. This means that the assumed model is incorrect and cannot be applied.

We can demonstrate this by means of an example from an investigation concerning the factors affecting the magnesium content of herbage. At first, a normal regression model was constructed and tested by observations out of an uncontrolled experiment. The diagram in Figure 5 shows the hypothetical model where magnesium content of the herbage is the dependent variable or effect. Further it is assumed that the factors magnesium, potash and humus content of the soil, the crude-protein content and the proportion of weeds in the herbage, will influence causally the magnesium content of the herbage. In the diagram these influences are marked by arrows ; the rate and the direction are calculated from the observations. Thus we assume that a change of the magnesium content of the soil only affects the magnesium content of the herbage but does not affect the crude-protein content and the proportion of weeds. However, we know from other investigations that this is not true ; the model is therefore not acceptable. Essentially we meet, in this case, a so-called chain process, which is not describable by means of one equation.

The diagram in Figure 6 describes a model of these relationships which are probably more in agreement with reality. The variables crude-protein content and the proportion of weeds are not only taken as independent variables ; both variables are now cause as well as effect. A change of the magnesium content of the soil affects the magnesium content of the herbage not only directly but also indirectly via the chain : proportion of weeds and crude-protein content. The model of Figure 5 without these chain processes

Figure 5

REGRESSION MODEL WITH MAGNESIUM CONTENT OF THE HERBAGE AS DEPENDENT VARIABLE, OTHER VARIABLES AS INDEPENDENT CAUSAL VARIABLES

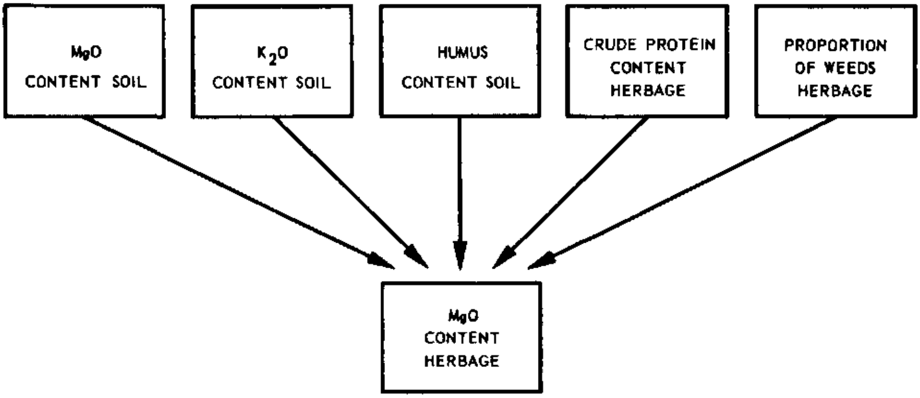
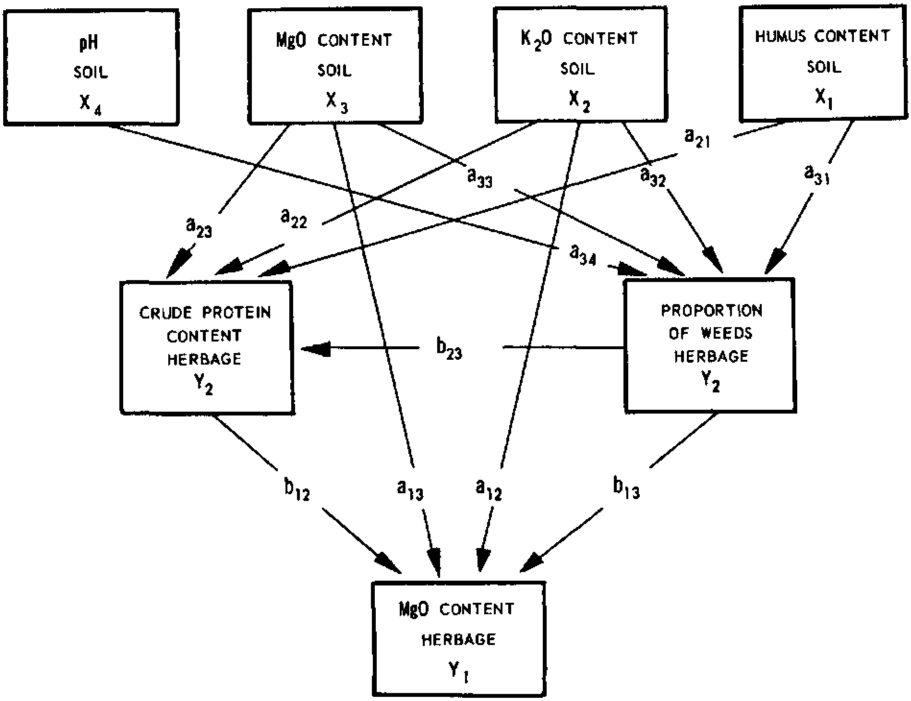


Figure 6

DIRECT AND INDIRECT INFLUENCES OF THE FOUR CAUSAL FACTORS ON THE MAGNESIUM CONTENT OF THE HERBAGE



can be represented by one equation :

$$y_1 = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5,$$

the second model needs a system of the following three equations :

$$y_1 = b_{12}y_2 + b_{13}y_3 + a_{12}x_2 + a_{13}x_3$$

$$y_2 = b_{23}y_3 + a_{21}x_1 + a_{22}x_2 + a_{23}x_3$$

$$y_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4$$

Such systems of equations can be solved by the method of path co-efficients. The term "path" is concerned with paths via which the influence is effected. By this method the hypothesis formulated in a model concerning these relationships is tested and quantified. The influence is represented by the path co-efficient, giving the rate and direction of the effect change for every unit change of the causal variable. Table 1 gives the results of the analysis of the model shown in Figure 6.

The general form of an equations system describing a chain process is as follows :

$$\begin{array}{l} b_{11}y_1 + \dots + b_{1M}y_M + a_{11}x_1 + \dots + a_{1L}x_L = u_1 \\ b_{21}y_1 + \dots + b_{2M}y_M + a_{21}x_1 + \dots + a_{2L}x_L = u_2 \\ \vdots \\ b_{M1}y_1 + \dots + b_{MM}y_M + a_{M1}x_1 + \dots + a_{ML}x_L = u_M \end{array}$$

It is clear that some path co-efficients 'a' and 'b' (a priori) may be assumed to be zero in real models. By means of this method it is also possible to investigate models in which feedback systems are included. In our opinion, such models are to be preferred to the normal regression models, especially by reason of their closer correspondence to the reality. It is possible to use non-linear functions in these systems. The method is closely related to the method of simultaneous equations as used in econometrics.

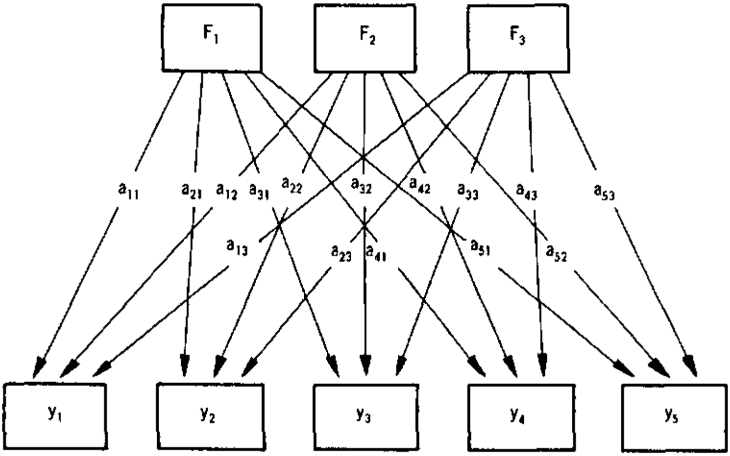
Table 1

Computed values of the twelve path co-efficients of the model, Figure 6

Cause / Effect	Humus content (x_1)	K_2O content soil (x_2)	M_gO content soil (x_3)	pH (x_4)	Proportion of weeds (y_3)	Crude protein content (y_2)
Proportion of weeds (y_3)	1,67	-0,23	-0,031	5,26		
Crude protein content (y_2)	-0,74	0,11	0,011		0,20	
MgO content of herbage (y_1)		-0,0038	0,0004		0,0041	0,0083

Figure 7

DIAGRAM OF A FACTOR-ANALYSIS MODEL



Extreme cases of such models are those on which factor analysis is based. The number of limiting conditions in a factor analysis model is small ; as a result the system of equations has become so-called unidentifiable - an exact solution cannot be obtained by mathematical arguments only. The schematic diagram of such a model is given in Figure 7. The causal x-variables, here named F, are unknown. Next the analysis tries to calculate these F-variables as so-called aspects. This factor analysis is not only important for the testing of such models, it can also be used to provide ideas for drawing up a more limited hypothesis. The possibilities of this analysis are many. The method is very suitable, for example, to indicate and to quantify the ecological properties of grasses grown under natural conditions. The starting point for the analysis is the matrix of correlation co-efficients between soil factors and sociological characteristics, in this case the frequency percentages of the grasses. The factor analysis with a following rotation of the aspect axes resulted in a number of aspects given in Table 2, of which the first aspect represents the grass reaction to water supply. The differences in numbers running from plus one to minus one are a measure of this reaction. The positive numbers show the hydrophile character, the negative ones the drought resistence. The most remarkable result is that it is obtained by a mathematical analysis followed by a rotation to the simple structure only. The choice of a rotation to the simple structure is based on the already mentioned phenomenon, that many factors have a small influence and only few factors have a great one. A rotation of the model to simple structure tries to reach the same situation. In the Netherlands, the Agricultural Economics Research Institute is using this factor analysis model in farm management research.

Table 2

Interdependences of soil factors and
frequency percentages of grasses;
aspect values after rotation to simple structure

Factor	Aspects			
	1	2	3	4
pH (KC1)	0,655	-0,246	-0,209	-0,074
Humus content	0,684	-0,098	0,003	-0,240
Silt content	0,811	-0,298	-0,113	0,003
Sand content	-0,881	0,242	0,074	0,110
Specific surface sand	0,671	-0,261	-0,258	-0,028
Magnesium content soil	0,575	-0,385	0,266	-0,152
Phosphate content soil (water)	-0,137	0,550	0,255	0,010
Phosphate content (citric acid)	0,650	0,184	0,112	0,243
Potash content	-0,049	0,691	0,396	-0,463
Copper content (Asp.)	0,647	-0,340	-0,096	0,055
Distance farm	0,318	0,029	-0,493	0,020
Depth clay-layer	0,197	0,360	-0,040	0,380
Thickness humus-layer	0,023	-0,004	-0,038	0,568
Moisture content	0,611	-0,124	-0,062	0,400
Groundwater level	0,626	-0,381	-0,075	-0,409
Fluctuation	-0,495	-0,214	0,059	-0,023
Nitrogen dressing	-0,151	0,352	0,357	0,320
Phosphate dressing	0,007	0,461	-0,037	0,059
Potash dressing	0,023	0,538	0,252	0,116
Poa pratensis L.	-0,401	0,052	0,103	0,248
Festuca rubra L.	0,383	-0,131	-0,412	0,298

	1	2	3	4
Agrostis tenuis Sibth.	-0,341	-0,259	-0,213	-0,446
Lolium perenne L.	-0,254	-0,217	0,282	-0,166
Poa annua L.	-0,219	0,252	0,563	-0,069
Alopecurus geniculatus L.	0,321	-0,301	0,336	-0,357
Agropyron repens P.B.	-0,246	0,075	0,372	0,326
Festuca pratensis Huds.	0,787	-0,024	-0,005	0,154
Poa trivialis L.	0,314	-0,726	0,034	-0,181
Agrostis stolonifera L.	-0,105	-0,186	-0,365	-0,107
Dactylis glomerata L.	-0,111	0,069	0,156	0,342
Achillea Millefolium L.	-0,270	0,072	0,227	0,181
Ranunculus repens L.	0,051	-0,427	-0,086	-0,218
Cardamine pratensis L.	0,346	-0,521	-0,025	0,004
Carex stolonifera Hoppe	0,716	0,099	-0,165	0,043
Glyceria maxima Holmb.	0,807	-0,018	-0,138	0,214
Ranunculus acer L.	0,297	-0,344	-0,489	-0,181
Rumex Acetosa L.	0,393	-0,152	-0,414	0,272
Holcus lanatus L.	0,274	-0,183	-0,568	-0,134
Anthoxanthum odoratum L.	0,172	-0,246	-0,676	-0,020
Centaurea Jacea L.	0,174	-0,201	-0,251	0,202
Bellis perensis L.	0,099	-0,482	-0,101	0,307
Cynosurus cristatus L.	0,046	-0,300	-0,232	-0,373
Alopecurus pratensis L.	0,141	-0,047	0,022	0,252
Luzula camprestis Lam. et D.C.	-0,137	0,033	-0,556	-0,023
Trifolium repens L.	-0,104	-0,203	-0,041	-0,545
Bromus mollis L.	-0,069	0,031	-0,216	0,337
Phleum pratense L.	0,024	0,011	0,309	0,124
Taraxacum officinale Web.	-0,116	-0,083	0,129	0,671
Leontodon autumnalis L.	-0,229	-0,199	0,256	0,080
Phalaris arundinacea L.	-0,207	0,176	0,062	-0,015
Quality figure grass	0,050	-0,196	0,266	-0,103

Summary

We have the experience that causation, especially in agricultura phenomena, is complex and that the method of analysis used in natur science is not satisfactory in all respects. Some suggestions for disentanging this complexity based on the following ideas are given.

The first point is connected with the notion that in agricultural research, with its applied character, the hypothesis expressed in a model and followed by a testing has to supply the main contribution to new knowledge. According to our experience this is frequently forgotten.

The second point is the idea that this testing can also be carried out with observational data from experiments without artificial change (uncontrolled experiments).

The third point is the knowledge that the research worker can choose between many models and functions. In this it is not necessary to confine the choice to functions with few factors and to regression models, in which the <u>ceteris-paribus</u> principle must be assumed.

Definite advice on which approach and which models and functions should be chosen cannot be given. Each problem must seek its own method of analysis and each research worker must go his own way and choose his own models.

Literature quoted

1. FERRARI, Th.J. Vergelijking tussen proeven met en zonder ingreep. Landbouwk. Tijdschr. 7 792–801 (1960).
2. FERRARI, Th.J. Causal soil-plant relationships and path co-efficients. Plant and Soil XIX, 81–96 (1963).
3. HEADY, E.O. and DILLON, J.L. Agricultural production functions. Ames 1961.
4. HEADY, E.O., SCHNITTKER, J.A. Milk production functions, hay/grain su stitution rates and economic optima in

JACOBSON, N.L. and BLOOM, S. dairy cow rations. Agr. Exp. Station, Iowa State College, Research Bull. 444, 1956.

5. HOFFMAN, E. and DORFELS, H. Die funktionale Betrachtungsweise bei der Auswertung von Feldversuchen. Landw. Versuchs-und Untersuchungsw., 9, 75-107 (1963).