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A 4^3 FACTORIAL DESIGN WITH CONFOUNDING IN 8×8 QUASI-LATIN SQUARES

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Introduction

Investigations, in which the shape of the curve illustrating the reaction to one or a few factors is important, may require the inclusion of 4 levels of each factor in the experimental design. If n factors are tested in all possible combinations, a 4^n factorial experiment is obtained. YATES [1]) has given rules by which a 4^n experiment can be formally transformed into a 2^{2n} experiment. Each factor at 4 levels is formally replaced by 2 quasi-factors at 2 levels. In this way the simple rules for 2^{2n} experiments are made applicable to 4^n experiments. Yet, a 4^n experiment is not identical to a 2^{2n} experiment. Especially by introduction of confounding some modifications become necessary.

An investigation concerning humification in the top-soil carried out by JAC. KORTLEVEN at the Agricultural Experiment Station and Institute for Soil Research T.N.O. at Groningen, required the construction of a 4^3 experiment with 4 replications. It was decided to use four 8×8 quasi-Latin squares with partial confounding. The types of confounding described in literature to be used in 2^6 experiments were unsuitable, since some main effects are formally interpreted as interactions.

New groups of interactions were designed and a set of eight groups was selected to be confounded with differences between rows or columns of the four squares.

Construction of confounding groups

For the sake of simplicity let the three factors be denoted by N, P and K. There are four levels of each factor:

$n_1, n_2, n_3, n_4 ; p_1, p_2, p_3, p_4 ; k_1, k_2, k_3, k_4.$

Formally these levels may be written as combinations of pairs of factors at two levels, one pair for each of the three factors N, P and K. Let these combinations correspond in the following order to the levels mentioned;

$-, a, b, ab ; -, c, d, cd ; -, e, f, ef.$

There are 9 degrees of freedom for the main effects: $N', N'', N''' ; P', P'', P''' ; K', K'', K'''$. The corresponding effects in the formal 2^6 design are $A, B, AB ; C, D, CD ; E, F, EF$. Thus some first order interactions of the 2^6 design are main effects in the 4^3 design, and these should not be included in any group of interactions that is to be confounded. Obviously there are 27 degrees of freedom for first order interactions: $N'P'$, $N'P''$, $N'P'''$, $N'K'$ .... and 27 for second order interactions: $N'P'K'$, $N'P'K''$, $N'P'K'''$, $N'P''K'$ ....

For confounding with differences between rows and columns in 8×8 quasi-Latin squares, groups of 7 interactions are required. These groups are determined by three independent interactions. The remaining four cannot be chosen freely, but can be found as single and multiple products of the first three, when the products are defined in the way described by FINNEY [2]). An example of such a group is:

$$\begin{array}{cccc} N'P'' & N'K'' & (P''K'') & N''P'K' \\ (N'''P'''K') & (N'''P'K''') & (N''P'''K''') & \end{array}$$

or

$$\begin{array}{cccc} AD & AF & (DF) & BCE \\ (ABCDE) & (ABCEF) & (BCDEF) & \end{array}$$

Such a group includes 3 first order interactions. The following procedure may be followed to produce a complete set.

Nine symbols $N', N'', N''' ; P', P'', P''' ; K', K'', K'''$ are available to denote the groups of 7 interactions required. Combinations including two symbols of the same letter are no separate entities ($N'N''' = N''$) and should be avoided. Likewise no combination of any two symbols should be used twice in one group. Schematically the constitution of a group is as follows:

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[1]) YATES, F., The design and analysis of factorial experiments, Imp. Bur. Soil Sci. Tech. Comm. 35 (1937).

[2]) FINNEY, D. J. Recent developments in the design of field experiments, J. Agr. Sci 36 (1946) 184–191.

where the dots are to be replaced by symbols in the following way:

Take one symbol having the letter N, say N':

N' . N' . .. .

Take another symbol having the letter P, say P'':

$N'P''$ N' . P'' . ...

Combine these with a third symbol with the letter K, say K'', thus completing the 3 first order interactions required:

$N'P''$ $N'K''$ $P''K''$ ... ...

The remaining symbols N'' and N''' are now used

$N'P''$ $N'K''$ $P''K''$ N'' . . N'' . .
N''' . . N''' . . .

These are combined with the two symbols P' and P''' still unused

$N'P''$ $N'K''$ $P''K''$ $N''P'$. $N''P'''$.
$N'''P'$. $N'''P'''$.

The last two symbols K' and K''' can be used in two different ways:

$N'P''$ $N'K''$ $P''K''$ $N''P'K'$ $N''P'''K'''$
$N'''P'K'''$ $N'''P'''K'$

or $N'P''$ $N'K''$ $P''P''$ $N''P'K'''$ $N''P'''K'$
$N'''P'K'$ $N'''P'''K'''$

There are 27 different groups of three first order interactions and each of these can be combined with two different groups of four second order interactions. The complete set includes 54 different groups. In the complete set any first order interaction occurs six times and any second order interaction eight times. A balanced design requires 27 replications.

The following eight groups were selected for confounding in the actual design:

1.	$N'''P'''$	$N'''K'$	$P'''K'$	$N'P'K''$	$N'P''K'''$	$N''P'K'''$	$N''P''K''$
2.	$N''P'$	$N''K''$	$P'K''$	$N'P''K'$	$N'P'''K'''$	$N'''P''K'''$	$N'''P'''K'$
3.	$N'''P'$	$N'''K'''$	$P'K'''$	$N'P''K''$	$N'P'''K'$	$N''P''K'$	$N''P'''K''$
4.	$N'P''$	$N'K'''$	$P''K'''$	$N''P'K''$	$N''P'''K'$	$N'''P'K'$	$N'''P'''K''$
5.	$N''P'''$	$N''K'''$	$P'''K'''$	$N'P'K''$	$N'P''K'$	$N'''P'K'$	$N'''P''K''$
6.	$N''P''$	$N''K'$	$P''K'$	$N'P'K'''$	$N'P'''K''$	$N'''P'K''$	$N'''P'''K'''$
7.	$N'''P''$	$N'''K''$	$P''K''$	$N'P'K'$	$N'P'''K'''$	$N''P'K'''$	$N''P'''K'$
8.	$N'P'''$	$N'K''$	$P'''K''$	$N''P'K'$	$N''P''K'''$	$N'''P'K'''$	$N'''P''K'$

The independent interactions are given in the first, second and fourth column. The interactions $N'P'$, $N'K'$, $P'K'$ and $N''P'''K'''$ are absent, whereas $N'P'K''$, $N'P''K'$, $N'P'''K'''$, $N''P'K'''$, $N''P'''K'$ and $N'''P'K'$ occur twice.

Construction of the plan

The plan was written down by applying FINNEY's [3]) rules to the formal 2^6 design. The principal block corresponding to some group of interactions, say

$N''P'$ $N''K''$ $(P'K'')$ $N'P''K'$
$(N'P'''K''')$ $(N'''P''K''')$ $(N'''P'''K')$

or transformed

BC BF (CF) ADE
$(ACDEF)$ $(ABDEF)$ $(ABCDE)$,

was found by writing the interaction contrasts BC, BF and ADE in the following way

$$
\begin{aligned}
BC &= (1 + a)\ (1 - b)\ (1 - c)\ (1 + d)\ (1 + e)\ (1 + f) \\
BF &= (1 + a)\ (1 - b)\ (1 + c)\ (1 + d)\ (1 + e)\ (1 - f) \\
-ADE &= (1 - a)\ (1 + b)\ (1 + c)\ (1 - d)\ (1 - e)\ (1 + f)
\end{aligned}
$$

and looking for three independent treatment combinations having the $+$ sign in all three expressions. The combinations *ad*, *ae* and *bcf* are suitable. The remaining combinations may be found by multiplication: *de*, *abcdf*, *abcef*, *bcdef*. In this way the principal row of the first square is found to be, after randomization:

ae, *de*, *abcdf*, *bcf*, *ad*, *bcdef*, *abcef*, –.

In a similar way the randomized principal column is obtained. The position of both the principal row and the principal column in the square is determined by the place of the (–) treatment in the randomized groups. The treatment combinations appearing in the principal column may be used as multipliers to produce the remaining rows from the principal row.

The first square completed is shown at the end of this paper, together with the transformation of the 2^6 design into the 4^3 design. The four squares forming the complete design are also given, the numbers denoting the levels of the three factors in the order n, p, k.

[3]) FINNEY, l.c.

Samenvatting

Bij het ontwerpen van een plattegrond voor een 4^3-proef met partiële „confounding" in „quasi-latin squares" van 8×8 vakjes bleken enige wijzigingen nodig in de methode, die bekend is voor 2^6-proeven. Er moesten nieuwe bij elkaar horende groepen van interacties worden samengesteld. Er werd een werkwijze gevonden voor het maken van deze groepen. Uit de groepen werd een geschikt stel gekozen. Deze werden gebruikt bij het samenstellen van de plattegrond.

Voor het invullen van de objecten in de plattegrond werd een bruikbare snelle werkwijze gevonden.

Interactions of group 1 and 2 confounded with columns and rows respectively.

acdef $n_2p_4k_4$	*cef* $n_1p_2k_4$	*ab* $n_4p_1k_1$	*bd* $n_3p_3k_1$	*acf* $n_2p_2k_3$	*be* $n_3p_1k_2$	*abde* $n_4p_3k_2$	*cdf* $n_1p_4k_3$
bcde $n_3p_4k_2$	*abce* $n_4p_2k_2$	*f* $n_1p_1k_3$	*adf* $n_2p_3k_3$	*bc* $n_3p_2k_1$	*aef* $n_2p_1k_4$	*def* $n_1p_3k_4$	*abcd* $n_4p_4k_1$
c $n_1p_2k_1$	*acd* $n_2p_4k_1$	*bdef* $n_3p_3k_4$	*abef* $n_4p_1k_4$	*cde* $n_1p_4k_2$	*abdf* $n_4p_3k_3$	*bf* $n_3p_1k_3$	*ace* $n_2p_2k_2$
bef $n_3p_1k_4$	*abdef* $n_4p_3k_4$	*cd* $n_1p_4k_1$	*ac* $n_2p_2k_1$	*bdf* $n_3p_3k_3$	*acde* $n_2p_4k_2$	*ce* $n_1p_2k_2$	*abf* $n_4p_1k_3$
ae $n_2p_1k_2$	*de* $n_1p_3k_2$	*abcdf* $n_4p_4k_3$	*bcf* $n_3p_2k_3$	*ad* $n_2p_3k_1$	*bcdef* $n_3p_4k_4$	*abcef* $n_4p_2k_4$	— $n_1p_1k_1$
df $n_1p_3k_3$	*af* $n_2p_1k_3$	*bce* $n_3p_2k_2$	*abcde* $n_4p_4k_2$	*ef* $n_1p_1k_4$	*abc* $n_4p_2k_1$	*bcd* $n_3p_4k_1$	*adef* $n_2p_3k_4$
abd $n_4p_3k_1$	*b* $n_3p_1k_1$	*acef* $n_2p_2k_4$	*cdef* $n_1p_4k_4$	*abe* $n_4p_1k_2$	*cf* $n_1p_2k_3$	*acdf* $n_2p_4k_3$	*bde* $n_3p_3k_2$
abcf $n_4p_2k_3$	*bcdf* $n_3p_4k_3$	*ade* $n_2p_3k_2$	*e* $n_1p_1k_2$	*abcdef* $n_4p_4k_4$	*d* $n_1p_3k_1$	*a* $n_2p_1k_1$	*bcef* $n_3p_2k_4$

One square of the plan.

Interactions of group 1 and 2 confounded with columns and rows respectively

244	124	411	331	223	312	432	143
342	422	113	233	321	214	134	441
121	241	334	414	142	433	313	222
314	434	141	221	333	242	122	413
212	132	443	323	231	344	424	111
133	213	322	442	114	421	341	234
431	311	224	144	412	123	243	332
423	343	232	112	444	131	211	324

Interactions of group 3 and 4 confounded with columns and rows respectively

334	223	144	422	131	413	341	212
442	111	232	314	243	321	433	124
214	343	424	142	411	133	221	332
311	242	121	443	114	432	324	233
231	322	441	123	434	112	244	313
122	431	312	234	323	241	113	444
143	414	333	211	342	224	132	421
423	134	213	331	222	344	412	141

Interactions of group 5 and 6 confounded with columns and rows respectively

131	314	412	322	233	241	424	143
442	223	121	211	344	332	113	434
312	133	231	141	414	422	243	324
234	411	313	423	132	144	321	242
413	232	134	244	311	323	142	421
124	341	443	333	222	214	431	112
221	444	342	432	123	111	334	213
343	122	224	114	441	433	212	331

Interactions of group 7 and 8 confounded with columns and rows respectively

413	121	243	132	331	444	214	322
332	244	122	213	414	321	131	443
123	411	333	442	241	134	324	212
231	343	421	314	113	222	432	144
114	422	344	431	232	143	313	221
242	334	412	323	124	211	441	133
424	112	234	141	342	433	223	311
341	233	111	224	423	312	142	434

The plan completed.