

A SIMULATION MODEL OF THE COMBINED TRANSPORT
OF WATER AND HEAT PRODUCED BY A THERMAL
GRADIENT IN POROUS MEDIA

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1. Introduction

During the past twenty years much attention has been given to the phenomenon of combined heat-moisture transfer in porous materials, particularly in soils (e.g. Philip and De Vries, 1957; Cary, 1965; Rose, 1968a,b; Letey, 1968; Hadas, 1968; Cassel et al., 1969; Fritton et al., 1970; and references cited by these authors). The fact that a temperature gradient can result in movement of soil water has been reported as early as 1915 (Bouyoucos; cited by Rose, 1968a).

Soils under natural field conditions are subjected to continuous temperature fluctuations and two significant changes of temperature in the soil profile can be distinguished. The first one is caused by the daily radiation cycle and results in a temperature wave which penetrates the soil up to a depth of about 35 cm, the most pronounced effects occurring in the upper 5-10 cm. The annual or seasonal radiation cycle gives rise to a second temperature wave penetrating the soil to a considerably greater depth than the diurnal wave: of the order of 10 meters. The thermal gradients brought about by these temperature waves tend to move soil moisture in both the vapor and the liquid phase, while the direction of movement changes every 12 hours and 6 months for the daily and seasonal waves respectively.

Lebedeff (1927; cited by Rose, 1968a, and Cassel et al., 1969) concluded from a field experiment in Russia that the temperature gradient associated with the annual wave can cause an upward moisture movement (vapor phase) of more than 6 cm of soil water in a winter period. This is a considerable amount of water moving through the soil profile, which most likely has to be partly related to freezing processes (density of saturated water vapor over ice is less than over liquid water of the same temperature).

Cary (1966) mentions four possible reasons why water flows in the liquid phase under the influence of a temperature gradient. A difference in the density of water vapor due to a temperature difference will also create a diffusive flow of water vapor. In general the flow of moisture occurs from warmer to cooler areas and the proportion of water movement as vapor to that as liquid will increase with decreasing soil water content. An increase of the moisture content in the cold region may eventually originate a flux in the opposite direction, i.e. from cold to hot, which will be predominantly in the liquid phase.

Quite a few laboratory experiments on this subject and related aspects have been conducted since 1950 (e.g. references cited by Philip and De Vries, 1957, and Rose, 1968a). The quantitative implications of all this research for soil moisture transport in the field environment, however, have been inadequately investigated and different conclusions have been drawn with respect to this matter (Rose, 1968a).

The importance of these combined processes of heat and moisture transfer must be looked at especially with reference to evapotranspiration of soils of the arid regions, temperature regulation of seed-beds, and such-like. If thermal vapor movement is of any agricultural significance, it will be under conditions of relatively low water contents (Rose, 1966). Results of work done by Rose (1968a, b, c) indicate that under field conditions where daytime radiation is high, and consequently high temperature gradients in the soil profile, the net vapor transfer (principally a thermal flow) is of comparable magnitude to net moisture transfer in the liquid phase (principally an isothermal flow) at suctions as low as 200-300 mbars. At suctions greater than about 5 bars vapor transfer was found to be the dominant mechanism of transport. According to Rose (1968b) vapor transport may be important in special circumstances in the water economy of plants, particularly in the germination and early establishment phases of plant growth. Moreover, the thermally induced moisture flow may significantly affect the net transfer of salts and plant nutrients (Cary, 1966; Rose, 1968b), either directly (thermal liquid flow) or indirectly by changing moisture content gradients (returning capillary flow).

The purpose of this report is to present a simulation model for the combined flow of moisture and heat through porous media such as soils, written in CSMP (Continuous System Modeling Program). Since in the system under consideration an imposed temperature difference will cause several properties to change simultaneously with time (transient-state), an analytical solution of the involved heat and moisture fluxes is very complicated if not impossible. In that case simulation may provide a means to solve the transient-state situation by assuming successively changing steady-state systems.

The described model is based upon an approach advanced by Philip and De Vries (1957). Before Philip and De Vries (1957) had developed their theory, some investigators used a modification of Fick's law of diffusion in describing soil water movement due to temperature gradients. In fact the Philip-De Vries theory is an extension of Fick's modified law.

In addition to the Philip -De Vries theory, Taylor and Cary (Taylor and Cary, 1960 and 1964; Cary and Taylor, 1962a, b; Cary, 1965) have proposed a theory based upon the thermodynamics of irreversible processes. Their equations are of the same general form as those developed by Philip and De Vries and so far the application of irreversible thermodynamics to the problem of interacting heat and moisture fluxes in soils did not produce any new information (Rose, 1968b).

Recently, Cassel et al. (1969) applied both the Philip-De Vries theory and the Taylor-Cary theory to their experimental results. Net water fluxes predicted by the Philip-De Vries theory agreed reasonably well with observed values (about equal), while application of the Taylor-Cary theory led to seriously underestimated values (off by a factor 10-40). Results obtained by Fritton et al. (1970) are also in favor of the Philip-De Vries theory.

The facts mentioned before seem to justify the use of the Philip-De Vries theory as a basis for the simulation model. The presented model must be viewed as a preliminary one, and will no doubt await further improvements and modifications. In order to apply to open systems, for example, an evaporation term needs to be included. It has been a first aim to simulate the combined transport processes occurring in the relatively simple case of a (for moisture) closed soil column of finite length.

2. Theory

A definition of the symbols used in this section is given separately in Appendix I.

Supposing the transport process in a porous medium to be a frictional flow with a frictional force proportional to the flow rate, one may assume that the flux (q_A) in a certain direction is proportional to the driving force (F) in this direction. Thus

$$q_{A(x)}^{\rightarrow} = B \cdot F_{(x)}^{\rightarrow} \quad (1)$$

The potential of the agent related to the driving force F is given by

$$\psi_F = - \int F_{(x)}^{\rightarrow} \cdot d\vec{x} \quad (2)$$

where the minus sign indicates that the potential increases if the direction of the driving force is opposed to the positive x -direction. The potential is a scalar quantity, whereas the driving force and the flux are vectors. The total potential of the agent is obtained by adding all its partial potentials.

From (2) one may substitute $F_{(x)}^{\rightarrow} (= - \text{grad } \psi_F)$ in (1), which gives

$$q_{A(x)} = - B \text{ grad } \psi_F \quad (3)$$

Denoting the amount of the agent A per unit of volume by A , ψ_F and the differential capacity of the medium for the agent $A(C_A)$ are formally combined by

$$C_A = \delta A / \delta \psi_F \quad (4)$$

Transformation of (3) and substitution of (4) yields

$$q_{A(x)} = - B (\delta \psi_F / \delta A) \text{ grad } A = - (B/C_A) \text{ grad } A \quad (5)$$

Defining the diffusivity D as the ratio of transport coefficient and capacity (B/C), one obtains

$$q_{A(x)} = - D \text{ grad } A \quad (6)$$

Applying the continuity condition (principle of the conservation of matter) leads to

$$\delta A / \delta t = - \text{grad } q_{A(x)} = \text{grad } (D \text{ grad } A) \quad (7)$$

For steady-state situations $q_{A(x)}$ is constant, and accordingly $\delta A / \delta t = 0$.

2.1. Vapor transfer

The steady-state equation of vapor diffusion in air (Fick's law) is quite similar to (6) and may be written as

$$q_v = - D_a \text{ grad } \rho \quad (8a)$$

Instead of taking the water vapor density gradient one may also develop different expressions by using the gradient of the partial pressure of water vapor in units of either dyne cm^{-1} (e') or mm Hg (e), which results in (Rose, 1968a)

$$q_v = - (D_a / R_w T) \text{ grad } e' = - 1.36 (D_a g / R_w T) \text{ grad } e \quad (8b)$$

Vapor diffusion in a porous material (e.g. soil) will be less than in air, because of a longer path length (resulting in a smaller gradient) and a volumetric air content $a < 1$. Further a mass-flow factor $v (= P / (P - e))$ may be introduced to allow for the mass-flow of water vapor due to a gradient of the total gas pressure. At normal soil temperatures, this factor is clearly quite close to unity. Therefore, for vapor diffusion in soils (8a) becomes

$$q_v = - (D_a v a a) \text{ grad } \rho \quad (9)$$

As pointed out by Philip and De Vries (1957), Fick's law of diffusion modified to apply for vapor movement in porous media (eq. (9)), denoted in their paper as the "simple theory", will underpredict water vapor transport occurring under thermal gradients. They examined some literature data and made quantitative comparisons, which generally yielded values for the ratio of observed transfer to the value predicted by the simple theory of about 3-10. Cassel et al. (1969) calculated this ratio from their

own experimental data and found values which are of the same order of magnitude: about 5.

For water vapor in the soil air in equilibrium with the water in the medium one may write the following relationship, neglecting possible osmotic influences (see, for example, Bolt et al., 1965, p. 26)

$$\psi' = (RT/18) \ln (e/e_s) = (RT/18) \ln (\rho/\rho_0) = R_w T \ln h \quad (10)$$

Expressing the water pressure in cm H₂O instead of erg g⁻¹, gives

$$\psi = (R_w T/g) \ln h$$

or (11)

$$h = \exp (\psi g/R_w T)$$

From (10) and (11) it follows that

$$\rho = \rho_0 h = \rho_0 \exp (\psi g/R_w T) \quad (12)$$

Differentiating equation (12), one finds

$$\text{grad } \rho = h \text{ grad } \rho_0 + \rho_0 \text{ grad } h \quad (13)$$

ρ_0 is independent of the water content of the medium and (for reasons discussed by Philip and De Vries, 1957) $\delta h/\delta T$ may be taken as zero in the full range of h . In other words, one may conclude that ρ_0 is a function of T only and h is a function of θ only. Thus (13) may be re-written as

$$\text{grad } \rho = h (\delta \rho_0/\delta T) \text{ grad } T + \rho_0 (\delta h/\delta \theta) \text{ grad } \theta \quad (14)$$

Taking the derivative of (11) with respect to θ

$$\delta h/\delta \theta = \exp (\psi g/R_w T) \cdot (g/R_w T) \cdot (\delta \psi/\delta \theta) = (gh/R_w T) \delta \psi/\delta \theta \quad (15)$$

and substituting ρ for $\rho_0 h$, equation (14) becomes

$$\text{grad } \rho = h (\delta \rho_0/\delta T) \text{ grad } T + (g\rho/R_w T) (\delta \psi/\delta \theta) \text{ grad } \theta \quad (16)$$

Inserting (16) into (9) and expressing the flux in cm sec^{-1} yields

$$q_v/\rho_l = - (D_a \alpha a/\rho_l) \{ h (\delta\rho_o/\delta T) \text{ grad } T + (g\rho/R_w T) (\delta\psi/\delta\theta) \text{ grad } \theta \} \quad (17)$$

$$= - D_{Tv} \text{ grad } T - D_{\theta v} \text{ grad } \theta$$

Equation (9) has now been separated into a thermally induced and an isothermally induced vapor flux, as represented by the first and second term on the right-hand side of equation (17) respectively.

Writing β for $\delta\rho_o/\delta T$ and $1/C_w$ for $\delta\psi/\delta\theta$, the vapor diffusivities are given by

$$D_{Tv} = D_a \alpha a h \beta / \rho_l \quad (18)$$

and

$$D_{\theta v} = D_a \alpha a g \rho / \rho_l R_w T C_w \quad (19)$$

So far, the equations refer to the simple theory of vapor diffusion with the assumption that no interaction of the vapor phase with either the liquid phase or the solid phase will take place.

At this point the further development of the theory as has been suggested by Philip and De Vries (1957) may be introduced. Their extended theory can be summarized as follows.

At (liquid) moisture content values $\theta_1 < \theta_K$, where θ_K is the value of θ_1 at which K becomes practically zero (no liquid continuity and consequently no significant moisture transfer in the liquid phase), moisture (vapor) transfer is not just restricted to the air-filled pore space, but can also proceed through the isolated regions of liquid between the soil particles, which are assumed to be paths of low resistance to vapor (Rose, 1968a), by condensing and evaporating processes on the upstream and downstream side respectively. In other words, regarding the flow as a series-parallel process, the entire porosity $\epsilon_T (= a + \theta)$, and not just the gas-filled part a , is effective in vapor diffusion under such moisture conditions. The value of θ_K will depend upon soil texture (Philip and De Vries, 1957), being greater and occurring at relatively high suction levels in finer-textured soils. Vapor transport becomes dominant at suctions of 5 to 15 bars (Slatyer, 1967; Rose, 1968b). Kramer (1969) mentions a suction of 15 bars as the point where the liquid continuity is broken.

A second modification arises from the fact that the thermal gradient across air-filled pores, $(\text{grad } T)_a$, which is effective in promoting vapor diffusion, can considerably exceed the mean overall temperature gradient, $\text{grad } T$, in the medium by a factor $\zeta \leq 1$. Note that one has about the same temperature difference over a shorter distance when considering only the air-filled pores (see for instance Figure 42 in Rose, 1966). Thus

$$\zeta = (\text{grad } T)_a / \text{grad } T \quad (20)$$

From (17) and (18) it follows that for a single air-filled pore ($\alpha a = 1$) the vapor flux due to a temperature gradient may be given by

$$q_v / \rho_1 = - (D_a \text{ v h } \beta / \rho_1) (\text{grad } T)_a \quad (21)$$

where $(\text{grad } T)_a$ represents the thermal gradient in the pore.

Re-defining $(\text{grad } T)_a$ as the average temperature gradient in all air-filled pores, (21) is also applicable to the vapor flux in the air-filled porosity of the medium, simply by multiplying with the factor \underline{a}

$$q_v / \rho_1 = - (D_a \text{ v a h } \beta / \rho_1) (\text{grad } T)_a = - D_{Tv} \text{ grad } T \quad (22)$$

the tortuosity factor α being included in $(\text{grad } T)_a$.

Assuming the liquid flow through the isolated wedges of liquid to equal the vapor flux through the air-filled pores, in accordance with the suggestions mentioned before, the revised theory for the total vapor flux density due to a temperature gradient is expressed by (for $\theta_1 < \theta_K$)

$$q_v / \rho_1 = - \{ (a + \theta) D_a \text{ v h } \beta / \rho_1 \} (\text{grad } T)_a = - D_{Tv} \text{ grad } T \quad (23)$$

Using (20), the thermal vapor diffusivity as predicted by the revised theory is thus

$$D_{Tv} = (a + \theta) D_a \text{ v h } \beta \zeta / \rho_1 \quad (24)$$

In the following the symbol D_{Tv} will be used in this sense to distinguish it from the value predicted by the simple theory (eq. (18)), unless otherwise noted.

Defining $(\text{grad } T)_i$ as the temperature gradient averaged over the fractional volume, X_i , occupied by the component i in the medium, the value of ζ can be calculated for a system made up of n constituents from (20) with (Philip and De Vries, 1957)

$$\text{grad } T = \sum_{i=0}^n X_i (\text{grad } T)_i \quad (25)$$

The actual method of calculating ζ as indicated by De Vries (1963) will be described under section 3. Generally the value of ζ will be in the range 1.0 - 2.5 depending, among other things, on total porosity, volumetric moisture content, and temperature (Philip and De Vries, 1957; Rose, 1968a). As will be shown in 4.1.4., a value of about 1.8 will usually be a good approximation for ζ . If specific data as moisture content and total porosity of the soil are available, a more accurate value may be estimated from Table 5.

The picture changes when $\theta_1 > \theta_K$, and the degree of liquid continuity increases from zero (at $\theta_1 = \theta_K$) on. Thus for moisture contents increasing from this point the degree of vapor continuity decreases, whereas liquid moisture transfer due to thermally induced capillary potential gradients becomes gradually more important. The decrease of the vapor-induced transfer through isolated liquid wedges mentioned before is not only due to a reduction in the number of islands and in the opportunity for vapor transfer (air-filled porosity decreases) but also to an increase in the radii of curvature of the menisci to the point where automatic adjustment to the vapor flux is no longer possible (Philip and De Vries, 1957). One may suppose, therefore, that the effective cross-section for the series-parallel vapor transfer (interacting with the liquid phase) will decrease as θ_1 increases from θ_K . Assuming as a first approximation a linear decrease, the term $(a + \theta)$ in equations (23) and (24) may be replaced by a term $\{ a + f(a)\theta \}$ with $f(a) = 1$ for $\theta_1 < \theta_K$ and $f(a) = 0$ for $\theta_1 = \epsilon_T$.

Rose (1968a) writes in his equations the factors $f(\epsilon)$ and ξ instead of αa and $\{ a + f(a)\theta \}/a$, but his approach is essentially the same as the one developed by Philip and De Vries (1957).

It should be noted that the possibility of interacting vapor transfer with the liquid phase, which is adsorbed as a thin layer of water molecules around the particles (at $\theta_1 < \theta_K$), has been neglected. It is expected, however, that the moisture flow by surface diffusion is small and unlikely to have much of an effect on the total transfer (Philip and De Vries, 1957).

2.2. Liquid transfer

In describing soil water movement under the influence of a temperature gradient, one should also give consideration to the moisture flow in the liquid phase.

Taking (3) as the basic steady-state equation, the liquid moisture transfer in unsaturated media (Darcy's law) expressed in units of cm sec^{-1} is given by (considering only the matric potential)

$$q_1/\rho_1 = -K \text{ grad } \psi \quad (26)$$

where $\psi = f(\theta, T)$.

Writing the differential of ψ as

$$d\psi = (\delta\psi/\delta T)_\theta + (\delta\psi/\delta\theta)_T \quad (27)$$

one has

$$\text{grad } \psi = (\delta\psi/\delta T)_\theta \text{ grad } T + (\delta\psi/\delta\theta)_T \text{ grad } \theta \quad (28)$$

In (28) $\text{grad } \theta$ and $\text{grad } T$ are measurable quantities, whereas $(\delta\psi/\delta\theta)_T$ can be found from the moisture characteristic of the porous medium.

Liquid flow ($K > 0$) begins at water contents corresponding to values 0.5 - 0.9 for the relative humidity. Philip and De Vries (1957) and Rose (cited by Jackson, 1964) suggest a value of about 0.6. Jackson (1964) mentions values of 0.5-0.7 and 0.8-0.9 for two loam soils. In the θ range where liquid flow occurs, capillary condensation is the important process in determining the value of ψ as opposed to the process of physical adsorption on solid surfaces at low values of h .

Since capillarity depends directly on the surface tension of water (σ), ψ will be directly proportional to σ . This relation is given by (e.g. Bolt et al., 1965, p. 5; Van Wijk and De Vries, 1963, p. 44).

$$\psi' = \psi/g = 2\sigma/R \quad \text{or} \quad \psi = 2g\sigma/R \quad (29)$$

The surface tension of water is a function of T and for a constant value of θ , one may write

$$\delta\psi/\delta T = (2g/R) (\delta\sigma/\delta T) = (\psi/\sigma) (\delta\sigma/\delta T) \quad (30)$$

Introducing $\gamma = (1/\sigma) (\delta\sigma/\delta T)$, (30) becomes

$$\delta\psi/\delta T = \gamma\psi \quad (31)$$

Hence, by combining (26), (28) and (31) the following expression is obtained

$$q_1/\rho_1 = - K\gamma\psi \text{ grad } T - K (\delta\psi/\delta\theta) \text{ grad } \theta \quad (32)$$

Defining the liquid diffusivities as

$$D_{T1} = K\gamma\psi \quad (33)$$

$$D_{\theta 1} = K (\delta\psi/\delta\theta) \quad (34)$$

and putting (33) and (34) into (32), one has

$$q_1/\rho_1 = - D_{T1} \text{ grad } T - D_{\theta 1} \text{ grad } \theta \quad (35)$$

Considering also the gravitational potential z , (35) becomes

$$q_1/\rho_1 = - D_{T1} \text{ grad } T - D_{\theta 1} \text{ grad } \theta - Kk \quad (36)$$

2.3. Total moisture transfer

Combining equations (17) and (24) with (35) or (36) the following expressions may be written for total moisture movement in the horizontal and vertical case, respectively.

$$q_m/\rho_1 = - D_T \text{ grad } T - D_\theta \text{ grad } \theta \quad (37)$$

$$q_m/\rho_1 = - D_T \text{ grad } T - D_\theta \text{ grad } \theta - Kk \quad (38)$$

where

$$D_T = D_{Tv} + D_{T1} \quad (39)$$

$$D_\theta = D_{\theta v} + D_{\theta 1} \quad (40)$$

The liquid diffusivities are dominant at high moisture contents, whereas the vapor diffusivities become more important at low moisture contents.

Application of the continuity requirement (see eq. (7)) yields the following second order partial differential equation of moisture transfer

$$\delta\theta/\delta t = \text{grad } (D_T \text{ grad } T) + \text{grad } (D_\theta \text{ grad } \theta) + \text{grad } K \quad (41)$$

Omitting the third term on the right-hand side of (41) gives the case of horizontal moisture movement.

The moisture gradients produced by thermal moisture transfer (liquid + vapor) will start a capillary return flow in the liquid phase, if moisture contents exceed the value of θ_K . So the net moisture transfer may be expected to increase with increasing moisture content until the point θ_K has been reached. A further increase of θ_1 beyond the value of θ_K will result in a rapid decrease of the net moisture transfer.

2.4. Heat transfer

The heat flux density due to a temperature gradient grad T may also be derived from (3)

$$q_h = - \lambda \text{ grad } T \quad (42)$$

The value of λ can be calculated from the physical properties of the different constituents of the porous system, using a weighted average according to De Vries (1963)

$$\lambda = \left(\sum_{i=0}^n k_i X_i \lambda_i \right) / \left(\sum_{i=0}^n k_i X_i \right) \quad (43)$$

where k_i is the ratio of the average temperature gradient in component i and the corresponding quantity in the continuous medium in which component i is dispersed (i.e. air or water). The method of calculating λ will be discussed in more detail under section 3.

In assessing the value of λ_i for air containing water vapor, the following should be taken into consideration.

Dividing equation (22) by \underline{a} , the mean vapor flux density in all air-filled pores is also given by (21). Due to transfer of latent heat by

vapor movement (distillation effect), this vapor flow produces an apparent increase of the thermal conductivity in the air-filled porosity by an amount

$$\lambda_v = L D_a v h \beta \quad (44)$$

Thus, the apparent thermal conductivity of air containing water vapor is given by

$$\lambda_{app} = \lambda_a + \lambda_v \quad (45)$$

where λ_a is the thermal conductivity of (dry) air due to normal heat conduction. This value of λ_{app} should be inserted into (43) for the conductivity of the air-filled pores.

Using continuity considerations (see eq. (7)) one obtains the second order partial differential equation in one dimension for the heat conduction in the medium

$$C_h (\delta T / \delta t) = \text{grad} (\lambda \text{ grad } T) \quad (46)$$

where C_h represents the volumetric heat capacity of the medium and the thermal distillation effect (eq. (44)) is included in λ . For soils, C_h may be calculated from (De Vries, 1963)

$$C_h = 0.46 (x_m + x_q) + 0.60 x_o + \theta \quad (47)$$

Taking also into account the transfer of latent heat in the vapor phase induced by moisture gradients, (46) becomes

$$C_h (\delta T / \delta t) = \text{grad} (\lambda \text{ grad } T) + L \text{ grad} (D_{\theta v} \text{ grad } \theta) \quad (48)$$

In most cases, however, the second term on the right-hand side will be small compared with the first one.

2.5. The equations describing combined moisture and heat transfer

Equations (41) and (48) derived in the previous sections apply to the simultaneous transfer of moisture and heat in porous media.

Assuming at every instant liquid water to be in equilibrium with water vapor, De Vries (1958) made a distinction between θ_1 and θ_v

$$\theta_v = (\epsilon_T - \theta_1)\rho/\rho_1 \quad (49)$$

Application of equation (49) leads to more extensive expressions for $\delta\theta/\delta t$ and $\delta T/\delta t$ (De Vries, 1958) which can be derived from his equations (9) and (19)

$$\delta\theta_1/\delta t = (HY - IZ)/(HJ - GI) \quad (50)$$

$$\delta T/\delta t = (GY - JZ)/(GI - HJ) \quad (51)$$

The terms added by this procedure, however, will be fully negligible under most circumstances (see also, for example, Rose 1968a). The values of G and I (see Appendix I) will be quite close to 1 and the value of C_h , respectively, whereas the terms HJ, HY and JZ can be neglected compared with remaining terms. Also the third term in Y is relatively small.

One may conclude, therefore, that in fact (50) and (51) are similar to (41) and (48) respectively. Thus, equations (41) and (48) may be expected to serve adequately as a basis for the simulation model described in the next paragraph. Except for very small values of θ_1 , values of θ_1 and θ ($= \theta_1 + \theta_v$) will be practically equal, so one may write $\delta\theta_1/\delta t$ and grad θ_1 instead of $\delta\theta/\delta t$ and grad θ .

3. The simulation program

The solution of a transient-state heat flow, for instance, in a porous medium in its simplest form (λ and C_h constant) can be found from tabulated functions (erfc functions). However, the second order partial differential equations (41) and (48) mentioned in the previous paragraph and which describe the heat and moisture fluxes in a porous medium due to a temperature gradient cannot be solved analytically, because λ , C_h , and moisture diffusivities do vary in space and time. Differential equations of this type can only be solved by means of numerical methods. Since calculating by hand would be a very time-consuming process, the use of a computer is often inevitable.

By using a computer language such as CSMP (Continuous System Modeling Program) one is able to solve the problem in question by (step-wise) numerical integration procedures. In this way one finds the (continuously) occurring changes of temperature and water content in time and space. The steps in time are performed by the computer, whereas the spatial variation is introduced by the programmer using a compartmentalized simulation model. For the purpose of the latter the porous medium is divided into a certain number of compartments, not necessarily of the same thickness. The heat content and water content of each compartment are calculated after a certain (often with time varying) time step Δt by adding the integral (heat content, water content) obtained at time t and the net (heat, moisture) flow for the compartment during Δt . In doing so, one assumes the net flow rate to be constant in the period Δt , which will be normally not the case. If the time steps are taken small enough, however, the errors being made will be negligible. Secondly, it is assumed that both heat and moisture are uniformly distributed in each compartment. It is easily seen, therefore, that also the thickness of the compartments is very important (temperature and moisture gradient terms).

Some examples of the application of CSMP to transport phenomena in soils have been given by Wierenga and De Wit (1970), De Wit and Van Keulen (1970), Van Keulen and Van Beek (1971), Stroosnijder et al. (1972) and Goudriaan and Waggoner (1972).

The simulation program for the combined flow of moisture and heat presented in this report has also been written in CSMP and is given as a whole in Table 1, pp. 31-36. A schematic representation of the compartmentalized model is given in Figure 1.

In the following sub-sections of this paragraph the model will be described in more detail. It should be noted, however, that in describing the model the NOSORT sequence (to be explained in 3.1.) of the cards has not always been followed. For a definition of the abbreviations used, one is also referred to Appendix II. If more information is needed on the various statements used in the program, one may consult the CSMP manual (Anon., 1968).

An important assumption underlying the model is the fact that the porous medium is considered to be homogeneous. As a first goal it is intended to present in this report a simulation program for the combined heat-moisture transfer in a homogeneous soil column of finite length. Modifications which apply to a soil column consisting of different (homogeneous) soil layers with their orientation perpendicular to the direction of flow will no doubt be possible in a similar way as indicated by Van Keulen and Van Beek (1971). In that case suction gradients rather than moisture content gradients will have to be considered.

In CSMP programs an initial and dynamic segment can be distinguished, the former one being optional. These two segments will be discussed successively in the sections 3.1. and 3.2.

3.1. The initial segment of the program

The initial segment of the program performs the computation of initial condition values (invariable geometry of the system, initial parameter values) and should start with an INITIAL statement. Although being optional, this segment is very useful for the problem in question, because otherwise the computation of initial values has to be repeated at every time step. In other words, computation time is saved by using an initial segment.

Because of the fact that arrays (= complete sets of quantities with one variable name, a particular quantity being indicated by an index) are used, a NOSORT section may be introduced only. This means that the sorting capability SORT of CSMP cannot be used and that the statements following the NOSORT card have to be given in the proper computational sequence.

The FIXED statement indicates that the listed variables (on the same card) are numerals and consequently fixed-point numbers (integers, i.e. whole numbers having no decimal point) instead of real (floating-point) numbers. I represents a counter which is used to perform the necessary calculations for all successive compartments. For this purpose use is made of the DO

statement, which makes it possible to carry out a section of the program repeatedly, with changes in the value of the fixed-point variable (I): "DO loops". The first number in the DO statement is a statement number, and I = 1, NL simply means that the statements following the DO statement are executed repeatedly, for I varying from 1 to NL, up to and including the line which starts with the statement number mentioned before. At each succeeding execution I is increased by 1. NL represents the total number of layers or compartments into which the column is divided. In this program a total of 25 compartments has been considered.

Arrays have to be declared by use of either a STORAGE label or a (FORTRAN) REAL statement, the latter having a slash (/) in the first column. The number within parentheses following the variable name must be at least the maximum number of indices used for this variable in order that all the indexed variables can be properly located. In a CSMP program a total of 10 such FORTRAN lines is allowed. The program described in this report contains 2 EQUIVALENCE statements (to be explained in 3.2.) and 6 REAL statements (see Table 1). No more than 25 variable names can be declared with STORAGE cards.

INITIAL

NOSORT

FIXED I, NL

STORAGE TCOM(25), IWC(25), ITEMP(25)

Data for the STORAGE locations may be entered by use of the TABLE data statement. This has been done for the thickness, the initial water content, and the initial temperature of all 25 compartments.

TABLE TCOM(1-25) =

TABLE IWC(1-25) =

TABLE ITEMP(1-25) =

The invariable geometry of the system is calculated with (see Figure 1)

DIST (1) = 0.5 * TCOM (1)

DPTH (1) = 0.5 * TCOM (1)

DO 1 I = 2, NL

DIST (I) = 0.5 * (TCOM (I) + TCOM (I-1))

DPTH (I) = DPTH (I-1) + DIST (I)

1 CONTINUE

The initial amount of water, the initial volumetric heat capacity (see eq. (47) in 2.4.), and the initial volumetric heat content are calculated from

$$\begin{aligned} \text{IAMW (I)} &= \text{IWC (I)} * \text{TCOM (I)} \\ \text{IVHCP (I)} &= 0.46 * \text{SOLC} + 0.60 * \text{OMC} + \text{IWC (I)} \\ \text{IVHTC (I)} &= \text{ITEMP (I)} * \text{IVHCP (I)} * \text{TCOM (I)} \end{aligned}$$

It should be kept in mind that a unit area is considered.

In this program 5 components of the soil system are distinguished: quartz (Q), other minerals (M), organic matter (OM), air (A), and moisture (W). The contents of Q, M and OM, and the porosity (POR) of the system may be found with the following statements

```
PARAM SOLC =      , OMC =
PARAM FRQ =
FRM = 1. - FRQ
QC = FRQ * SOLC
MC = FRM * SOLC
POR = 1. - SOLC - OMC
```

where the PARAM data statement assigns numeric values to the various parameters and SOLC (solid content) is the sum of quartz and other minerals.

The remainder of the statements given in the initial segment are used in connection with the calculation of λ and ζ in the dynamic segment, and will be explained under section 3.2.

3.2. The dynamic segment of the program

Before proceeding with the actual combined heat-moisture flow, it will be indicated in 3.2.1. and 3.2.2. how the values for the thermal conductivity (λ) and the ratio of the average temperature gradient across air-filled pores to the mean overall temperature gradient (ζ) may be evaluated.

3.2.1. The calculation of the overall thermal conductivity

As pointed out by De Vries (1963), the thermal conductivity of a porous system can be calculated from eq. (43) (see 2.4.). In applying this equation to soils it is assumed that the soil system may be visualized as a continuous medium (air or water) in which soil particles (e.g. quartz, other minerals, organic matter) are homogeneously dispersed.

The values of λ_i for the different components are given in the initial segment and expressed in units of $\text{cal cm}^{-1} \text{ day}^{-1} \text{ }^\circ\text{C}^{-1}$.

PARAM CONQ = , CONM = , CONH = , CONW = , CONA =

where CONA refers to the conductivity of dry air. In the simulation model it is assumed that the conductivities are independent of the temperature.

The k_i values are calculated from the expression (De Vries, 1963)

$$k_i = 1/3 \sum_{a,b,c} \left[\frac{1}{1 + \left(\frac{\lambda_i}{\lambda_0} - 1 \right) g_a} \right] \quad (52)$$

where λ_0 stands for the thermal conductivity of the continuous medium. Assumptions inherent in (52) are (i) an ellipsoidal shape of the soil granules, (ii) a random orientation of the granules, and (iii) no mutual influences between the granules. The factor g_a is called the depolarisation factor of the ellipsoid in the direction of the a-axis and depends on the shape of the ellipsoid only and not on its size.

Soil particles can be approximately considered as spheroids (condition (i) above) which implies (De Vries, 1963)

$$g_a = g_b \quad (53a)$$

The value of g_c can be found from

$$g_a + g_b + g_c = 1 \quad (53b)$$

In the special case of spherical granules, one has $g_a = g_b = g_c = 1/3$.

Condition (iii) mentioned above does not hold for soils. However, De Vries (1952, 1963) has shown that (52) may still be applied to soils. If necessary a simple correction can be introduced.

For quartz the value of k_i with water and dry air as the continuous medium, respectively, is calculated in the initial segment of the program according to (52) with

PARAM GA =

GC = 1. - 2. * GA

KQW = 1./3. * (2./(1. + (CONQ/CONW - 1.) * GA) +
1./(1. + (CONQ/CONW - 1.) * GC))

$$KQA = 1./3. * (2./(1. + (CONQ/CONA - 1.) * GA) + 1./(1. + (CONQ/CONA - 1.) * GC))$$

For other minerals and organic matter the same procedure is followed using for g_a and g_c the same values as in the case of quartz.

$$KMW =$$

$$KMA =$$

$$KHW =$$

$$KHA =$$

In calculating the overall thermal conductivity of the medium three moisture ranges have been distinguished (De Vries, 1963):

- (a) the range where it is no longer allowed to consider water as the continuous medium
- (b) the range where water can be considered as the continuous medium, subdivided into
 - (b1) the range where the relative humidity h does not differ appreciably from unity and, consequently, λ_v in eq. (45) is equal to the value at saturation, and
 - (b2) the range where h diminishes rapidly at decreasing moisture contents.

De Vries (1963) suggests 0.03 for coarse textured soils and 0.05 to 0.10 for fine textured soils as the transitional θ -value between ranges a and b, whereas the wilting percentage (which corresponds with a suction of about 15.000 mbar) may be taken as the boundary value between the ranges b1 and b2.

The method of calculating λ , as outlined by De Vries (1963), comes down to the application of eq. (43) to the ranges b1 and b2, and a linear interpolation procedure to range a.

Before being able to apply (43) in the wet range (water as continuous medium) still another factor is needed, viz. k_i for air. The factor g_a for "air particles" dispersed in water will vary with water content. Following De Vries (1963) one may write for the effective value of g_a of the air-filled pores as an approximation

$$g_a = 0.333 - \{a/(a + \theta)\} \times 0.298 \quad (54a)$$

and

$$g_a = 0.013 + (\theta/\theta_w) \{g_a(h) - 0.013\} \quad (54b)$$

in the ranges b_1 and b_2 , respectively, where θ_w represents the volumetric moisture content at wilting point and $g_a(h)$ is the value of g_a at θ_w calculated with (54a). The values of g_a and g_c for the air-filled pores are computed in the program using an IF statement nested in a DO loop.

```

      IF (WC(I) - WCH) 60, 70, 70
60   GAA(I) = 0.013 + WC(I)/WCH * (GAAH - 0.013)
      GAC(I) = 1. - 2. * GAA(I)
      GOTO 4
70   GAA(I) = 0.333 - A(I)/POR * 0.298
      GAC(I) = 1. - 2. * GAA(I)
4     CONTINUE

```

If the value of the expression between parentheses following IF is negative, statement 60 and following statements are executed; if zero or positive, statement 70 (and following) is executed. GAAH is calculated in the initial segment.

```

PARAM WCH =
      AH = POR - WCH
      GAAH = 0.333 - AH/POR * 0.298

```

The value of k_1 for air with water as continuous medium is now computed with (recalling eq. (44) and (45) in 2.4.)

```

      CVAP(I) = L * DATM(I) * V * H(I) * B
      APCA(I) = CONA + CVAP(I)
      KAW(I) = 1./3. * (2./(1. + (APCA(I)/CONW - 1.) * GAA(I)) +
                      1./(1. + (APCA(I)/CONW - 1.) * GAC(I)))

```

For range a a linear interpolation procedure is suggested between the value of λ for θ equal to zero (dry soil) and the λ corresponding with a boundary value of θ , which depends on the soil texture. The thermal conductivity for dry soil (dry air as continuous medium) can be found also from (43), but now a correction factor 1.25 has to be introduced in the right-hand side of the equation (De Vries, 1963). This is done in the initial segment of the program.

```

      TCND = 1.25 * (POR * CONA + KQA * QC * CONQ + KMA * MC * CONM +
                   KHA * OMC * CONH) / (POR + KQA * QC + KMA * MC
                                       + KHA * OMC)

```

The thermal conductivity for the ranges a, b1 and b2 is now computed in the dynamic segment of the program with an IF statement nested in a DO loop (see Table 1)

```

      IF (WC(I) - CWC) 40, 50, 50
40 -
      -
      -
      -
      TCNL(I) = (CWC * CONW + ..... + KAWX * (POR-CWC) * APCAX)/
                (CWC + ..... + KAWX * (POR-CWC))
      TCN(I) = TCND + WC(I)/CWC * (TCNL(I) - TCND)
      GOTO 14
50 TCN(I) = (WC(I) * CONW + ..... + KAW(I) * A(I) * APCA(I))/
                (WC(I) + ..... + KAW(I) * A(I))
14 CONTINUE

```

CWC ("critical water content") represents the transitional value of θ between the ranges a and b and is given in the initial segment on a PARAM card. The 40 and 50 statements are executed if the water content of the compartment is in range a and b, respectively. The first 7 statements in 40 are needed to calculate TCNL, i.e. the value of λ at $\theta = \text{CWC}$. The variable names with an X refer to $\theta = \text{CWC}$, GAAX and GACX being given in the initial segment.

3.2.2. The calculation of ζ

Combining eq. (20) and (25) the value of ζ , i.e. the ratio of the average temperature gradient across the air-filled pores to the overall temperature gradient, follows from (Philip and De Vries, 1957)

$$\zeta = \frac{(\text{grad } T)_a}{a(\text{grad } T)_a + \theta(\text{grad } T)_w + x_q(\text{grad } T)_q + x_m(\text{grad } T)_m + x_o(\text{grad } T)_o} \quad (55)$$

when the temperature gradients are considered, averaged over the volumes occupied by air (a), water (w), quartz (q), other minerals (m), and organic matter (o).

Regarding air and water as the continuous medium for $\theta < \text{CWC}$ (see 3.2.1.) and $\theta \geq \text{CWC}$, respectively, ζ may be computed by dividing both

numerator and denominator in (55) by $(\text{grad } T)_a$ for $\theta < \text{CWC}$ and by $(\text{grad } T)_w$ for $\theta \geq \text{CWC}$. Thus, one obtains

$$\zeta = 1 / \left(\sum_{i=0}^n k_i X_i \right) \quad (56a)$$

and

$$\zeta = k_i(\text{air}) / \left(\sum_{i=0}^n k_i X_i \right) \quad (56b)$$

for $\theta < \text{CWC}$ and $\theta \leq \text{CWC}$, respectively.

The values of k_i for $\theta \geq \text{CWC}$ have been calculated already in order to find λ (see 3.2.1.). For $\theta < \text{CWC}$, k_i is found in a similar way, but now with air (containing water vapor) as the continuous medium.

The computation of ζ (ZETA) is programmed by means of an IF construction (see Table 1).

```

IF (WC(I)-CWC) 20, 30, 30
20 KWVA(I) = 1./3. * (2./(1. + (CONW/APCA(I) - 1.) * GA) +
                    1./(1. + (CONW/APCA(I) - 1.) * GC))
KQVA(I) =
KMVA(I) =
KHVA(I) =
ZETA(I) = 1./(A(I) + WC(I) * KWVA(I) + QC * KQVA(I) +
              MC * KMVA(I) + OMC * KHVA(I))
....
GOTO 8
30 ZETA(I) = KAW(I)/(A(I) * KAW(I) + WC(I) + QC * KQW
              + MC * KMW + OMC * KHW)
....
8 CONTINUE

```

3.2.3. The thermal and isothermal moisture diffusivities

The thermal liquid diffusivity

The thermal liquid diffusivity - DTL - may be computed in the dynamic segment according to eq. (33)

$$\text{DTL}(I) = \text{WCN}(I) * \text{GAM} * (-P(I))$$

GAM is given on a PARAM card, whereas the hydraulic conductivity (WCN) and the suction (P) are found from FUNCTION tables, taking into account the influence of the viscosity of water on the hydraulic conductivity.

The hydraulic conductivity and the viscosity of water are related by

$$K = K_i / \eta \quad (57)$$

where K_i denotes the intrinsic permeability of the medium. When the FUNCTION table for the hydraulic conductivity applies to a temperature of 20°C, the hydraulic conductivity at $t^\circ\text{C}$, K_t , is found from

$$K_t = (\eta_{20} / \eta_t) \times K_{20} \quad (58)$$

The variation with temperature of η for water (ETA) is also given in a FUNCTION table. The volumetric heat capacity (VHCP) of the medium is calculated with eq. (47) (see 2.4.).

FUNCTION WCONTB =

FUNCTION PTB =

FUNCTION ETATB =

WC(I) = AMW(I)/TCOM(I)

VHCP(I) = 0.46 * SOLC + 0.60 * OMC + WC(I)

TEMP(I) = VHTC(I)/(TCOM(I) * VHCP(I))

WCON(I) = AFGEN(WCONTB,WC(I))

ETA(I) = AFGEN(ETATB,TEMP(I))

WCN(I) = N/ETA(I) * WCON(I)

P(I) = AFGEN(PTB,WC(I))

The AFGEN term refers to an arbitrary function generator and provides a linear interpolation procedure between consecutive points given in a FUNCTION table. On a FUNCTION table card the data points of $y = \text{FUNCTION}(x)$ are given in pairs: (x_1, y_1) , (x_2, y_2) , etc. The first and second number of the pair always indicate the value of the independent and dependent variable, respectively. The numbers have to be separated by commas, whereas the parentheses may be omitted.

The isothermal liquid diffusivity

The isothermal liquid diffusivity - DWL - follows from (34) and is computed with

$$DWL(I) = WCN(I) * SP(I)$$

SP represents the slope of the suction-water content curve at a certain water content WC and is approximately calculated as follows (taking ΔWC as 2 % of WC): :

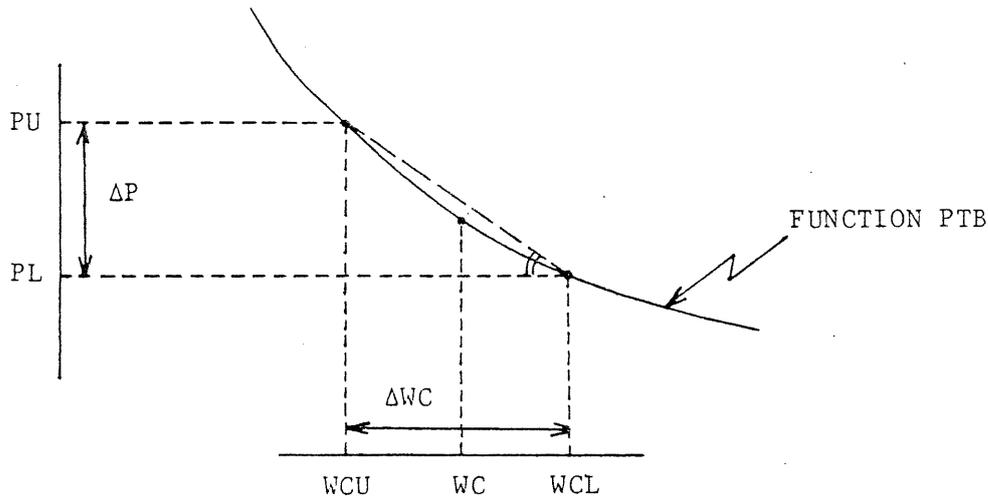
$$WCU(I) = 0.99 * WC(I)$$

$$WCL(I) = 1.01 * WC(I)$$

$$PU(I) = AFGEN(PTB, WCU(I))$$

$$PL(I) = AFGEN(PTB, WCL(I))$$

$$SP(I) = (PU(I) - PL(I)) / (WCL(I) - WCU(I))$$



The thermal vapor diffusivity

Using eq. (24) the thermal vapor diffusivity DTV can be calculated with

$$DTV(I) = (A(I) + FA(I) * WC(I)) * DATM(I) * V * H(I) * B * ZETA(I) / WDEN$$

$V(=v)$, $B(=\beta)$, and $WDEN(=\rho_1)$ are given on a PARAM card.

The air content (A) of the medium is defined as

$$A(I) = 1. - WC(I) - SOLC - OMC$$

FA, the factor which has to be introduced to account for the change in the effective cross-section (for vapor transfer) with changing water content, can be found from a FUNCTION table

FUNCTION FATB =

$$FA(I) = AFGEN(FATB, WC(I))$$

The diffusion coefficient of water vapor in air ($\text{cm}^2 \text{ day}^{-1}$), DATM, and the fractional relative humidity, H, are computed with

$$\begin{aligned} \text{ATMP(I)} &= \text{TEMP(I)} + 273. \\ \text{DATM(I)} &= 86400. * (4.42 \text{ E-}4 * \text{ATMP(I)} ** 2.3/\text{PRES}) \\ \text{H(I)} &= \text{EXP} (-\text{P(I)} * \text{G}/(\text{R} * \text{ATMP(I)})) \end{aligned}$$

The expression for H follows from (11), whereas DATM is given by Krischer and Rohnalter's relationship (1940; in Philip and De Vries, 1957)

$$D_a = 4.42 \times 10^{-4} T^{2.3} / P \quad (\text{cm}^2 \text{ sec}^{-1}) \quad (59)$$

PRES (total gas pressure, mm H_g), G(=g), and R(=R_w) are given on a PARAM card.

The isothermal vapor diffusivity

The isothermal vapor diffusivity DWV is computed from eq. (19)

$$\text{DWV(I)} = \text{DATM(I)} * \text{V} * \text{ALFA} * \text{A(I)} * \text{G} * \text{VAPD(I)} / (\text{WDEN} * \text{R} * \text{ATMP(I)} * \text{CAP(I)})$$

where the density of water vapor (VAPD) can be found using (12). The saturated vapor density (VPDS) is given as a FUNCTION table.

FUNCTION VPDSTB =

$$\begin{aligned} \text{VPDS(I)} &= \text{AFGEN}(\text{VPDSTB}, \text{TEMP(I)}) \\ \text{VAPD(I)} &= \text{H(I)} * \text{VPDS(I)} \end{aligned}$$

The differential water capacity of the medium (CAP) is found by taking the reciprocal value of SP

$$\text{CAP(I)} = 1./\text{SP(I)}$$

3.2.4. Flow of moisture

As described by eq. (41), two flows of moisture can be distinguished in case of a horizontal column, viz. one flow due to a moisture gradient and another due to a temperature gradient. They may be calculated with

$$\begin{aligned} \text{WFLW(I)} &= \text{AVDW(I)} * (\text{WC(I-1)} - \text{WC(I)}) / \text{DIST(I)} \\ \text{WFLT(I)} &= \text{AVDT(I)} * (\text{TEMP(I-1)} - \text{TEMP(I)}) / \text{DIST(I)} \end{aligned}$$

where WFLW(I) and WFLT(I) represent the fluxes between the two adjacent compartments I-1 and I (see Figure 1). If one regards the flow to occur from

the middle of one compartment to the middle of the adjacent one, the distance between these centers (DIST) will enter into the gradient term.

For the diffusivity term an average diffusivity has to be determined between the two compartments I-1 and I. Several averaging methods may be applied (De Wit and Van Keulen, 1970). Here, a (weighted) arithmetic average of the diffusivities is computed

$$\begin{aligned} \text{AVDW}(I) &= (\text{DIFW}(I-1) * \text{TCOM}(I-1) + \text{DIFW}(I) * \text{TCOM}(I)) / (2. * \text{DIST}(I)) \\ \text{AVDT}(I) &= (\text{DIFT}(I-1) * \text{TCOM}(I-1) + \text{DIFT}(I) * \text{TCOM}(I)) / (2. * \text{DIST}(I)) \end{aligned}$$

where, in accordance with (39) and (40),

$$\begin{aligned} \text{DIFW}(I) &= \text{DWL}(I) + \text{DWV}(I) \\ \text{DIFT}(I) &= \text{DTL}(I) + \text{DTV}(I) \end{aligned}$$

Considering the relatively simple case of a (for moisture) closed soil column, one may write

$$\begin{aligned} \text{WFLW}(1) &= 0. \\ \text{WFLW}(26) &= 0. \\ \text{WFLT}(1) &= 0. \\ \text{WFLT}(26) &= 0. \end{aligned}$$

When the flow due to gravitational forces is also taken into account (vertical flow), a term AWCN for the average hydraulic conductivity has to be added to the right-hand side of the WFLW expression. This is most conveniently done by adding

$$\dots\dots\dots + \text{AWCN}(I) * \text{GRAV}$$

where

PARAM GRAV = 1. (vertical column) or
PARAM GRAV = 0. (horizontal column)

AWCN is calculated in the same way as AVDW and AVDT

$$\text{AWCN}(I) = (\text{WCN}(I-1) * \text{TCOM}(I-1) + \text{WCN}(I) * \text{TCOM}(I)) / (2. * \text{DIST}(I))$$

Defining the net water flow for compartment I as the flow into compartment I minus the flow into compartment I+1, one has

$$\begin{aligned} \text{NWF}(I) &= \text{WFLW}(I) - \text{WFLW}(I+1) \\ \text{NWFT}(I) &= \text{WFLT}(I) - \text{WFLT}(I+1) \end{aligned}$$

and for the total net flow

$$\text{TNWF}(I) = \text{NWFT}(I) + \text{NWF}(I)$$

At any moment the amount of water in a compartment follows from an integral

$$AMW = \text{INTGRL} (IAMW, TNWF)$$

If 25 compartments are considered, this formal statement is written as an integrator array

$$AMW1 = \text{INTGRL} (IAMW1, TNWF1, 25)$$

In this array the indexed variable names have to be written without parentheses. In order to indicate that AMW1 and AMW(1) (etc.) refer to the same variable, these variable names can be assigned to the same storage location using an EQUIVALENCE statement (FORTRAN card with a slash in the first column)

```
/ EQUIVALENCE (AMW(1),AMW1),(IAMW(1),IAMW1),(TNWF(1),TNWF1)
```

The variable names used in an integrator array may be declared only with a REAL statement

```
/ REAL IAMW(25),AMW(25),TNWF(25)
```

The integration is performed with the fourth-order Runge-Kutta METHOD with variable integration interval using Simpson's Rule for error estimation. The METHOD RKS needs no special specification, so that this card is an optional one.

After each time step the computer starts on the next computational sequence with a new value for the water content (obtained from the new value of the integral)

$$WC(I) = AMW(I)/TCOM(I)$$

3.2.5. Flow of heat

For the flow of heat expressions similar to those which have been developed for the moisture flow may be derived. The heat fluxes due to a temperature gradient and a moisture gradient are given by (see eq. (48))

$$HFLT(I) = ATCN(I) * (TEMP(I-1)-TEMP(I))/DIST(I)$$

$$HFLW(I) = ADWV(I) * L * (WC(I-1)-WC(I))/DIST(I)$$

where

$$ATCN(I) = (TCOM(I-1)+TCOM(I))/(TCOM(I-1)/TCN(I-1)+TCOM(I)/TCN(I))$$

(De Wit and Van Keulen, 1970)

and

$$ADWV(I) = (DWV(I-1) * TCOM(I-1) + DWV(I) * TCOM(I)) / (2. * DIST(I))$$

A temperature gradient imposed on the system may be introduced by defining the boundary conditions, i.e. the temperature of the surface (or left side) of the column, TS, and the heat flow out of the 25th compartment, HFLT(26).

In order to be able to introduce both sinusoidal and non-sinusoidal temperature variations, the following expression for TS has been used

$$TS = TAV + TAMP * SIN(6.2832 * RPER * TIME)$$

where TAV, TAMP, and RPER are given on a PARAM card.

The heat flow out of the 25th compartment is

$$HFLT(26) = TCN(25) * (TEMP(25) - TEMP(26)) / DIST(26) * FUDGE$$

with

$$DIST(26) = 0.5 * TCOM(25)$$

$$TEMP(26) = ITEMP(25) * FUDGE$$

In case of a constant temperature (here the initial temperature) at the end of the column, one may write

$$PARAM FUDGE = 1.$$

If the end of the column can be considered to be isolated, one has

$$PARAM FUDGE = 0.$$

Since the column is closed for water:

$$HFLW(1) = 0.$$

$$HFLW(26) = 0.$$

The net heat fluxes and total net heat flow are obtained with

$$NHFT(I) = HFLT(I) - HFLT(I+1)$$

$$NHFW(I) = HFLW(I) - HFLW(I+1)$$

$$TNHF(I) = NHFT(I) + NHFW(I)$$

The integrator array for the volumetric heat content is given by

$$VHTC1 = INTGRL(IVHTC1, TNHF1, 25)$$

$$/ \quad EQUIVALENCE(VHTC(1), VHTC1), (IVHTC(1), IVHTC1), (TNHF(1), TNHF1)$$

$$/ \quad REAL IVHTC(25), VHTC(25), TNHF(25)$$

In the next time step the new temperature of compartment I is calculated with

$$TEMP(I) = VHTC(I) / (TCOM(I) * VHCP(I))$$

3.2.6. The output_control

The output may be controlled with FORTRAN WRITE and CSMP PRTPLT (print plot) and PRINT statements. For an explanation of the WRITE-FORMAT statements the reader is referred to a FORTRAN manual. Suffice it to mention that the WRITE capability is used at times PRDEL, 2 x PRDEL, 3 x PRDEL, and so on, with

```
X = IMPULS(0.,PRDEL)
IF (X * KEEP.LT.0.5) GOTO 18
-
-
-
-
-
-
```

```
18 CONTINUE
```

With the IMPULS function X equals zero at times $\neq i \times PRDEL$ and is equal to 1 at times $i \times PRDEL$ ($i = 0,1,2,\dots$). KEEP is an internal CSMP variable being equal to 1, when the actual rates of changes of the integrals are calculated, and zero in all other conditions. Thus, only when both X and KEEP equal 1, the WRITE routine is used. Otherwise, no output is requested ($X * KEEP$ Less Than 0.5) and the calculation continues.

Using FORTRAN output routines has the advantage that the arrays do not have to be undimensionalized. If use is made of CSMP PRTPLT and PRINT routines, the requested output has to be undimensionalized with

```
T1 = TEMP(1)
WC1 = WC(1)
etc.
```

The PRINT and PRTPLT statements are used to specify which variables are to be printed and print-plotted at time intervals PRDEL and OUTDEL, respectively. On a TIMER card PRDEL and OUTDEL are given together with the finish time (FINTIM).

```
TIMER FINTIM = ,PRDEL = ,OUTDEL =
```

Time units have to be the same as for the diffusivities and conductivities.

PROBLEM INPUT STATEMENTS

TITLE COMBINED TRANSPORT OF WATER AND HEAT IN POROUS MATERIALS

***** AREA IS SET TO UNITY *** CM**2 *****

INITIAL
 NOSORT
 FIXED I, NL
 PARAM NL=25
 PARAM SOLC=0.54,OMC= 0.
 PARAM FRQ= 0.4
 PARAM CONQ=1762.6,CONM=604.8,CONH=51.8,CONW=122.7,CONA=5.3
 PARAM GA=0.125
 PARAM WCH=0.15,CWC=0.06
 STORAGE TCOM(25),IWC(25),ITEMP(25)
 TABLE TCOM (1-25)= 25 * 0.8
 TABLE IWC(1-25)= 25 * 0.20
 TABLE ITEMP(1-25)=25 * 15.

DIST(1)=0.5 * TCOM(1)
 DPTH(1)=0.5 * TCOM(1)

DO 1 I=2,NL
 DIST(I)=0.5 *(TCOM(I)+TCOM(I-1))
 DPTH(I)=DPTH(I-1)+DIST(I)
 1 CONTINUE

DO 2 I=1,NL
 IAMW(I)=IWC(I)*TCOM(I)
 2 CONTINUE

FRM=1.-FRQ
 QC=FRQ*SOLC
 MC=FRM*SOLC
 POR=1.-SOLC-OMC
 AH=POR-WCH
 GAAH=0.333-AH/POR*0.298
 GAAX=0.013+CWC/WCH*(GAAH-0.013)
 GACX=1.-2.*GAAX

DO 3 I=1,NL
 IVHCP(I)=0.46*SOLC + 0.60*OMC + IWC(I)
 IVHTC(I)=ITEMP(I)*IVHCP(I)*TCOM(I)
 3 CONTINUE

GC=1.-2.*GA
 KQW=1./3.*(2./[1.+(CONQ/CONW-1.)*GA]+1./[1.+(CONQ/CONW ...
 -1.)*GC])
 KMW=1./3.*(2./[1.+(CONM/CONW-1.)*GA]+1./[1.+(CONM/CONW ...
 -1.)*GC])
 KHW=1./3.*(2./[1.+(CONH/CONW-1.)*GA]+1./[1.+(CONH/CONW ...
 -1.)*GC])

Table 1

```

KQA=1./3.*(2./(1.+(CONQ/CCNA-1.)*GA)+1./(1.+(CCNG/CCNA ...
-1.)*GC))
KMA=1./3.*(2./(1.+(CCNM/CCNA-1.)*GA)+1./(1.+(CCNM/CCNA ...
-1.)*GC))
KHA=1./3.*(2./(1.+(CCNH/CCNA-1.)*GA)+1./(1.+(CCNH/CCNA ...
-1.)*GC))
TCND=1.25*(POR*CCNA + KQA*QC*CONQ + KMA*MC*CCNM + KHA ...
*QMC*CONH)/(POR + KQA*QC + KMA*MC + KHA*QMC)

```

DYNAMIC

NOSORT

PARAM TAV=25.,TAMP=0.,RPER=0.,FUDGE=1.

PARAM GRAV=0.

PARAM PRES=760.,WDEN=1.,G=981.,R=4.615E6,L=586.,A=1.005

PARAM GAM=-2.09E-3,V=1.024,B=1.05E-6,ALFA=0.67

FUNCTION WCONTB=0.,0.,0.03,1.5E-10,0.17,1.5E-10,...

0.18,6.35E-5,0.19,8.72E-5,0.20,0.0002,0.21,...

4.85E-4,0.22,8.12E-4,0.23,0.0011,0.24,0.0017,0.25,0.0024,...

0.26,0.0062,0.27,0.0151,0.28,0.0188,0.29,0.0324,0.30,0.0535,...

0.31,0.08,0.32,0.1261,0.33,0.1814,0.34,0.2618,0.35,0.3681,...

0.36,0.5685,0.37,0.7344,0.38,0.864,0.39,1.27,0.40,1.96,0.41,...

2.42,0.42,2.88,0.43,3.75,0.44,4.16,0.45,4.20,0.46,4.24

FUNCTION PTB=0.0,3.5E6,0.015,7.E5,0.02,2.5E5,0.025,5.E4,0.03,...

39935.,0.06,33485.,0.09,27035.,0.12,20585.,0.15,14135.,...

0.19,7.7E3,0.19,5100.,0.20,3900.,0.21,3190.,0.22,2600.,0.23,...

2100.,0.24,1675.,0.25,1100.,0.26,850.,0.27,665.,0.28,550.,...

0.29,450.,0.30,331.,0.31,258.,0.32,212.,0.33,175.,0.34,143.,...

0.35,116.,0.36,94.,0.37,75.,0.38,59.,0.39,45.,0.40,36.,0.41,...

28.,0.42,21.,0.43,15.,0.44,10.,0.45,5.,0.46,0.

FUNCTION FATB=0.,1.0,0.17,1.0,0.46,0.

FUNCTION VPDSTB=0.,4.85E-6,5.,6.80E-6,10.,9.40E-6,15.,12.85E-6,...

20.,17.30E-6,25.,23.05E-6,30.,30.38E-6,35.,39.63E-6,40.,51.1E-6

FUNCTION ETATB=0.,1.80,5.,1.52,10.,1.31,15.,1.14,20.,1.005,25.,...

0.89,30.,0.80,35.,0.72,40.,0.66

***** MCISTURE TRANSFER *****

DO 4 I=1,NL

WC(I)=AMW(I)/TCOM(I)

VHCP(I)=0.46*SOLC + 0.60*CMC + WC(I)

TEMP(I)=VHTC(I)/(TCOM(I)*VHCP(I))

WCON(I)=AFGEN(WCONTB,WC(I))

ETA(I)=AFGEN(ETATB,TEMP(I))

WCN(I)=N/ETA(I)*WCON(I)

P(I)=AFGEN(PTB,WC(I))

FA(I)=AFGEN(FATB,WC(I))

A(I)=1.-WC(I)-SOLC-DMC

IF (WC(I)-WCH) 60,70,70

60 GAA(I)=0.013+WC(I)/WCH*(GAAH-0.013)

GAC(I)=1.-2.*GAA(I)

GOTO 4

70 GAA(I)=0.333-A(I)/POR*0.298

GAC(I)=1.-2.*GAA(I)

4 CONTINUE

Table 1(continued)

```

DO 5 I=2,NL
AWCN(I)=(WCN(I-1)*TCCM(I-1)+WCN(I)*TCOM(I))/(2.*DIST(I))
5 CONTINUE

DO 6 I=1,NL
ATMP(I)=TEMP(I) + 273.
6 CONTINUE

DO 7 I=1,NL
DTL(I)=WCN(I)*GAM*(-P(I))
7 CONTINUE

DO 8 I=1,NL
H(I)=2.718**(-P(I)*G/(R*ATMP(I)))
DATM(I)=86400.*(4.42E-4*ATMP(I)**2.3/PRES)
CVAP(I)=L*DATM(I)*V*H(I)*R
APCA(I)=CONA+CVAP(I)
KAW(I)=1./3.*(2./(1.+(APCA(I)/CONW-1.)*GAA(I))+1./(1.+ ...
(APCA(I)/CONW-1.)*GAC(I)))

IF (WC(I)-CWC) 20,30,30
20 KWVA(I)=1./3.*(2./(1.+(CONW/APCA(I)-1.)*GA)+1./(1.+...
(CONW/APCA(I)-1.)*GC))
KQVA(I)=1./3.*(2./(1.+(CONQ/APCA(I)-1.)*GA)+1./(1.+...
(CONQ/APCA(I)-1.)*GC))
KMVA(I)=1./3.*(2./(1.+(CONM/APCA(I)-1.)*GA)+1./(1.+...
(CONM/APCA(I)-1.)*GC))
KHVA(I)=1./3.*(2./(1.+(CONH/APCA(I)-1.)*GA)+1./(1.+...
(CONH/APCA(I)-1.)*GC))
ZETA(I)=1./(A(I)+WC(I)*KWVA(I)+QC*KQVA(I)+MC*KMVA(I))+...
OMC*KHVA(I))
DTV(I)=(A(I)+FA(I)*WC(I))*DATM(I)*V*H(I)*B*ZETA(I)/WDEN
GOTO 8
30 ZETA(I)=KAW(I)/(A(I)*KAW(I)+WC(I)+QC*KQW+MC*KMW+OMC*KFW)
DTV(I)=(A(I)+FA(I)*WC(I))*DATM(I)*V*H(I)*B*ZETA(I)/WDEN

8 CONTINUE

DO 9 I=1,NL
WCU(I)=0.99*WC(I)
WCL(I)=1.01*WC(I)
PU(I)=AFGEN(PTB,WCU(I))
PL(I)=AFGEN(PTB,WCL(I))
SP(I)=(PU(I)-PL(I))/(WCL(I)-WCU(I))
DWL(I)=WCN(I)*SP(I)
CAP(I)=1./SP(I)
9 CONTINUE

DO 10 I=1,NL
VPDS(I)=AFGEN(VPDSTB ,TEMP(I))
VAPD(I)=H(I)*VPDS(I)
DWV(I)=DATM(I)*V*ALFA*A(I)*G*VAPD(I)/(WDEN*R*ATMP(I)*CAP(I))
10 CONTINUE

```

Table 1(continued)

```

DO 11 I=1,NL
DIFT(I)=DTL(I) + DTV(I)
DIFW(I)=DWL(I) + DWV(I)
11 CONTINUE

```

```

WFLW(1)=0.
WFLT(1)=0.

```

```

DO 12 I=2,NL
AVDT(I)=(DIFT(I-1)*TCOM(I-1)+DIFT(I)*TCOM(I))/(2.*DIST(I))
AVDW(I)=(DIFW(I-1)*TCOM(I-1)+DIFW(I)*TCOM(I))/(2.*DIST(I))
WFLT(I)=AVDT(I)*(TEMP(I-1)-TEMP(I))/DIST(I)
WFLW(I)=AVDW(I)*(WC(I-1)-WC(I))/DIST(I) + AWCN(I)*GRAV
12 CONTINUE

```

```

WFLW(26)=0.
WFLT(26)=0.

```

```

DO 13 I=1,NL
NWFT(I)=WFLT(I)-WFLT(I+1)
NWFV(I)=WFLW(I)-WFLW(I+1)
TNWF(I)=NWFT(I)+NWFV(I)
13 CONTINUE

```

***** HEAT TRANSFER *****

```

DO 14 I=1,NL
IF (WC(I)-CWC) 40,50,50
40 PX=AFGEN(PTB,CWC)
HX=2.718**(-PX*G/(R*ATMP(I)))
DATMX=86400.*(4.42E-4*ATMP(I)**2.3/PRES)
CVAPX=L*DATMX*V*HX*B
APCAX=CONA+CVAPX
KAWX=1./3.*(2./(1.+(APCAX/CONW-1.)*GAAX)+1./(1.+(APCAX/CONW ...
-1.)*GACX))
TCNL(I)=(CWC*CONW + KQW*QC*CONQ + KMW*MC*CONM + KHW ...
*OMC*CONH + KAWX*(POR-CWC)*APCAX)/(CWC + KQW*QC + KMW ...
*MC + KHW*OMC + KAWX*(POR-CWC))
TCN(I)=TCND+WC(I)/CWC*(TCNL(I)-TCND)
GOTO 14
50 TCN(I)=(WC(I)*CONW + KQW*QC*CONQ + KMW*MC*CONM + KHW ...
*OMC*CONH + KAW(I)*A(I)*APCA(I))/(WC(I) + KQW*QC + KMW*MC ...
+ KHW*OMC + KAW(I)*A(I))

```

```

14 CONTINUE

```

```

TS=TAV+TAMP*SIN(6.2832*PPER*TIME)
HFLT(1)=TCN(1)*(TS-TEMP(1))/DIST(1)

```

```

DO 15 I=2,NL
ATCN(I)=(TCOM(I-1)+TCOM(I))/(TCOM(I-1)/TCN(I-1)+TCOM(I)/...
TCN(I))
HFLT(I)=ATCN(I)*(TEMP(I-1)-TEMP(I))/DIST(I)
15 CONTINUE

```

Table 1(continued)

```

DIST(26)=0.5*TCOM(25)
TEMP(26)=ITEMP(25)*FUDGE
HFLT(26)=(TCN(25)*(TEMP(25)-TEMP(26))/DIST(26))*FUDGE
HFLW(1)=0.

```

```

DO 16 I=2,NL
ADWV(I)=(DWV(I-1)*TCCM(I-1)+DWV(I)*TCOM(I))/(2.*DIST(I))
HFLW(I)=ADWV(I)*L*(WC(I-1)-WC(I))/DIST(I)
16 CONTINUE

```

```
HFLW(26)=0.
```

```

DO 17 I=1,NL
NHFT(I)=HFLT(I)-HFLT(I+1)
NHFW(I)=HFLW(I)-HFLW(I+1)
TNHF(I)=NHFT(I)+NHFW(I)
17 CONTINUE

```

```

AMW1=INTGRL(IAMW1,TNWF1,25)
VHTC1=INTGRL(IVHTC1,TNHF1,25)

```

```

/ EQUIVALENCE (AMW(1),AMW1),(IAMW(1),IAMW1),(TNWF(1),TNWF1)
/ EQUIVALENCE (VHTC(1),VHTC1),(IVHTC(1),IVHTC1),(TNHF(1),TNHF1)
/ REAL IAMW(25),AMW(25),IVHTC(25),VHTC(25),TNWF(25),TNHF(25)
/ REAL DIST(30),DPTH(30),IVHCP(30),VHCP(30),TEMP(30),ATMP(30)
/ REAL WC(30),WCN(30),P(30),DTL(30),A(30),FA(30),H(30),DATM(30)
/ REAL DTV(30),CAP(30),DWL(30),VPDS(30),VAPD(30),DIFT(30),ADWV(30)
/ REAL DIFW(30),DWV(30),AVDT(30),AVDW(30),WFLT(30),HFLT(30),GAA(30)
/ REAL WFLW(30),AWCN(30),WCON(30),ETA(30)
STORAGE NHFT(30),NHFW(30),TCN(30),ATCN(30),GAC(30)
STORAGE HFLW(30),NHFT(30),NHFW(30),CVAP(30),APCA(30),TCNL(30)
STORAGE KHVA(30),KWVA(30),KQVA(30),KMVA(30),KAW(30),ZETA(30)
STORAGE PU(30),PL(30),WCU(30),WCL(30),SP(30)

```

```
***** OUTPUT CONTROL *****
```

```

X=IMPULS(0.,PRDEL)
IF (X*KEEP.LT.0.5) GOTO 18
100 FORMAT (15F7.2/10F7.2)
WRITE (6,101)
101 FORMAT (1H ,6H DEPTH)
WRITE (6,100) (DPTH(I),I=1,NL)
102 FORMAT (15F7.4/10F7.4)
WRITE (6,103)
103 FORMAT (1H ,34H WATERCONTENT FOR DIFFERENT DEPTHS)
WRITE (6,102) (WC(I),I=1,NL)
104 FORMAT (15F7.3/10F7.3)
WRITE (6,105)
105 FORMAT (1H ,33H TEMPERATURE FOR DIFFERENT DEPTHS)
WRITE (6,104) (TEMP(I),I=1,NL)
18 CONTINUE

```

```

METHOD RKS
TIMER FINTIM=0.5,OUTDEL=0.025,PRDEL=0.025,DELT=1.E-6

```

Table 1(continued)

```
T1=TEMP(1)
T2=TEMP(2)
T5=TEMP(5)
T10=TEMP(10)
T15=TEMP(15)
T20=TEMP(20)
T25=TEMP(25)
```

```
PRTPLT TS,T1,T2,T5,T10,T15,T20,T25
```

```
WC1=WC(1)
WC2=WC(2)
WC3=WC(3)
WC4=WC(4)
WC5=WC(5)
WC10=WC(10)
WC15=WC(15)
WC20=WC(20)
WC25=WC(25)
```

```
PRTPLT WC1,WC2,WC3,WC4,WC5,WC10,WC15,WC20,WC25
```

```
DTL1=DTL(1)
DTL2=DTL(2)
DTV1=DTV(1)
DTV2=DTV(2)
DWV1=DWV(1)
DWV2=DWV(2)
DWL1=DWL(1)
DWL2=DWL(2)
H1=H(1)
H2=H(2)
DATM1=DATM(1)
DATM2=DATM(2)
KAW1=KAW(1)
KAW2=KAW(2)
SP1=SP(1)
SP2=SP(2)
WFLT2=WFLT(2)
WFLW2=WFLW(2)
TCN1=TCN(1)
TCN2=TCN(2)
ATCN2=ATCN(2)
HFLT1=HFLT(1)
HFLT2=HFLT(2)
ADWV2=ADWV(2)
HFLW2=HFLW(2)
```

```
PRINT DTL1,DTL2,DTV1,DTV2,DWV1,DWV2,DWL1,DWL2,H1,H2,DATM1,DATM2,...
KAW1,KAW2,SP1,SP2,WFLT2,WFLW2,TCN1,TCN2,ATCN2,HFLT1,HFLT2,...
ADWV2,HFLW2
```

```
END
STOP
ENDJOB
```

Table 1(continued)

4. Results

Before proceeding with some simulation results it may be wise to make a few general remarks. First, the model has not been tested in detail with respect to its sensibility to changes of parameter values and functions. Secondly, it should be kept in mind that in this report the relatively simple case has been simulated of a homogeneous soil column, which is closed for water. Further, water potentials other than matric or gravitational and matric suction changes due to hysteresis have been omitted.

4.1. Heat-moisture flow in a soil column due to a sudden increase (or decrease) of the temperature at one side of the column for different initial moisture contents

The simulation program has been applied for the case of a horizontal soil column of 20 cm length, divided into 25 compartments of the same thickness. Thus (see also Table 1)

$$\text{GRAV} = 0., \text{NL} = 25, \text{TCOM} = 0.8$$

Hydraulic conductivity and suction curves (see Figures 2 and 3) are taken from Van Keulen and Van Beek (1971; their FUNCTION COTB5 and FUNCTION SUTB5 for an unplowed light humous sandy soil), while the tabulated functions of the saturated vapor density (VPDS) and the viscosity of water (ETA) are derived from data of Van Wijk (1963) and a handbook of physical constants, respectively. Other constants adopted are (see also Philip and De Vries, 1957)

PRES = 760. (mm Hg)	N = 1.005 (centipoises)
WDEN = 1. (g cm ⁻³)	GAM = -2.09 x 10 ⁻³ (°C ⁻¹)
G = 981. (cm sec ⁻²)	V = 1.024
R = 4.615 x 10 ⁶ (erg g ⁻¹ °C ⁻¹)	B = 1.05 x 10 ⁶ (g cm ⁻³ °C ⁻¹)
L = 586. (cal g ⁻¹)	ALFA = 0.67

Values of the thermal conductivity at 20°C were used

CONQ = 1762.6 (cal cm ⁻¹ day ⁻¹ °C ⁻¹)	
CONM = 604.8	"
CONH = 51.8	"
CONW = 122.7	"
CONA = 5.3	"

As a consequence the thermal conductivity of the soil, TCN, is only slightly temperature dependent via the apparent thermal conductivity of the air. It should be noted that all transport coefficients and diffusivities are expressed in time units of days. The following system constants were chosen

$$\text{SOLC} = 0.54 \text{ (cm}^3 \text{ cm}^{-3}\text{)}$$

$$\text{OMC} = 0.$$

$$\text{FRQ} = 0.4 \text{ (i.e. 40 \% of the solid content consists of quartz)}$$

$$\text{CWC} = 0.06$$

$$\text{WCH} = 0.15$$

$$\text{GA} = 0.125$$

Changes in the value of CWC affect all the TCN values calculated for moisture contents $<$ CWC. Calculated DTV and ZETA values are only affected over the θ -range where CWC is changed. Changes in DTV and ZETA at the point $\theta = \text{CWC}$ which are too abrupt may possibly occur and should be avoided. According to De Vries (1952, 1963), a value of 0.125 for GA is reasonable for soils. An FA-function was introduced as

$$\text{FUNCTION FATB} = (0.,1.0),(0.17,1.0),(0.46,0.)$$

assuming an initial moisture content and initial temperature, uniformly distributed throughout the column

$$\text{IWC} = 0.05 \text{ (cm}^3 \text{ cm}^{-3}\text{)}$$

$$\text{ITEMP} = 15. \text{ (}^\circ\text{C)}$$

Three more simulation runs were made with IWC values of 0.10, 0.20, and 0.35, respectively. An average temperature gradient of $0.5 \text{ }^\circ\text{C cm}^{-1}$ was imposed on the soil column with

$$\text{TAV} = 25. \text{ (}^\circ\text{C)}$$

$$\text{TAMP} = 0.$$

$$\text{FUDGE} = 1.$$

Integration was performed with METHOD RKS.

4.1.1. Initial moisture content

The temperature distribution in the soil column at time 0.025 day and 0.50 day is shown in Figure 4. The distribution at time 0.025 is approximately the same for all four moisture contents, which means that in this case the

value of the thermal heat diffusivity ($= \lambda/C_h$) is independent of the moisture content.

The moisture distribution in the soil column is given in Table 2 for different time intervals up to 0.50 day and in Figure 5 at time 0.50 day.

As expected, water will move from the warm side to the cold end of the column, while the amount of water being moved is greatest at the lower moisture contents. Of course, the total net movement for a closed soil column will have to be equal to zero. For $\theta = 0.05$ and $\theta = 0.10$, the effect is more pronounced at both ends of the column, whereas at the higher moisture contents the moisture distribution occurs more gradually over the whole column. This can be easily explained by the fact that at the higher water contents the liquid diffusivities become more important (see Table 4). The system will try to build up a moisture gradient which counteracts the temperature gradient. From Table 2 it is evident that at the highest moisture content ($\theta = 0.35$) a steady-state situation has been established at time 0.30 day, while for the other water contents the moisture distribution is still changing at time 0.50 day.

4.1.2. Temperature gradient

To get an impression of the effect of the magnitude of the temperature gradient, a simulation run was made at $\theta = 0.05$ with

$$\text{ITEMP} = 5., \text{TAV} = 35.$$

which results in an average temperature gradient of $1.5 \text{ }^\circ\text{C cm}^{-1}$.

Figure 6 and Table 3a show the effect of the (three times) increased temperature gradient. The amount of water that has been moved after 0.10 day also increased by a factor of about 3, whereas the shape of the curve has remained unchanged.

4.1.3. Hydraulic conductivity and matric suction

To investigate the influence of the hydraulic conductivity and the matric suction two other functions (see Figures 2 and 3) were taken from Van Keulen and Van Beek (1971; their FUNCTION COTB1 and FUNCTION SUTB1 for a plowed light humous sandy soil). The constants 0.06 and 0.03 were adopted for WCH and CWC, respectively, while FA was given as

$$\text{FUNCTION FATB} = (0., 1.0), (0.09, 1.0), (0.46, 0.)$$

The results of the simulation runs at $\theta = 0.05$ (time = 0.10 day) and $\theta = 0.20$ (time = 0.05 day) are given in Table 3b and Figure 6.

At $\theta = 0.05$ the suction is about the same for both the plowed and unplowed soil, but the hydraulic conductivity of the plowed soil is a factor 10^4 higher, resulting in higher liquid diffusivities (see Table 4) and, consequently, less water movement and a more gradual moisture distribution in the column.

At $\theta = 0.20$ the suction is about a factor 20 lower for the plowed soil, while the hydraulic conductivity is a factor 10 higher. Results are only obtained for time = 0.05 day, but it seems likely that there will be no striking differences between the two soil types (for $\theta = 0.20$). Comparing the diffusivities of both soils at $\theta = 0.20$ shows similar orders of magnitude (see Table 4).

4.1.4. The calculation of λ (TCN), ζ (ZETA), and the thermal moisture diffusivities (DTV,DTL) in the model

To calculate the thermal conductivity λ , the value of ζ , and the thermal moisture diffusivities (DTV,DTL) at 20°C and at different water contents, for the soil types mentioned before, separate runs were made with a part of the program using the following cards

```
RENAME  TIME = WC, FINTIM = FINWC  
METHOD  RECT  
TIMER   FINWC = 0.46, DELT = 0.01, OUTDEL = 0.01, PRDEL = 0.01
```

Now constant water content steps of 0.01 (METHOD RECT, DELT = 0.01) instead of variable time steps are taken by the computer. The results are given in the Figures 7 through 10 and Table 5.

Figure 7 shows the thermal conductivity as calculated by the program. Two other functions for a sandy soil and a clay soil are given for comparative purposes. The values of the computed functions are rather high, especially in the lower water content range.

Values computed for ζ at different porosities and moisture contents (see Table 5) are in good agreement with data of Philip and De Vries (1957).

The thermal diffusivities, DTV and DTL, are presented in Figures 8 and 9, respectively, while values of the total thermal diffusivity (DIFT) can be read from Figure 10. Data for Yolo light clay are taken from Philip and De Vries (1957).

At all moisture contents the thermal vapor diffusivities are of comparable order of magnitude for the different soil types, the differences being greatest at the lower water contents. Thermal liquid diffusivities become important at the higher moisture contents and show more pronounced differences between the three soil types. For the coarser-textured sandy soils the values of DTL are much higher and the maximum value of DTL for these soils will exceed the maximum of DTV and dominate the shape of the DIFT-curve, as expected (Philip and De Vries, 1957). The relative constancy of DIFT for Yolo light clay arises from the fact that DTV and DTL for this soil are of the same order of magnitude.

In the model, DTL is only slightly temperature dependent via the effect of the viscosity of water on the hydraulic conductivity. Also the effects of temperature changes on DTV, as far as it has been put in the program, are fairly small. Both diffusivities are positively correlated with temperature.

4.1.5. Thermal conductivity

Two simulation runs were made to investigate the effect of the magnitude of the thermal conductivity on the heat-moisture flow (hydraulic conductivity and suction curves not the same as in 4.1.1. or 4.1.3.). In the first run λ was given as an arbitrary λ - θ function, while in the second one this factor was calculated as indicated (see 3.2.1. and also Figure 7). The runs were made at an initial volumetric moisture content of 5.0 %.

From Table 6 one may conclude that the lower the thermal conductivity, the greater the amount of water that will be moved, whereas at the higher thermal conductivity more water will accumulate at the cold end of the soil column. This can be explained easily by looking at Figure 11. For relatively low values of the thermal conductivity the temperature gradient will be steeper in the hot end of the column, while a reverse situation occurs at the cold side: a higher temperature gradient at high thermal conductivity values. A steady state temperature distribution at time 0.50 day has not yet been established in case of a low thermal conductivity. It should be noted that the value of τ was not the same for both runs.

Simulation runs made at $\theta = 0.20$ (λ : 242 and 293 respectively) showed no large differences.

4.1.6. Integration method

When dealing with very small changes of the integrals, the choice of the integration method may be important.

It appeared that with METHOD MILNE an integral is "emptying" itself, even if the rate of change is zero. This emptying occurs at a very small rate, so in most cases one does not have to be concerned about this fact. With METHOD RKS emptying did not occur, although still a small negative total net water flow remained for the soil column as a whole (see Table 7).

As is shown in Table 7, METHOD MILNE will not present any difficulties at low moisture contents. For long simulation periods and at higher moisture contents, however, METHOD RKS is preferable to METHOD MILNE.

4.1.7. The simulation of an experiment

Oomen and Staring (1972) performed an experiment with a closed soil column of 8.6 cm length and examined the moisture distribution after a sudden temperature drop (10°C) at one side of the column (average temperature gradient of about $1.2^{\circ}\text{C cm}^{-1}$). Hydraulic conductivity and suction curves for their soil are shown in Figures 2 and 3. Values for the hydraulic conductivity were computed in the model as indicated by Rijtema (1969).

$$\text{WCN} = 7. \times P \times \times (-1.4)$$

and

$$\text{WCN} = 80. \times \text{EXP} (-0.07 \times P)$$

for $P > 150$ and $P \leq 150$, respectively.

Simulation runs were made using 20 compartments of 0.43 cm at 3 initial volumetric moisture contents

$$\text{NL} = 20., \text{TCOM} = 0.43, \text{IWC} = 0.045, 0.09 \text{ or } 0.16$$

Other parameters and functions adopted were

$$\text{SOLC} = 0.42$$

$$\text{ITEMP} = 23.$$

$$\text{TAV} = 13.$$

$$\text{ZETA} = 1.8$$

$$\text{FUNCTION FATB} = (0., 1.0), (0.15, 1.0), (0.46, 0.)$$

This FA function had been introduced by mistake. A better function in this case would have been

FUNCTION FATB = (0.,1.0),(0.05,1.0),(0.58,0.)

The thermal conductivity was given as a function (arbitrary function in Figure 7). The results of the simulation are presented in Figures 12 through 16 and Table 8.

Temperature distribution at time 0.01 day and 0.50 day are shown in Figure 12. In this case thermal heat diffusivities do depend on moisture content.

Figures 13, 14 and 15 show the moisture distribution in the column after different time intervals (up to 0.50 or 3.00 days) for initial volumetric moisture contents of 4.5 %, 9.0 % and 16.0 %, respectively. Because the integrations were still performed with METHOD MILNE, a considerable amount of water has been lost after 0.50 day at the highest water content (see 4.1.6.).

In Figure 16 the moisture distributions at time 0.50 day for all three moisture contents are given. Apart from the effect of the integration method, it is evident that there are no large differences between the three water contents. This may be explained by the fact that the thermal and isothermal moisture diffusivities are of the same order of magnitude in all three cases (see Table 8). As contrasted with the values of Table 4, the value of DTL shows only a slight increase, while DWL even decreases. Obviously the product of K and ψ (see eq. (33)) decreases in this moisture content range.

In Figure 17 some experimental results of Oomen and Staring (1972) have been presented together with the simulated results for $\theta = 0.045$. Because of several uncertainties regarding the experimental and simulation procedures, drawing definite conclusions from this Figure would not be very wise. The experimental results, for example, show a considerable error in moisture balance for the highest moisture content. At this water content, water is even accumulating at the hot end of the column.

4.2. Heat-moisture flow in a soil column due to a sinusoidal temperature variation at one side of the column for different initial moisture contents

A sinusoidal temperature variation of 10°C can be simply introduced with

$$\text{ITEMP} = 15., \text{TAV} = 15., \text{TAMP} = 10.$$

Hydraulic conductivity and suction curves used are the same as in 4.1, while the length of the soil column is taken again as 20 cm, divided into 25 compartments of 0.8 cm. Two periods of the temperature wave have been applied: 0.01 day (RPER = 100.) and 0.5 day (RPER = 2.). In Figures 18 through 20 some simulation results are shown for a wave period 0.5 day and a volumetric moisture content of 12.0 %. It should be noted that these integrations were still performed with METHOD MILNE.

The moisture content variation is very significant in the first compartment and shows a fairly good "feed-back" with temperature variation (only a small phase delay): an increase of the temperature is accompanied with a moisture content decrease and vice versa (see Figure 18). In case of a constant temperature (equal to TAV) at the other end of the column (FUDGE = 1.) the maximum moisture content value that will be reached is less than in case of an isolated column (FUDGE = 0.).

For the second compartment the effect on moisture content is already much less pronounced (small net water movement for this compartment) and changes in moisture content are positively correlated with the temperature variation (see Figure 19). Differences between FUDGE = 0. and FUDGE = 1. are for this compartment negligible.

In the last compartment (Figure 20) there is still a considerable temperature variation in case of FUDGE = 0., but the moisture content stays almost constant at its initial value. The small decrease in maximum temperature and average moisture content can probably be attributed to the "emptying" of the heat and moisture content integrals, a property inherent in the METHOD MILNE (see 4.1.6.). With FUDGE = 1. the situation is quite different. Now, the sinusoidal temperature variation has almost completely disappeared, whereas the moisture tends to accumulate to some extent.

Other simulation runs were made with temperature waves of period 0.01 day, a constant temperature at the other column end (FUDGE = 1.), and 3 different volumetric moisture contents (5 %, 12 % and 20 %). Temperature and moisture content variations for different compartments are given in Table 9. It is obvious from this table that only in the first compartment at low moisture contents a noticeable water content variation occurs. But, on the average, moisture contents in all compartments stay at their initial value.

These results are in flat contradiction with the experimental results of Hadas (1968), who found no significant changes in moisture distribution,

i.e., a steady-state situation, after the second cycle of the applied heat waves (see Figure 21). He worked with a silty loam soil and columns of 5 cm diameter and 20 cm length. Heat waves applied had a sinusoidal shape with an amplitude of 6°C and periods of 4, 8, 16 and 32 min., and the moisture content distribution was measured after 1, 2, 6 and 16 cycles. It is hard to understand why the water would flow in one direction only, as it did in his experiments, under the influence of a sinusoidal temperature variation, creating alternately positive and negative temperature gradients.

5. Discussion

A few limitations of the model have been indicated incidentally in the text already.

As mentioned earlier, only matric and gravitational components of the water potential are considered. Osmotic potential gradients, however, may have a reducing effect on the moisture transfer. Jackson et al. (1965) found the greatest net water movement in a closed soil column under a temperature gradient in the absence of salt, possibly because the salt acted as a sink for water vapor at the hot end of the column.

Also evaporation of moisture out of the column to the atmosphere is not taken into consideration. It seems likely that under field conditions a more severe drying of the upper layer of the soil will occur and that the combined heat-moisture transfer will be less in the field environment compared with a closed soil column (same conditions) as both the heat flow into the soil (due to the loss of latent heat by evaporation) and the thermal heat diffusivity of the soil decrease (Hadas, 1968).

The heat flow equation for the soil (eq. (48)) does not account for the transfer of sensible heat due to the "mass flow" of moisture (vapor and liquid phase). These and other refining aspects of both the heat and moisture flow equations are discussed in more detail by De Vries (1958), although for the simulation model and practical purposes these refinements will in general be of little importance. For soils under natural conditions, the dominant terms in (48) are often $C_h(\delta T/\delta t)$ and $\text{grad}(\lambda \text{ grad } T)$.

When checking the simulation model with an experiment, it will be necessary to get accurate ψ - θ and K - θ relationships for the soil(s) used in that particular experiment, especially for the lower moisture content range.

Apart from all this, the question arises whether simulating these combined heat-water transport processes makes any sense from an agricultural point of view. What value do these processes of combined heat-water transport really have for agriculture? In the introduction it has been stated already that existing literature gives no clear opinion about this matter. Net moisture transfer due to a thermal gradient will be greatest under circumstances of relatively low moisture contents (see 2.3.) and high temperature gradients, i.e. more arid conditions. Under natural field conditions, however, arid and semi-arid soils will have little agricultural value (Buringh, 1968). For environmental conditions other than arid or

semi-arid, the present authors are strongly inclined to give a negative answer to the above posed question. Asking this question is also important in view of the financial aspects of simulating. With respect to this last point it may be recommendable to make at first a run with a part of the program at a certain (average) temperature for the calculation of λ and ζ at different water contents (see 4.1.4.), after which ζ and λ can be put into the model as parameter and function, respectively, thus saving computation time.

6. Summary

A preliminary simulation program for the combined heat-water flow in porous materials, based on the theory of Philip and De Vries (1957) is described and the influence of a constant temperature gradient or a sinusoidal temperature variation on the moisture movement in a (for water) closed and homogeneous soil column (horizontal) is simulated for different conditions (initial volumetric moisture content, soil type, temperature conditions).

Some limitations of the model are indicated.

Definite conclusions regarding the predicting value of the model cannot be drawn yet.

Appendix I

Definition of symbols

B	transport coefficient or conductivity of the medium for the agent
C_A	= $\delta A / \delta \psi_F$, differential capacity of the medium for agent A
C_h	volumetric heat capacity of the medium ($\text{cal cm}^{-3} \text{ } ^\circ\text{C}^{-1}$)
C_w	differential water capacity of the medium ($\text{cm H}_2\text{O}^{-1}$)
D	= B/C , diffusivity of the medium for the agent ($\text{cm}^2 \text{ sec}^{-1}$)
D_a	molecular diffusion coefficient of water vapor in air ($\text{cm}^2 \text{ sec}^{-1}$)
D_p	diffusion coefficient of water vapor in the porous medium ($\text{cm}^2 \text{ sec}^{-1}$)
D_T	= $D_{Tv} + D_{Tl}$, thermal moisture diffusivity ($\text{cm}^2 \text{ sec}^{-1} \text{ } ^\circ\text{C}^{-1}$)
D_{Tl}	thermal liquid diffusivity ($\text{cm}^2 \text{ sec}^{-1} \text{ } ^\circ\text{C}^{-1}$)
D_{Tv}	thermal vapor diffusivity ($\text{cm}^2 \text{ sec}^{-1} \text{ } ^\circ\text{C}^{-1}$)
D_θ	= $D_{\theta v} + D_{\theta l}$, isothermal moisture diffusivity ($\text{cm}^2 \text{ sec}^{-1}$)
$D_{\theta l}$	isothermal liquid diffusivity ($\text{cm}^2 \text{ sec}^{-1}$)
$D_{\theta v}$	isothermal vapor diffusivity ($\text{cm}^2 \text{ sec}^{-1}$)
$F(x)$	driving force on agent (in x-direction)
G	= $1 + (D_{\theta v} / \alpha \nu D_a) - (\rho / \rho_1)$ (nondimensional)
H	= $(\epsilon_T - \theta_1) h \beta / \rho_1$ ($^\circ\text{C}^{-1}$)
I	= $C_h + L(\epsilon_T - \theta_1) h \beta$ ($\text{cal cm}^{-3} \text{ } ^\circ\text{C}^{-1}$)
J	= $(L \rho_1 D_{\theta v} / \alpha \nu D_a) - L \rho + \{\rho_1 g(\psi - T \gamma \psi) / j\}$ (cal cm^{-3})
K	unsaturated hydraulic or capillary conductivity (cm sec^{-1})
L	latent heat of vaporization of water (cal g^{-1})
P	total gas pressure (mm Hg)
R	universal gas constant ($\text{erg g}^{-1} \text{ } ^\circ\text{C}^{-1}$) or radius of curvature of the liquid surface in the pore (= "effective" pore radius; cm)
R_w	gas constant of water vapor (= $4.615 \times 10^6 \text{ erg g}^{-1} \text{ } ^\circ\text{C}^{-1}$)
T	absolute temperature ($^\circ\text{K}$)
grad T	temperature gradient ($^\circ\text{C cm}^{-1}$ or $^\circ\text{K cm}^{-1}$)
X_i	fractional volume of component i in the medium ($\text{cm}^3 \text{ cm}^{-3}$)
Y	= grad ($\lambda \text{ grad T}$) + $L \rho_1 \text{ grad } (D_{\theta v} \text{ grad } \theta_1) + \rho_1 c_1 \{ (D_{\theta l} \text{ grad } \theta_1) + D_{Tl} \text{ grad T} + Kk \} \text{ grad T}$ ($\text{cal cm}^{-3} \text{ sec}^{-1}$)
Z	= grad ($D_\theta \text{ grad } \theta_1$) + grad ($D_T \text{ grad T}$) + grad K (sec^{-1})

a	= ϵ , volumetric air content of the medium ($\text{cm}^3 \text{cm}^{-3}$)
c_l	specific heat of liquid water ($\text{cal g}^{-1} \text{ } ^\circ\text{C}^{-1}$)
e, e'	partial vapor pressure of water in units of mm Hg and dyne cm^{-2} respectively
e_s	saturation vapor pressure of water (mm Hg)
g	acceleration due to gravity ($= 981 \text{ cm sec}^{-2}$)
grad	gradient (cm^{-1})
h	fractional relative humidity (nondimensional)
j	mechanical equivalent of heat ($= 4.18 \times 10^7 \text{ erg cal}^{-1}$)
k	unit vector in the vertical upward direction
k_i	ratio of the average temperature gradient in component i and the corresponding quantity in the continuous medium, in which component i is dispersed (i.e. air or water, nondimensional)
$q_{A(x)}$	flux density of agent A (in x-direction, units of transported agent $\text{cm}^{-2} \text{sec}^{-1}$)
q_h	heat flux density ($\text{cal cm}^{-2} \text{sec}^{-1}$)
q_l	flux density of water in the liquid phase ($\text{g cm}^{-2} \text{sec}^{-1}$)
q_m	$= q_v + q_l$, total moisture flux density ($\text{g cm}^{-2} \text{sec}^{-1}$)
q_v	flux density of water vapor ($\text{g cm}^{-2} \text{sec}^{-1}$)
t	time (sec)
x	horizontal coordinate (cm)
x_o	fractional volume of organic matter ($\text{cm}^3 \text{cm}^{-3}$)
x_m	fractional volume of soil minerals other than quartz ($\text{cm}^3 \text{cm}^{-3}$)
x_q	fractional volume of quartz particles ($\text{cm}^3 \text{cm}^{-3}$)
z	vertical coordinate, positive upwards (cm)

α	tortuosity factor allowing for the extra path length (nondimensional)
β	$= \delta\rho_o / \delta T$ ($\text{g cm}^{-3} \text{ } ^\circ\text{C}^{-1}$)
γ	$= (1/\sigma) (\delta\sigma/\delta T)$ ($^\circ\text{C}^{-1}$)
ϵ	$= a$, volumetric air content of the medium ($\text{cm}^3 \text{ cm}^{-3}$)
$f(z)$	$= D_p/D_a$, ratio of diffusion coefficients (nondimensional)
ϵ_T	total porosity of the medium ($\text{cm}^3 \text{ cm}^{-3}$)
ζ	$= (\text{grad } T)_a / \text{grad } T$, ratio of average temperature gradient in air-filled pores to the overall temperature gradient (nondimensional)
θ	$= \theta_v + \theta_l$, total volumetric moisture content of the medium ($\text{cm}^3 \text{ cm}^{-3}$)
θ_K	value of θ_l at which "liquid continuity" fails ($\text{cm}^3 \text{ cm}^{-3}$)
θ_l	volumetric liquid content of the medium ($\text{cm}^3 \text{ cm}^{-3}$)
θ_v	volumetric vapor content of the medium ($\text{cm}^3 \text{ cm}^{-3}$)
λ	overall thermal conductivity of the medium ($\text{cal cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{C}^{-1}$)
λ_a	thermal conductivity of dry air ($\text{cal cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{C}^{-1}$)
λ_{app}	$= \lambda_a + \lambda_v$, apparent thermal conductivity of air containing water vapor ($\text{cal cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{C}^{-1}$)
λ_i	thermal conductivity of component i in the medium ($\text{cal cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{C}^{-1}$)
λ_o	thermal conductivity of the continuous medium ($\text{cal cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{C}^{-1}$)
λ_v	apparent increase of the thermal conductivity of air due to vapor diffusion ($\text{cal cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{C}^{-1}$)
v	$= P/(P-e)$, a mass flow factor to allow for the mass flow of water vapor arising from a difference in boundary conditions for the air and water vapor components in the medium (mm Hg^{-1})
ξ	factor ≥ 1 arising from vapor flux enhancement mechanisms (nondimensional)
ρ	density of water vapor (g cm^{-3})
ρ_l	density of (liquid) water (g cm^{-3})
ρ_o	density of saturated water vapor (g cm^{-3})
σ	surface tension of water (dyne cm^{-1})
ψ, ψ'	matric potential of the moisture in the medium, in units of cm H_2O and erg g^{-1} respectively (negative in unsaturated media)
ψ_F	partial potential of the agent related to driving force F

* A, AIR CONTENT, CM**1/CM**3
 * AH, AIR CONTENT CORRESPONDING WITH WCH
 * ALFA, TORTUOSITY FACTOR
 * (I)AMW, (INITIAL) AMOUNT OF WATER, CM**3/CM**2=CM
 * APCA, APPARENT THERMAL CONDUCTIVITY OF AIR,
 * CAL/CM, DAY, CENTIGRADE
 * ATMP, ABSOLUTE TEMPERATURE, DEGREES KELVIN
 * B, = D(VAPDS)/D(ATEMP)
 * CAP, CAPACITY, 1/CM
 * CONQ, THERMAL CONDUCTIVITY OF QUARTZ,
 * CAL/CM, DAY, CENTIGRADE
 * CONH, IDEM FOR ORGANIC MATTER
 * CONM, IDEM FOR MINERALS OTHER THAN QUARTZ
 * CONW, IDEM FOR WATER
 * CONA, IDEM FOR DRY AIR
 * CVAP, THERMAL CONDUCTIVITY OF AIR DUE TO VAPOR MOVEMENT,
 * CAL/CM, DAY, CENTIGRADE
 * CWC, CRITICAL WATERCONTENT (FOR WATERCONTENT VALUES SMALLER
 * THAN CWC AIR INSTEAD OF WATER IS CONSIDERED AS THE
 * CONTINUOUS MEDIUM (CALCULATION OF TCN AND ZETA)),
 * CM**3/CM**3
 * DATM, MOLECULAR DIFFUSIVITY OF WATER VAPOR IN AIR, CM**2/DAY
 * DPTH, DISTANCE BETWEEN SOIL SURFACE AND CENTER COMPARTMENT, CM
 * DIFT, THERMAL DIFFUSIVITY OF WATER (TOTAL), CM**2/DAY, CENTIGRADE
 * DIFW, ISOTHERMAL DIFFUSIVITY OF WATER (TOTAL), CM**2/DAY
 * DIST, DISTANCE BETWEEN CENTERS OF COMPARTMENTS, CM
 * DTL, THERMAL LIQUID DIFFUSIVITY, CM**2/DAY, CENTIGRADE
 * DTV, THERMAL VAPOR DIFFUSIVITY, CM**2/DAY, CENTIGRADE
 * DHL, ISOTHERMAL LIQUID DIFFUSIVITY, CM**2/DAY
 * DAV, ISOTHERMAL VAPOR DIFFUSIVITY, CM**2/DAY
 * ETA, VISCOSITY OF WATER, CENTIPOISES
 * FA, FACTOR DEPENDING ON WATERCONTENT
 * FRQ, FRACTION OF THE SOLID CONTENT WHICH CONSISTS OF QUARTZ
 * FRM, IDEM FOR MINERALS OTHER THAN QUARTZ
 * G, ACCELERATION DUE TO GRAVITY, CM/SEC**2
 * GAA, GAC AND GAAH, ANISOTROPY FACTORS NEEDED TO CALCULATE KAW
 * GA AND GC, ANISOTROPY FACTORS NEEDED TO CALCULATE
 * KQW, KMW, KHW, KQA, KMA, KHA, KCVA, KMVA, KHVA AND KWVA
 * GAM, MULTIPLIER NEEDED TO CALCULATE DTL, 1/CENTIGRADE
 * H, FRACTIONAL RELATIVE HUMIDITY
 * HFLT, THERMAL HEAT FLOW, CAL/DAY, CM**2
 * HFLW, HEAT FLUX BY DISTILLATION, CAL/DAY, CM**2
 * KQW, RATIO OF THE AVERAGE TEMPERATURE GRADIENT IN QUARTZ AND
 * THE CORRESPONDING QUANTITY IN THE MEDIUM (WATER)
 * KMW, IDEM FOR MINERALS OTHER THAN QUARTZ
 * KHW, IDEM FOR ORGANIC MATTER
 * KAW, IDEM FOR AIR
 * KQA, RATIO OF THE AVERAGE TEMPERATURE GRADIENT IN QUARTZ AND
 * THE CORRESPONDING QUANTITY IN THE MEDIUM (DRY AIR)
 * KMA, IDEM FOR MINERALS OTHER THAN QUARTZ
 * KHA, IDEM FOR ORGANIC MATTER

Appendix II

Symbolic names simulation model

* KWVA, RATIO OF THE AVERAGE TEMPERATURE GRADIENT IN WATER AND
 * THE CORRESPONDING QUANTITY IN THE MEDIUM (AIR(+WATER
 * VAPOR))
 *
 * KQVA, IDEM FOR QUARTZ
 * KMVA, IDEM FOR MINERALS OTHER THAN QUARTZ
 * KHVA, IDEM FOR ORGANIC MATTER
 * L, LATENT HEAT OF VAPORIZATION OF WATER, CAL/GRAM
 * MC, CONTENT OF MINERALS OTHER THAN QUARTZ, CM**3/CM**3
 * N, VISCOSITY OF WATER AT 20 DEGREES CELCIUS, CENTIPOISES
 * OMC, ORGANIC MATTER CONTENT, CM**3/CM**3
 * P, SUCTION, CM
 * POR, POROSITY, CM**3/CM**3
 * PRES, TOTAL GAS PRESSURE, MM HG
 * PU AND PL, SUCTION VALUES USED IN THE CALCULATION OF THE SLOPE
 * OF THE SUCTION-WATERCONTENT CURVE
 * R, GAS CONSTANT OF WATER VAPOR, ERG/GRAM, KELVIN
 * RPER, RECIPROCAL VALUE OF THE PERIOD OF THE SINUSOIDAL
 * TEMPERATURE VARIATION, 1/DAY
 * QC, QUARTZ CONTENT, CM**3/CM**3
 * SOLC, SOLID CONTENT, CM**3/CM**3
 * SP, SLOPE OF THE SUCTION-WATERCONTENT CURVE, CM**4/CM**3=CM
 * TAMP, AMPLITUDE OF THE TEMPERATURE WAVE, CENTIGRADES
 * TAV, AVERAGE TEMPERATURE OF THE TEMPERATURE WAVE, CENTIGRADES
 * TCOM, THICKNESS COMPARTMENT, CM
 * TCN, THERMAL CONDUCTIVITY, CAL/CM, DAY, CENTIGRADE
 * TCND, THERMAL CONDUCTIVITY OF DRY SOIL (WATERCONTENT
 * EQUALS ZERO), CAL/CM, DAY, CENTIGRADE
 * TCNL, LOWER LIMIT THERMAL CONDUCTIVITY RANGE CALCULATED
 * WITH WATER AS THE CONTINUOUS MEDIUM
 * (I)TEMP, (INITIAL) TEMPERATURE, CENTIGRADES
 * TNHF, TOTAL NET HEAT FLOW, CAL/DAY, CM**2
 * TNWF, TOTAL NET WATER FLOW, CM/DAY
 * V, MASS-FLCW FACTOR
 * VAPD, DENSITY OF WATER VAPOR, GRAM/CM**3
 * VPDS, DENSITY OF SATURATED WATER VAPOR
 * (I)VHCP, (INITIAL) VOLUMETRIC HEAT CAPACITY,
 * CAL/CM**3, CENTIGRADE
 * (I)VHTC, (INITIAL) VOLUMETRIC HEAT CONTENT,
 * CAL/CM**2
 * (I)WC, (INITIAL) WATERCONTENT, CM**3/CM**3
 * WCH, WATERCONTENT WHERE THE VALUE FOR THE RELATIVE HUMIDITY
 * IS ABOUT 0.990 (P ABOUT 15000 MBAR)
 * WCN, HYDRAULIC CONDUCTIVITY, CM/DAY
 * WCON, HYDRAULIC CONDUCTIVITY AT 20 DEGREES CELCIUS, CM/DAY
 * WCU AND WCL, WATERCONTENT VALUES USED IN THE CALCULATION OF THE
 * SLOPE OF THE SUCTION-WATERCONTENT CURVE
 * WDEN, DENSITY OF (LIQUID) WATER, GRAM/CM**3
 * WFLT, THERMAL WATER FLOW, CM/DAY
 * WFLW, ISOTHERMAL WATER FLOW, CM/DAY
 * ZETA, RATIO OF TEMPERATURE GRADIENT IN AIR-FILLED PCRES
 * TO OVERALL TEMPERATURE GRADIENT

Appendix II (continued)

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initial volumetric moisture (%) content	time (days)	absolute change [*]) in initial volumetric moisture content (%) for compartment												
		1	2	3	4	5	10	15	20	21	22	23	24	25
		(-)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)			(+)
5.00	0.025	.19	.04	.03	.03	.02	.01	x	x	x	x	x	x	x
	0.05	.29	.04	.03	.03	.03	.01	.01	x	x	x	x	x	x
	0.075	.36	.05	.03	.03	.03	.02	.01	x	x	x	x	x	.01
	0.10	.43	.06	.04	.03	.03	.02	.01	.01	x	x	x	x	.03
	0.20	.65	.09	.04	.04	.03	.02	.02	.01	.01	.01	x	x	.15
	0.30	.86	.13	.03	.04	.03	.03	.02	.01	.01	.01	x	x	.30
	0.40	1.06	.18	.03	.04	.03	.03	.02	.01	.01	.01	x	x	.45
0.50	1.28	.24	.02	.04	.03	.03	.02	.01	.01	.01	x	x	.61	
		(-)	(+)	(+)	(+)	(+)	(+)	(+)						(+)
10.00	0.025	.15	.03	.02	.02	.02	.01	x	x	x	x	x	x	x
	0.05	.23	.03	.03	.02	.02	.01	x	x	x	x	x	x	x
	0.075	.28	.03	.03	.02	.02	.01	.01	x	x	x	x	x	.01
	0.10	.33	.03	.03	.03	.02	.02	.01	x	x	x	x	x	.03
	0.20	.49	.03	.03	.03	.02	.02	.01	x	x	x	x	x	.14
	0.30	.63	.03	.03	.03	.02	.02	.01	x	x	x	x	x	.26
	0.40	.77	.03	.03	.03	.02	.02	.01	x	x	x	x	x	.38
0.50	.90	.03	.03	.03	.02	.02	.01	x	x	x	x	x	.51	

*) (-) = decrease
 (+) = increase
 x = no change

Table 2 Spatial and temporal variation of the volumetric moisture content after imposing a temperature gradient (25°C-15°C) for different initial water contents (homogeneous soil column having a length of 25 cm). Suction and hydraulic conductivity curves used are those of an unplowed light humous sandy soil (van Keulen and van Beek, 1971).

initial volumetric moisture (%) content	time (days)	absolute change* in initial volumetric moisture content (%) for compartment												
		1	2	3	4	5	10	15	20	21	22	23	24	25
20.00	0.025	(-)	(-)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)
		.10	.02	.01	.02	.02	.01	x	x	x	x	x	x	x
				(-)										
	0.05	.12	.05	.01	.01	.02	.01	x	x	x	x	x	x	x
	0.075	.13	.07	.02	x	.02	.02	.01	x	x	x	x	x	.01
					(-)									
	0.10	.13	.08	.04	.01	.01	.02	.01	x	x	x	x	.01	.02
					(-)									
	0.20	.14	.10	.07	.04	.02	.02	.01	.01	.01	.01	.02	.04	.07
	0.30	.15	.12	.09	.06	.04	.01	.01	.01	.02	.03	.04	.07	.11
0.40	.17	.14	.10	.08	.05	.01	.01	.02	.03	.05	.07	.10	.14	
0.50	.18	.15	.12	.09	.07	x	.01	.03	.05	.06	.09	.12	.17	
35.00	0.025	(-)	(-)	(-)	(-)	(-)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)
		.05	.04	.03	.02	.01	.02	.01	x	x	x	x	x	x
	0.05	.06	.05	.04	.03	.02	.01	.02	.01	.01	.01	.01	.01	.01
	0.075	.06	.05	.04	.04	.03	x	.01	.02	.02	.02	.02	.02	.02
							(-)							
	0.10	.06	.05	.04	.04	.03	.01	.01	.02	.03	.03	.03	.03	.03
	0.20	.06	.05	.05	.04	.04	.01	.01	.03	.03	.04	.04	.05	.05
	0.30	.06	.05	.05	.04	.04	.02	.01	.03	.03	.04	.04	.05	.05
	0.40	.06	.05	.05	.04	.04	.02	.01	.03	.03	.04	.04	.05	.05
	0.50	.06	.05	.05	.04	.04	.02	.01	.03	.03	.04	.04	.05	.05

*) (-) = decrease
 (+) = increase
 x = no change

Table 2 (continued)

	initial volumetric moisture content (%)	time (days)	absolute change*) in initial volumetric moisture content (%) for compartment												
			1	2	3	4	5	10	15	20	21	22	23	24	25
(a)	5.00	0.025	(-)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)	(+)
		0.05	.54	.12	.09	.07	.06	.02	x	x	x	x	x	x	x
		0.075	.83	.15	.10	.09	.08	.04	.01	x	x	x	x	x	.01
		0.10	1.05	.17	.11	.10	.09	.05	.02	.01	.01	.01	.01	x	.04
(b)	5.00	0.025	(-)	(-)	(+)	(+)	(+)	(+)	(+)	(+)				(+)	(+)
		0.05	.10	.01	.02	.02	.02	.01	x	x	x	x	x	x	x
		0.075	.12	.04	x	.02	.02	.01	x	x	x	x	x	x	x
		0.10			(-)	.01	.01	.02	.01	.01	x	x	x	x	.01
(b)	20.00	0.025	(-)	(+)	(+)	(+)	(+)	(+)							
		0.05	.12	.01	.02	.02	.02	.01	x	x	x	x	x	x	x
			(-)	.02	.02	.02	.01	x	x	x	x	x	x	x	

*) (-) = decrease
 (+) = increase
 x = no change

Table 3 Spatial and temporal variation of the volumetric moisture content after imposing a thermal gradient.

(a) Temperature gradient: 35°C-5°C

Suction and hydraulic conductivity curves are the same as for Table 2.

(b) Temperature gradient: 25°C-15°C

Suction and hydraulic conductivity curves refer to a plowed humous sandy soil (van Keulen and van Beek, 1971).

initial volumetric moisture content (%)	soil type	thermal vapor diffusivity DTV (cm ² day ⁻¹ °C ⁻¹)	thermal liquid diffusivity DTL (cm ² day ⁻¹ °C ⁻¹)	isothermal vapor diffusivity DWV (cm ² day ⁻¹)	isothermal liquid diffusivity DWL (cm ² day ⁻¹)
5	unplowed light humous sandy soil	2.4 x 10 ⁻²	1.1 x 10 ⁻⁸	2.2 x 10 ⁻²	3.2 x 10 ⁻⁵
	plowed light humous sandy soil	2.0 x 10 ⁻²	3.7 x 10 ⁻⁴	9.5 x 10 ⁻¹	13.5
10	unplowed light humous sandy soil	2.2 x 10 ⁻²	0.9 x 10 ⁻⁸	2.1 x 10 ⁻²	3.6 x 10 ⁻⁵
	"	2.1 x 10 ⁻²	1.7 x 10 ⁻³	8.5 x 10 ⁻³	25.0
20	plowed light humous sandy soil	1.9 x 10 ⁻²	1.1 x 10 ⁻³	1.6 x 10 ⁻⁴	6.4
35	unplowed light humous sandy soil	1.2 x 10 ⁻²	0.1	7.5 x 10 ⁻⁵	1000

Table 4 Rough values (at 25°C) for the thermal and isothermal moisture diffusivities at different volumetric moisture contents.

Values of ζ for $a + \theta =$

θ	0.3		0.46		0.5		0.7					
0.0	3.03*)	3.0**)	3.2***)	2.07*)	1.9**)	2.0***)	1.40*)	1.4**)	1.4***)			
0.01	2.96	3.04		2.05	2.08		1.40	1.41				
0.02	2.75	2.98		1.97	2.07		1.39	1.41				
0.03	2.76	2.35		1.97	1.83		1.39	1.35				
0.04	2.76	2.31		1.99	1.82		1.40	1.36				
0.05	2.81	2.28		2.01	1.81		1.41	1.36				
0.06	2.29	2.25		1.82	1.80		1.37	1.36				
0.07	2.27	2.24		1.82	1.80		1.37	1.36				
0.08	2.26	2.23		1.82	1.80		1.38	1.37				
0.09	2.25	2.23		1.83	1.81		1.38	1.38				
0.10	2.24	2.22	2.0	2.7	1.83	1.81	1.7	1.9	1.39	1.38	1.3	1.4
0.15	2.21				1.83				1.41			
0.20	2.22				1.84				1.43			
0.25	2.25				1.86				1.45			
0.30	2.30	2.1	2.9		1.88		1.7	2.0	1.47		1.5	1.6
0.35					1.91				1.49			
0.40					1.94				1.51			
0.45					1.99				1.53			
0.50							1.8	2.2	1.55		1.5	1.6
0.55									1.58			
0.60									1.60			
0.65									1.64			
0.70									1.67		1.6	1.8

*) values computed in simulation program for the porous system described (solid content: 40% quartz, 60% other minerals); in the dry range ($\theta = 0.0-0.10$) the value of ζ will vary depending on suction (relative humidity!) and the θ value for CWC, while the variation will be greater the lower the porosity (first and second column: CWC 0.06 and 0.03, respectively, and different suctions)

**) values computed by Philip and de Vries (1957) (solid content: 100% minerals other than quartz)

***) values computed by Philip and de Vries (1957) (solid content: 100% quartz)

Table 5 Values of ζ at 20°C for different porosities and moisture contents.

λ (cal cm^{-1} day $^{\circ}\text{C}^{-1}$)	ξ	time interval (days)	absolute change*) in % volumetric moisture for compartment						
			1	2	3	4	5	20	25
47.5	1.80		(-)	(+)	(+)	(+)	(+)	(+)	(+)
		0.01	.08	.05	.02	.01	x	x	x
		0.1	.51	.09	.08	.07	.06	x	x
		0.5	1.25	.06	.10	.09	.09	.02	0.16
			(-)	(+)	(+)	(+)	(+)	(+)	(+)
		0.01	.07	.02	.02	.01	.01	x	x
173	2.07	0.1	.34	.03	.03	.03	.03	x	.02
		0.5	.88	.02	.03	.03	.03	.01	.46

*) (-) = decrease
 (+) = increase
 x = no change

Table 6 Moisture distribution corresponding with Figure 11.

Sum totals (expressed as absolute volumetric % of the compartment of 0.8 cm) of decreases (-) and increases (+) of volumetric moisture content throughout the column due to a thermal gradient (25°C-15°C) after 0.5 day of simulation for initial volumetric moisture content (%)=

Integration method	5.00			10.00			20.00			35.00		
MILNE	-1.25	+1.24	$\frac{\Delta}{-0.01}$	-1.15	+0.97	$\frac{\Delta}{-0.18}$	-0.74	+0.41	$\frac{\Delta}{-0.33}$	-	-	$\frac{\Delta}{-}$
RKS	-1.25	+1.25	-	-0.77	+0.70	-0.07	-0.60	+0.52	-0.08	-0.40	+0.31	-0.09
	-1.28	+1.27	-0.01	-0.90	+0.83	-0.07	-0.72	+0.64	-0.08			

Table 7 Comparison of METHOD MILNE with METHOD RKS

Initial volumetric moisture content (%)	DTV (cm ² day ⁻¹ °C ⁻¹)	DTL (cm ² day ⁻¹ °C ⁻¹)	DWV (cm ² day ⁻¹)	DWL (cm ² day ⁻¹)
4.5	2.7 x 10 ⁻²	9.5 x 10 ⁻⁴	3.6 x 10 ⁻³	12.3
9.0	2.6 x 10 ⁻²	1.5 x 10 ⁻³	6.6 x 10 ⁻⁴	13.0
16.0	2.5 x 10 ⁻²	3.3 x 10 ⁻³	1.1 x 10 ⁻⁴	10.0

Table 8 Rough values (at 23°C) for the thermal and isothermal moisture diffusivities at different volumetric moisture contents.

compartment	initial volumetric moisture content: 5% simulated period: 0.1 day		initial volumetric moisture content: 12% simulated period: 0.1 day		initial volumetric moisture content: 20% simulated period: 0.5 day	
	temperature (°C)					
	minimum	maximum	minimum	maximum	minimum	maximum
1	5.70	24.36	7.40	22.91	7.57	22.73
2	6.99	23.23	10.58	20.06	10.85	19.78
5	10.28	20.34	14.15	16.47	14.27	16.27
10	14.22	16.43	14.99	15.25	14.96	15.19
15	15.00	15.24	15.00	15.09	15.00	15.07
20	14.99	15.07	15.00	15.04	15.00	15.03
25	15.00	15.01	15.00	15.00	14.99	15.01
	volumetric moisture content (%)					
	minimum	maximum	minimum	maximum	minimum	maximum
	1	4.75	5.17	11.95	12.01	19.96
2	4.93	5.04	11.99	12.02	19.97	20.01
5	4.97	5.03	11.99	12.00	19.98	20.00
10	4.99	5.01	12.00	12.00	19.98	20.00
15	5.00	5.00	12.00	12.00	19.99	20.00
20	5.00	5.00	12.00	12.00	19.99	20.00
25	4.99	5.00	12.00	12.00	19.99	20.00

Table 9 Temperature and moisture content variation generated in soil column by a sinusoidal variation at the left side for different initial volumetric moisture contents. Initial temperature: 15°C; average temperature sinusoidal wave: 15°C; amplitude of temperature wave: 10°C; period of temperature wave: 0.01 day; right side of column kept at the initial temperature (FUDGE = 1.).

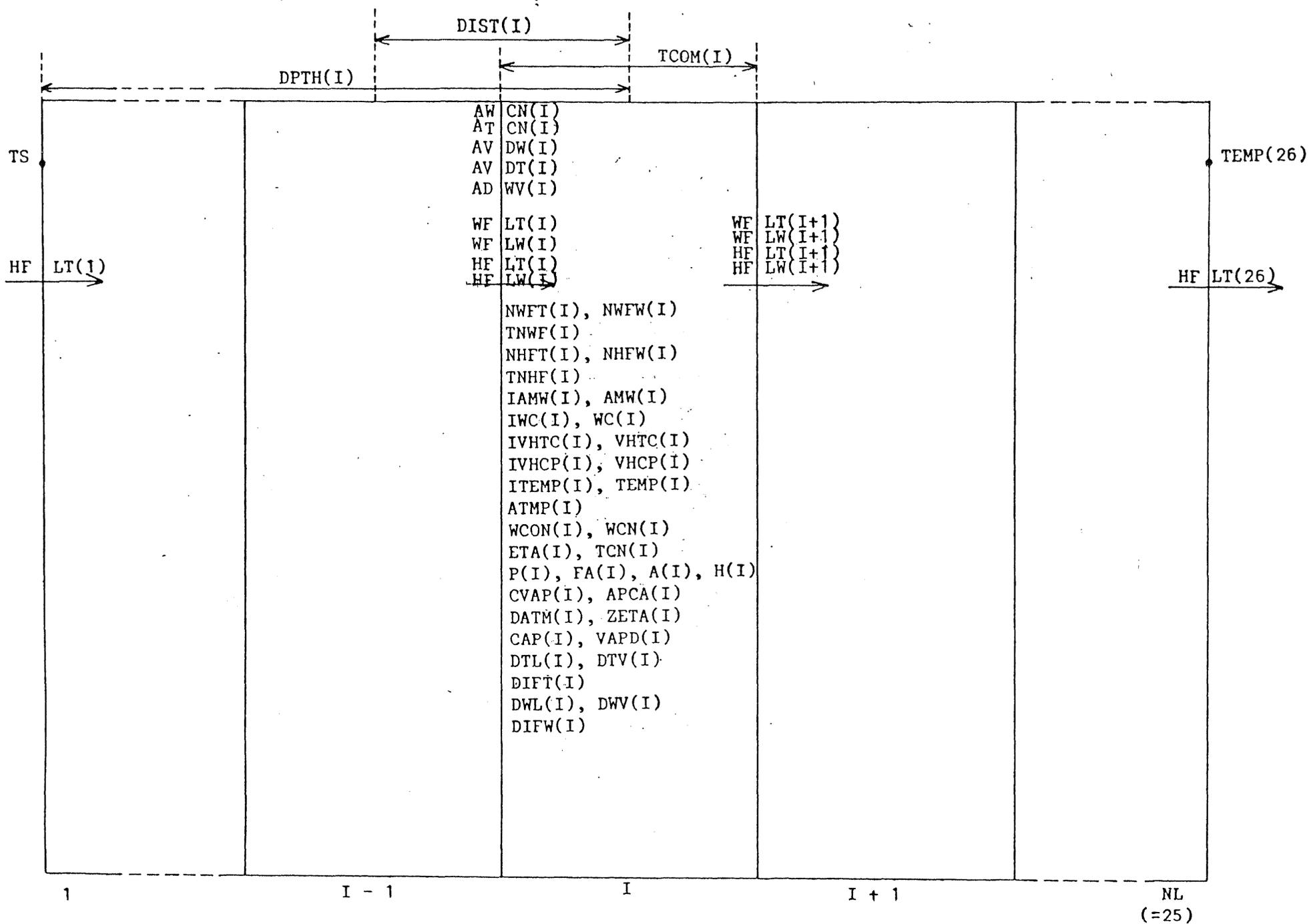
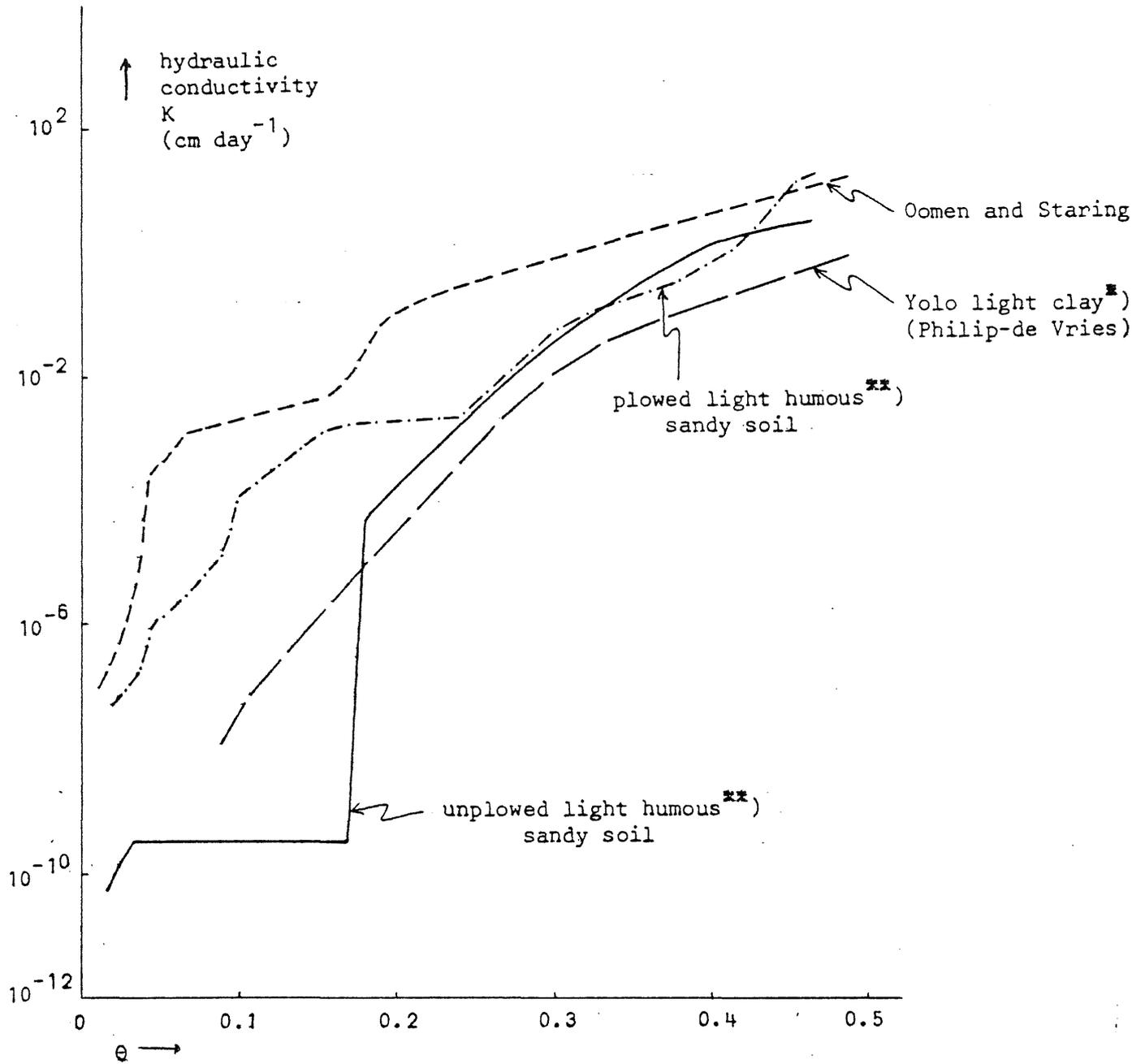


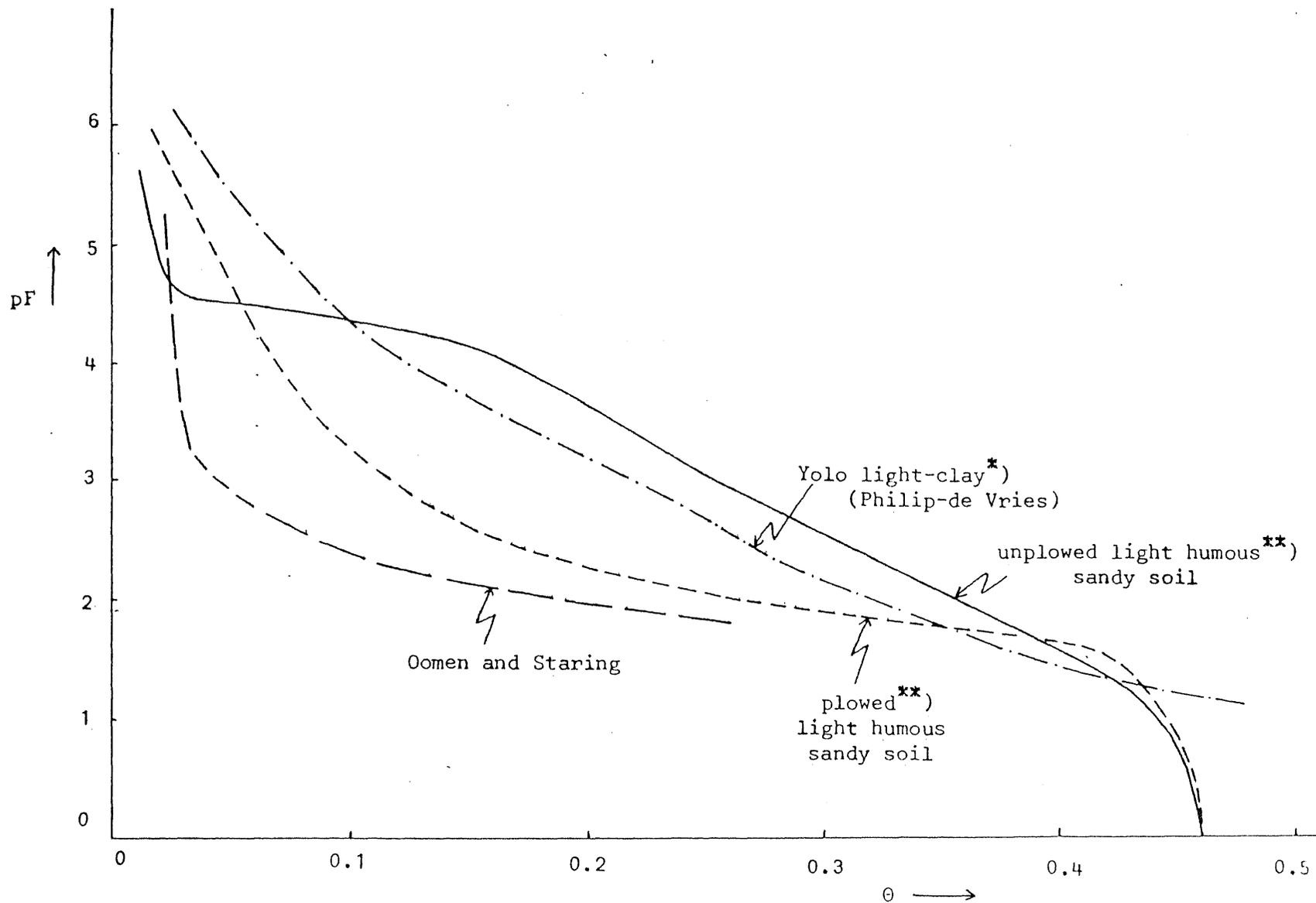
Fig. 1 Schematic representation of the geometry of the system (homogeneous porous column) and main symbols used in the simulation program for the combined flow of water and heat.



^{*}) data from Philip (in Slatyer, 1967)

^{**}) data from Van Keulen and Van Beek (1971)

Fig. 2 Relationship between hydraulic conductivity, K, and volumetric moisture content, θ , for the soils discussed in this report.



*) data from Philip (in Slatyer, 1967)

**) data from van Keulen and van Beek (1971)

Fig. 3 Relationship between the negative logarithm of the matric potential, pF, and volumetric moisture content, θ , for the soils discussed in this report.

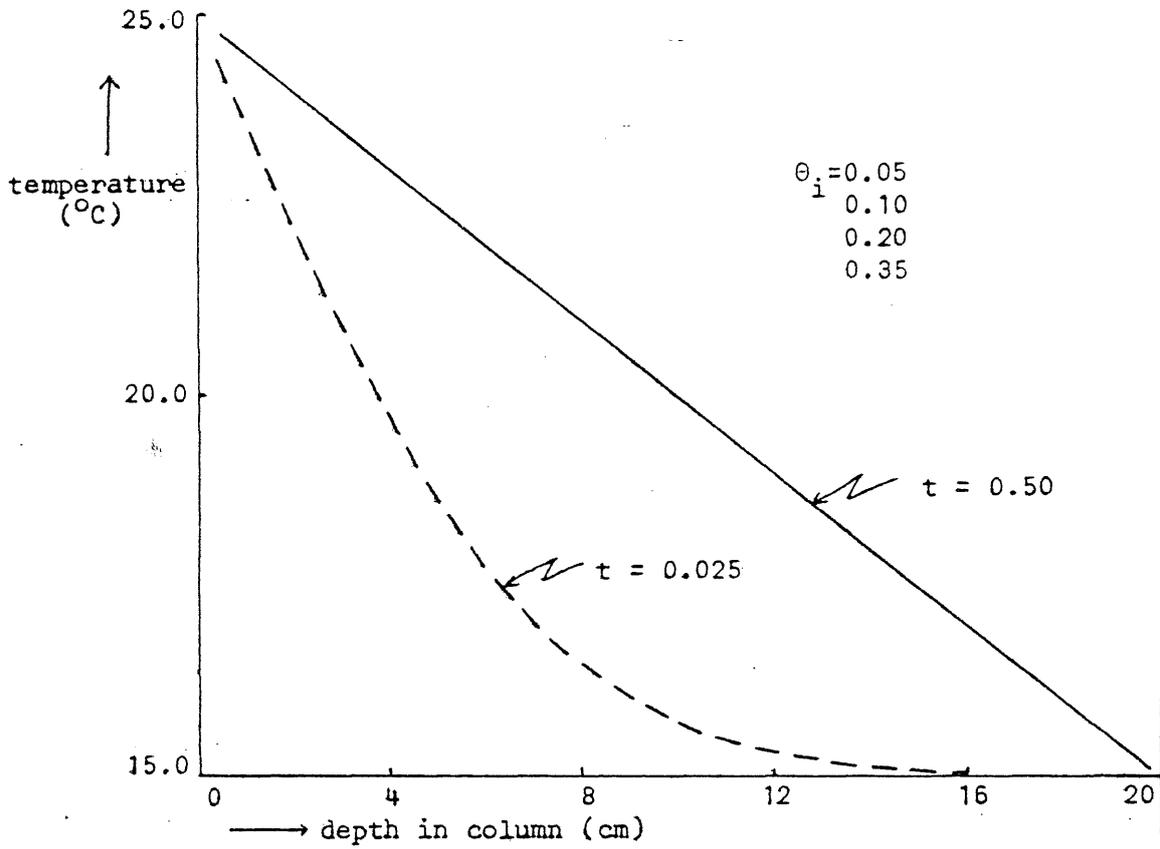


Fig. 4 Temperature distribution in soil column, 0.025 and 0.50 day after a sudden rise from 15°C to 25°C at the left side; the temperature at the right side is kept constant at the initial value.

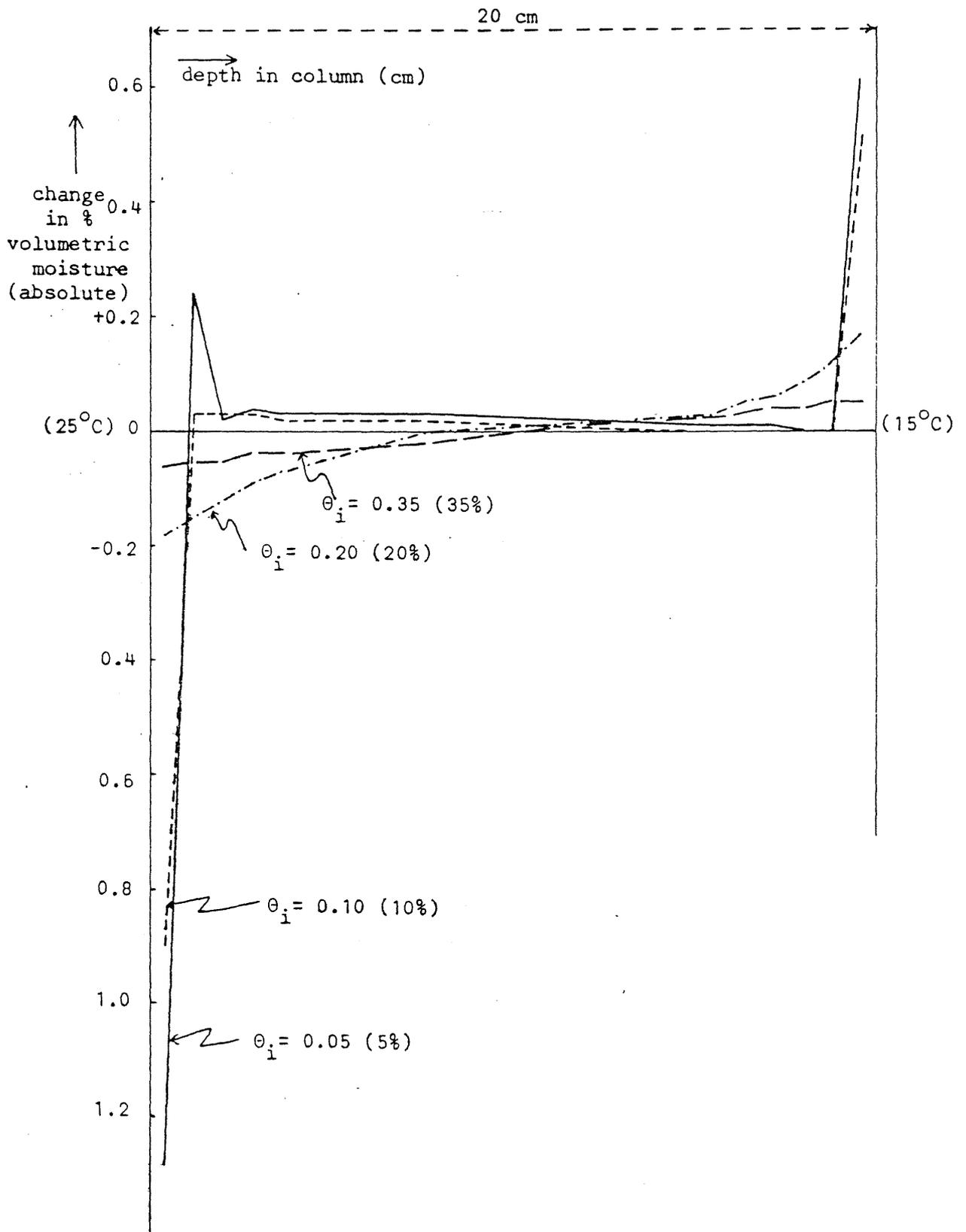


Fig. 5 Moisture distribution in soil column, 0.50 day after a sudden rise from 15°C to 25°C at the left side, for different initial moisture contents. Curve indicates the absolute change of initial volumetric moisture content percentage throughout the column. The temperature at the right side is kept constant at the initial value.

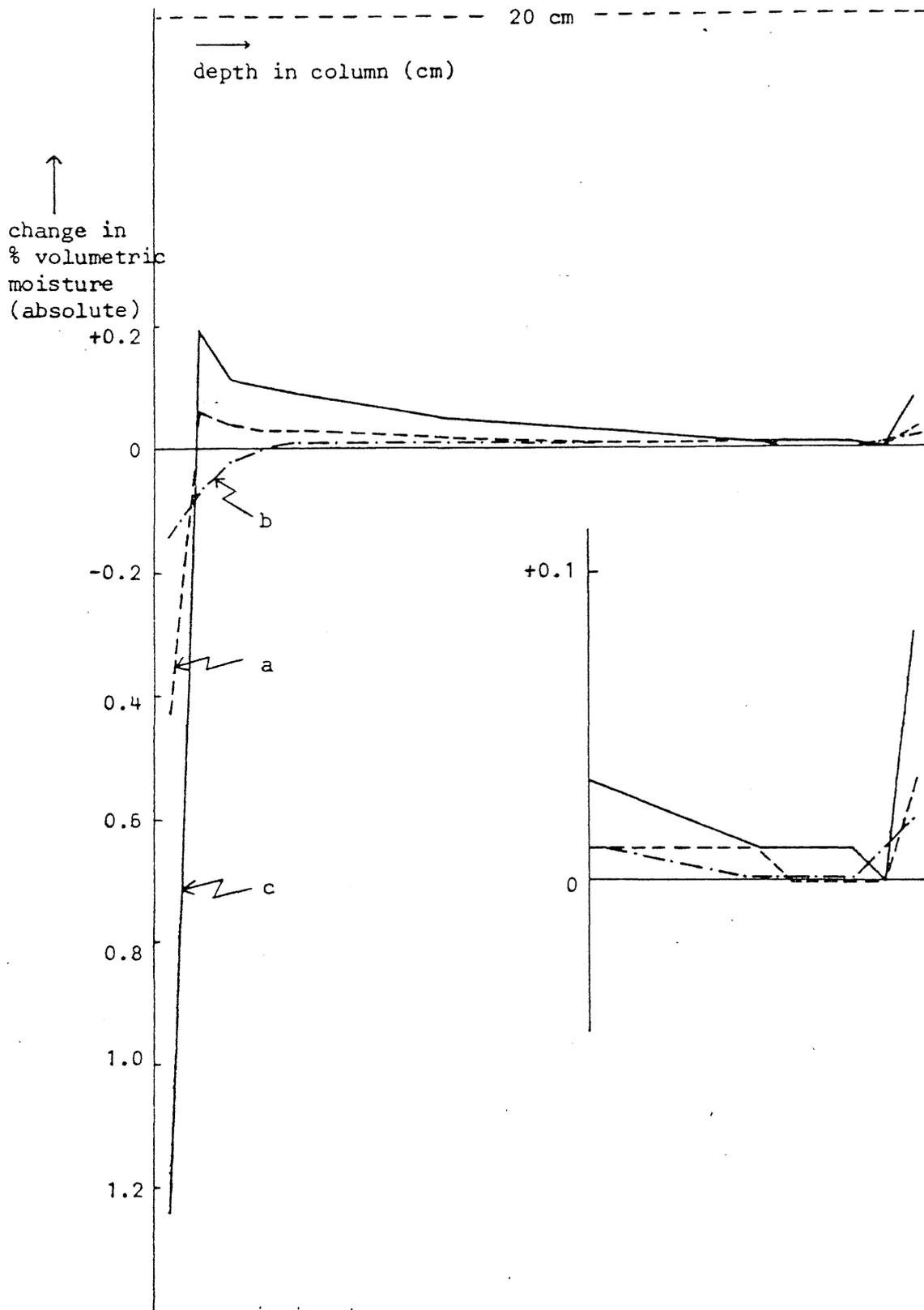


Fig. 6 Moisture distribution in soil column, 0.10 day after a sudden rise from 15°C to 25°C (curve a and b) or from 5°C to 35°C (curve c) at the left side. Curve indicates the absolute change of initial volumetric moisture content percentage throughout the column. The temperature at the right side is kept constant at the initial value.
 $\theta_i = 0.05$ (5%)
a and c: unplowed humous sandy soil
b: plowed humous sandy soil

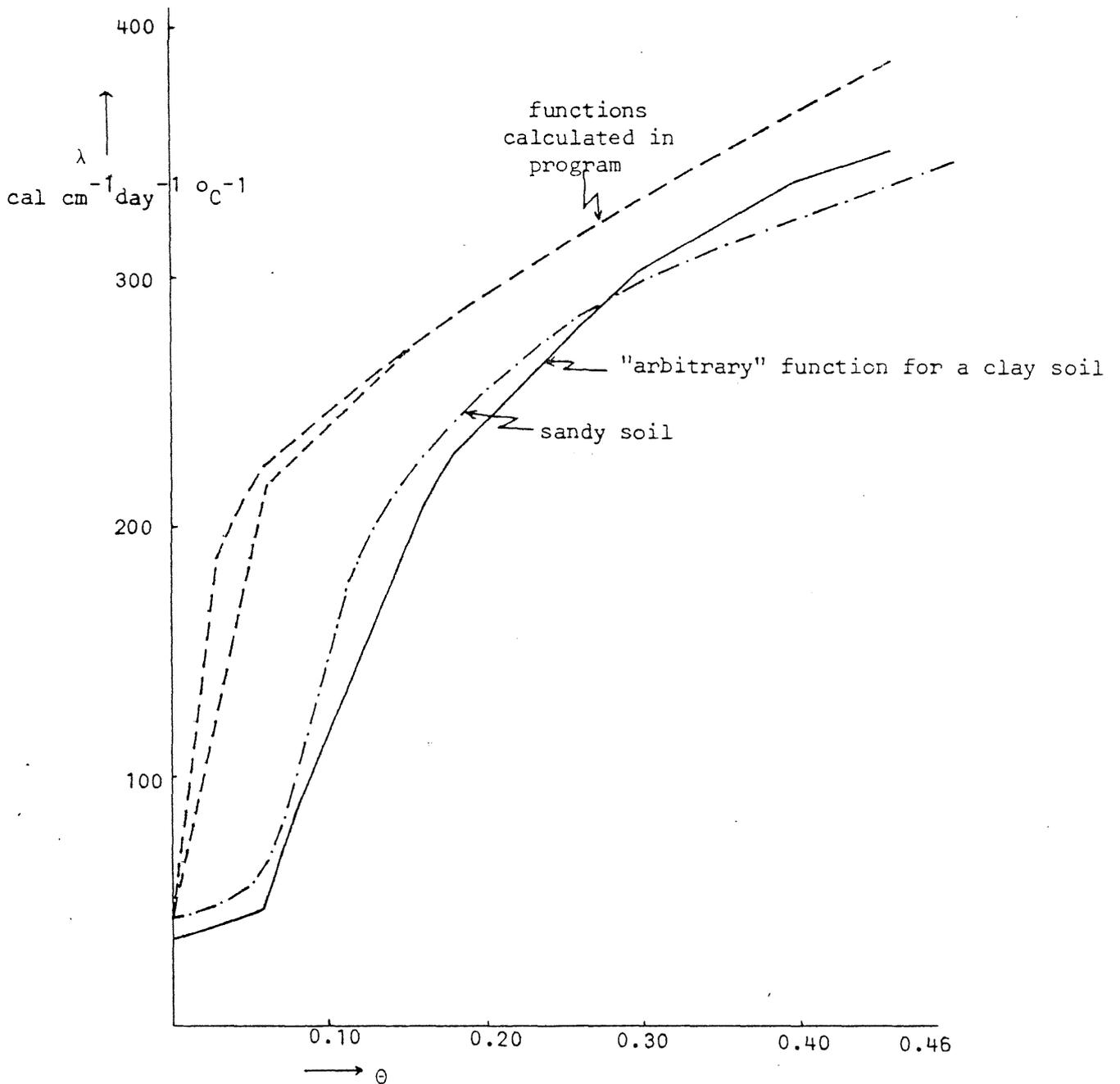


Fig. 7 Thermal conductivity (λ) as a function of the volumetric moisture content. Arbitrary function derived from data of de Vries (1963) for a clay soil; function of the sandy soil (3.5% organic matter, porosity 50%) is taken from Bolt et al. (1965).

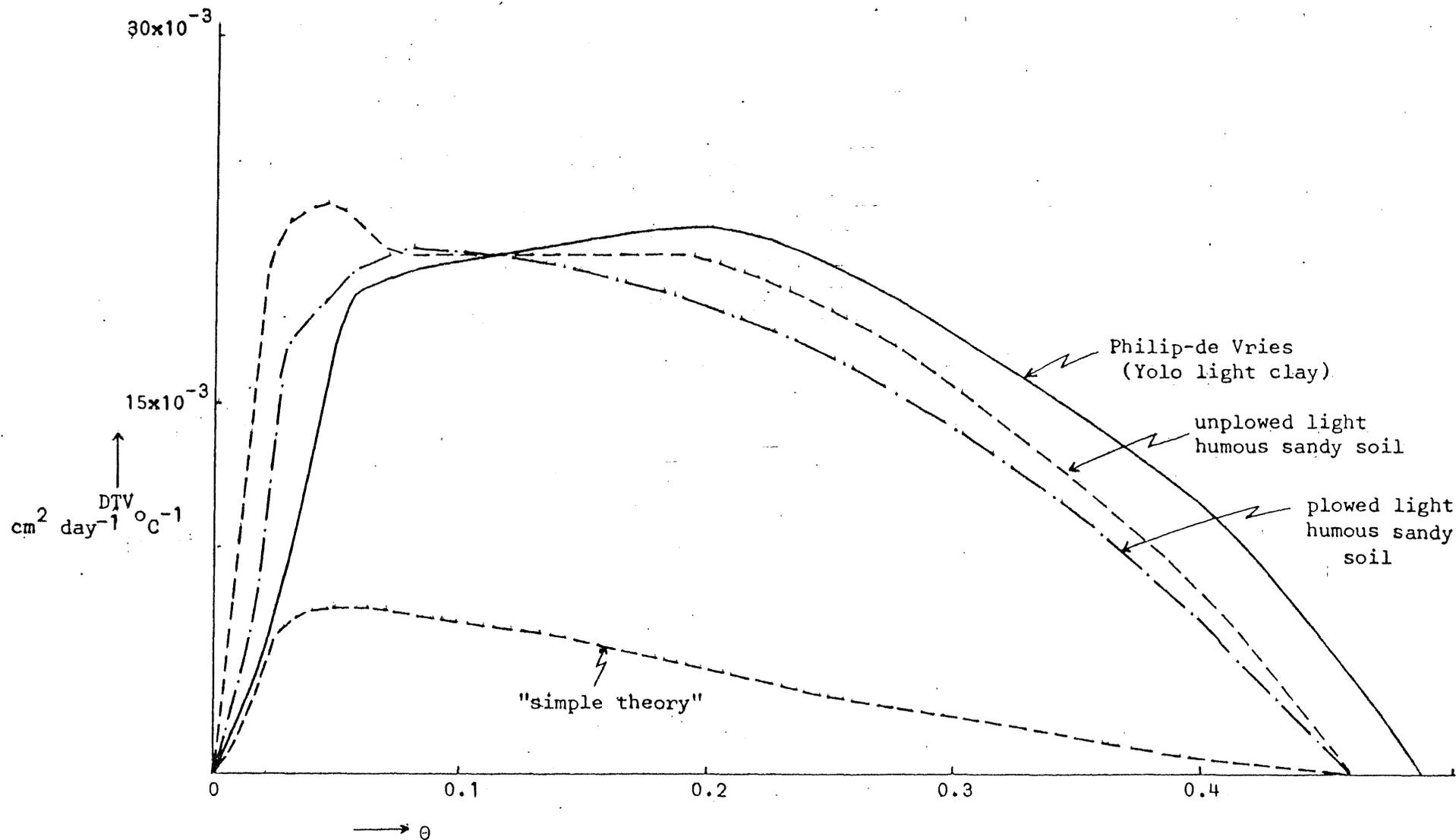


Fig. 8 Thermal vapor diffusivity, DTV, both according to the "simple theory" and according to the Philip-de Vries theory, as function of volumetric moisture content at 20° C for different soils.

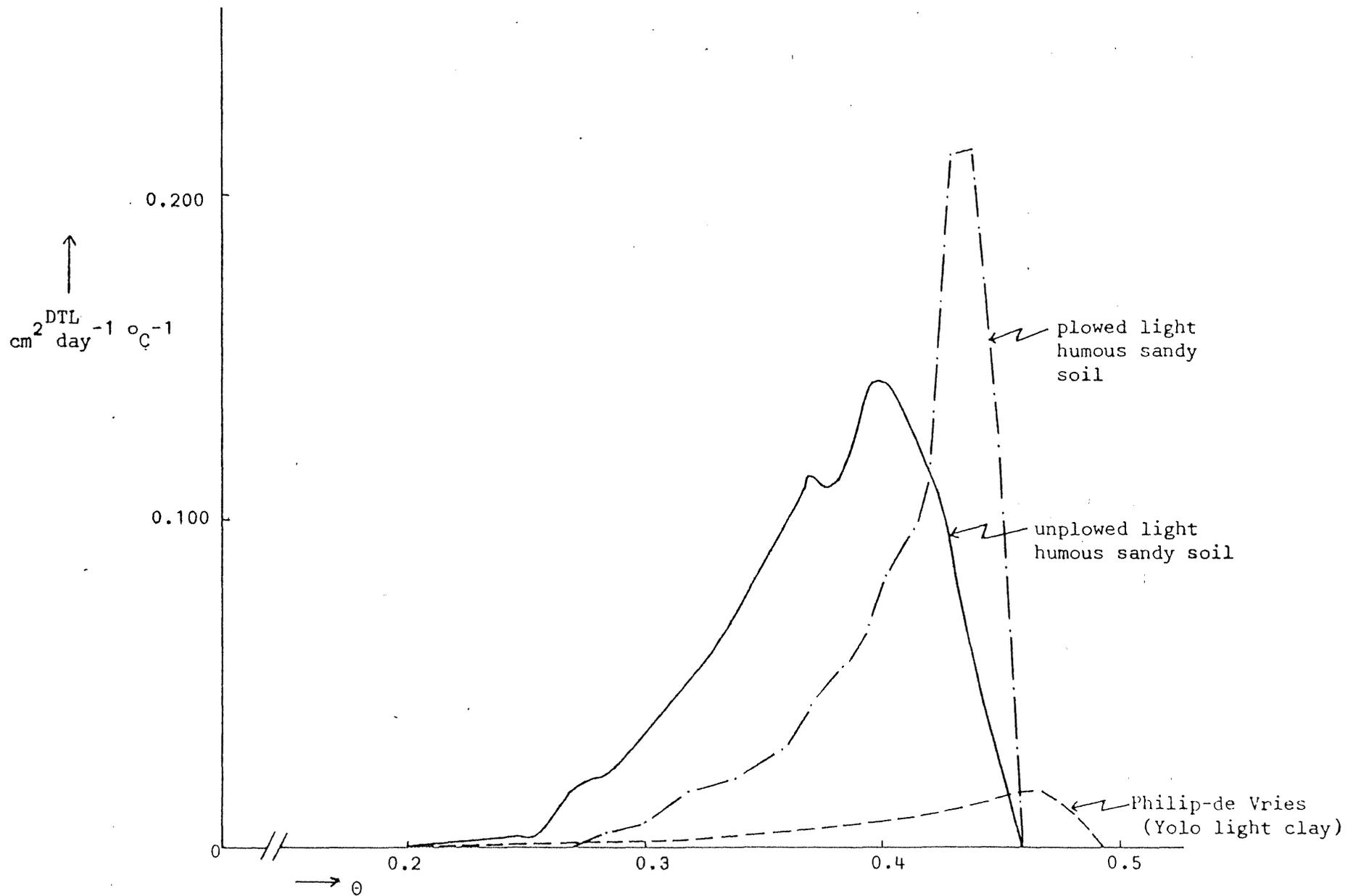


Fig. 9 Thermal liquid diffusivity, DTL, as a function of volumetric moisture content at 20°C for different soils.

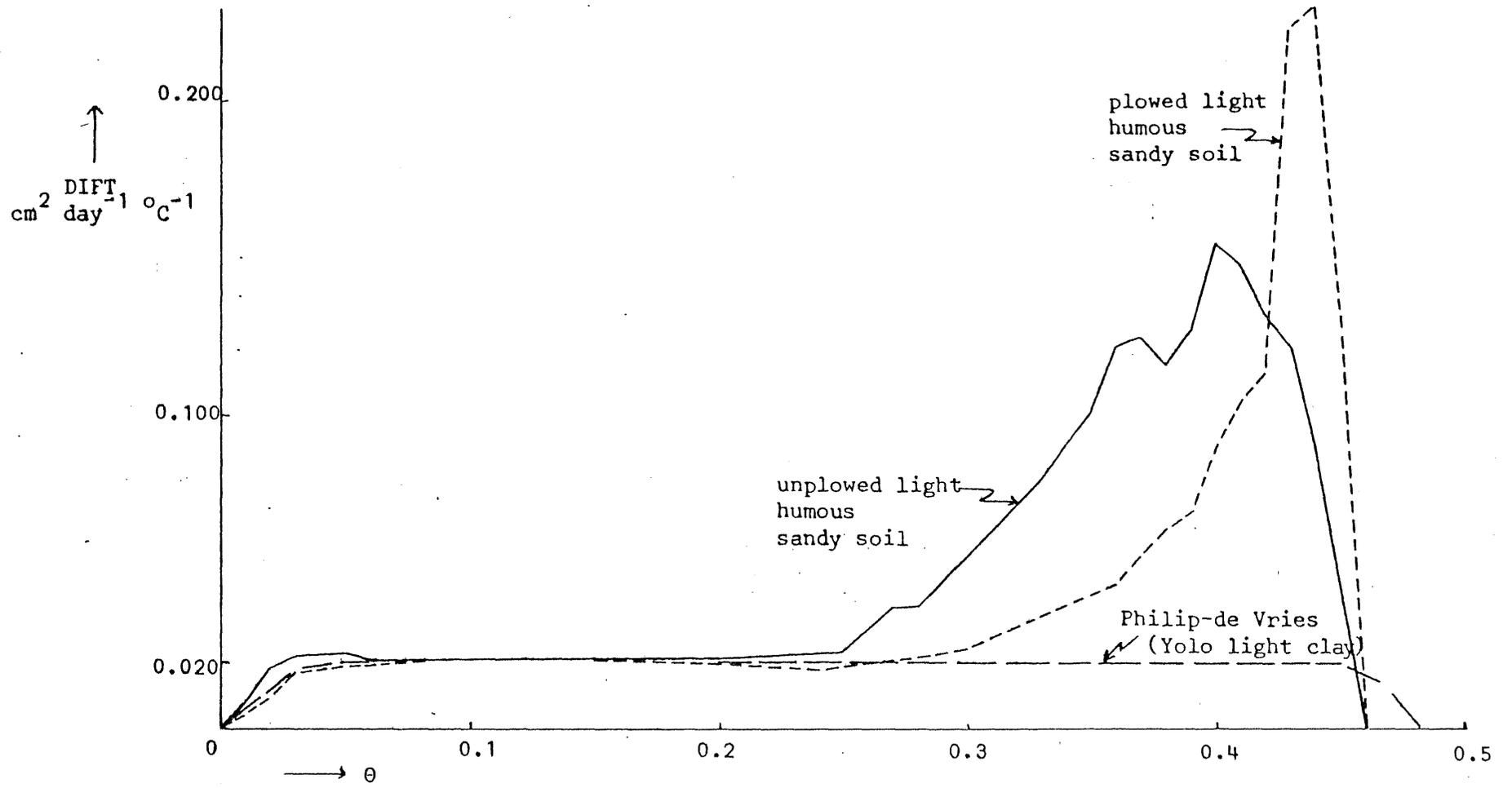


Fig. 10 Thermal moisture diffusivity, DIFT (=DTV + DTL), as a function of volumetric moisture content at 20°C for different soils.

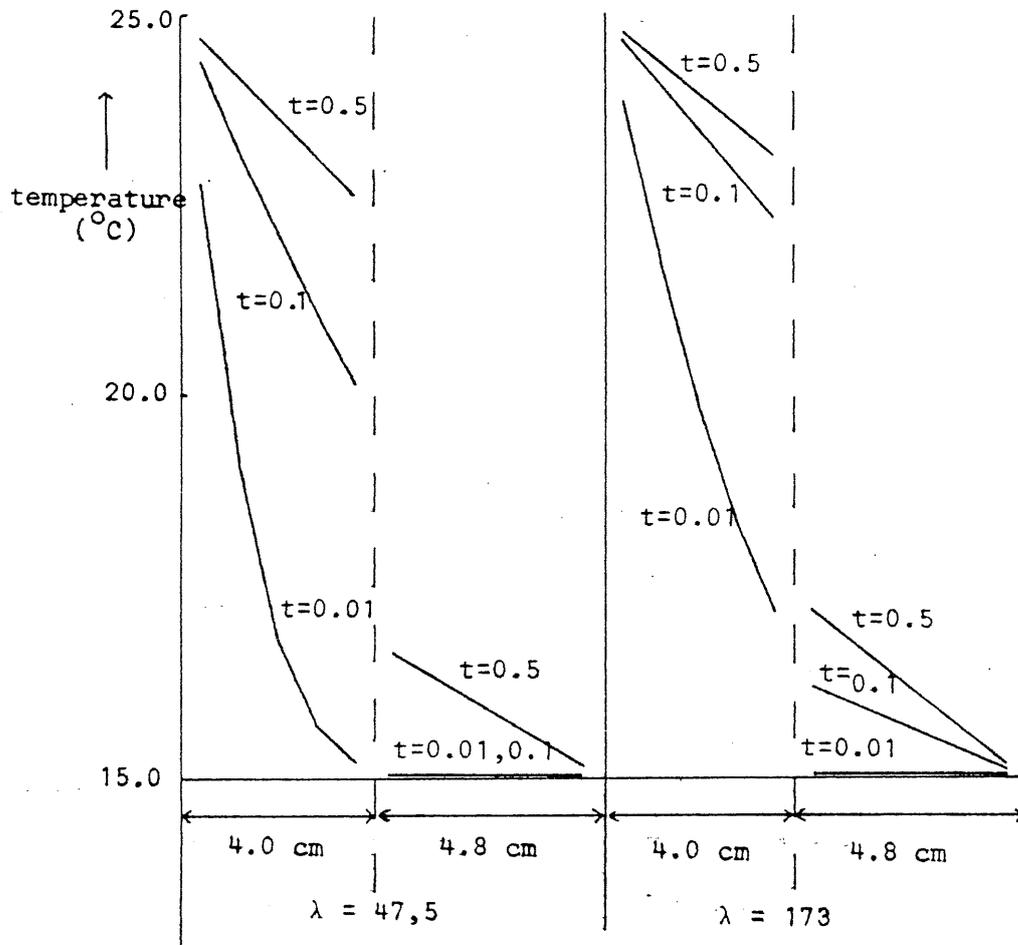


Fig. 11 Temperature distribution in the first 4.0 cm and the last 4.8 cm of a soil column (total length 20 cm, initial temperature 15°C) for different time intervals after application of a thermal gradient (25°C - 15°C) and for two thermal conductivities. Initial volumetric moisture content of the column: 5.00%.

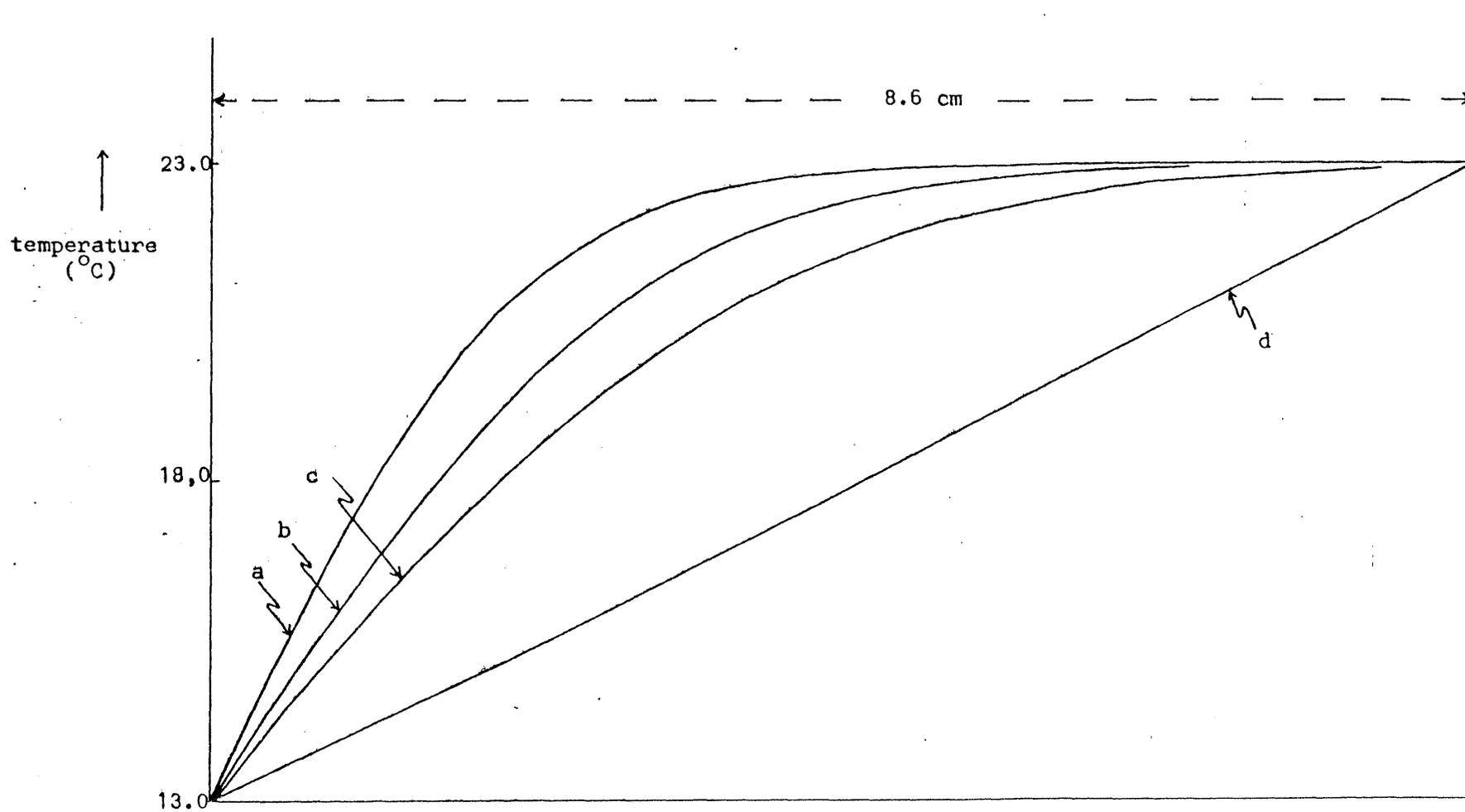


Fig. 12 Temperature distribution in soil column, 0.01 day (curve a, b and c) and 0.50 day (curved) after a sudden drop from 23°C to 13°C at one side.
 Curve a, b and c: initial volumetric moisture content of 4.5%, 9.0%, and 16.0%, respectively.
 Curve d: 4.5%, 9.0%, and 16.0% initial volumetric moisture.

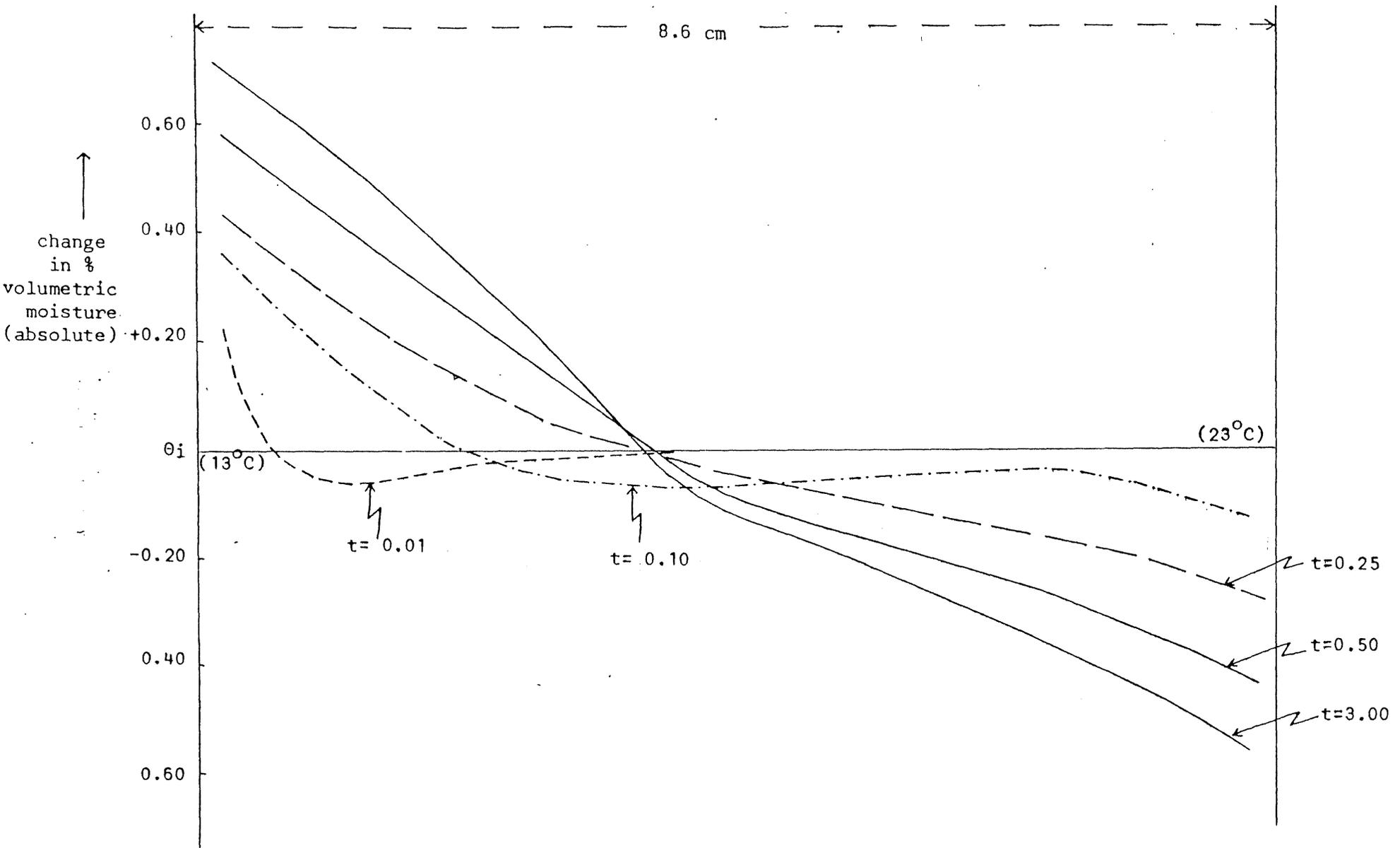


Fig. 13 Moisture distribution in soil column after a sudden temperature drop from 23°C to 13°C at one side for different time intervals.
Initial volumetric moisture content = 4.50%.

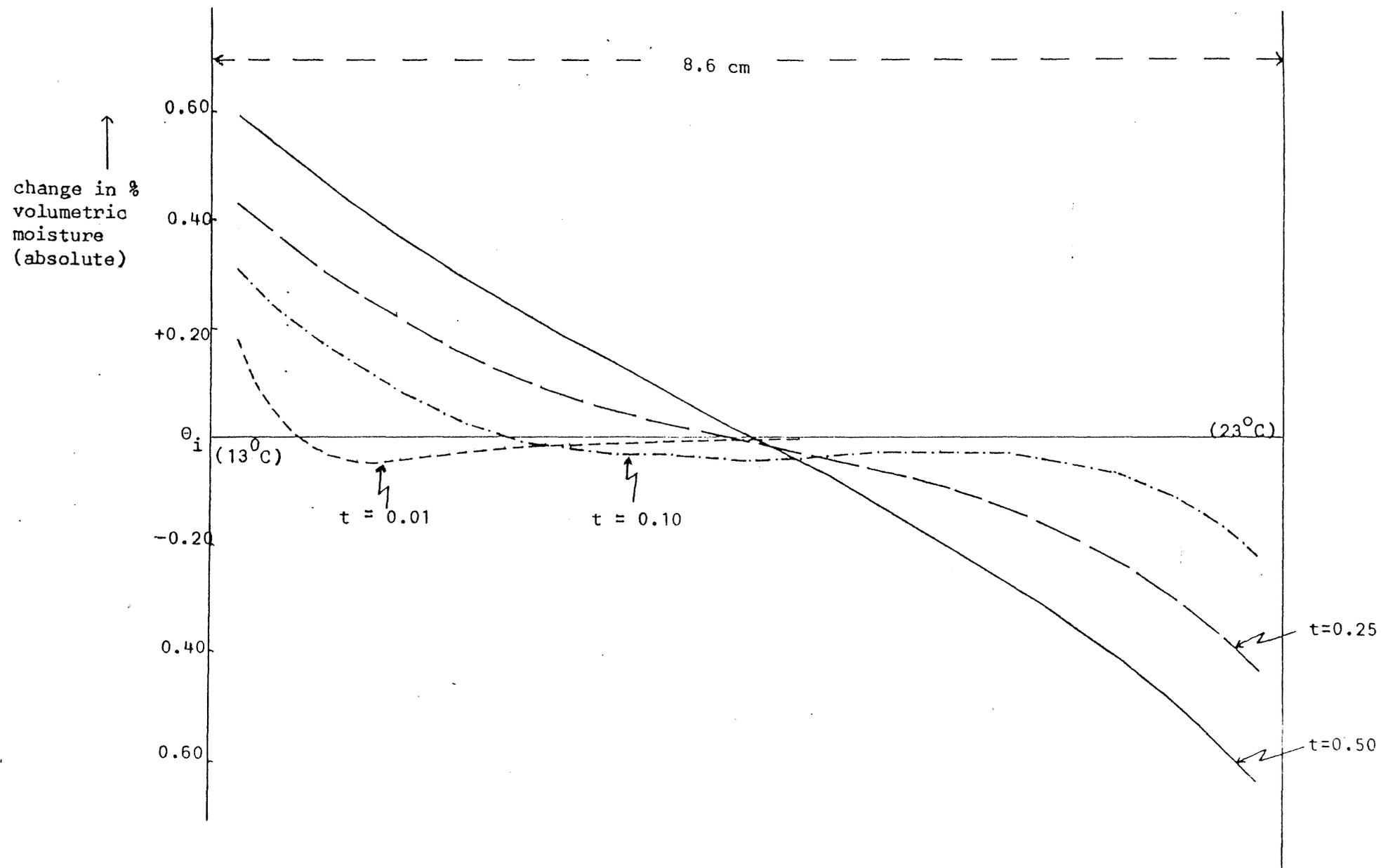


Fig. 14 Same as Figure 13 but now for initial volumetric moisture content = 9.0%.

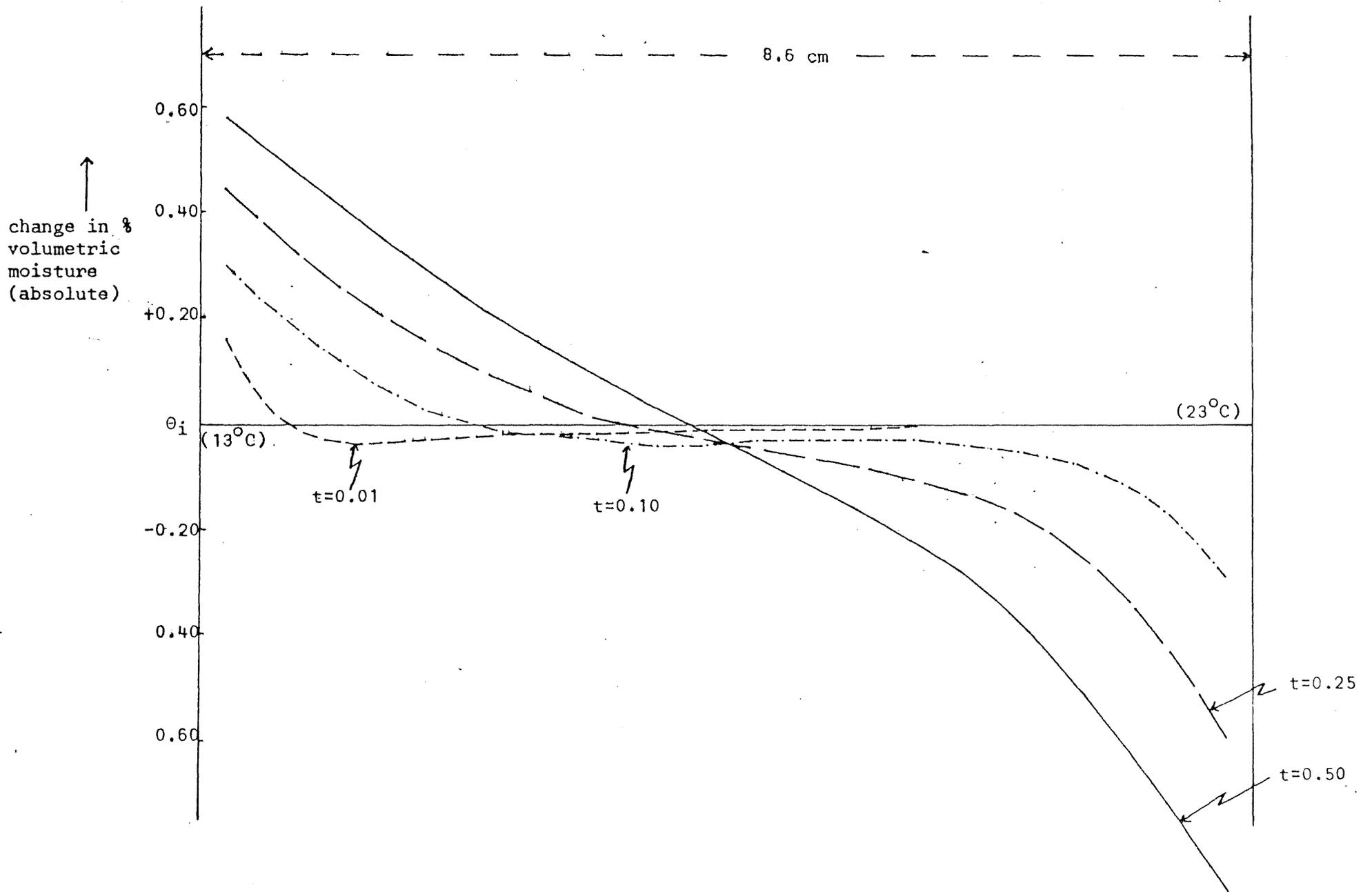


Fig. 15 Same as Figure 13 but now for initial volumetric moisture content = 16.0%.

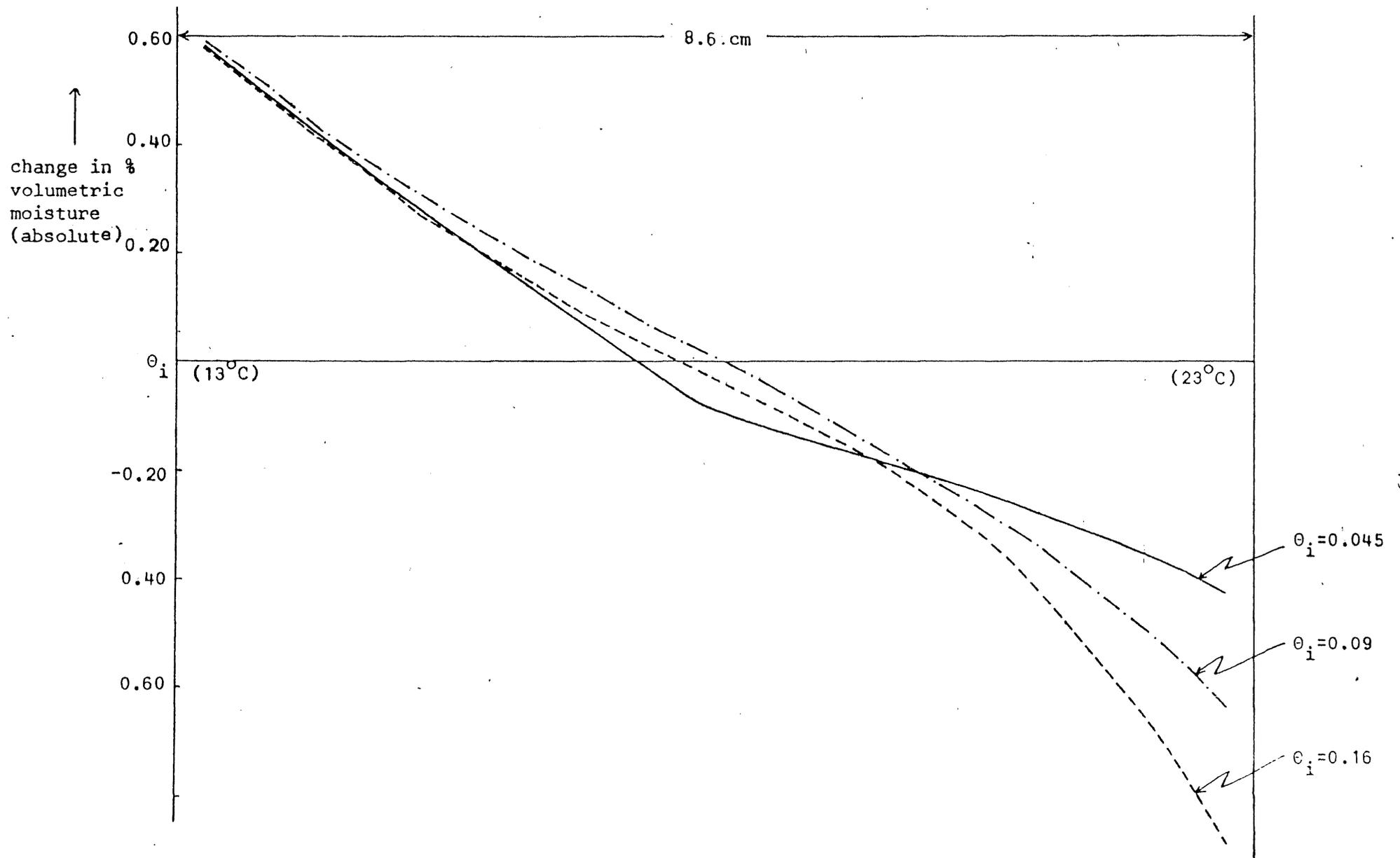


Fig. 16 Moisture distribution in soil column 0.5 day after a sudden temperature drop from 23°C to 13°C at one side for different initial volumetric moisture contents.

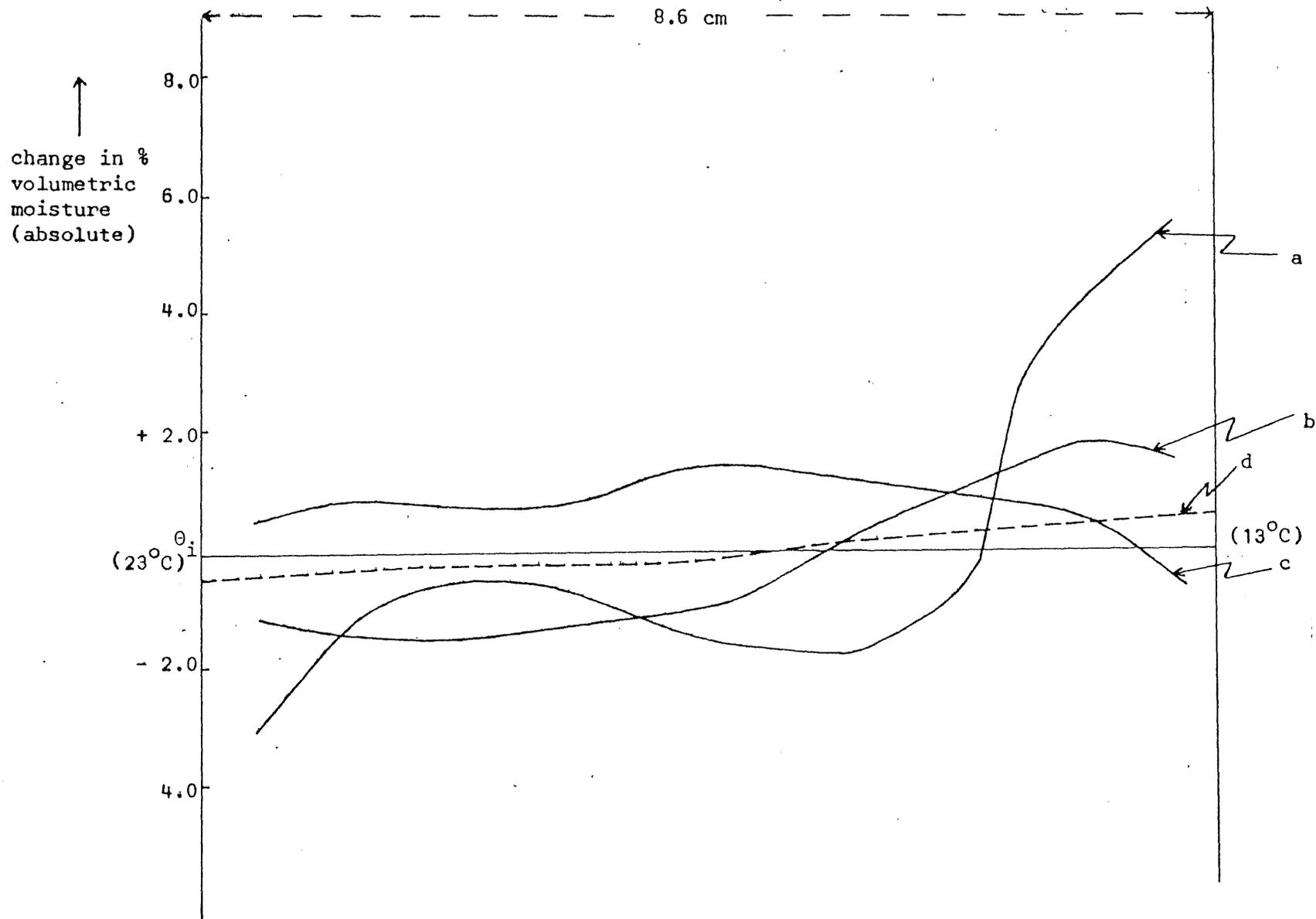


Fig. 17 Moisture distribution in soil column (total length 8.6 cm), 1.0 day (curve a, b and c) and 0.5 day (curve d) after a sudden temperature drop from 23°C to 13°C at one side. Curve a, b and c: experimental data from Oomen and Staring (1972) for initial volumetric moisture contents of 5.10%, 9.05% and 15.75%, respectively. Curve d: simulation for an initial volumetric moisture content of 4.50 %.

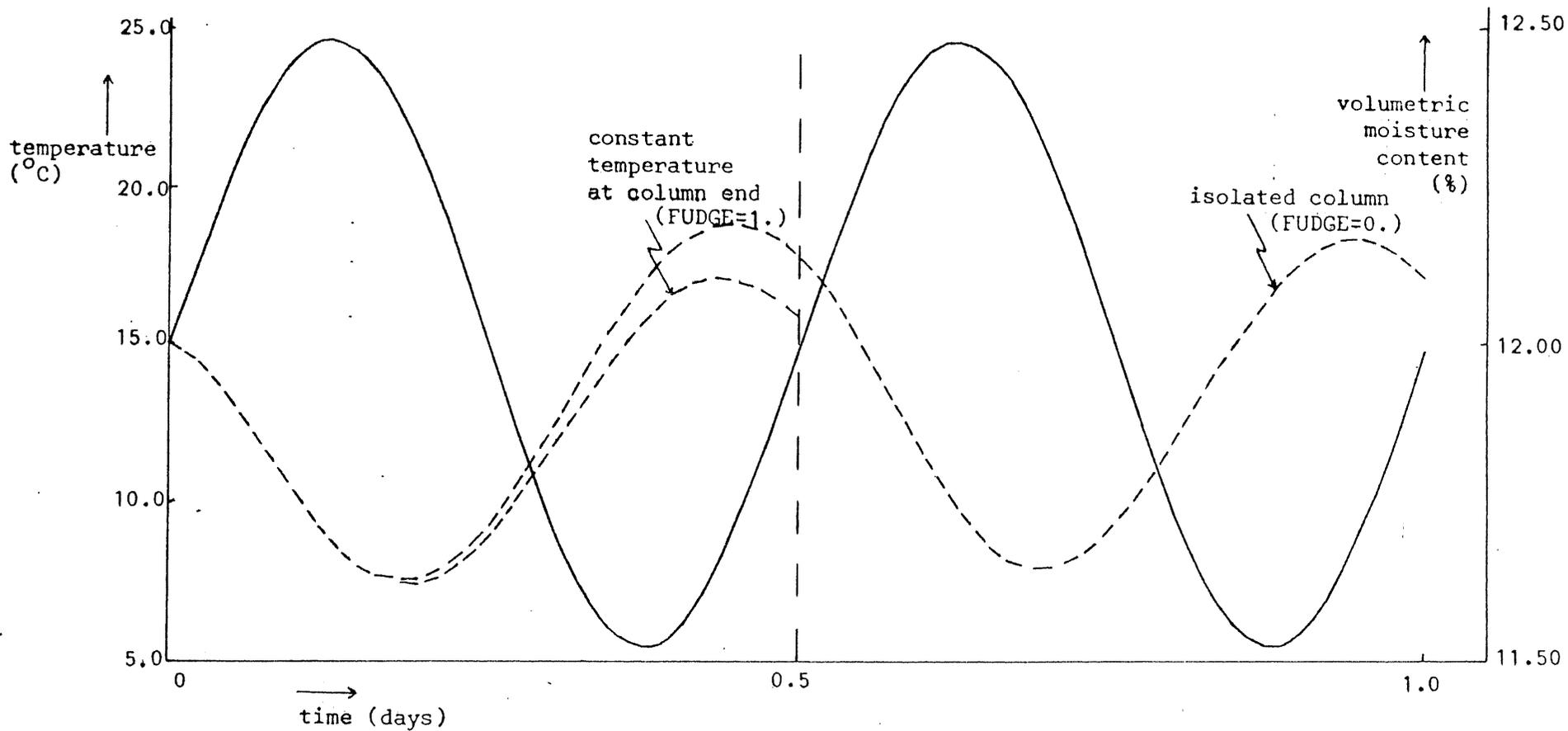


Fig. 18 Temperature (-) and moisture content (---) variation generated in the first compartment (0.8 cm) of a horizontal soil column (total length 20 cm) by a sinusoidal variation at the left side of the column. Initial temperature and volumetric moisture content: 15°C and 12%. Sinusoidal variation: average temperature 15°C amplitude 10°C period 0.5 day

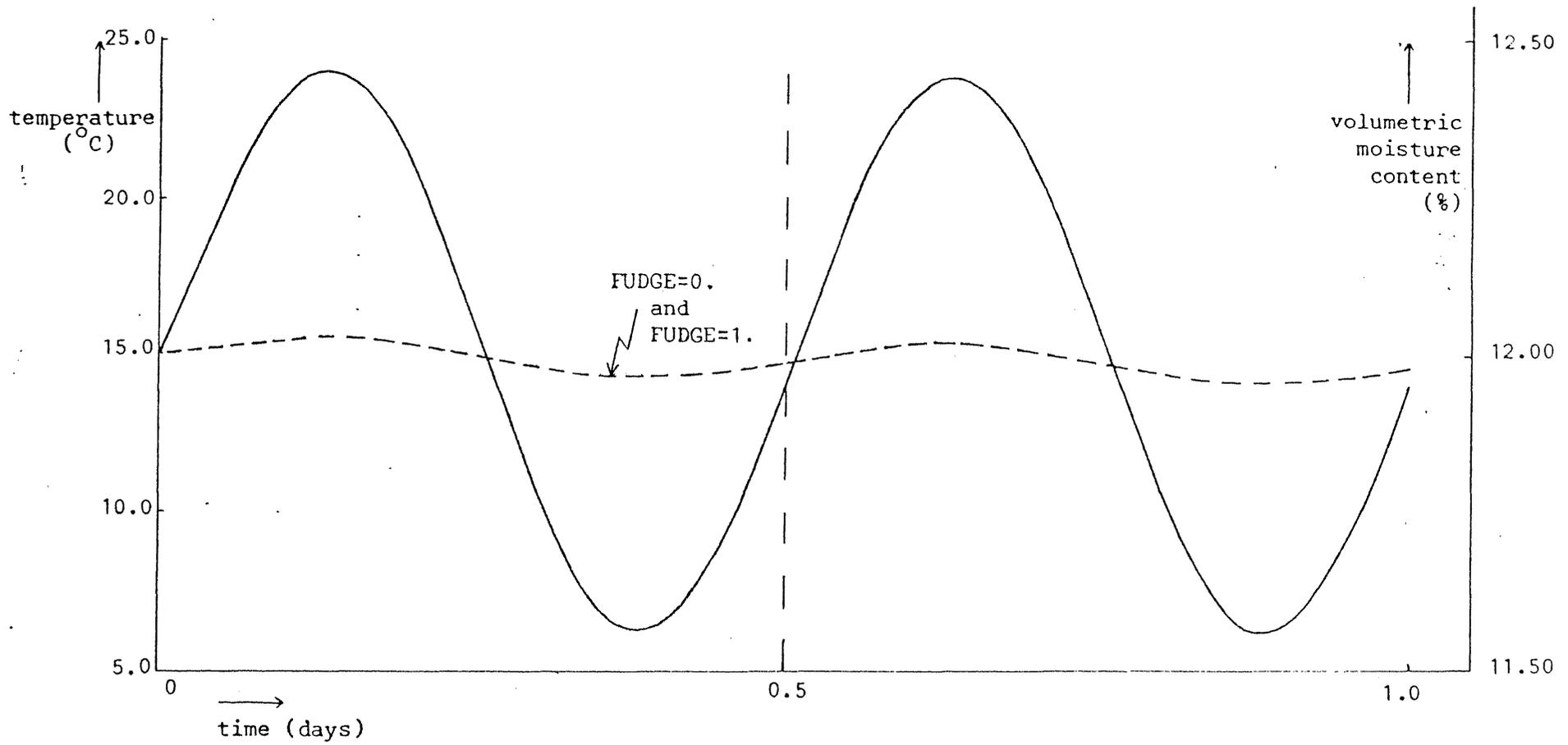


Fig. 19 Same as Figure 18 but now for the second compartment of the soil column.

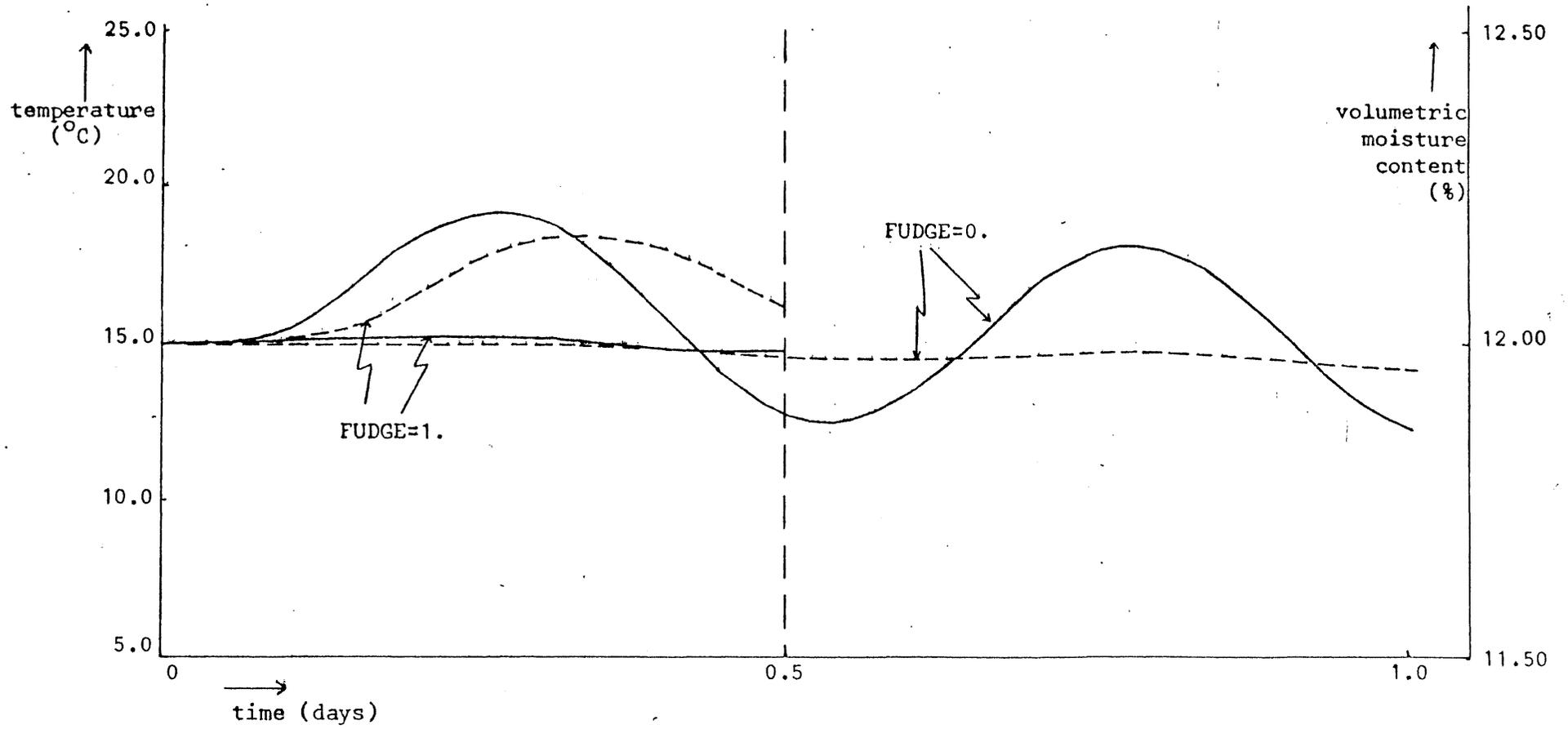


Fig. 20 Same as Figure 18 but now for the last (25th) compartment of the soil column.

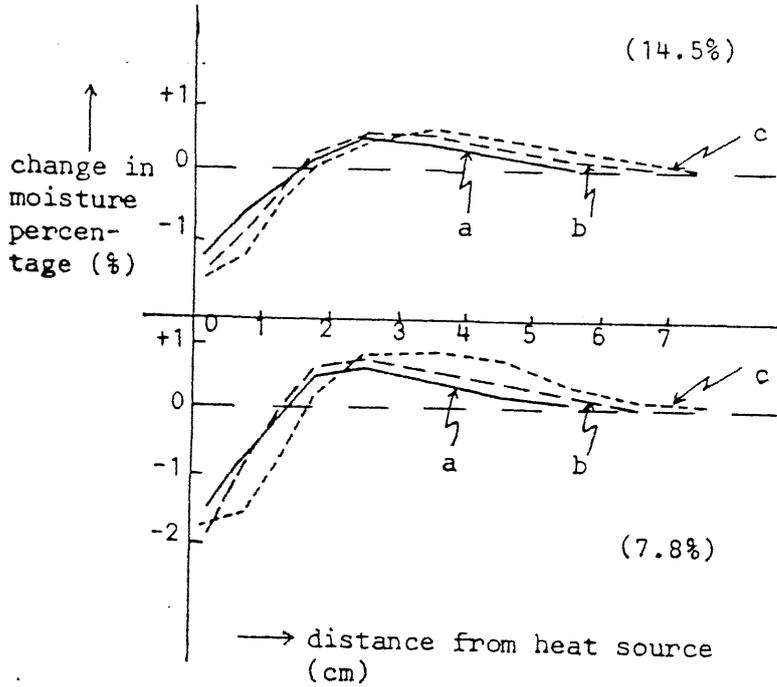


Fig. 21 The change in moisture percentage in respect to distance from the heat source after two cycles (of the applied heat wave), for two initial moisture contents (% by weight).

- a: period heat wave 8 min.
- b: " " " 16 min.
- c: " " " 32 min.

(data from Hadas, 1968).