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CRITICAL REMARKS CONCERNING THE VALIDITY OF THE MITSCHERLICH EFFECT LAW *)

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(Mit deutscher Zusammenfassung)

A few years ago Mitscherlich[3]) and Gericke[1]) published a verification of the validity of the Mitscherlich Effect Law based upon the results of more than 27000 field tests with graded amounts of P_2O_5, K_2O and N, which have been laid down in the pre-war years on the principal soil types of Germany, using the principal crops as test plants.

The Mitscherlich Law is represented by the following equation:

$$\log (A - y) = \log A - c(x + b),$$

meaning that the increase of the yield of any crop under the influence of increasing amounts of any growth factor x is proportional to the difference between a partial yield y obtained at any stage and a certain maximum possible yield A; c is a constant that depends on the nature of the growth factor and determines the slope of the yield curve. The quantity b represents the amount of x originally present in the soil.

The use of average results from large numbers of experimental fields has the advantage that the influence of accidental deviations and errors is to a large degree eliminated.

According to both these investigators the validity of the equation was confirmed. Consequently the relation between yield and the amounts of any fertilixer may be represented by a logarithmic curve which is identical for all crops under investigation. The constant c is approximately equal in all cases with the same nutrient. This result

*) W. Stollenwerk in a recent paper [5]) criticizing the work of Mitscherlich and Gericke but employing different methods attained approximately similar results to those of the present paper.

is considered by Gericke to be an average one, from which individual deviations occur. Mitscherlich's conclusion agrees with that contained in former publications and is very definite: "Das Wirkungsgesetz der Wachstumsfaktoren als solches besteht und ist richtig" *).

From the constancy of c it is deduced by Mitscherlich that the need for any plant nutrient of all crops is quite similar: "Alle unsere Kulturpflanzen reagieren hinsichtlich der Nährstoffausnutzung zur Ertragsbildung ganz in gleicher Weise! Es gibt so z.B. keine Kulturpflanze, die vielleicht die Bodenphosphorsäure besser aufschlösse als eine andere! Diese Ansicht gehört in das Reich der Fabeln".

It is no wonder that the results obtained by these well-known German investigators has evoked interest elsewhere.

Willcox [7]) has drawn them to the attention of English readers and emphasizes the outstanding importance of this work. He refrains however from any criticism of the methods applied by Mitscherlich and Gericke.

Since many of the present readers may not be well acquainted with the extensive work of Mitscherlich and the widespread criticisms of it (Rippel [4]), Meyer [2], de Vries [6]) a.o.), it seems worth while to criticize in detail the method of calculation and the interpretation of the results.

CALCULATION AND INTERPRETATION OF RESULTS

The procedure of the authors was to take all the series of tests with each of the different crops and to compute the average yields of the treated and the untreated plots. Accepting the values $c = 0.6$ for P_2O_5, $c = 0.4$ for K_2O and $c = 0.6$ for N which values were derived from previous investigations, the maximum value A was calculated from the differences between the average yields of the untreated and those of each of the treated plots.

This method of computation of A gave 4 different values of A, from which the average was calculated. This is statistically unwarranted. The yield of the untreated plot is used 4 times, viz. in each of the (4) equations with the yields of the other plots. Consequently a

*) There is a law governing the effect of growth factors which is exact.

possible error in the yield of the untreated plot has too heavy an influence on the value of A.

An improved method of calculation is given below.

It was supposed by Mitscherlich and Gericke that the averages of the plots from a large number of experiments treated in a similar way can be used for an evaluation of the validity of the law. A mathematical verification of this assumption can be given, assuming that c is really a constant for each crop.

Suppose that the following equation is valid for a certain crop in a certain case:

$$\log (A_i - Y_i) = \log A_i - c(x + b_i)$$

The index i indicates that the values concerned deal with the results of an experiment i of the series.

Then

$$\frac{A_i - Y_i}{A_i} = 10^{-c(x+b_i)}$$

$$Y_i = A_i - A_i\, 10^{-c(x+b_i)} = A_i - A_i\, 10^{(-cb_i)}\, 10^{-cx}$$

Substitute for $A_i\, 10^{(-cb_i)}$, which is a constant in any given experiment B_i, then the equation can be written as follows

$$Y_i = A_i - B_i\, 10^{-cx}$$

Obviously this equation will not describe exactly the results of a given experiment since the yield is subject to accidental fluctuations. Actually the equation must be written

$$y_i = A_i - B_i\, 10^{-cx} + \varepsilon_i$$

ε_i is the difference between the yield y_i found and the theoretical yield $Y_i = A_i - B_i\, 10^{-cx}$.

If the average yield $\bar{y}$ is computed for a distinct amount of fertilizer x from a large number of experiments, then

$$\bar{y} = \bar{A} - \bar{B} . 10^{-cx} + \bar{\varepsilon}$$

from which it follows that the Mitscherlich Law for constant value of c is also valid for the averages.

It is further evident from this equation, that if y_i (or $\bar{y}$) is plotted against 10^{-cx} a straight line is obtained, from which an equation, since it is a regression line, can be computed in the usual way. This computation also gives an estimate of s for the value of the standard error $\sigma = \sqrt{\Sigma(\varepsilon)^2/n}$ of the deviations from the regression line

$$s = \sqrt{\frac{\Sigma\,(\bar{y} - \overline{Y})^2}{n-2}} \simeq \sigma.$$

It is obvious too that the mean value of b indicating the amount of nutrient originally in the soil can be deduced from $\overline{B}$ by means of the formula

$$\overline{B} = \overline{A}\; 10^{-c\bar{b}} \;\text{ or }\; \bar{b} = \frac{\log \overline{A} - \log \overline{B}}{c}.$$

The results have been examined by Gericke by computing the standard error of A determined in the manner critized above (p. 98). These errors appear small. Hence the conclusion is drawn that the Mitscherlich Law fits the data.

According to Mitscherlich the good agreement between the curve calculated from the results by means of the equation and the results really obtained is the proof of the validity of the Law. This judgment which is based upon a curve derived from the actual data is not very clear.

CRITICISM ON THE WAY OF JUDGMENT

In the opinion of the author it is not only important that the deviations should appear small but they should really be small in respect of the number of experiments involved. It must further be shown that the deviations found by applying the postulated values of c are smaller indeed than those found by applying other values of c.

The error in A is presented by Gericke in the form of the mean error which is an unusual statistic (Table I, *a*). We have calculated the standard error with the aid of the formula $s_A = \sqrt{\Sigma d^2/n - 1}$ *) using the data of Gericke. The percentage value of s: $100 s_A/\overline{A}$ is given (*b*). The standard error of y (as a percentage, $100 s/\bar{y}$) is also calculated from the deviations of the points from the curves calculated by Mitscherlich, using the formula $s = \sqrt{\Sigma d^2/n - 2}$ ** (as two constants A and b are adopted, it is necessary to divide by $n - 2$ (table I, *c*).

*) $d = A - \overline{A}$ **) $d = y - Y$.

TABLE I

Percentage errors found by different methods of computation	rye	oats	wheat	barley	sugar beet	mangold	potato	hay	clover
a. mean error found by Gericke	1.5	1.3	1.3	2.7	0.9	0.9	0.7	2.9	5.0
b. standard error of A $\frac{100\, s_A}{\bar{A}}$	2.0	1.8	2.1	3.8	1.3	1.4	0.9	3.4	7.8
c. standard error of y comp. from data of Mitscherlich $\frac{100\, s}{\bar{y}}$	2.5	2.2	2.6	4.7	1.6	1.7	1.1	4.2	9.6
d. $\frac{100\, s_A}{\bar{A}}$, own calcul.	1.0	1.0	1.4	1.4	—	0.8	0.4	2.0	2.5
e. $\frac{100\, s}{\bar{y}}$, own calcul.	1.3	1.3	1.4	1.8	—	0.9	0.5	2.7	3.3

As a result of the use of a curve less adapted to the points (resulting from the incorrect method of calculation mentioned above) the standard errors of A and y are decidedly higher than those deduced from our calculations (table I, *d* and *e*).

Should these standard errors (also the lower ones found by our calculation) really be considered negligibly small as has been asserted by Mitscherlich, Gericke and Willcox? This may be seriously doubted considering the number of experiments. If the factors phosphate, potash and nitrogen do not interact with other factors as postulated by Mitscherlich and if the effect factor c is equal in all experiments, then purely accidental errors would be reduced to the $1/\sqrt{n}$ part (that is to 1/41 of the original value in the case of potatoes, to 1/39 in the case of hay, to 1/35 in the case of rye etc. as these experiments have been performed 1642, 1535 and 1218 times respectively). This conclusion, assuming that c is a constant, follows from the justification of taking the average given above (p. 99). Consequently the standard errors $\sigma_i = \sqrt{\Sigma\varepsilon_i^2/n}$ of the single fields must have been extremely high. According to our calculations they would amount to 44% for rye, 40% for oats, 31% for wheat, 34% for barley, 20% for potatoes, 25% for mangolds, 104% for hay and 38% for clover. Thus it proves that no satisfactory confirmation of the Mitscherlich Law has been obtained.

COMPUTATION OF THE STANDARD ERROR USING OTHER VALUES OF c IN THE CASE OF PHOSPHATE EXPERIMENTS

Mitscherlich and Gericke have assumed a constant value of c. However, it ought to be the purpose of the investigation to trace in how far this assumption is permissable. The computations have therefore been repeated with different values of c. The values of c giving minimum values of s are determined by means of a calculation of A and b according to the method described above.

The phosphate experiments are most suited to this purpose as 5 plots dressed with different amounts of fertilizer (0, 30, 60, 90, 120 kg/ha P_2O_5) are available. The experiments with sugar beet where only 4 different amounts of phosphate were used are omitted, as the number of observations is too low for a reliable determination of a regression line. The potash experiments consisting of 4 dressed plots only are not considered here for the same reason. The nitrogen experiments are complicated by the occurrence of yield depressions.

A further objection must be made against the use of long term experiments for the present purpose, when the design of the experiments was similar in successive years. Phosphate manuring usually causes residual effects in following years. It follows that larger amounts of phosphate are responsible for the effects observed than the quantities actually applied in a certain year. Plotting the latter instead of the unknown larger amounts (added quantity + residual phosphate) against the yield results in a steeper slope of the curve which corresponds to a higher value of c. The exact values of c therefore are probably lower than those actually found.

The calculated values of c for minimum values of s are to be found in table II. With one exception these values of c are lower than 0.6 and are appreciably variable. The average value of c is 0.44 ± 0.09. The minimum values of s are considerably lower than the values of s found if $c = 0.6$. Again the percentage value of s using the best fitting value of c is lower than the corresponding value if $c = 0.6$ given in table I. The standard errors of the single fields presented in the last column of table II are generally lower than those found before. Nevertheless our conclusion based on the well-known variability in field trials that these errors are by far too high is also confirmed for varying values of c.

An exception must however be made in the case of potatoes. The standard error of 2.3 corresponds to the order of the errors usually found in our own experiments performed with this crop. The value is even rather low. Thus it may be concluded that the Effect Law is satisfactorily confirmed for this crop, but that the value of c is 0.40.

The elimination of many secondary reactions in the case of potatoes as a result of the use of comparable plant material, number of plants etc., which are less readily realized for cereals (occurrence of tillering, fructification, maturation) may be suggested as reasons for the far better results obtained with potatoes. In addition it is possible that the value of c is more subject to variations with other crops.

TABLE II

Crop	c for min. s	minimum s	s if $c = 0.6$	$\frac{\text{min}.s}{\bar{y}}$ in %	$\frac{\text{min}.s}{\bar{y}} x\sqrt{n}$ in %
Rye	0.40	0,250	0.299	1.05	36.6
Oats	0.50	0.338	0.351	1.23	38.1
Wheat . . .	0.50	0.395	0.404	1.42	31.7
Barley . . .	0.60	0.468	0.468	1.77	33.7
Potatoes . .	0.40	0.144	1.270	0.06	2.3
Mangolds .	0.35	4.265	6.695	0.59	14.5
Hay	0.35	1.800	2.058	2,33	91.4
Clover. . .	< 0.10	—	2.744	—	—
mean . . .	0.44 ± 0.09				

The value of c found for mangolds is not particularly high. Yet this value is decidedly higher than which is usual for this crop. What has been said for potatoes is probably true of mangolds. However, the value of c is smaller again ($c = 0.35$) than that postulated by Mitscherlich.

No value of c has been determined for clover. A minimum value of s was not even reached with $c = 0.10$. It is biologically unlikely that c has a still lower value. Besides the results obtained with this crop are irregular.

Our conclusion is that the value of c in single experiments is subject to large variations (and this would also be true if experimental errors could have been avoided) or that considerable deviations of the logarithmic slope of the yield curve occur. According to our own experience the first possibility may be true in many cases.

COMPUTATION OF THE AMOUNT OF AVAILABLE PHOSPHATE IN THE SOIL

Accepting $c = 0.6$ for phosphate G e r i c k e has calculated the value b (in quintals/ha P_2O_5) for various crops. These values vary between 0.82 for hay and 1.46 for sugar beets. We have found (table III) a variation between 0.82 (hay) and 1.31 (potatoes), and for arable crops only 1.01 — 1.31. It is however clear that this calculation has lost its theoretical foundation if the suggested constancy of c is not confirmed. Larger variations are found when using the correct values of c for minimum s (table III).

TABLE III

crop	b in the case of		c for s minimal
	$c = 0.6$	value of c if s = minimal	
Rye	1.01	1.11	0.40
Oats	1.05	1.21	0.50
Wheat	1.16	1.34	0.50
Barley	1.16	1.16	0.60
Mangold	1.31	1.83	0.35
Potato	1.22	1.78	0.40
Hay	0.82	1.19	0.35
Clover	0.89	—	< 0.10

It follows from table III that the available amount of phosphate varies rather significantly (for arable crops only 1.11 — 1.83). Of course there is no a priori objection to the creation of a conventional method of soil testing by means of the application of a postulated constant value of c for all crops. However, such a method (the M i t s c h e r l i c h pot culture method) is not principally distinguished from any other conventional method of soil testing. It has no greater merit than other methods from a physiological point of view as has been claimed by M i t s c h e r l i c h. This method can only be evaluated by means of field tests. It has no value in itself as a physiological foundation is absent.

It is clear from the foregoing that the conclusion of G e r i c k e with regard to the different phosphate contents of the German soil types, on which the various crops were grown, are erroneous.

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SUMMARY

A criticism is made of the evaluation of the Mitscherlich Effect Law on the basis of more than 27000 field experiments with graded amounts of fertilizers by Mitscherlich and Gericke. Accidental errors are eliminated to a large degree by the use of large numbers of field tests. According to these investigators (also Willcox) a very good confirmation of the law has been obtained, and the discrepancies found should be small and without any real significance.

In contradiction to this view it is shown in the present paper that the discordance is important considering the large number of experiments. An exception should be made in the case of potatoes in which a confirmation of the Law was obtained. The principal cause of these discrepancies is probably the variability of the effect factor c of the fertilizer involved in single experiments.

The constancy of $c = 0.6$ as for phosphate suggested by Mitscherlich was not confirmed for various crops. Better fitting results were obtained with different values of c. These values varied in the case of phosphate between 0.35 for mangolds and 0.60 for barley with an average value for all crops of 0.44 ± 0.09.

It is probable that the values of c found are somewhat too high as a result of the residual effects involved in long term phosphate experiments.

The determination of the amount of phosphate b in the soil gave rather discrepant results. The method of soil testing adopted by Mitscherlich based upon the assumption of a constant value of c lacks a physiological foundation. Any advantage of this method over other conventional methods of soil testing is denied.

ZUSAMMENFASSUNG

Kritische Bemerkungen zur Gültigkeit des Mitscherlichschen Gesetzes der Wachstumsfaktoren

Es wird die Prüfung der Gültigkeit des Mitscherlichschen Gesetzes, wie diese anhand von 27000 Feldversuchen von Mitscherlich und Gericke durchgeführt wurde, kritisiert. Der Vorteil einer Benutzung größerer Zahlen von Versuchen ist daß zufällige Fehler eliminiert werden können. Nach der Meinung beider Förscher (auch Willcox) wurde eine recht befriedigende Bestätigung des Gesetzes erhalten. Die gefundenen Abweichungen würden klein und ohne wesentliche Bedeutung sein.

Im Gegensatz zu dieser Auffassung wird aber gezeigt, daß die Abweichungen unter Berücksichtigung der großen Zahl der Versuche beträchtlich sind (Tabelle II). Eine Ausnahme macht nur die Kartoffel, mit welcher eine Bestätigung des Gesetzes gefunden wurde. Die Hauptursache der Abweichungen ist wahrscheinlich die Inkonstanz des Wirkungsfaktors c in den verschiedenen Versuchen.

Die Konstanz von $c = 0.6$, wie diese Mitscherlich als erwiesen betrachtet, konnte für verschiedene Gewächse nicht gefunden werden. Genauere Angleichung der Daten wurde mit wechselnden Werten für c erhalten. Diese Werte lagen zwischen 0.35 für Futterrüben und 0.60 für Gerste, der mittlere Wert betrug 0.44 ± 0.09. Es ist aber wahrscheinlich daß die gefundenen Werte von c bei den mehrjährigen Feldversuchen infolge der Nachwirkung der Phosphatdüngung mehr oder weniger zu hoch sind.

Die Bestimmung der im Boden vorhandenen wirksamen Phosphorsäuremenge b ergab ziemlich auseinanderliegende Werte. Die Mitscherlichsche Methode der Bodenuntersuchung, welche sich auf die Annahme konstanter Wirkungsfaktoren stützt, entbehrt einer physiologischen Grundlage. Die Vorteile dieser gegenüber den chemischen Methoden werden verneint.

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