

FACTOR ANALYSIS IN AGRICULTURAL RESEARCH ¹⁾

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1 INTRODUCTION

Statisticians have developed a group of analytical methods for the purpose of psychological research which would make it possible to estimate quantitatively, by the results of tests, the properties underlying the psyche. These methods, which in English publications are usually termed *factor analysis*, *factorial analysis* or *multiplefactor analysis*, have recently been applied and developed further in other sciences as well.

Among psychologists themselves there is no agreement on how to answer the query as to whether it is possible to give a quantitative description of mental abilities, and assuming this to be possible, whether the above-mentioned methods can be used (cf. Uppsala Symposium on psychological factor analysis, 17-19 March 1953). It is difficult to check the results of an analysis as psychological qualities do not generally lend themselves very well to direct measurement. THURSTONE has increased confidence in this method by conducting an analysis of the correlations between a number of stereometric properties of blocks of varying shapes. The treatment did, in fact, bring out the fundamental properties of length, breadth and height. The advocates of factor analysis assume, by analogy, that the method is also capable of giving a meaningful analysis of psychological properties in fundamental groups (the so-called factors).

In recent years agricultural research workers have been employing this method in order to solve problems of soil fertility and agricultural ecology and economy. It is anticipated, of course, that the method will be of value for this purpose; on the other hand these applications may be important for psychology since when their practical use has been demonstrated in agriculture there may be an increase in confidence in their suitability for psychological research. The results of such an analysis with agricultural data are often more easy to understand as one may have to deal with concisely defined characteristics. Moreover, the practical use of the method can be tested by comparing the results with knowledge gained in another way.

The background of this method is probably best described with reference to an agricultural example. We have chosen for this purpose the investigation into the factors determining the botanical composition of grassland. This is, in fact, a subject to which we think factor analysis may be profitably applied (DE VRIES, 1954).

The botanical composition of grassland may be described by means of the incidence of a number of species of grasses and weeds. The percentage by weight may be used, for instance, as an index of the frequency with which a specific species of grass occurs. Experience and research have shown that this percentage by weight depends, among other things, on the water supply, the chemical richness of the soil, etc. The species vary greatly as regards the

¹⁾ Received for publication April 8, 1957.

degree of reaction to the water supply, for example. The species with a marked reaction to a growth factor are termed indicator plants. In the simplest case one species reacts to one growth factor only, and possibly several species to the same factor. The agreement in the reaction of these species is reflected in the closeness of the correlations between the percentages by weight determined in a number of fields. Vice versa, when species occur of which the percentages by weight are highly correlated to each other, this may be due to the fact that they react to the same growth factor. Viewed in this way, the correlation coefficient gives the *communality*.

In reality things are much more complicated than this because most species react not only to one factor but to several. In many cases the reactions will vary in intensity and sometimes even take the opposite direction. The correlation coefficients, as the result of all these effects, are no longer easy to comprehend and to use as an index of the communality. A technique has now been evolved in factor analysis which would also be capable of indicating the common effects in these complex cases and of classifying them according to intensity.

Factor analysis, as applied in psychology, attempts to describe the test results as a function of hypothetical factors. The term "factor" does not presumably lead to any great misunderstanding in psychology; at any rate, the term *component* introduced by HOTELLING has never found much favour. We believe, however, that the term "factor" may cause confusion in agricultural research. Thus in soil fertility research the term is already employed for each of the effects such as ground water level, moisture retention of the profile and precipitation; together they indicate the extent of the plant's water supply. But from the point of view of plant physiology the origin of the water is of minor significance, and it is only the total quantity of water available that is important. In factor analysis the object is to trace and quantify such "hypothetical" factors if present. In order to prevent confusion with the ground water level, for instance, which is designated as a "factor", we think it preferable to employ the term *aspect* for the fundamental effect found, viz. in this case the water supply aspect²).

Aspects which are important for more than one species of grass are termed common aspects (common factors). The *specific* aspects (specific factors) are only important for a single species of grass. The common aspects are therefore used to explain the correlations between the percentages by weight. The specific aspect mainly corresponds to the part that is not reflected in the correlations. The analysis then attempts to give an answer to the question as to how many aspects are responsible for the correlations found between the percentages by weight of the grass species, and also to what extent these grass species react to these aspects.

2 MATHEMATICAL MODEL

It is possible to construct a mathematical model on the basis of the commonly known facts described in chapter 1, and also on the requirement that

²) In view of the term polyfactor analysis (regression analysis) which is commonly used in agricultural research in the Netherlands it would also probably be preferable to speak of aspect analysis.

the description with aspects should be simpler than the data. Which function is acceptable in this case? The simplest mathematical function suitable is the linear one. This function is employed in factor analysis.

Let us denote by F_1, F_2, \dots, F_m the m common aspects, and by S_1, S_2, \dots, S_n the specific aspects (for each of the n grass species or variables). The complete linear function for a variable z_j ($j = 1, 2, \dots, n$) may then be written

$$z_j = a_{j1}F_1 + a_{j2}F_2 + \dots + a_{jm}F_m + a_jS_j \quad (1)$$

Starting from this model it is also clear that there are n of these equations (1), i.e. for each of the variables j . In our case the variable z_j indicates the percentage by weight of grass species j . According to (1) the percentage by weight z_j in a field i ($i = 1, 2, \dots, N$) is then

$$z_{ji} = a_{j1}F_{1i} + a_{j2}F_{2i} + \dots + a_{jm}F_{mi} + a_jS_{ji} \quad (2)$$

From this it follows that N equations (2) may be constructed corresponding to the N pastures covered by the investigation.

In psychological terminology z_{ji} represents the test result of individual i in test j , F_1, F_2, \dots, F_m are then the fundamental psychological qualities which cannot be directly measured. This immeasurability is the reason why the comparatively much simpler regression analysis cannot be applied to these problems. In regression analysis it is, however, necessary to make postulations regarding the number and nature of the F_s .

The basic problem of factor analysis is to estimate the value of the coefficients a_{js} ($s = 1, \dots, m$) of the common aspects. The value of the coefficient a_{j1} (viz. the loading) thus denotes the weight of aspect F_1 in the variable z_j . Assuming that in our example F_1 denotes the water supply aspect, then the value of a_{j1} denotes the intensity with which the percentage by weight of grass species j reacts to water. For a specific species of grass this value may be equal to zero. This means that the grass species has no indicator value; in this case the aspect F_1 has no influence in equation (2). A high indicator value goes with a high value of a_{js} . The maximum value of a_{js} is 1; the coefficient may also be positive and negative. Such statistical limits are imposed on the aspects and variables that the sum of the quadrates of the coefficients a_{js} and a_j in equation (1) is equal to 1. This means that a_{j1}^2 , for instance, denotes that part of the variance of z_j which may be attributed to the effect of the first aspect.

The value of z_{ji} shown in equation (2) does not represent the value found but the expectation thereof. The value found will differ because deviations occur owing to various causes. These deviations are included by introducing an empirical term T_{ji} , so that equation (2) becomes

$$z_{ji} = a_{j1}F_{1i} + a_{j2}F_{2i} + \dots + a_{jm}F_{mi} + a_jS_{ji} + T_{ji} \quad (3)$$

Usually, however, it is not possible to make any distinction between S and T . In that case equation (2) remains unchanged and S_{ji} is not assumed to be a constant (unlike F_{si}).

Starting from equation (2), it is possible to indicate how the common aspects are held responsible for the correlations between the variables z_j . It can be shown that the correlation coefficient r_{jk} between the variable z_j and z_k is obtained by expressing all data in standard units and subsequently multi-

plying the elements from the equation for z_{ji} by the corresponding elements from the equation for z_{ki} , and then summing the result over the fields N and dividing by N . Assuming there are no correlations between the aspects, the result is

$$r_{jk} = a_{j1}a_{k1} + a_{j2}a_{k2} + \dots + a_{jm}a_{km} \quad (4)$$

In the above equations the only terms known are the z_{ji} and the r_{jk} to be calculated from it. Clearly these data are insufficient for determining the coefficients a_{js} ($j = 1, 2, \dots, n$; $s = 1, 2, \dots, m$) in an unambiguous way, nor, in fact, are the aspect values F_s , for the various fields known either. If these were known it would be possible to arrive at the solution by means of the normal regression analysis. The solution is obtained by giving the aspects limiting properties and starting from the correlation coefficients to be calculated. There is no need to discuss this method of calculation here in greater detail. The solution found is always closely related to the limitations imposed which are partly of a statistical kind. Various solutions are described in the literature on the subject. In the example discussed below a modified centroid method of THURSTONE is applied ³⁾.

The result of a factor analysis is usually shown in tabular form. Table 1 is an example of this. This table is essentially a shortened form of equation (1) with 4 aspects, the specific aspect, the F 's and the plus signs being omitted for the sake of simplicity, and the numerals in columns 2 to 5 being the loadings a_{js} . The numeral in the last column denotes that part of the variance of the variable which is explained by the 4 common aspects.

A geometrical description of the mathematical model of the factor analysis may also help towards a better understanding of the results obtained. The n variables z_j may in this case be represented by n vectors lying in an m -dimensional space; m is the number of common aspects from equation (1). The length of a vector is denoted by h_j which is then equal to the root of the sum of quadrates of the coefficients a_{js} ($s = 1, 2, \dots, m$). The quadrate of this length is shown in the table in the last column under h^2 , this number again being the part explained. The correlation r_{jk} between 2 variables j and k is equal to $h_j h_k \cos \varphi_{jk}$, in which φ_{jk} is the angle between the two vectors.

A variable may also be shown by the m co-ordinates of the end point of the vector with respect to the co-ordinate axes F_s which are vertical in relation to each other. These co-ordinates are the coefficients a_{js} given in the table. It can be inferred from $r_{jk} = h_j h_k \cos \varphi_{jk}$ that this frame of axes may be rotated about the origin without the correlation coefficients changing; the length and the position of the vectors z_j and z_k remain unchanged in relation to each other. The indeterminateness of the solution obtained hinges on this fact. The correlation coefficient may also be shown according to equation (4). It can be seen from this equation that the values of a_{js} and a_{ks} are determined by the position of the aspect axes. These coefficients change when the frame of axes rotates about the origin.

A rotation of this kind, in which F_s changes to F'_s , is often used as an aid to the psychological, agricultural, etc. interpretation of the results ob-

³⁾ The modifications relating to the computational method were suggested to us by DR. G. HAMMING.

tained. (See also chapter 3). For this purpose THURSTONE recommends the rotation to the *simple structure*, as this would bring out the deeper background of the results obtained. A simple structure is theoretically obtained when: a. each row of the table has at least 1 zero (i.e. the relevant aspect has no significance), b. each column has at least m zeroes, and c. in each pair of columns there are at least m variables which in one column have coefficients equal to zero, but not in the other.

3 AGRICULTURAL INTERPRETATION

The psychological and agricultural interpretability is not guaranteed by the solution obtained. This interpretability may possibly be obtained without further study, but there is no certainty as regards this. Methods exist, however, by which it is thought possible to give a meaningful interpretation of the aspects obtained in the manner described above. Owing to the nature of the subject it is difficult in psychology to test the accuracy of these definitions. With the use of factor analysis in agricultural research there is a possibility of interpreting in a meaningful manner an agricultural definition and the aspects obtained. The value of factor analysis becomes doubtful when there is no agreement between the results obtained thereby and the knowledge already acquired in another manner.

The first possibility of agricultural interpretation and testing depends on the properties of equation (2) in which the linear function is expressed for the various fields. In these equations the z_{ji} are known first of all since they are determined. The analysis has also provided an estimate of the coefficients a_{js} . With the aid of equation (2) it is now possible to give an estimate of the value of the common aspects F_{ji} in field i . As our example we again take the aspect of the water supply F_1 . The F_{1i} may be worked out according to equation (2) for all N fields. It is now clear that these calculated aspect values should show a close mean agreement with the factors, to be determined for each separate field, that determine the extent of the water supply. In this connection we have in mind the ground water level and the moisture retention in the profile. Vice versa, of course, it is possible to arrive at an interpretation of the aspect as the water supply aspect from a relation found between the aspect values calculated for each separate field and the observations of ground water level and moisture retention.

A second possibility of agricultural interpretation rests on the following method. In the foregoing we always started from the assumption that only so-called dependent factors such as yield, percentages by weight, etc., are used as variables z_j . But it is not essential to limit ourselves to dependent factors, to the exclusion of the so-called independent factors. Thus it is also possible to supplement the series of dependent factors, viz. in our example the percentages by weight of the various grass species, by a number of independent factors such as soil factors. From a mathematical point of view these supplementary factors make no difference as in this case the correlations between the dependent and independent factors are also determined. From the agricultural point of view such a supplement is all the more attractive because interest is centred on the significance of the soil factors for the composition of the sward. As regards method, these additions are actually significant in that they afford us a better agricultural interpretation of the aspects

obtained. As it is, the coefficients a_{j1} of the water supply aspect in the factors of ground water level and moisture retention should have a comparatively high value in the water supply aspect. Vice versa, high levels of these coefficients may help us in interpreting the aspect from the agricultural point of view.

There is another application of factor analysis which is closely connected with the above. In regression analysis the starting point is a model in which the number of independent factors and the nature thereof are assumed to be known. In regression analysis one is often faced with the difficulty of having to make a choice from among a large number of independent factors. In this case, moreover, it is often impossible to make a choice on theoretical grounds only. A choice must be made, however, as it is impossible to include every factor in the regression analysis. If the variables z_j are properly selected, by studying the result of a factor analysis it is possible to make a fairly justified choice of the independent factors which can be included in the subsequent regression analysis.

4 AN APPLICATION OF FACTOR ANALYSIS TO THE SPRING GROWTH OF GRASS

In this factor analysis use was made of the grass yields in 1951, 1952 and 1953 on 50 experimental sites in the Guelderland Valley. The yield which had grown before about 15th May was taken as the spring yield. Every year there were great differences in yield from field to field. It is important to be able to identify the factors responsible for these differences since according to KEMP (1952) this is the very period during which approximately $\frac{1}{3}$ of the annual yield is obtained. An investigation of the factors which might influence this spring growth shows that they are comparatively large in number. We have tried to obtain an idea of the mutual significance of the various factors with the aid of a factor analysis. We afterwards carried out a regression analysis with the most acceptable factors.

The following variables are included in the factor analysis:

| | |
|-----------------------------------|--------------------------------------|
| 1 pH(KCl) | 13 nitrogenous fertilisation |
| 2 content of organic matter | 14 phosphatic fertilisation |
| 3 content of clay particles | 15 potassic fertilisation |
| 4 content of fine sand | 16 grade of quality pasture |
| 5 U figure (specific surface) | 17 N content of grass |
| 6 P-citr (P status) | 18 P_2O_5 content of grass |
| 7 K-HCl (K status) | 19 K_2O content of grass |
| 8 MgO content (Mg status) | 20 MgO content of grass |
| 9 thickness of humus layer | 21 spring yield in % of annual yield |
| 10 distance of farm (management) | 22 spring yield |
| 11 ground water level | 23 annual yield |
| 12 fluctuation ground water level | |

In the case of most of the variables the choice is obvious enough, but there are a few whose inclusion may possibly require some explanation. In this connection we would observe that it may be useful to include in the factor analysis variables denoting approximately the same thing, e.g. content of clay particles, content of fine sand and U figure (specific surface). The content figures of the grass were used for testing the effect of corresponding soil factors and fertilisations. It may also be anticipated that factors influencing

spring growth will be important for the annual yield. The variables 21 and 23 are included in order to investigate the relation between spring yield and annual yield. The botanical composition of the sward will no doubt also be significant.

The results of the factor analysis, together with the data for 1951, are shown in Table 1; the other years give corresponding results (Fig. 1). We can only discuss a few points with regard to this table.

Table 1 The coefficients a_{js} of the modified centroid solution.

| Variable \ Aspect | F ₁ | F ₂ | F ₃ | F ₄ | h ² |
|-------------------|----------------|----------------|----------------|----------------|----------------|
| 1 | 0.20 | 0.24 | 0.41 | -0.09 | 0.27 |
| 2 | -0.38 | 0.75 | 0.02 | -0.15 | 0.73 |
| 3 | -0.10 | 0.80 | 0.04 | -0.20 | 0.69 |
| 4 | 0.17 | 0.68 | -0.13 | 0.13 | 0.53 |
| 5 | -0.04 | 0.88 | -0.14 | 0.13 | 0.81 |
| 6 | 0.42 | 0.08 | 0.45 | -0.20 | 0.43 |
| 7 | 0.62 | -0.04 | 0.06 | -0.04 | 0.39 |
| 8 | 0.27 | 0.61 | 0.44 | 0.02 | 0.64 |
| 9 | 0.43 | -0.11 | -0.02 | 0.22 | 0.25 |
| 10 | -0.34 | 0.05 | -0.09 | -0.34 | 0.24 |
| 11 | 0.52 | -0.39 | -0.09 | 0.01 | 0.42 |
| 12 | -0.29 | 0.59 | -0.05 | 0.17 | 0.46 |
| 13 | 0.59 | 0.05 | 0.45 | 0.11 | 0.57 |
| 14 | 0.37 | 0.01 | 0.84 | 0.00 | 0.84 |
| 15 | 0.43 | 0.18 | 0.71 | 0.04 | 0.72 |
| 16 | 0.57 | -0.09 | -0.04 | 0.20 | 0.37 |
| 17 | 0.36 | -0.02 | 0.00 | 0.63 | 0.53 |
| 18 | 0.47 | -0.13 | 0.14 | 0.58 | 0.59 |
| 19 | 0.68 | -0.13 | -0.08 | 0.30 | 0.58 |
| 20 | -0.03 | -0.05 | 0.22 | 0.15 | 0.07 |
| 21 | 0.90 | -0.01 | -0.09 | -0.31 | 0.91 |
| 22 | 0.98 | 0.03 | -0.02 | -0.11 | 0.97 |
| 23 | 0.75 | 0.07 | 0.03 | -0.38 | 0.71 |

On studying this table the first thing that strikes us is that great differences exist between the variables as regards the part of the variance which is explained by the four aspects (column h^2). Of the yield variables 21, 22 and 23 the greater part is explained, viz. 91%, 97% and 71% respectively. This is in contrast to the MgO content of the grass of which the variance is only explained as to 7%. Between these two extremes there are all kinds of intermediates.

It is also noticeable that the aspects are of varying significance for the explanation of the variance. In the yield variables aspect 1 is the most important feature, and the other aspects have little if any significance. In view of the high values for the coefficients this aspect might therefore be termed the yield aspect. Aspect 2 is chiefly responsible for explaining the variance of the variable 2, 3, 4 and 5. It may also happen that more than one aspect is found which provides an explanation. This is the case with the fertilisation variables 13, 14 and 15; the variance is chiefly explained by the aspects 1 and 3, and so on.

The investigation was conducted in order to establish the factors on which

spring growth depends. For this reason we will confine ourselves mainly to the first aspect. For a proper understanding of this a rotation to the simple structure is generally desirable. Such a rotation effects no improvement in the present case, one reason being that the coefficients for the yield variables are already very high.

By making use of equation (4) it is also possible to determine from Table 1 which factors are mainly responsible for high correlations. Here we observe that a low coefficient (loading) reduces the significance of a high coefficient in the same aspect for the correlation coefficient. In the case of the yield variables in our example, this means that the significance of the different variables for the yield can be read directly from the first aspect, since the coefficients of the yield variables in aspects 2, 3 and 4 are either zero or small. It can now be seen that several variables are correlated to the spring yield. The highest correlations denote the potassium level, the potassium content of the grass, the nitrogenous fertilisation, the grade of quality and the ground water level. The P-citr, MgO content, phosphatic and potassic content may also be mentioned. Thus the potassic fertilisation is of less significance than the potash in the soil. Here, however, we should observe that these data are not capable of showing what share in the yield is possessed by each separate variable. This is owing to the fact that there are also correlations between the independent factors which cannot be eliminated.

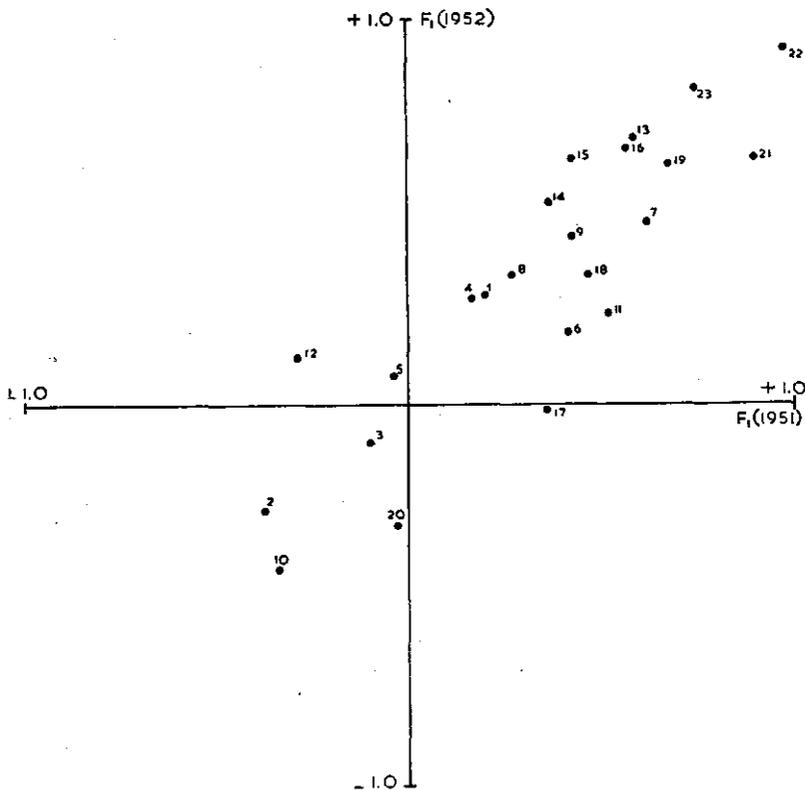


FIG. 1 THE AGREEMENT BETWEEN THE COEFFICIENTS (LOADINGS) a_{j1} FOUND FOR 1951 AND THOSE FOUND FOR 1952.

Figure 1 gives a clear idea of the degree to which the different variables (factors) are correlated to the spring yield. In this graph the coefficients of the first aspect, obtained from the 1951 data, are plotted against the corresponding 1952 coefficients. This graph also shows the extent to which the 1951 results agree with those for 1952. It can be seen from the correlation between 1951 and 1952 that there is a good agreement. The same applies to 1953.

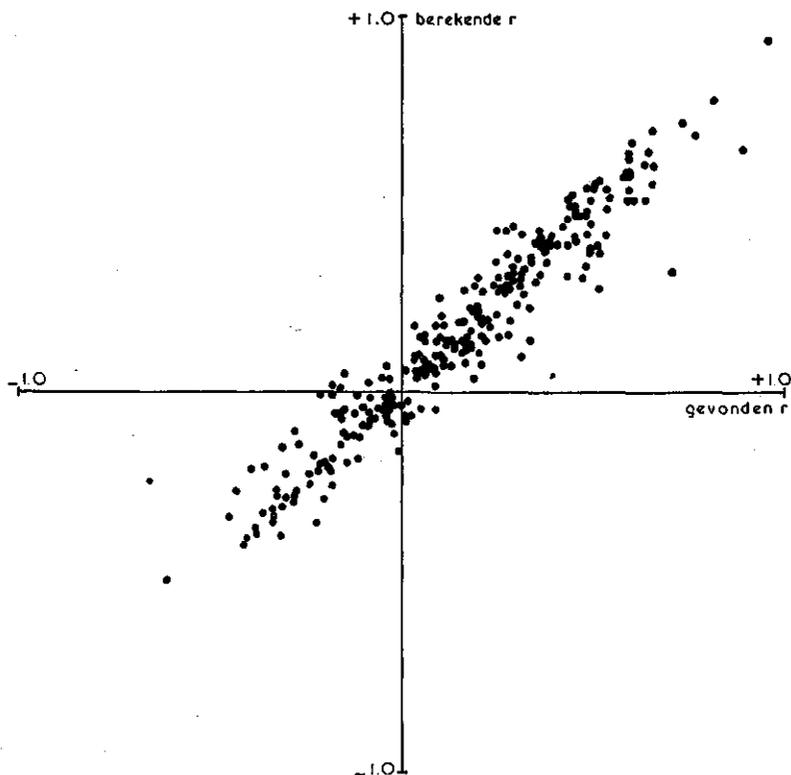


FIG. 2 THE AGREEMENT BETWEEN THE CALCULATED (ORDINATE) AND FOUND (ABSCIS) CORRELATION COEFFICIENTS IN 1951.

The graph in Figure 2 shows to what extent the data from Table 1 are capable of describing the correlations. The correlation coefficients found between each pair of variables are plotted against the corresponding coefficients which are calculated from the table with the data. Equation (4) is employed for this calculation. The calculated correlation coefficient between P-citr and phosphatic fertilisation is thus equal to $0.42 \times 0.37 + 0.08 \times 0.01 + 0.45 \times 0.84 - 0.20 \times 0.00 = 0.54$; the correlation coefficient found is 0.50, etc. Considering the inevitable errors of measurement, etc., the agreement is good. It consequently follows from this that the description of the data by means of Table 1 is acceptable. However, as we have seen, there are other solutions which provide an equally acceptable description of the correlations.

The results obtained were also employed for making a justified choice of the factors which can be used in a numerical – graphical regression analysis (FERRARI and SLUIJSMANS, 1955). The following factors were included in the regression analysis in order to explain the spring growth: P-citr, K-HCl, potassic fertilisation, nitrogenous fertilisation, ground water level and moisture retention of the profile in mm; the data for the last factor were not initially available and could not be included in the factor analysis (FERRARI, VAN DER SCHANS and SONNEVELD, 1957). Summarised briefly, the results for the three years are as follows.

The ground water level had a marked effect in every year. The deeper the ground water level the better is the spring growth. There is some difference in the effect in 1953 since with ground water levels deeper than 60–70 cm below the surface there is no longer a rise in yield, but even a slight fall. The ground water level had the greatest effect in 1951 when about 4.5 kg dry matter per diem were produced per 50 cm difference in ground water level. As an approximate guide the period of growth may be estimated to be about 45 days.

In general an increase in the moisture retention of the profile is favourable up to a certain limit, but in 1951 and 1953 an excessive moisture retention (over 100 mm) was unfavourable. In the case of 1953 this does not correspond to the effect that was found for the ground water level.

The influence of the phosphate level is not so clear; a high phosphate level was favourable in 1951 and 1953, but unfavourable in 1952. An improvement in the potash level has a favourable effect in every year. On the other hand the effect of the potassic fertilisation also varies; in 1952 it was favourable, in 1951 unfavourable, and in 1953 there was no appreciable effect.

The nitrogenous fertilisation had the greatest effect on the spring growth; in all years the effect may be represented by a substantially straight line. The effect is somewhat divergent; in 1953 the growth per diem per hectare is about 12 kg, and in 1951 and 1952 7.5 kg dry matter per 100 kg N.

Table 2 Statistical significance of the effect of the various factors. One, two and three crosses represent the attained levels of significance at the 5%, 1% and 0.1% points respectively (? : almost significant).

| Factor Year | P-citr | K-HCl | Potassic fertilisation | Nitrogenous fertilisation | Ground water level | Moisture retention |
|----------------|--------|-------|---------------------------|------------------------------|--------------------------|-----------------------|
| 1951 | ++ | +++ | ++ | +++ | ? | ++ |
| 1952 | | | | +++ | + | ++ |
| 1953 | | | | +++ | + | + |

The statistical significance of the found effect of the factors is shown in Table 2. The effect of the ground water level, the moisture retention and the nitrogenous fertilisation is significant in all cases. In the case of the other factors this significance is lacking in two out of the three years. It is noticeable that it was only during the first experimental year that it was possible to calculate a significance for these factors.

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