Agricultural Water Management, 4 (1981) 63—82
Elsevier Scientific Publishing Company, Amsterdam — Printed in The Netherlands

RESIDENCE TIMES OF WATER AND SOLUTES WITHIN AND BELOW THE ROOT ZONE

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ABSTRACT

Raats, P.A.C., 1981. Residence times of water and solutes within and below the root zone. Agric. Water Manage., 1981.

The progress of a parcel of water or solute in the course of time can be determined by integrating its speed along its path. This basic information can be used to describe the fate of collections of parcels of water forming a surface or occupying a region and to formulate input/output relationships characterizing transport across a region. It is shown that within the root zone the speed of the water or solute depends primarily on the components of the overall water balance, the average water content, and the distribution of the water uptake. Particular attention is given to recent attempts to infer water uptake from salinity data. Transport to drains, ditches, or streams induced by an input distributed uniformly over the soil surface is discussed in detail. If the ratio of the half-spacing between drains, ditches, or streams and the depth to the impermeable layer is larger than about five, then i) the isochrones are horizontal, except close to the outlets, and ii) the transit time density distribution is approximately exponential, i.e., the system approximates an apparently well-mixed system. Methods for determining transit time density distributions for more complicated flow patterns are discussed briefly. Estimates are also given for the retardation due to adsorption, for the influence of reactions, and for, the often small, influence of dispersion.

I INTRODUCTION

Traditionally the main concern of research related to water management has been to determine the quantities of water being transported and the distribution of pressure head and water content. But lately the quality of water is of at least as much interest. One possible approach to the management of water quality is to split the problem in two parts:

* find the space-time trajectories of parcels of water;
* determine the changes in quality of these parcels.

The space-time trajectories of parcels of water can be calculated from:

$$t-t_0 = \int_{s_0}^{s} v^{-1} ds, \qquad (1)$$

where $t-t_0$ is the time it takes for a parcel to travel from s_0 to s; and v is the speed along its path. The speed v can, in principle, be determined by solving the appropriate flow equation.

The second objective has many aspects:

* the change in the concentration of a parcel of water due to evaporation at or near the soil surface and due to selective uptake of water by plant roots;
* the gain or loss of solutes by parcels of water as a result of diffusive and dispersive mixing with their surroundings;
* the retardation of solutes relative to the water resulting from adsorption;
* changes due to precipitation or dissolution and due to decay or production.

At any point in the soil, the balance of mass for the water may be written as:

$$\partial\theta/\partial t = -\nabla.(\theta\underset{\sim}{v}) - \lambda T, \qquad (2)$$

where t is the time; ∇ is the vector differential operator; θ is the volumetric water content; $\underset{\sim}{v}$ is the velocity of the water; T is the rate of transpiration; and λ is the spatial distribution function for the uptake of the water. The flux, $\theta\underset{\sim}{v}$, of the water is given by Darcy's law:

$$\theta\underset{\sim}{v} = - k\nabla h + k\nabla z, \qquad (3)$$

where h is the tensiometer pressure head; k is the hydraulic conductivity; and z is the position in the gravitational field.

Also at any point in the soil, the balance of mass for a solute may be written as:

$$\frac{\partial}{\partial t}\theta c = - \nabla.\underset{\sim}{F}_s - \frac{\partial}{\partial t}\mu_a - \frac{\partial}{\partial t}\mu_f - \lambda_s N, \qquad (4)$$

where c is the concentration of the solute in the aqueous phase; $\underset{\sim}{F}_s$ is the flux of the solute; μ_a and μ_f are the densities of the solute per unit volume in the adsorbed and immobile phases; N is the rate of uptake by plant roots; and λ_s is the uptake distribution function. The flux $\underset{\sim}{F}_s$ is assumed to be the sum of a convective component $\theta\underset{\sim}{v}c$ and a diffusive component $- \mathcal{D}\nabla c$:

$$\underset{\sim}{F}_s = \theta\underset{\sim}{v}c - \mathcal{D}\nabla c. \qquad (5)$$

Combining equations (4) and (5) and using (2) gives:

$$\frac{\partial}{\partial t}c + \underset{\sim}{v}.\nabla c = \frac{\partial}{\partial t}c\Big|_{\text{parcel of water}}$$

$$= \left\{ \lambda Tc + \nabla . D \nabla c - \frac{\partial}{\partial t} \mu_a - \frac{\partial}{\partial t} \mu_f - \lambda_s N \right\} / \theta. \qquad (6)$$

On the left hand side of equation (6) appears a material time derivative, i.e., a time derivative following the motion of a parcel of water. On the right hand side appear five possible causes for change of the concentration of a parcel of water. They are all proportional to the dryness θ^{-1}, implying that for a given cause the absolute value of the rate of change of c is largest in sands, intermediate in loams, and smallest in clays.

In using equation (1) to describe the space-time trajectories of parcels of water and equation (6) to describe the change of the concentration of these parcels, I emphasize one of many possible approaches to the analysis of simultaneous transport of water and solutes. My aim is to demonstrate that this approach gives good qualitative insight in the fate of solutes within the root zone and in the region between the water table and the drains, ditches or streams.

2 THE ROOT ZONE

2.1 Depth-time Trajectories for Downward Flow of Parcels of Water

Within the root zone the time averaged velocity of the water at depth z is found by integrating equation (2) (Raats, 1975):

$$v = \theta v/\theta = \left\{ R+I-E-T \int_0^z \lambda dz \right\} / \theta = \left\{ D/T + \int_z^\infty \lambda dz \right\} T/\theta, \qquad (7)$$

where R is the rate of rainfall; I is the rate of irrigation; E is the rate of evaporation from the soil surface; T times the integral of λ from o to z represents the cumulative rate of uptake above depth z; and D = R+I-E-T is the rate of drainage. Below the root zone $\int_0^z \lambda dz = 1$ and the velocity v approaches the constant value:

$$v = v_\infty = (R+I-E-T)/\theta = D/\theta. \qquad (8)$$

Introducing equation (7) into equation (1) gives:

$$t - t_i = \int_{z_i}^{z} v^{-1} dz = (\theta/T) \int_{z_i}^{z} \left\{ D/T + \int_z^\infty \lambda dz \right\}^{-1} dz. \qquad (9)$$

Equation (9) describes depth-time trajectories of parcels of water starting at time t_i at depth z_i. Below the root zone the trajectories approach straight line asymptotes with a slope equal to the velocity D/θ defined by equation (8). The intercept of these asymptotes with the z-axis is given by:

$$d_i = z_i + \int_{z_i}^{\infty} (1 - v_\infty/v) dz. \qquad (10)$$

Introducing (7) into (10) shows that d_i can be expressed entirely in terms of z_i, D/T and λ:

$$d_i = z_i + \int_{z_i}^{\infty} \left\{ \int_z^{\infty} \lambda dz \right\} \left\{ D/T + \int_z^{\infty} \lambda dz \right\}^{-1} dz. \quad (11)$$

If $\lambda = o$ for $z \geqslant \delta$ then δ may be regarded as the rooting depth. If δ is finite then equation (9) implies:

$$t_\delta - t_i = \int_{z_i}^{\delta} v^{-1} dz = (\theta/T) \int_{z_i}^{\delta} \left\{ D/T + \int_z^{\delta} \lambda dz \right\}^{-1} dz, \quad (12)$$

while equation (11) reduces to:

$$d_i = \delta - v_\infty (t_\delta - t_i). \quad (13)$$

If $z_i = z_0 = 0$ then the time interval $t_\delta - t_i = t_\delta - t_0$ represents the residence time in the root zone. Fig. 1 shows a graphical interpretation of equation (10). At any depth the integrand $(1-v_\infty/v)$ is the fraction of the flux which will be taken up below that depth.

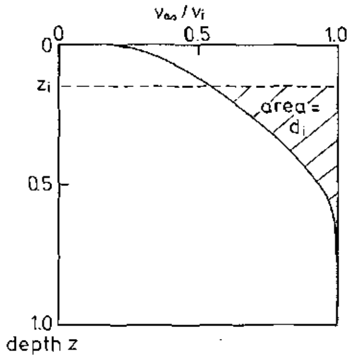

Fig. 1. Graphical interpretation of equation (10)

The second terms on the right hand sides of equations (10) and (11) represent the cumulative displacement induced by water adsorbed by roots at some distance below the soil surface. A large rooting depth induces a relatively rapid leaching of the soil solution over a large depth even if the rate of drainage is small. The role of uptake of water by plant roots in restricting the salinity near the soil surface was already clearly understood by Hilgard and Loughridge (1895, 1906). Fig. 2 shows the distribution in March at the end of the wet season. Most roots of the native spring growth of herbs and flowers were found in the top 40 cm. Near the end of the dry season in September the distribution of the salts had hardly changed from that shown in Fig. 2.

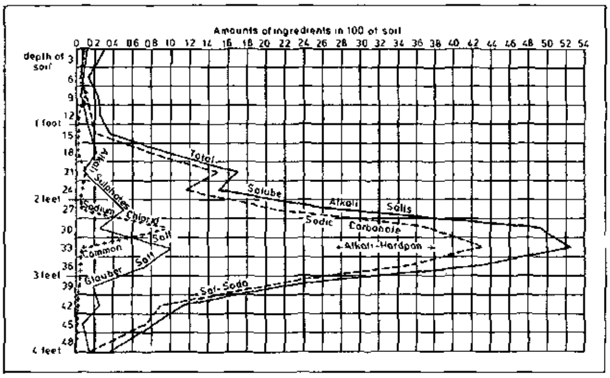

Fig. 2. Distribution of salts in March 1895 for unirrigated, black alkali land of Tulare Experiment Substation, California (after Hilgard and Loughridge, 1895, 1906).

Ancient Mesopotamian farmers may already have profited from water extraction over large depth. Jacobson and Adams (1958) wrote:

"In spite of almost proverbial fertility of Mesopotamia in antiquity, ancient control of the water table was based only on avoidance of overirrigation and on the practice of weed-fallow in alternate years. As was first pointed out by J.C. Russel, the latter technique allows the deep-rooted *shoq (Proserpina stephanis)* and *agul (Alhagi maurorum)* to create a deep-lying dry zone against the rise of salts through capillary action. In extreme cases longer periods of abandonment must have been a necessary, if involuntary, feature of the agricultural cycle. Through evapotranspiration and some slow draining they could eventually reduce an artifically raised water table to safe levels".

2.2 Change of the Solute Concentration of Parcels of Water

If only changes in solute concentration due to selective uptake of water by plant roots are considered then equation (6) reduces to:

$$\partial c/\partial t + v\partial c/\partial z = \left.\frac{dc}{dt}\right|_{parcel} = \frac{\lambda T}{\theta} c. \qquad (14)$$

Thus the rate of increase of the concentration is proportional to the rate of uptake of water λT, the dryness θ^{-1}, and the current concentration of the parcel of water. Integration of equation (14) gives:

$$c_{parcel} = c_i \exp \int_0^t (\lambda T/\theta)dt,$$

$$= c_i \exp \int_{z_i}^{z} (\lambda T/\theta v)dz,$$

$$= c_i \exp \int_{z_i}^{z} \lambda \left\{D/T + \int_z^\infty \lambda dz\right\}^{-1} dz, \qquad (15)$$

where c_i is the concentration at time t_i.

The dependence on the initial solute distribution enters in equations (9) and (15) through (t_i, z_i, c_i). Setting $z_i/\delta = 0$ in equation (9) and replacing in the resulting equation = by < defines the region in which the distribution of the concentration at time t is determined by the time course of the concentration at the soil surface for times $t > t_i$:

$$t-t_i < \int_0^z v^{-1} dz = (\theta/T) \int_0^z \left\{ D/T + \int_z^\infty \lambda dz \right\}^{-1} dz. \qquad (16)$$

In particular, if the concentration at the soil surface has a constant value c_o then in the region defined by the inequality (16) a time-invariant distribution of the concentration will have been reached. An explicit expression for this concentration profile valid for an arbitrary uptake distribution function λ is obtained by setting $c_i = c_o$ and $z_i = 0$ in equation (15):

$$c = c_o \exp \int_0^z \lambda \left\{ D/T + \int_z^\infty \lambda dz \right\}^{-1} dz. \qquad (17)$$

The steady profile for some ratio $(D/T)_i$ and uptake distribution λ_i may serve as an initial state for a transition to another ratio D/T and another uptake distribution λ. The space time trajectories will be given by:

$$t-t_o = (\theta/T)_i \int_{z_o}^{z_i} \left\{(D/T)_i + \int_{z_i}^\infty \lambda_i dz\right\}^{-1} dz$$

$$+ (\theta/T) \int_{z_i}^{z} \left\{D/T + \int_z^\infty \lambda dz\right\}^{-1} dz, \qquad (18)$$

and combining equations (15) and (17) will give:

$$c/c_0 = \exp\left\{\int_0^{z_i} \frac{\lambda_i}{(D/T)_i + \int_z^{\infty} \lambda_i dz} dz + \int_{z_i}^{z} \frac{\lambda}{(D/T) + \int_z^{\infty} \lambda dz} dz\right\} \quad (19)$$

More generally, gradual changes of D/T and λ could be treated in a similar manner by incremental extensions of equations (18) and (19):

$$t-t_0 = \sum_{n=1}^{N} (\theta/T)_n \int_{z_{n-1}}^{z_n} \left\{ (D/T)_n + \int_{z_n}^{\infty} \lambda_n dz \right\}^{-1} dz, \quad (20)$$

$$c/c_0 = \exp \sum_{n=1}^{N} \int_{z_{n-1}}^{z_n} \lambda_n \left\{ (D/T)_n + \int_z^{\infty} \lambda_n dz \right\}^{-1} dz. \quad (21)$$

2.3 Results for Specific Uptake Distributions

The theory presented above applies to any uptake distribution function λ. In the literature various aspects of the theory have been worked out in detail for two special cases:

1. Step function uptake distribution:

$$\lambda = \delta^{-1}, \quad 0 < z < \delta, \quad (22)$$

$$\lambda = 0, \quad z \geqslant \delta, \quad (23)$$

where δ is the rooting depth. This assumption was used in a pioneering paper by Gardner (1967), in a recent review by Parlange (1980), and in an analysis of supply of water and nutrients in soilless culture (Raats, 1980c).

2. Exponentially decreasing uptake distribution:

$$\lambda = \delta_e^{-1} \exp - z/\delta_e \quad (24)$$

where δ_e corresponds to the depth of an equivalent, uniform root system with the same rate of uptake at the soil surface and rate of transpiration T. Elsewhere I have presented in detail various implications of equation (24). Rawlins (1973) and Jury et al. (1977) used equation (24) for $0 < z < \delta$ and equation (23) for $z \geqslant \delta$.

Fig. 3 shows depth-time trajectories for the water under an orange tree based upon equations (9) and (24) and data that will be discussed later on (cf. Fig. 15 of Van Schilfgaarde, 1977). Fig. 4 shows steady salinity profiles calculated from equations (17) and (24) for leaching fractions L = 0.2 and 0.05

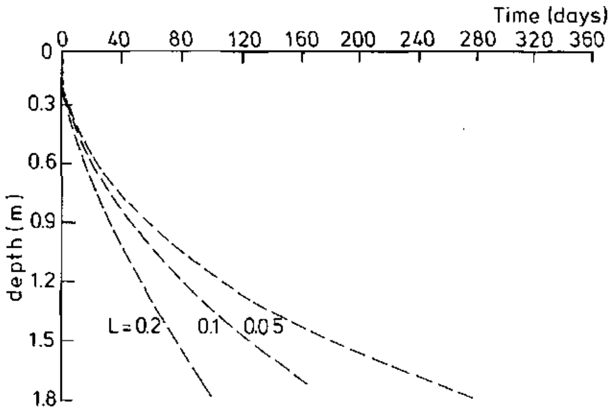

Fig. 3. Depth-time trajectories of parcels of water under an orange tree, based on equation (24) with δ_e = 0.4 m for L = D/(R+I) = 0.05, 0.1, and 0.2.

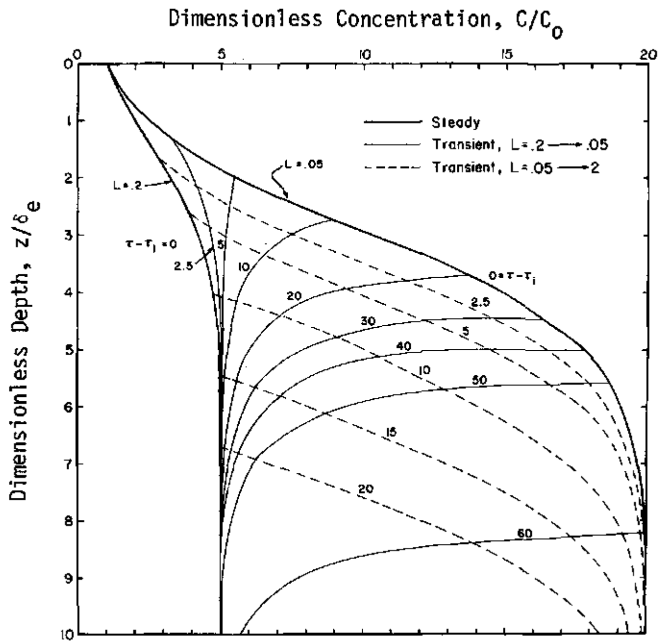

Fig. 4. Steady salinity profiles for L = D/(R+I) = 0.05 and 0.2, and transient salinity profiles for various dimensionless times $\tau = \frac{E + T}{\theta \delta_e} t$ (after Raats, 1975).

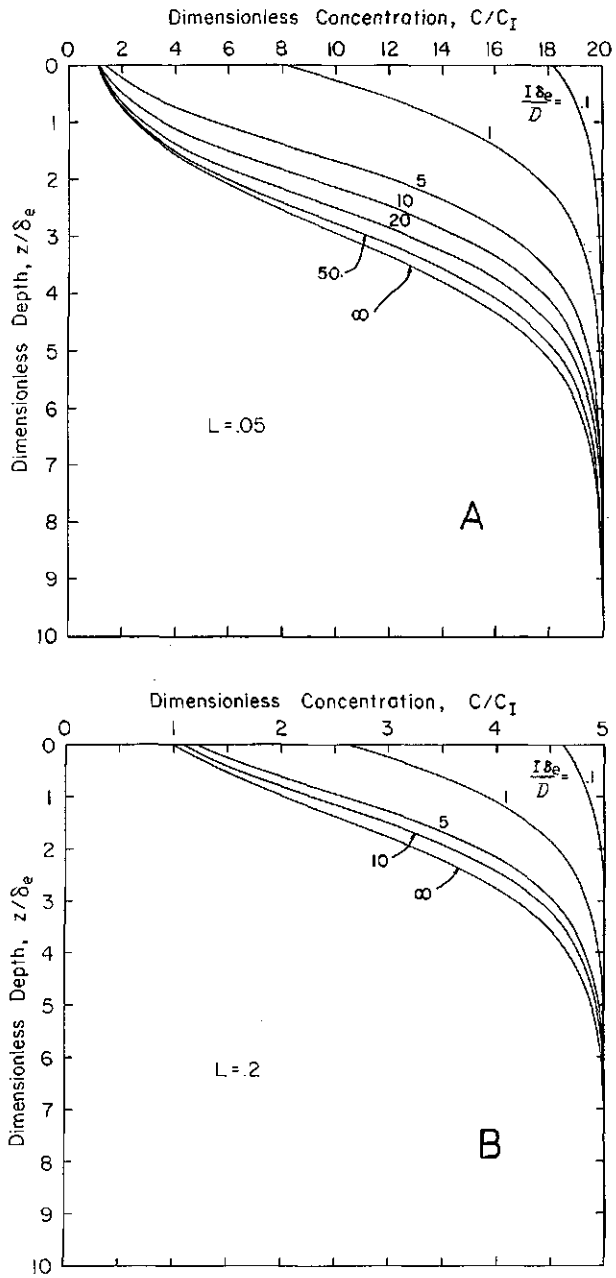

Fig. 5. Influence of dispersion upon steady salinity profiles
A : L = 0.05, B : L = 0.2.

(Raats, 1974a, b; 1975). Also shown are transient salinity profiles at various times calculated from equations (18), (19), and (24) for transitions from L = 0.05 to 0.2 and the reverse (Raats, 1975). In going from L = 0.05 to 0.2 the "old" salinity profile experiences virtually a parallel downward displacement. In going from L = 0.2 to 0.05, the salinity profile at later times develops a bulge, whose front becomes steeper in the course of time. The salinity profiles reported by Peck (1975, 1977) have shapes similar to these transient profiles, but their origin is perhaps more complicated (Peck et al., 1981).

Diffusion and dispersion will counteract the steepening due to selective uptake of water. Figs. 5a and b show the influence of diffusion and dispersion upon steady salinity profiles (Raats, 1977). The salinity profiles are functions of the leaching fraction L and the Peclet number $I\delta_e/D$. For a given leaching fraction, the Peclet number is a measure of the relative importance of convective and diffusive transports.

2.4 Lognormal Distribution of the Concentration Below the Root Zone

The one-dimensional, vertical transport model formulated above implies that, at any time, the concentration at any depth is uniform. However, wide variations of the concentrations of individual samples taken at a certain depth in the lower part of the root zone have been reported independently by Oster and Wood (1977) and by Wierenga and Sisson (1977). In both cases the distribution of log c was found to be normal; in other words the distribution of c was found to be lognormal. It is well known that lognormal distributions can be generated by a process in which the change of the random variable at any step of the process is a random proportion of the previous value of the random variable. This type of genesis of lognormal distributions was first formulated in 1903 by the Dutch astronomist Kapteyn and is now known as the theory of proportionate effect (Aitchison and Brown, 1957). If $\lambda T/\theta$ is a random variable, then according to equation (14) the change of c with a given change of t will be a random proportion of the previous value of c. Given the likely random nature of λ, θ, and T, it is not surprising that the distribution of c tends to being lognormal.

2.5 Inference of the Distribution of Water Uptake from Salinity Data

The uptake of the water does affect the distribution of the solute. Roughly, as water is taken up and solute is excluded by the plant roots, the concentration of the solute increases. Thus the space-time distribution of the solute will in part reflect the distribution of the uptake of the water. This suggests that the distribution of the uptake of the water can perhaps be inferred from the distribution of the solute. In the following the transport of water and solutes will be assumed to be one-dimensional (see also Raats, 1980b). For one-dimensional transport, solving equation (5) for the water flux θv gives:

$$\theta v = F_s/c + D\frac{\partial}{\partial z}\ln c. \tag{25}$$

Introducing equation (25) into equation (2) and solving for λT gives:

$$\lambda T = -\frac{\partial}{\partial t}\theta - \frac{\partial}{\partial z}F_s/c - \frac{\partial}{\partial z}D\frac{\partial}{\partial z}\ln c. \tag{26}$$

Integration of the mass balance for the solute expressed in equation (4) gives an expression for F_s:

$$F_s = F_{so} - \int_{z_0}^{z}\left\{\lambda_s N + (\theta c + \mu_a + \mu_f)\right\}dz, \tag{27}$$

$$= F_{so} - \frac{d}{dt}\int_{z_0}^{z}(\theta c + \mu_a + \mu_f)dz - N\int_{z_0}^{z}\lambda_s dz. \tag{28}$$

Equations (27) and (28) simply show that the flux of solute at depth z is equal to the flux of solute at depth z_0 minus the time rate of change of storage between z_0 and z. Introducing equation (28) into equation (26) gives:

$$\lambda T = -\frac{\partial}{\partial t}\theta - F_{so}\frac{\partial}{\partial z}c^{-1} - \frac{\partial}{\partial z}D\frac{\partial}{\partial z}\ln c$$

$$+ \frac{\partial}{\partial z}c^{-1}\left\{\frac{d}{dt}\int_{z_0}^{z}(\theta c + \mu_a + \mu_f)dz + N\int_{z_0}^{z}\lambda_s dz\right\}. \tag{29}$$

If the flow is steady and the effects of dispersion, adsorption, precipitation and uptake are negligible, then equation (29) reduces to:

$$\lambda T = -F_{so}\frac{d}{dz}c^{-1}. \tag{30}$$

According to equation (30), the rate of water uptake may be calculated as the product of the salt flux F_{so} and the negative of the slope of the dilution profile, $-dc^{-1}/dz$. Gardner (1967) appears to have been the first to realize this. He wrote: "Equation (5) (= (30) above) gives us a relation between the water uptake pattern with depth and the concentration distribution. Since it is easier to measure the concentration than to measure the flux directly, the concentration gradient may give a better measure of w (= λT above) than the divergence of the flux density. Furthermore, the lower limit of the water uptake can be ascertained from the depth at which the concentration becomes constant".

It appears that equation (30) was not noticed for the following seven years (Raats, 1974a, b). Oster et al. (1974) applied the method to brome grass under high frequency irrigation in outdoor lysimeters. The cumulative water uptake distribution estimated from chloride data was 60, 80 and 90% for depths of 15, 30, and 45 cm, respectively. Evaporation losses in the 0-1 cm depth interval accounted for about half of the applied water. Plots of the log of the cumulative uptake as a function of depth were approximately linear. This meant that the distribution of the rate of uptake could be approximated by equation (24).

For two different lysimeters the rooting depth parameters δ_e turned out to be 8.5 cm and 9.6 cm, respectively. Assuming a dispersion coefficient of 0.05 $cm^2 day^{-1}$, the third term on the right hand side of equation (29) had a negligible effect on the estimate of δ_e. Fig. 5 shows that if the leaching fraction is small and the Peclet number is rather small, then the influence of dispersion will be noticeable. If electrical conductivity data are used as a basis, then dissolution/precipitation described by the last term on the right hand side of equation (12) also needs to be considered. In the lysimeters the sum of the mineral equilibria and diffusion corrections to the rate of uptake was zero to the 15 cm depth. At greater depths the mineral equilibria correction was dominant and increased the calculated rate of uptake by as much as 30%.

The steady state distribution of chloride was also used to estimate the distribution of the water uptake under an orange tree (Van Schilfgaarde, 1977; Dirksen et al., 1979). The cumulative relative water uptakes were 64, 86, 93, 97, and 98%, respectively for depths of 0.3, 0.6, 0.9, 1.2, and 1.5 m, respectively, corresponding roughly to an equivalent rooting depth δ_e of 0.4 m. This information can in turn be used to calculate depth-time trajectories of parcels of water. Assuming $\theta = 0.5$, $T = 7$ mm day^{-1}, and $\delta_e = 0.4$ m, Fig.3 shows such time courses for leaching fractions 0.05, 0.1, and 0.2 (cf. Fig. 15 of Van Schilfgaarde, 1977).

In a laboratory study of space/time distributions of matric and osmotic potentials of daily irrigated alfalfa, Dirksen et al. (1980) estimated the distribution of the water uptake on two different days from hydraulic data and from the salt flux and salinity sensor readings on another day. The agreement between the two estimates was good. Between 80 and 90% of the uptake occurred above 0.50 m.

Jury et al. (1978a, b, c) used soil salinity sensor and chloride data to estimate fractional water uptake above a depth of 5 cm and in the layer 0-20 cm, respectively, in a lysimeter experiment with wheat and sorghum. Corrections for precipitation were made by using the chemical equilibrium model of Oster and Rhoades (1975) and calculating the electrical conductivity of the resulting mixed salt solutions by the method of McNeal et al. (1970). It turned out that 50% or more of the water was evaporated or was taken up within 5 cm from the soil surface.

Thus far, only one-dimensional flows have been discussed in this section. To infer anything about the distribution of the water uptake from the distribution of the salinity, one must have separate information about the flow pattern. For example, it can be shown that the proper generalization of equation (30) to multi-dimensional flows is given by :

$$\lambda T = - (A_0/A)\, F_{so} \frac{\delta c}{\delta s}^{-1}, \tag{31}$$

where $\delta/\delta s$ is the directional derivative along a streamline; A is the cross sectional area of a stream tube; and the subscript o indicates a reference point along the same stream tube. A logical next step would be to consider experiments involving localized irrigation or uptake patterns under trees and sufficiently detailed measurements of the salinity distribution and the flow pattern so that equation (31) can be used.

3 THE SATURATED ZONE

3.1 The Input-Output Relationship

Steady, multi-dimensional convective transport of solutes was discussed in detail in two recent papers (Raats, 1978a, b), the first paper dealing with the general theory, the second paper with specific flow problems. The time required for a parcel of water to move from one point to another along a streamline can be determined and this basic information can then be used to describe collections of parcels of water forming a surface. For any geometry and boundary conditions, the cumulative transit time distribution function, q, is defined as the fraction of the stream tubes with transit times smaller than τ. The transit time density distribution is defined as the derivative of q with respect to τ and may be regarded as the transfer function, $T[\tau]$, for the flow system:

$$T[\tau] = dq/d\tau, \tag{32}$$

where the square brackets denote functional dependence. The general relationship between the input I and the output O can be written as:

$$O[t] = \int_{t_0}^{t} T[\tau]\, I[t-\tau]\, d\tau + \text{a contribution of solutes present at time } t_0. \tag{33}$$

The function q can be determined by measuring the concentration of an ideal tracer in the output following a step change of the concentration in the input. The function $dq/d\tau$ can be determined by measuring the output resulting from a pulse distributed uniformly in the input.

To determine the travel time density distribution one must use equation (1) to calculate the transit times along the different stream lines. For steady flow, equation (1) is equivalent to the kinematical result:

$$\tau = t - t_\alpha = (\theta v)_\alpha^{-1} \; A_\alpha^{-1} \int_{s_\alpha}^{s} \theta A ds, \tag{34}$$

where A is the cross-sectional area of an infinitesimal stream tube and the subscript α refers to the input surface. If the flow pattern and the distribution of θ are known then the right hand side can be evaluated by graphical and numerical procedures. Other methods are based on an equation resulting from substituting Darcy's law into equation (1) and transforming from integration with respect to s to integration with respect to the total head H:

$$\tau = t - t_\alpha = - \int_{H_i}^{H} (\theta/k) \; |\nabla H|^{-2} \; dH. \tag{35}$$

For a few problems the right hand side of equation (35) has been evaluated analytically. If the distributions of θ, k, and H are known from an analytical or a numerical solution of the flow problem or from measurements, then the right hand side of equation (35) can always be evaluated numerically.

3.2 Apparently well mixed systems

The flow pattern shown in Fig. 6 is induced by a uniform input at the water table. The region in which the flow occurs is assumed to be a rectangle. A drain,

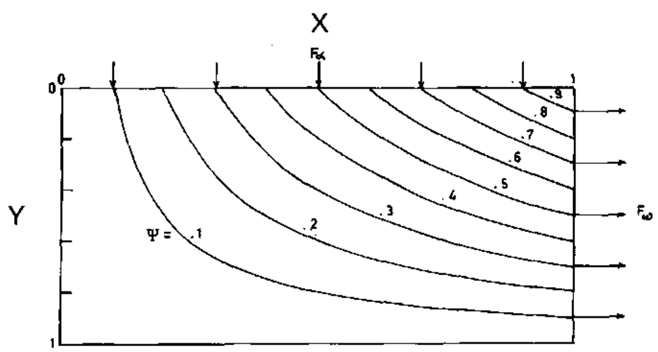

Fig. 6. Flow pattern with uniformly distributed input and output.

ditch, or stream is assumed to be located on the right hand side. For the time being the output is assumed to be uniformly distributed. On the left hand side is the midplane between drains, ditches, or streams. The bottom is assumed to be impermeable. In other words, at steady state the input at the water table is equal to the output at the drain, ditch or stream.

The turnover time $\bar{\tau}$ of the system is equal to the volume θXY of water in the flow system divided by the flux $F_\alpha X$:

$$\bar{\tau} = \frac{\theta XY}{F_\alpha X} = \frac{\theta Y}{F_\alpha} . \tag{36}$$

The turnover time is the characteristic time for convective transport of solutes through the system. The horizontal and vertical components of the velocity are given by:

$$v_x = x/\bar{\tau} \tag{37}$$

$$v_y = (Y - y)/\bar{\tau}. \tag{38}$$

Both components are inversely proportional to the turnover time $\bar{\tau}$. They are shown in Figs. 7 and 8. The horizontal component of the velocity increases linearly from zero at the midplane to its maximum at the drain, ditch or stream.

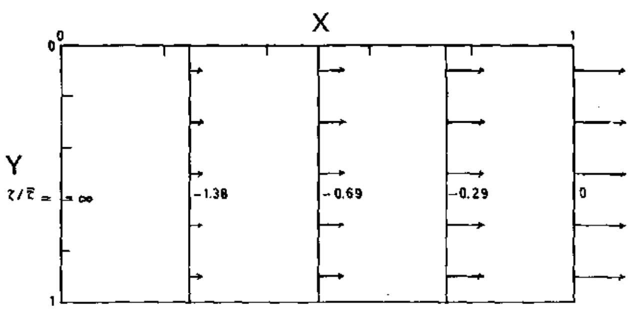

Fig. 7. Horizontal component of the velocity and associated vertical isochrones.

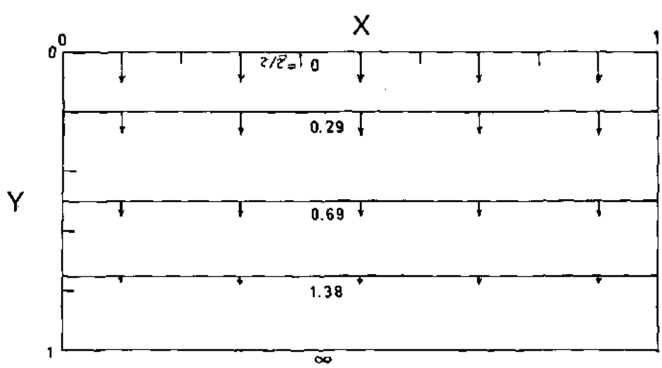

Fig. 8. Vertical component of the velocity and associated horizontal isochrones.

The vertical component of the velocity decreases linearly from its maximum at the soil surface to zero at the impermeable base. Since for this particular flow the vertical component of the velocity at a certain depth does not depend on the distance from the drain, ditch, or stream, parcels of water situated in a horizontal plane will jointly move down and remain in a horizontal plane. Integration of equation (37) shows that a parcel introduced into the system at $x = x_0$ will arrive at a drain, ditch, or stream located at $x = X$ after a time interval τ given by :

$$\tau = \bar{\tau} \ln X/x_0. \qquad (39)$$

The implications of equation (39) can be best understood by considering a pulse of solutes in the input and the resulting output. Fig. 9 shows the distribution of the fraction of the solute remaining in the flow system at successive times. The remainder of the solute is uniformly distributed. It is as if such a band of solute is elastic and is being stretched uniformly. The output is largest at time zero and decreases exponentially with time. In other words, the transit time density distribution is given by :

$$dq/d\tau = \bar{\tau}^{-1} \exp -\tau/\bar{\tau}, \qquad (40)$$

and the cumulative transit time distribution is given by :

$$q = 1 - \exp - \tau/\bar{\tau}. \qquad (41)$$

This means that the system behaves as an apparently well mixed system. It is as if there is a steady flux $F_\alpha X$ of water through a perfectly stirred reservoir with a volume —XY. Of course, in reality the model is based on piston displacement and the transit-time density distribution is entirely dictated by the flow pattern shown in Fig. 6.

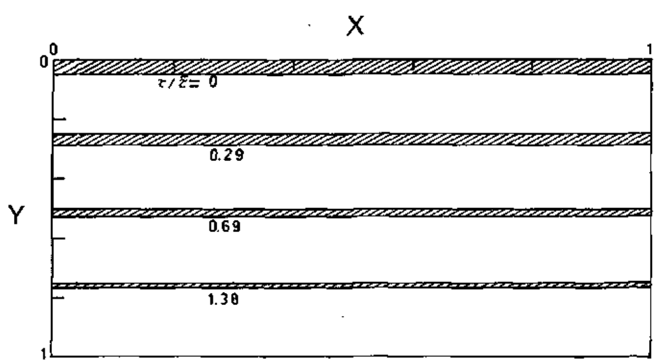

Fig. 9. Fate of a horizontal layer of solute.

It appears that an exponential distribution of arrival times was considered by Eriksson (1958). It was used later without further justification in theoretical discussions of input/output relationships (e.g. Eriksson, 1971; Nir and Lewis, 1975), and in attempts to fit experimental data on tritium in river basins (Eriksson, 1963), leaching of solutes from a laboratory model (Peck, 1973) and the chloride balance of some farmed and forested catchments in south-western Australia (Peck and Hurle, 1973). The relationship between the exponential travel time density distribution and the flow pattern shown in Fig. 6 gradually emerged in papers by Eldor and Dagan (1972), Ernst (1973), and Gelhar and Wilson (1974). Elsewhere I have discussed in detail implications and limitations of this important special case of the general theory (Raats, 1977, 1978b, 1980a; see also Rice and Raats, 1980). Here I note only that for $X/Y < 5$ the deviations from the flow pattern shown in Fig. 6 due to convergence to the drain, ditch, or stream must be taken into account (cf. Ernst, 1973 and Raats, 1978b, Fig. 7).

3.3 More Complex Flow Problems

Some of the literature on transit time density distribution for more complicated geometries was discussed elsewhere (Raats 1977, 1978b). Other examples, related to regional flow problems, are given by Nelson (1978). Here I will give just one example. If in the flow problem discussed in detail in subsection 3.2 the depth $Y \to \infty$, then the turnover time $\bar{\tau}$ is no longer useful. Ernst (1973) showed that in the limit $Y \to \infty$ the transit times are distributed according to:

$$\tau/\bar{\tau}_\infty = (1-x/X)\cot\frac{\pi x}{2X}, \tag{42}$$

where:

$$\bar{\tau}_\infty = \theta X/F_\alpha. \tag{43}$$

The theory discussed thus far applies to convective transport of a solute distributed uniformly over the input surface, and not subject to diffusion and dispersion, to adsorption, and to production or decay. For the flow pattern shown in Fig. 6, Eldor and Dagan (1972) have shown that diffusion and dispersion have only a very small influence upon the distribution of the solute in the horizontal direction and thus any resulting deviations from the transit time density distribution defined by equation (40) would be expected to be small. Linear adsorption can be accounted for by multiplying characteristic times such as $\bar{\tau}$ and $\bar{\tau}_\infty$ defined by equations (36) and (43) by a retardation factor $(\theta+k)/\theta$, where k is the adsorption constant (Raats, 1980a). If the adsorption is nonlinear, then the speed of the parcels of solute depends on the concentration and a detailed numerical calculation will generally be necessary, except for one aspect. For nonlinear adsorption with so-called favorable adsorption isotherms concentration profiles along streamlines will have a tendency to be compressed to shocks on the upstream side and the c-profile will have a tendency to be stretched on the downstream side. For unfavorable isotherms the reverse will occur. If c_α and c_∞ are the concentrations on the two sides of the shock then the average adsorption capacity over the range c_α to c_β serves as a retardation factor for the velocity of shock waves.

To accommodate non-uniform distribution of solute over the input surface and linear production or decay, the transfer function T [τ] is generalized to (Raats 1978a, 1980a):

$$T = dq/d\tau . r . \exp(\alpha t), \qquad (44)$$

where r is the input density distribution and α is a rate constant. Linear production and decay with a rate constant(> 0 for production, and <0 for decay) is accounted for by a factor exp αt.

4 CONCLUDING REMARKS

The principles governing the transport of solutes in the natural environment are quite well understood. Nevertheless quantifying a particular field situation remains a difficult task. Fortunately, in many cases it is possible to estimate the residence time within and below the root zone from the mass balance and flow pattern for the water and some information about the interaction of the solute with its environment.

5 REFERENCES

Aitchison, J. and Brown, J.A.C., 1957. The lognormal distribution. University Press, Cambridge.

Dirksen, C., Oster, J.D. and Raats, P.A.C., 1979. Water and salt transport, water uptake, and leaf water potential during regular and suspended high frequency irrigation of citrus. Agric. Water Manage., 2: 241-256.

Dirksen, C., Raats, P.A.C. and Shalhevet, J.S., 1980. Interaction of alfalfa with matric and osmotic soil water potentials nonuniform in space and time. II. Daily irrigation. Soil Sci. Soc. Am. J.,

Eldor, M. and Dagan, G., 1972. Solutions of hydrodynamic dispersion in porous media. Water Resour. Res., 8: 1316-1331.

Eriksson, E., 1958. The possible use of tritium for estimating groundwater storage. Tellus, 10: 472-477.

Eriksson, E., 1963. Atmospheric tritium as a tool for the study of certain hydrological aspects of river basins. Tellus, 15: 303-308.

Eriksson, E., 1971. Compartment models and reservoir theory. Ann. Rev. Ecol. Syst.,2: 67-84.

Ernst, L.F., 1973. De Bepaling van de Transporttijd van het Grondwater bij Stroming in de Verzadigde Zone. Nota ICW 755, 42 pp.

Gardner, W.R., 1967. Water uptake and salt-distribution patterns in saline soils. *In*: Isotope and Radiation Techniques in Soil Physics and Irrigation Studies, IAEA Proceedings Series, 335-341.

Gelhar, L.W. and Wilson, J.L., 1974. Ground water quality modeling. Ground Water, 12: 399-408.

Hilgard, E.W. and Loughridge, R.H., 1895. The distribution of the salts in alkali soils. Calif. Agric. Exp. Sta. Bull., 108, 14 pp.

Hilgard, E.W. and Loughridge, R.H., 1906. Nature, value, and utilization of alkali lands, and tolerance of alkali by cultures. Revised reprints of Calif. Agric. Exp. Sta. Bulls. 128 and 133, 73 pp.

Jacobson, T. and Adams, R.M., 1958. Salt and silt in ancient Mesopotamian Agriculture. Science, 128: 1251-1258.

Jury, W.A., Fluhler, H. and Stolzy, L.H., 1977. Influence of soil properties, leaching fraction, and plant water uptake on solute concentration distribution. Water Resour. Res., 13: 645-650.

Jury, A., Frenkel, H., Fluhler, H., Devitt, D. and Stolzy, L.H., 1978a. Use of saline irrigation waters and minimal leaching for crop production. Hilgardia, 46: 169-192.

Jury, W.A., Frenkel, H. and Stolzy, L.H., 1978b. Transient changes in the soil-water system from irrigation with saline water: 1. Theory. Soil Sci. Soc. Am. J., 42: 579-585.

Jury, W.A., Frenkel, H., Devitt, D. and Stolzy, L.H., 1978c. Transient changes in the soil-water system from irrigation with saline water: II. Analysis of experimental data. Soil Sci. Soc. Am. J., 42: 585-590.

McNeal, B.L., Oster, J.D. and Hatcher, J.T., 1970. Calculation of electrical conductivity from solution composition data as an aid to in situ estimation of soil salinity. Soil Sci., 110: 405-414.

Nelson, R.W., 1978. Evaluating the environmental consequences of groundwater contamination: 1, 2, 3, and 4. Water Resour. Res., 14: 409-450.

Nir, A. and Lewis, S., 1975. On tracer theory in geophysical systems in the steady and nonsteady state. Part I. Tellus, 27: 372-383.

Oster, J.D. and Rhoades, J.D., 1975. Calculated drainage water compositions and salt burdens resulting from irrigation with river waters in the Western United States. J. Environ. Qual., 4: 73-79.

Oster, J.D. and Wood, J.D., 1977. Hydro-salinity models: sensitivity to input variables. In: J.P. Law and G.V. Skogerboe (Eds). Proc. Nat. Conf. Irrig. Return Flow Quality Management. U.S. Environmental Protection Agency and Colorado State University, p. 253-259.

Oster, J.D., Raats, P.A.C. and Dirksen, C., 1974. Calculation of water uptake distribution from observed steady salinity profiles. Agron. Abstr., p. 14.

Parlange, J.Y., 1980. Water transport in soils. Ann. Rev. Fluid Mech., 12: 77-102.

Peck, A.J., 1973. Analysis of multidimensional leaching. Soil Sci. Soc. Am. Proc., 37: 320.

Peck, A.J., 1975. Effect of land use on salt distribution in the soil. In: A. Poljakoff Mayber and J. Gale (Eds) Ecological Studies. Analysis and Synthesis. Vol. 15 Plants in Saline Environments. Springer, Berlin.

Peck, A.J., 1977. Development and reclamation of secondary salinity. In: J.R. Russell and E.L. Greacen (Eds) Soil factors in crop production in a semi-arid environment. Univ. of Queensland Press in association with the Australian Society of Soil Science Inc. St. Lucia, Queensland, 327 pp.

Peck, A.J. and Hurle, D.H., 1973. Chloride balance of some farmed and forested catchments in south-western Australia. Water Resour. Res., 9: 648-657.

Peck, A.J., Johnston, C.D. and Williamson, D.R., 1981. Analysis of solute distributions in deeply weathered soils. Agric. Water Manage., 4: 83-102

Raats, P.A.C., 1974a. Movement of water and salts under high frequency irrigation. Proc. 2nd Int. Drip Irrig. Congr. San Diego, Calif, July 7-14, 1974.

Raats, P.A.C., 1974b. Steady flow of water and salt in uniform soil profiles with plant roots. Soil Sci. Soc. Am. Proc., 38: 717-722.

Raats, P.A.C., 1975. Distribution of salts in the root zone. J. Hydrol., 27: 237-248.

Raats, P.A.C., 1977. Convective transport of solutes in and below the root zone. Proc. Int. Conf. on "Managing Saline Water for Irrigation: Planning for the future". Lubbock, Texas: 290-298.

Raats, P.A.C., 1978a. Convective transport of solutes by steady flows. I. General theory. Agric. Water Manage., 1: 201-218.

Raats, P.A.C., 1978b. Convective transport of solutes. II. Specific flow problems. Agric. Water Manage., 1: 219-232.

Raats, P.A.C., 1980a. Multidimensional transport of solutes in saturated and ir unsaturated soils. Neth. J. Agric. Sci., 28: 7-15.

Raats, P.A.C., 1980b. The distribution of uptake of water by plants: inference from hydraulic and salinity data. AGRIMED seminar on the movement of water and salts as a function of the properties of the soil under localized irrigation, 6-9 November 1979, Bologna (It.). (In press).

Raats, P.A.C., 1980c. The supply of water and nutrients in soilless culture. Proc. 4th Int. Congr. on Soilless Culture.

Rawlins, S.L., 1973. Principles of managing high frequency irrigation. Soil Sci. Soc. Am. Proc.,37: 626-629.

Rice, R.C. and Raats, P.A.C., 1980. Underground travel of renovated wastewater. J. Environmental Engineering Division of the American Society of Civil Engineers. 106: 1079 - 1098.

Van Schilfgaarde, J., 1977. Minimizing salt in return flow by improving irrigat efficiency. In: J.P. Law and G.V. Skogerboe (Eds). Proc. Nat. Conf. Irrigation Return Flow Quality Management, U.S. Environmental Protection Agency and Colorado State University.

Wierenga, P.J. and Sisson, J.B., 1977. Effects of irrigation on soil salinity and return flow quality. In: J.P. Law and G.V. Skogerboe (Eds). Proc. Nat. Conf. Irrig. Return Flow Quality Management. U.S. Environmental Protection Agency and Colorado State University. p. 115-121.