

THE BOUNDARY LAYER DEVELOPMENT
ON A V-SHAPED BROAD-CRESTED WEIR

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SUMMARY

REFERENCES

LIST OF SYMBOLS

APPENDIX I Velocity distribution profiles

I. INTRODUCTION

In hydrology, in water management and in irrigation, the accurate measurement of flow rates is a central problem. Critical flow structures of various types have been designed for this purpose.

Broad-crested weirs are popular for their sturdiness and for their ability to pass through floating debris and sediment load during floods. On the other hand friction losses are not insignificant as some length of crest and wall is exposed to the flowing masses. Such losses are accounted for by the introduction of a discharge coefficient into the discharge equation which is derived under the assumptions of a constant energy level and critical flow in the cross section over straight and parallel streamlines.

It is known that the discharge coefficient is not a constant and studies have been carried out in order to determine its dependence on the changing geometry of the flow pattern. (ref. 5) It is important to realize that the assumption (under which the discharge formula was derived) such as critical flow conditions and straight streamlines is an approximation of the reality.

The essence of friction losses should be studied in the development of boundary layers as a function of the flow pattern and the surface roughness of the measuring structure.

In this study a review is given of boundary layer development and its influence on the discharge relationship of a broad-crested weir with a triangular control section ($\alpha = 90^\circ$).

This type of measuring structure has already a high degree of accuracy at low flow rates as compared with a measuring structure with a rectangular control section.

Ippen introduced the concept of the boundary layer displacement thickness on the crest of a weir (with a rectangular control section). He proposed a discharge equation for the broad-crested weir with rounded off nose and rectangular control section in which the original specific head is corrected by the displacement thickness which exists at the section with critical flow conditions (D_c and u_c).

In fact this concept is applied to determine the discharge of weirs with arbitrary shapes. (v. Kalkwijk, ref. 6)

Therefore an experiment was set-up to study the boundary layer development and its influence on the discharge over the V-shaped broad-crested weir with a bottom angle of 90 degrees.

A general method is given to derive the boundary layer displacement thickness (δd) on the crest from measured velocity profiles. The results are compared with those obtained from the boundary layer development on flat plates in infinite fluids.

From the boundary layer displacement values the discharge coefficients are derived and compared with experimental data, obtained from the laboratory model.

The experiment was carried out by C.J.M. Bastiaansen in co-operation with the laboratory staff.

II. ANALITICAL DERIVATION OF THE DISCHARGE RELATIONSHIP FOR BROAD-CRESTED WEIRS WITH A TRIANGULAR CONTROL SECTION ($\alpha = 90^\circ$)

Neglecting the friction losses in a relatively short and abrupt transition in an open canal as compared to the internal conversion of energy (acceleration), a unique head-discharge relationship can be derived for the modular range of flow.

The specific energy-head (H_o) above the crest, assuming straight and parallel streamlines, can be defined as follows:

$$H_o = D + (\alpha) \cdot \frac{\bar{u}^2}{2g} = D + \alpha \cdot \frac{Q^2}{2g.A^2} \quad \dots\dots (1)$$

(with $\alpha \approx 1,0$)

in which

D = the waterdepth

\bar{u} = the average velocity in the considered cross-section

g = acceleration due to gravity

α = the energy velocity distribution coefficient

Q = discharge rate

A = wet cross-section

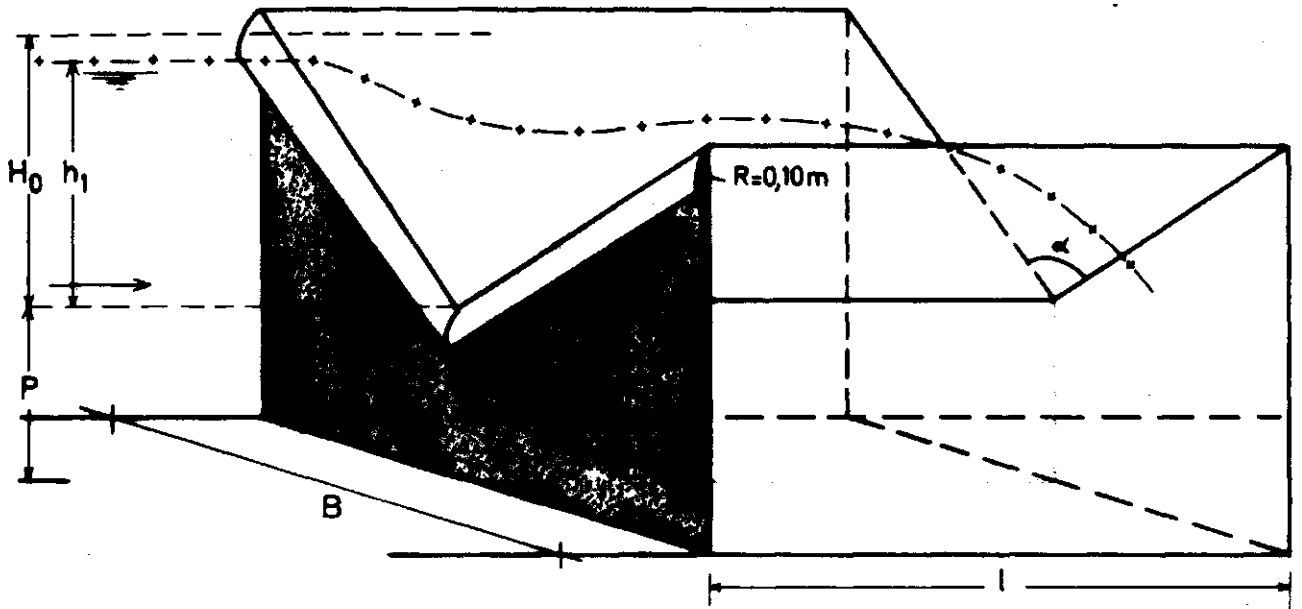


Fig. 1 V-shaped broad crested weir

Applying the principle of minimum energy whereby a constant flow rate Q has to be discharged:

$$\frac{dH_o}{dD} = 1 + \frac{d}{dD} \cdot \frac{Q^2}{2g \cdot A^2} = 0$$

For a triangular control section with $\alpha = 90^\circ$, A equals D^2

$$\frac{dH_o}{dD} = 1 + \frac{d}{dD} \cdot \frac{Q^2}{2g \cdot D^4} = 1 - 2 \frac{Q^2}{g \cdot D^5}$$

For the minimum value of H_o the critical depth D_c occurs;

$$D_c = \sqrt[5]{\frac{2Q^2}{g}} \quad \dots\dots (2)$$

From equation 1 and 2 it follows that for $H_o(\min)$ holds:

$$H_o(\min) = 5/4 D_c \text{ and } D_c = 4/5 H_o \quad \dots\dots(2a)$$

Hence:

$$\frac{\bar{U}_c^2}{2g} = H_o(\min) - D_c = \frac{1}{4} D_c \quad \dots\dots (3)$$

$$\bar{U}_c = \sqrt{\frac{1}{2} g \cdot D_c}$$

If the crest-section becomes critical it is possible to determine the relationship between the theoretical discharge rate Q_{th} and the specific energy head H_o :

$$Q_{th} = D_c^2 \cdot U_c = \sqrt{\frac{1}{2} g \cdot D_c} \cdot D_c^{5/2} \dots\dots (4)$$

or by substitution of D_c by H_o :

$$Q_{th} = \sqrt{\frac{1}{2} g} \cdot (4/5)^{5/2} \cdot H_o^{5/2} = \sqrt{2/5 \cdot g} \cdot 16/25 \cdot H_o^{5/2} \dots\dots (4a)$$

To obtain a practical head-discharge equation a C_d and C_v -coefficient are introduced in which the effects of viscosity and flow curvature are incorporated.

The discharge equation for viscous fluids reads as:

$$Q = C_v \cdot C_d \cdot 16/25 \cdot \sqrt{2/5 \cdot g} \cdot h_1^{5/2} \dots\dots (5)$$

$$Q = C_d \cdot 16/25 \cdot \sqrt{2/5 \cdot g} \cdot H_o^{5/2} \dots\dots (5a)$$

When the approach velocity is sufficiently low, C_v equals almost unity ($C_v = (H_o/h_1)^{5/2} \approx 1$).

The C_d -coefficient is to adjust for the effects of friction forces of the viscous fluid and the curvature of streamlines overhead the crest.

Experiments on the same weir as described in figure 1 were done by W. Boiten in 1976. His results are given in figure 2 and ref. 5 is made to lit.5. The average C_d -coefficient was 0.956.

For $0.10 < h/L < 0.50$ the C_d -value varies between 0.94 to 0.98.

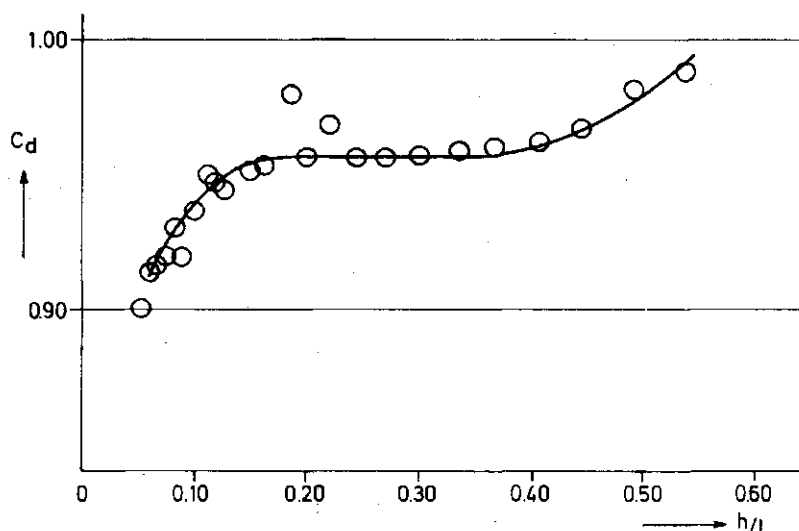


Fig. 2 Results on the estimation of the C_d -value on the broad crested weir with $\alpha = 90^\circ$ and $L = 0.9$ m. $R = 0.10$ m; less than full, (after W. Boiten, Ref. 5)

III. THE BOUNDARY LAYER DEVELOPMENT ON THE CREST

Near the boundary a layer of fluid is decelerating because of resistance to flow caused by the shear stresses at the wall. Therefore no undisturbed potential flow will occur above the crest.

This relatively thin layer in which the velocity deviates from the undisturbed velocity distribution is called boundary layer and it can develop in either laminar or turbulent flow.

Many investigators studied the boundary layer development on a flat plate, placed parallel to the streamlines in an infinite fluid with a constant ambient velocity (\bar{u}). Near the flat plate a velocity gradient ($\delta u / \delta y$) will develop. The gradient depends on the wall-roughness of the plate (in cases of turbulent boundary flow) and the degree of turbulence in the main stream. Near or on the plate the velocity becomes zero. The outer edge of the thus formed boundary layer can be defined arbitrarily as the location where the local velocity equals 99% of the constant velocity (\bar{U}) in the main stream. But in practice it will be difficult to determine the numerical value of the boundary layer thickness (δ). The distance to the point where the influence of the boundary is negligible can never be measured exactly.

In order to quantify the characteristics of the boundary layer in a more convenient way than by its thickness δ alone, the concept of the boundary layer displacement thickness (δd) was introduced.

The displacement thickness is defined as the distance over which the

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wall has to be (theoretically) displaced in order to discharge the same amount of fluid as in the case of undisturbed potential flow. See figure 3.

To meet the requirement of continuity according to the above mentioned definition, one can write:

$$\int_0^{\delta} u(y) \cdot dy = U(\delta - \delta d) \quad \dots\dots (6)$$

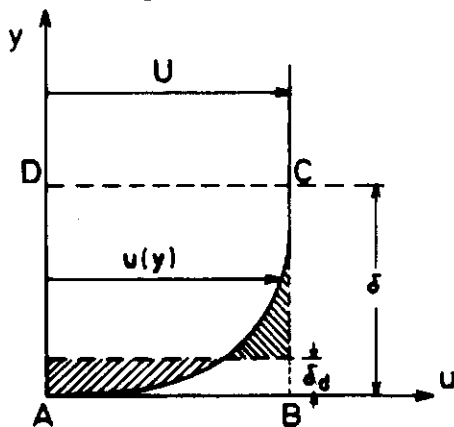


Fig. 3 Boundary layer displacement thickness

Since U is constant outside the boundary layer, it follows from equation 6

$$\delta - \delta d = \frac{1}{U} \int_0^{\delta} u(y) \cdot dy \quad \dots\dots (6a)$$

Hence:

$$\delta d = \delta - \frac{1}{U} \int_0^{\delta} u(y) \cdot dy = \frac{U\delta - \int_0^{\delta} u(y) \cdot dy}{U} \quad \dots\dots (6b)$$

$$\delta d = \frac{\text{area ABC}}{\text{basis AB}} \quad \dots\dots (7)$$

The boundary layer displacement thickness equals the area ABC divided by the undisturbed velocity U or length AB.

Note: the area ABC can be computed with numerical integration according to the Simpson rule.

In the computation of δd the greatest problem was the estimation of the δ -value. In figure 3 this value can be found by drawing the vertical tangent to the velocity distribution profile curve ($du/dy = 0$).

If there is no $u_{(max)}$ but a continuously increasing velocity, the

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vertical tangent cannot be drawn. Sometimes the velocity distribution profile curve has a point of inflexion instead of a maximum. This point can be found mathematically by $(d^2u/dy^2 = 0)$ the second derivative of $u(y)$ with respect to y .

In practice, regarding the velocity profiles App. I, the estimation of δ was not always possible and drawing the vertical tangent to the u -profile was often done arbitrarily.

IV. THE DISCHARGE RELATIONSHIP IN WHICH THE BOUNDARY LAYER EFFECTS ARE TAKEN INTO ACCOUNT

It is assumed, that outside the boundary layer on the weir, straight and parallel streamlines exist. The velocity U outside this layer can be written as:

$$U = [2g.(H_o - D)]^{\frac{1}{2}} \quad \dots\dots (8)$$

Continuity principle and assumption of crest displacement of δd , shown as follows:

$$Q = (A - W.\delta d).U \quad \dots\dots (9)$$

Inwhich W = wetted perimeter

δd = average displacement thickness

From substitution of equation 8 into equation 9 it follows:

$$Q = (A - W.\delta d). [2g.(H_o - D)]^{\frac{1}{2}} \quad \dots\dots (10)$$

For a triangular control section with $\alpha = 90^\circ$ yields:

$$A = D^2 \quad W = 2\sqrt{2}.D$$

thus:

$$Q = (D^2 - 2\sqrt{2}.D.\delta d). [2g.(H_o - D)]^{\frac{1}{2}} \quad \dots\dots (10a)$$

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Therefore one can write for the specific energy head H_o :

$$H_o = \frac{Q^2}{2g (D^2 - 2\sqrt{2}.D.\delta d)^2} + D \quad \dots\dots (10^b)$$

Application of the principle of minimum energy ($dH/dD = 0$) and the assumption that the flow becomes critical at the crest section, equation 10b yields:

$$\frac{dH_o}{dD} = 1 + \frac{Q^2}{2g} \left[\frac{d}{dD} (D_c^2 - 2\sqrt{2}.D_c.\delta d)^{-2} \right] = 0 \quad D = D_c \quad \dots\dots (10^c)$$

$$\frac{Q^2(2D_c - 2\sqrt{2}.\delta d)}{g(D_c^2 - 2\sqrt{2}.D_c.\delta d)^3} = 1 \quad \dots\dots (11)$$

The suffix c denotes the critical state of flow occurring at a certain cross-section. Substituting equation 11 into equation 10^b gives:

$$H_o = 5/4 D_c - 1/4 \sqrt{2}.\delta d \quad \dots\dots (12^a)$$

$$D_c = 4/5 H_o + 1/5 \sqrt{2}.\delta d \quad \dots\dots (12^b)$$

$$H_o - D_c = 1/5 H_o - 1/5 \sqrt{2}.\delta d \quad \dots\dots (12^c)$$

Substitution of D_c and $(H_o - D_c)$ in equation 10^a yields

$$Q = 16/25 (2/5 g)^{1/2} . (H_o - 2\frac{1}{4}.\sqrt{2}.\delta d)(H_o + \frac{1}{4}\sqrt{2}.\delta d)(H_o - \sqrt{2}.\delta d)^{1/2} \quad \dots\dots (13)$$

In deriving equation (13) it was assumed that δd does not depend on the waterdepth D , which is only justified for turbulent boundary layers along hydraulic rough walls. In that case δd does not depend on the velocity U outside the boundary layer and hence not on the waterdepth.

If the C_d value in equation 5 depends only on the boundary layer development and thus on the friction losses, its value can be derived from the boundary displacement thickness.

Combining equation 5 and equation 13 (and) with a C_v value equal to unity one can write for C_d :

$$C_d = (1 - 2,25 \sqrt{2} \cdot \delta d / H_0)(1 + 0,25 \sqrt{2} \cdot \delta d / H_0)(1 - \sqrt{2} \cdot \delta d / H_0)^{\frac{1}{2}} \dots\dots (14)$$

$$C_d \approx (1 - \sqrt{2} \cdot \frac{\delta d}{H_0})^{5/2} \approx (1 - 5/2 \sqrt{2} \cdot \frac{\delta d}{H_0}) \dots\dots (14^a)$$

Thus by estimating the boundary layer displacement thickness, it is possible to calculate the C_d -coefficient (when straight and parallel flow over the crest occurs).

When this equation is derived for a triangular control section for different bottom angles, it reads:

$$C_d = (1 - \frac{\delta d}{L} \cdot \frac{L}{H} \cdot \frac{1}{\sin \frac{1}{2}\alpha})^{\frac{1}{2}} \cdot (1 + \frac{\delta d}{L} \cdot \frac{L}{H} \cdot \frac{1}{4 \sin \frac{1}{2}\alpha}) (1 - \frac{\delta d}{L} \cdot \frac{L}{H} \cdot \frac{9}{4 \sin \frac{1}{2}\alpha}) \dots\dots (15)$$

V. EXPERIMENTAL SET-UP AND RESULTS

V-1 Description of experimental set-up and measuring equipment

The experimental model equipment consisted of a V-shaped broad-crested weir with an angle (α) of 90 degrees. The height of the crest above the bottom of the approach canal, with a width of 0.80 m, was 0.25 m. The weir was made of p.v.c.-material with a crest length of 0.90 m and a wall roughness of 2.10^{-5} m. It has a rounded off nose with a radius of 0.10 m (see figure 1).

Velocity profiles were measured with a so called "Wall Pitot" tube which was attached to a sledge to move it along and perpendicular to the direction of flow. The "Wall Pitot" tube (figure 4) consists of two tubes, a dynamic and a static one, both connected to a sensitive differential membrane manometer (d.m.m.). The signal of the d.m.m. was electronically amplified and reproduced by a digital voltmeter.

Near the bottom, in the boundary layer, the d.m.m. reacts very quickly to the changes in pressure due to changes in velocity of flow. Thus it was necessary to filter the electrical signal in order to reduce the fluctuations in the registration.

The electrical signal was also brought into a frequency convertor to enable the summons over a certain time-period (Δt).

In this way the electronic recording equipment reproduced the mean pressure difference (Δp) between the dynamic and static tube over a differential time (Δt). The velocity is computed by:

$$u = c \sqrt{2g \cdot \Delta p} \quad \dots\dots (16)$$

in which:

$$g = 9.80665 \text{ m/sec}^2$$

Δp = average difference of pressure head registered by the d.m.m.
in m.w.k. (metres water column)

c = a correction factor for energy losses of the "Wall Pitot" tube

To know the exact value of u it is necessary to calibrate the "Wall Pitot" tube. The calculation of δd however is not influenced by the

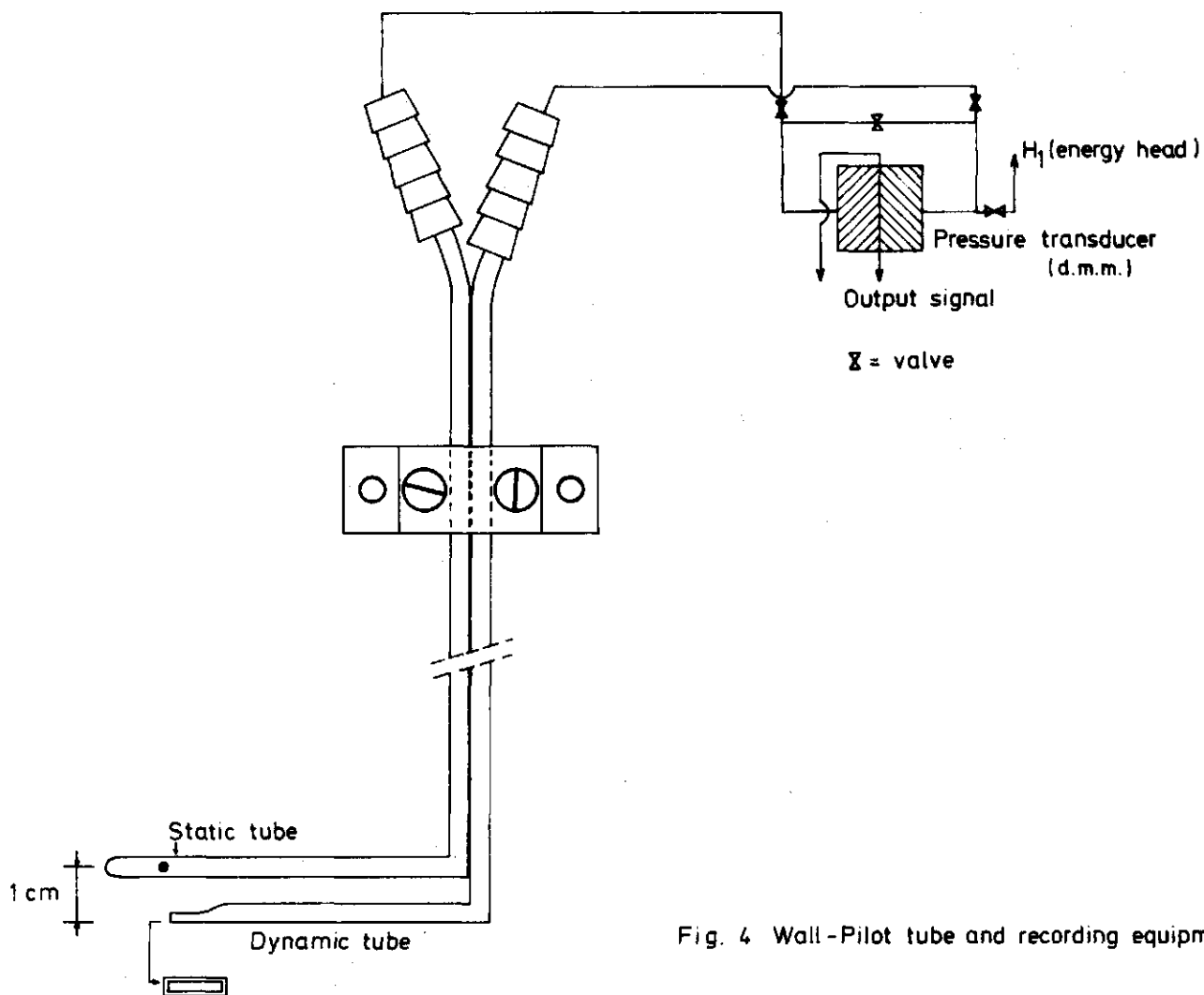


Fig. 4 Wall-Pilot tube and recording equipment

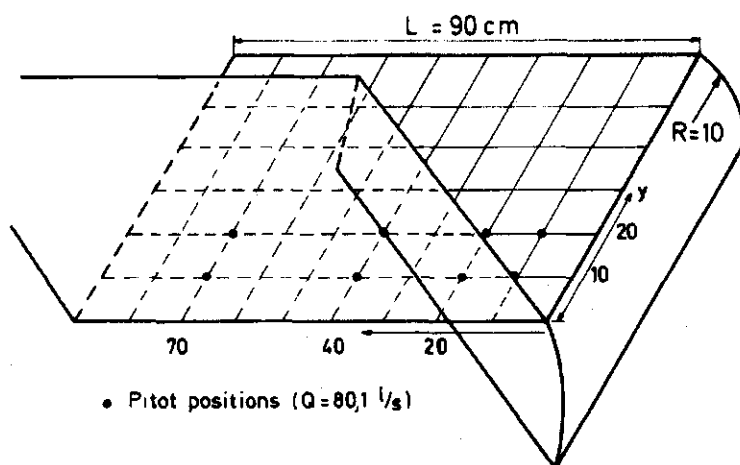


Fig. 5 Network on the crest

application of adjusted velocities as can be seen from equation 7. Multiplying the length of AB by a constant (correction) factor has no influence on the result of equation 7.

The distance between the dynamic and the static tube of the "Wall Pitot" tube is 0.01 m. In fact the static tube does not give the static pressure at the position of the dynamic tube, but just 0.01 m above that point. Therefore to correct the difference in Δp it is necessary to study the piezometric head distribution.

From the piezometric head distribution also the effect of curvature of flow and other flow disturbances can be studied.

Therefore the static tube and a water column with a constant head were connected to the d.m.m. $(R - P/\rho g)_y$.

The local pressure $(P/\rho g)$ can be found by subtracting the counter pressure (R) .

V-2 Results

On the weir-crest a network of squares is fitted so that each location of measurement can be indicated by two co-ordinates (x, y_b) . A location on the left side of the crest is provided with a suffix.

From figure 5 the different locations, where the measurement have been done, can be found.

The x-figure indicates the distance from the beginning of the crest and the y-figure the distance to the bottom edge, measured along the wall.

The wall is placed under an angle of 45 degrees with the vertical.

The distance between the lines of the network on the weir crest is 0.10 m.

V-2.1 The static pressure distribution diagrams

After some preliminary measurement it was recognised that the piezometric head distribution in the cross-section was not constant. In order to get insight in the piezometric head distribution $(y + P/\rho g)$ measurements were performed.

The static pressure tube was connected to the d.m.m. against the reference level H_o . $[H_o - (y + P/\rho g)]$.

The local pressure can be found by subtracting the measured value from

the counter pressure H_0 .

The measurements were done at a discharge rate of about $0.08 \text{ m}^3/\text{sec}$ at x values of 0.1 m, 0.2 m, 0.4 m and 0.7 m. In this way 4 piezometric head distribution diagrams were obtained (see figures 6, 7, 8 and 9). From these figures it can be concluded that the piezometric head distribution is not constant ($y + P/\rho g \neq \text{constant}$) but that there is a head-gradient in the vertical direction and also perpendicular to the bottom of the weir-crest.

When the watersurface had a convex shape the piezometric head decreased with the depth. The piezometric head increased with depth when the watersurface had a concave shape. In the x -direction the watersurface changed from a convex to a concave shape and back again to convex at the end of the crest.

In the x -direction the piezometric head decreased. This means that there is a pressure gradient along the boundary in the x -direction which has an influence on the boundary layer development (D , the waterdepth decreases and thus the flow velocity increases in the x -direction).

From the diagrams the corrections on the measured velocity profiles can be found (see section 3 of this chapter).

From the figures 6-9 it can be seen that the left and right side of the weir-crest are fairly symmetric.

V-2.2 Determination of the position of the critical section on the V-shaped broadcrested weir.

Assuming non curvature of streamlines (straight and parallel streamlines) the critical section occurs at a waterdepth D_c equal to 0.8 of the specific energy head H_0 (Figure 10).

The waterdepth over the weircrest was measured in the centre line of the crest. This is not correct because the shape of the watersurface is not flat but convex or concave of shape.

The critical section, in which the flow changes from sub-critical to super critical is also characterized by a Froude-number equal to unity.

In table 1 the determination of the Froude-number has been done for the two discharge rates: $Q = 28.6 \text{ l/s}$ and 80.1 l/s .

In table 2 the same calculation is done but the A -value, area of the

cross-section is measured on the model so that the objection mentioned above, is avoided.

Boiten ref. 5 found that the critical section occurred at $x \approx 0.6 H_0$, based on the value of Froude number equal to unity.

It is however difficult to indicate the critical section.

On the weir crest the flow at minimum specific energy head shows the usual variations of depth. Streamlines are not quite straight and parallel so that the transition from sub-critical to super-critical flow cannot be clearly indicated. But it may be obvious that the transition takes place already in the upper half of the weir crest.

In table 2 also the total energy head (H_x) over the whole cross-section for different x-values is computed.

From these calculations some conclusions can be drawn:

Table 1. Estimate of the Froude number

A) $Q = 28.61$ l/sec.

x(m)	y(cm)	A(m ²)	\bar{u} (m/sec)	Fr
.00	20.1	0.040	0.708	0.72
.10	18.7	0.035	0.818	0.86
.20	17.1	0.029	0.978	1.08
.40	16.7	0.028	1.026	1.14
.50	17.2	0.029	0.967	1.06
.60	16.6	0.028	1.038	1.16
.80	15.2	0.023	1.238	1.44

B) $Q = 80.1$ l/sec

x(m)	y(cm)	A(m ²)	\bar{u} (m/sec)	Fr
.00	30.8	0.095	0.844	0.70
.10	29.1	0.085	0.946	0.79
.20	27.1	0.073	1.091	0.96
.40	24.4	0.060	1.345	1.24
.60	23.9	0.057	1.402	1.30
.80	23.4	0.055	1.463	1.38

$$Fr = \frac{\bar{u}}{\frac{1}{\alpha} \sqrt{g \left(\frac{A}{B}\right)}} \quad \text{or} \quad Fr = \frac{\bar{u}}{\frac{1}{\alpha} \sqrt{g \left(\frac{y}{2}\right)}} \quad (\alpha = 1.01)$$

Table 2. Computation of $H_o(\text{min})$ and Fr-number for the different cross-sections with a discharge rate of 80.1 and 28.6 l/sec.

Q (l/s)	x (m)	H_o (m)	A (m ²)	\bar{y} (m)	\bar{u} (m/s)	H_x (m)	Fr (-)	H_x/H_o (-)	$(H_x/H_o)^{1/2}$ (-)	C_d
80.1	0.10	0.338	0.080	0.283	1.00	0.334	0.85	0.978	0.968	
80.1	0.20	0.338	0.073	0.271	1.09	0.332	0.94	0.981	0.953	0.952
80.1	0.40	0.338	0.061	0.248	1.31	0.334	1.20	0.984	0.960	
80.1	0.70	0.338	0.058	0.240	1.39	0.338	1.29	0.996		
28.6	0.10	0.224	0.033	0.181	0.87	0.220	0.93	0.98	0.952	0.950

Remarks: $\alpha \approx 1.00$ (1.01)

$$H_o = R_1 - 33.2 + u_1^2/2g = y + P/\rho g + u_1^2/2g$$

R = referentie level (left)

$$\bar{u}_1^2/2g \text{ for } Q = 80.1 \text{ l/s} = 1.5 \cdot 10^{-3} \text{ m}$$

$$Q = 28.6 \text{ l/s} = 0.3 \cdot 10^{-3} \text{ m}$$

$$H_x = \bar{D} + \frac{\bar{u}^2}{2g} \quad (\text{with } \alpha = 1.01)$$

C_d = calculated with the basic discharge equation

- the ratio of H_x/H_o to the power 2.5 at the transitional section or zone gives a " C_d -value" or "discharge coefficient value" which corresponds with values found in ref. 5; see also figure 2 of this paper.
- the value of H_x increases after the flow has changed to super-critical. The waterdepth decreases and the flow velocity increases.

V-2.3 Measurement of velocity profiles

For this purpose the static and dynamic tube were connected to the d.m.m. From measurements inside the boundary layer, the pressure fluctuations were averaged over a differential time of 100 and later 50 seconds. Outside the boundary layer the fluctuations in pressure are considerably less and an average over 20 seconds proved to be sufficient. To find the piezometric head variation in the same vertical, the piezometric head measured against H_0 , an average over 20 or even 10 seconds proved to be sufficient.

From the velocity measurement the velocity distribution profiles are derived and from the profiles the boundary layer displacement thickness (δd) can be calculated, as stated in Section III of this paper.

It should be noted here that the shape of the velocity-distribution profiles at the up- and downstream-end of the crest do not show a constant velocity outside the boundary layer due to flow curvature. For more details of this phenomenon reference see ref. 1, pages 43 to 47.

A second remark is that the velocity profiles are measured perpendicular to the bottom of the weir which means, with the triangular controlsection of the crest, at an angle of 45 degrees with the vertical.

In the first test run velocity distribution profiles at a discharge rate of 80.1 l/s were measured at $y = 0.10$ and 0.20 m with $x = 0.10, 0.20, 0.40$ and 0.70 m. Appendix I; figures I-1 to I-6.

Next the discharge rate was changed to 28.6 l/s and velocity profiles were measured at $y = 0.10$ m with $x = 0.14; 0.25; 0.40; 0.55; 0.70$ and 0.85 m. Appendix I; figures I-7 to I-11.

The computed boundary layer displacement thicknesses δd are given in table 3 and graphically presented in figure 11.

The results of the computed δd are compared with the boundary layer development on a flat plate in an infinite fluid according to Harrison, ref. 3. In the computation of the corresponding δd -value on a flat plate the following assumptions were made:

- wall roughness, k , equals $2 \cdot 10^{-5}$ m.
- for the average velocity the critical velocity is taken (u_c)
- the average temperature is 20° C so that the kinematic viscosity equals 10^{-6} m²/sec.
- for the transition from laminar to turbulent boundary flow a

- Reynolds number ($R_t = \frac{Uxt}{\nu}$) of (A). 3.10^5 and (B). 10^6 is assumed;

A and B correspond with those in table 4.

In table 4 the computation of the δd on the flat plate is given and graphically represented in figure 11. From figure 11 it can be seen that the δd -values on the weir-crest of the V-shaped weir do not differ much from the δd -values found on a flat plate in an infinite fluid, such as computed according to Harrison (ref. 3).

Statements given by Nikuradse and Delleur (ref. 2) show that the boundary layer on the crest would develop more slowly than on a flat plate in an infinite fluid because of the negative pressure gradient towards the downstream end of the crest (acceleration of flow).

From the computed δd -values of the experimental estimated velocity distribution profiles it appeared that transition to turbulent boundary flow did not occur. This could be expected because the (R_x) Reynolds number is about 3.10^5 to 10^6 and on the crest acceleration of flow occurred which may have suppressed possible turbulence.

The maximum value for δd occurred at a x-value of about 0.70 m. The flow velocity at $x = 0.70$ m for $Q = 80.1$ l/sec is about 1.43 m/sec. The Reynold number therefore is about 10^6 so that the boundary flow could be just in the transitional zone.

For different values of y different δd are found. Therefore new series of measurements were set-up to find the average δd -value in a cross-section.

This was done at a discharge rate of $Q = 28.6$ l/sec for $x = 0.14$ m and 0.70 m. It is assumed that $x = 0.14$ m lies within the critical flow section. The different velocity profiles are given in appendix I, figure I-13 to I-22, and the resulting δd -values in table 5. The graphical representation of these experiments is given in figure 12.

For $x = 0.70$ m the δd -value was computed for both sides of the weircrest (figures a and b in the appendix I; 13a to 16b).

Table 3. The boundary layer displacement thickness on the weircrest at discharge rates of 80.1 l/sec and 28.6 l/sec for different x-values (δd in mm).

Q = 80.1 l/sec

x(m)	.10	.20	.40	.70	
y .10 m	0.69	1.05	1.32	1.46	o *
20	-	0.65	0.88	0.79	•

Q = 28.6 l/sec

x(m)	.14	.25	.40	.55	.70	.85	
y .10 m	0.67	0.90	1.12	1.26	1.30	1.09	x *

Table 4. The computation of the boundary layer displacement thickness on a flat plate according to Harrison.

Q = 28.6 l/sec \rightarrow $u_c = 0.88$ m/sec(2) and Q = 80.1 l/sec \rightarrow $u_c = 1.04$ m/s (1)

$R_t = 3.10^5$ (A) and 10^6 (B) and $k = 2.10^{-2}$ mm.

x(m)	.10	.20	.40	.60	.80	1.00	
x/k	5000	10,000	20,000	30,000	40,000	50,000	
R_x (1)	1.04	2.08	4.16	6.24	8.32	10.4	x 10^5
(2)	0.88	1.76	3.52	5.28	7.04	8.8	x 10^5
δd A ₁	0.54	0.74	0.96	1.26	1.84	2.40	□ *
(mm) A ₂	0.57	0.80	1.08	1.20	1.76	2.30	△
B ₁	0.54	0.74	1.08	1.32	1.52	1.70	■
B ₂	0.57	0.80	1.12	1.38	1.60	1.80	▲

* these signs refer to figure 11.

Table 5. The boundary layer displacement thickness in (mm) by
 $Q = 28.6$ l/sec and $x = 0.14$ m and 0.70 m

$y \approx$	0.025	0.05	0.10	0.15	0.19	$\bar{\delta d}$
$x = 0.14$ m	0.83	0.79	0.61	0.66	0.61	0.70
$x = 0.70$ m						
y	1.64	1.28	1.28	1.46	1.73	} 1.42
y^1	1.57	1.27	1.29	1.42	-	

From the obtained δd -value from the left and right side of the weir-crest it may be concluded that the flow is sufficiently symmetric. The average $\bar{\delta d}$ -value at $x = 0.14$ and 0.70 m are comparable with the computed δd -value on a flat plate in an infinite fluid with $\bar{u} = u_c$ and $k = 2.10^{-2}$ mm. The latter values are for $x = 0.14$ m about 0.7 mm and for $x = 0.70$ m about 1.5 mm as can be read from table 4 and figure 11.

From the average δd -value the discharge coefficient can be found by using equation 14 of section IV. The C_d -values obtained for $Q = 28.6$ l/sec are:

$$\text{for } x = 0.14 \text{ m } (x_c) \quad C_d = 0.9889$$

$$\text{for } x = 0.70 \text{ m} \quad C_d = 0.9776$$

The C_d -value calculated from the formula:

$$Q = C_d \cdot C_v \cdot \frac{16}{25} \left(\frac{2}{5} g\right)^{\frac{1}{2}} \text{tg} \frac{\theta}{2} h_1^{2.5}$$

for $Q = 28.6$ l/sec and $\theta = 90^\circ$ was $C_d = 0.950$

and for $Q = 80.1$ l/sec the C_d -value was 0.952 .

The C_d -value according to figure 2, ref. 5 was 0.956 .

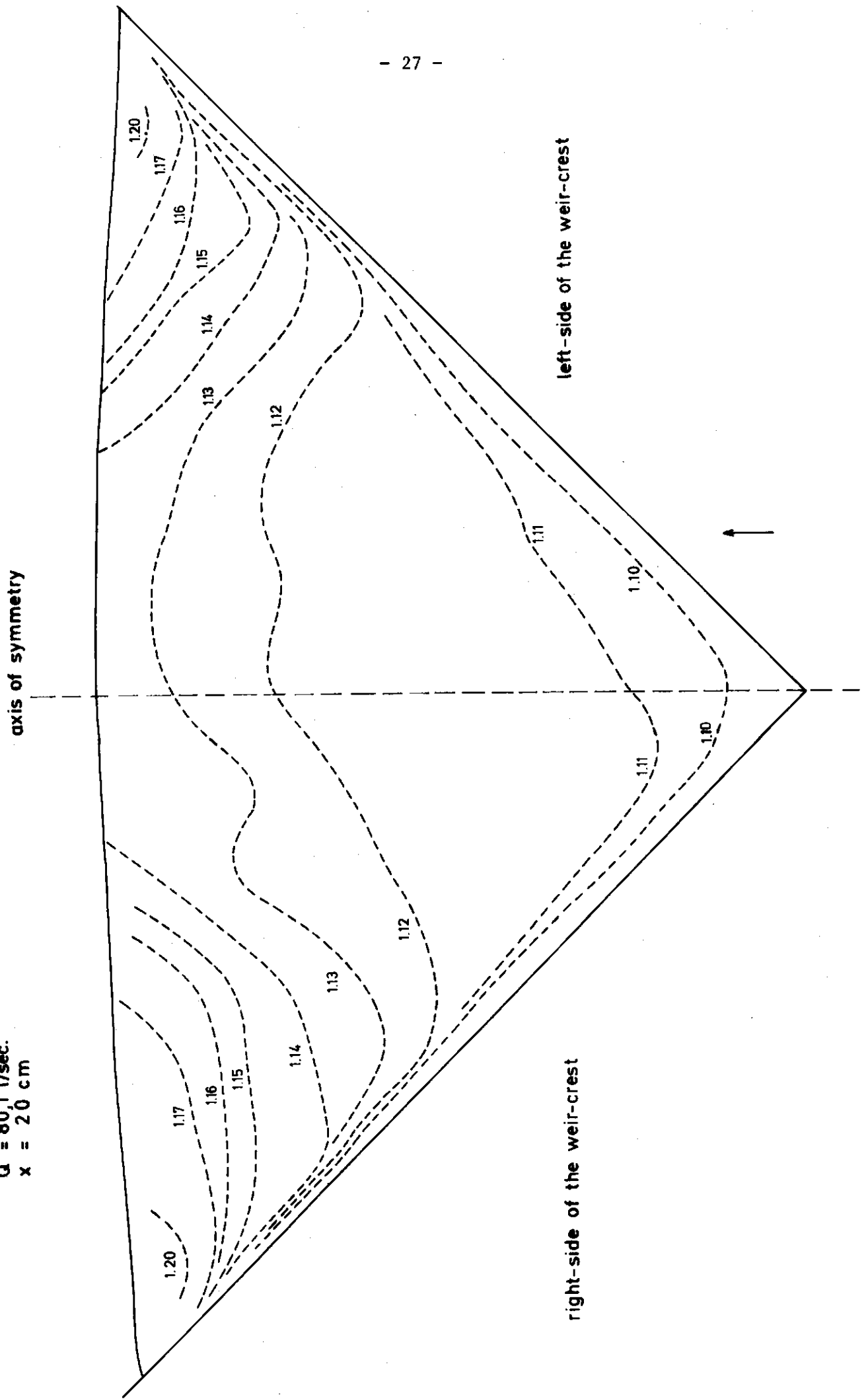
As stated before in section IV-2-2, the super-critical flow condition has no influence on the discharge relationship. Therefore it can be said that the boundary layer thickness at $x = 0.70$ m has no influence on the C_d -value. Only the boundary layer displacement thickness at a x -value where critical flow occurs influences the discharge coefficient.

It may be concluded here that not only the development of the

Fig. 13 Velocity distribution ; u (m/sec.)

$Q = 80,1$ l/sec.

$x = 20$ cm



boundary layer on the weircrest (at the critical section) can be held responsible for the C_d -value of about 0.956 but that also other phenomena are taking part into it. Another important fact is, that the velocity outside the boundary layer is not constant but increasing with the height above the bottom (to the side slopes of the weircrest). This is shown in the velocity profiles in App. I for all the profiles with x-value smaller than about 0.60 m. Examples are given by the figures I-7 and 18 to 22 for which x equals 0.14 m.

V-2.4 The velocity distribution in a cross section

Because of the variation in the piezometric head distribution and in the shape of the velocity distribution profiles it was interesting to investigate the velocity distribution in a cross-section as a whole. Specially in a section, in which critical flow occurs, the velocity distribution should be studied.

Therefore the measurements were done at $x = 0.20$ m with a discharge rate of 80.1 l/sec. The results are given in figure 13. This figure shows that even outside the boundary layer the velocity is not constant. In the outer edges the velocity is higher than in the centre of the triangular cross-section. This is in agreement with the piezometric head distribution diagram of figure 7 which shows a lower value in the outer edges than in the centre. This means that $u^2/2g \neq \text{constant}$ and $(y + p/\rho g)_y \neq \text{constant}$.

It is obvious that at the upstream end of the weircrest ($x = 0.2$ m) curvature of streamlines, specially in the outer edges, occurred.

VI DISCUSSION OF THE RESULTS AND CONCLUSIONS

From the analysis of the experimental data it can be seen that the boundary layer displacement thickness (δd) on the weircrest is increasing gradually from the beginning of the crest and that it reaches its maximum value for $0.5 l < x < 0.8 l$, depending on the discharge rate (figure 10).

The resolved values of δd prove to be in the same order as the δd values found on a plate in an infinite fluid with a Reynolds number (R_x) of about $3.0 \cdot 10^5$ to 10^6 , as computed by Harrison (ref. 3). Table 4 and figure 10. Others, like Nikuradse and Delleur (ref. 2) stated that the δd -value on a crest will develop more slowly than in an infinite fluid because of the negative pressure gradient ($dD/dx < 0$) over the measuring weircrest (acceleration of flow).

M. Vierhout (lit. no.1; 1973) found on a broadcrested weir with rectangular controlsection a δd -value of two (to three) times as high as the corresponding δd -value on a plate in an infinite fluid ($R_x = 3 \cdot 10^5$ and $k = 10^{-5}$ m).

The computed δd -values in one cross-section were not constant but depend on the position at the crest. The highest values were observed at the watersurface and towards the centre of the crest.

From the piezometric head distribution and velocity distribution diagrams it can be concluded that the streamlines over the crest are not straight and parallel. Specially in the upstream part of the weir flow curvature occurred. In the outer edges the flow velocity is higher and the piezometric head is lower than in the centre of the cross-section. These observations were made for $Q = 80$ l/sec and $x = 0.2$ m.

There was no special point of x where the transition of subcritical to supercritical flow occurred. It took place over a certain distance in which sub- and supercritical flow occurs in one cross-section. The transitional part or zone was situated in the upstream part of the weircrest, between 0.10 m to 0.30 m from the beginning of the flat part of the weircrest.

This means that the development of the boundary layer for $x > 0.30$ m.

has no significant influence on the discharge-relationship, c.q. on the discharge coefficient C_d .

At $x = 0.14$ m the δd -value is still very small; $\delta d = 7.10^{-4}$ m at a discharge rate of 28.6 l/sec.

The computed C_d -value with the Ippen-equation amounts 0.989 for $x = 0.14$ m. For $x = 0.70$ m the calculated C_d amounts 0.978. The C_d -value, found by the experiments of Boiten on the same weir crest, is 0.956 (ref. 5).

Therefore it is clear that also other phenomena must be of influence on the value of the C_d -coefficient. The curvature of flow at the entrance of the weircrest can be mentioned here.

The computed H_x -values (specific energy head at point x) at the transitional part of the crest, divided by the H_0 -value and brought to the power 5/2, results in a value equal or close to the C_d -value, given by the head-discharge relationship (table 2). It should be noted that the shape of the approach channel to the V-shaped weir is rectangular instead of triangular. Therefore some "contraction" at the inflow side of the weir may occur! With the increase of x , for $x > 0.25$ m, also H_x increased again. This means: - that the flow is supercritical. Also the Froude-number increased with x .

- the equation $H_0 = y + P/\rho g + \bar{u}^2/2g = \text{constant}$ has no validity.

At the entrance of the weircrest a fast drop of the watersurface takes place. Apart from the variation of velocity also the direction of flow of two particles of fluid at different positions in the cross-section, is not the same and not parallel to the bottom of the weir crest. The velocity in the upper layer of flow and outer edges of the weir crest were found to be higher than the velocity in the centre of the flow. Above the weir crest the shape of the watersurface changed from convex to concave and back again to convex at the end of the weir crest. This means that there exist also a flow-component perpendicular to the mainflow-direction.

All the phenomena mentioned above, show that the streamlines on the crest are not parallel and straight but that rotation of flow and other disturbances take place, especially at the entrance of the weir crest.

Apart from experimental results it is noted here that in order to obtain an accuracy of about 1% in the computed discharge, it is necessary to determine the zero-level of the gauge upstream of the weir crest exactly. The crest should be exactly horizontal to be sure of a reliable zero-level for the complete measuring range. Specially with low discharges the computed discharge is very sensitive to an error in the installed zero-level of the gauge (power $5/2$ in the equation).

SUMMARY

The aim of this study was to evaluate the boundary layer development on the crest of a V-shaped broadcrested weir. The boundary layer displacement thickness was computed to evaluate its influence on the discharge coefficient as given by the Ippen concept which can also be derived for a weir with a triangular control section instead of a rectangular control section.

From the experiments two important conclusions can be drawn:

- a. the boundary layer displacement thickness was in accordance with δd values found on a flat plate in an infinite fluid
- b. the transition of flow, for this kind of measurement structure, from sub- to supercritical flow occurred in the upper part of the weir ($x_{\text{crit.}} \approx 0.6 H_0$ for $Fr = 1$). At this distance the boundary layer displacement is too small to be fully responsible for the value of the discharge coefficient $C_d = 0.956$.

With the Ippen concept, which gives the derivation of the discharge relation in the C_d -coefficient as a function of the boundary layer displacement thickness δd , a C_d -value of 0.989 at $x = 0.14$ m and of 0.978 at $x = 0.70$ m (at a discharge rate of 28.6 l/sec) was found.

At $x = 0.70$ m the flow can be considered supercritical and therefore of no influence on the discharge relationship and the C_d -value.

It is obvious that not only the development of a boundary layer and thus of energy losses by friction between wall and fluid, is responsible for the discharge coefficient C_d of 0.956. Other phenomena are playing a role, like the sharp curvature of streamlines at the inflow side of the weir (a constant velocity and a constant piezometric head distribution outside the boundary layer did not exist).

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LIST OF SYMBOLS

The following symbols were adopted for use in this paper

A	area of wet cross-section	m^2
C_v	correction coefficient in the discharge relationship to measured head over the weir	-
C_d	discharged coefficient (standard)	-
D	depth of flow on the crest	m
D_c	depth of flow in the critical flow section or critical depth	m
Fr	Froude number	-
f	friction sign	
g	acceleration due to gravity	m/sec^2
H_o	specific energy level or head above the crest, measured at a small distance upstream of the weir table	m or mm
H_x	specific energy head at distance x from the weir table entrance	''
h	measured head upstream of weir crest, not considering the approach velocity	''
k	roughness of wall	mm
l	length of the weir crest	m
p	crest-height above the bottom of the approach canal	m
P	local pressure	N/cm^2
Q	discharge rate	$m^3/sec-l/sec$
Q_{th}	theoretical discharge rate	$m^3/sec-l/sec$
R	radius of rounded nose at entrance of weir	m
R	hydraulic radius	
R_e	Reynolds number	
R_x	length Reynolds number	
R_t	transition Reynolds number	
U	free-stream velocity outside the boundary layer	m/sec
u	point (local) velocity in the direction of the mainflow	m/sec
\bar{u}	average velocity in a cross-section	m/sec
u_c	critical velocity in the transition-section	m/sec
W	wet perimeter	m
x	co-ordinate in the direction of flow, measured along the boundary	m

x_{crit}	x co-ordinate on weircrest where critical flow conditions occur	m
y	distance from the bottom edge, measured along the wall. Together with x the co-ordinate system of the square network on the weircrest	
δ	boundary layer thickness	mm
δd	boundary layer displacement thickness	mm
ν	kinematic viscosity	m^2/sec
ρ	density of fluid	kgf/m^3

APPENDIX I Velocity distribution profiles

In this appendix the measured velocity profiles are given with the indication of discharge rate, co-ordinates and the computed δd -value. It should be noted that a part of the velocity distribution graph is not given e.g. the x-value mostly begins at 0.80 m/sec.

There are several sources of errors in the estimation of δd such as the influence of:

- 1) placing the dynamic tube on the crest bottom
(0.1 mm fault gives a relative error of 5 to 10% in the computation of δd , depending on the δd -value itself)
- 2) the drawing of the vertical tangent or the estimation to where influence of the boundary layer extends.

Other less important sources of errors are:

- 3) the zero-value of the d.m.m. (was sometimes not constant, specially in the beginning of the measuremently period)
- 4) in the boundary layer the electric signal was filtered to reduce the fluctuations. This gives an error when more fluctuations are below the average value than above this value.

Besides this an other error was made by the method to average first the Δp -value after which u was calculated by equation 16. In the equation the square root of Δp is taken. Therefore it should be better to take these roots or to calculate u before the average u -value is computed. But the errors caused by the procedure, followed in this paper, are relatively small, specially in the value δd computed from the velocity distribution profiles.