Wetting Moisture Characteristic Curves Derived from Constant-rate Infiltration into Thin Soil Samples[1]

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ABSTRACT

Although procedures are available for obtaining the wetting moisture characteristics of soils, they often involve lengthy equilibration periods. A simple and quick laboratory method is described for obtaining the wetting moisture characteristic using constant flux infiltration into a thin section of soil. A theoretical analysis of the process is tested by comparing predicted and observed times for pressure heads to develop in a fine sand sample. The technique is demonstrated using a fine sand, a loam, and a silty clay loam, and comparison is made with data obtained by other methods.

Additional Index Words: **uniform rate of wetting, linearization of Darcy's law.**

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THE relationship between tensiometer pressure h and volumetric water content θ is a key element in studies of equilibrium and transport of water in unsaturated soils. Buckingham (1907) determined the wetting curve by maintaining a water table at the base of a long, vertical soil column, and, after equilibrium was believed to have been reached, sampling the water content profile. Buckingham's method is restricted to long, homogeneous soil columns. In an extension of Buckingham's theory, Haines (1930) determined $\theta(h)$ by exposing the base of relatively thin samples of soil to water at a sequence of increasing pressures and measuring the initial water content, the uptake of water for each increment of pressure, and the water content at saturation.

This paper describes a rapid and precise method for obtaining a wetting curve by supplying a small constant flux of water to the top of a thin section of soil fully occupying a cell of known volume, and measuring the pressure head at the bottom with a pressure transducer. The method depends upon the notion that the increase in moisture content of the soil can be inferred from the influx, provided no water is lost from the sample. If the sample is thin and the rate of application of water is low enough, the pressure head difference across the sample will tend toward zero. This condition implies that the measured pressure head and a mean water content are applicable to the whole sample.

THEORY

One-dimensional, vertical transport of water in unsaturated soils is governed by the balance of mass equation:

$$\partial\theta/\partial t = -\,\partial v/\partial z, \qquad [1]$$

and Darcy's law:

$$v = -k(h)\frac{\partial h}{\partial z} + k(h), \qquad [2]$$

where t is time, z is the vertical coordinate defined positive downwards, v is flux, and $k(h)$ is the hydraulic conductivity pressure head relation. We seek the solution of Eq. [1] and Eq. [2] for a sample of finite depth L, subject to an initial condition:

$$h = h_n - z, \quad t = 0, \quad 0 \leq z \leq L, \qquad [3]$$

where h_n is the initial pressure head at the soil surface, and flux boundary conditions at both ends of the sample:

$$v = v_o, \quad t > 0, \quad z = 0, \qquad [4]$$

$$v = 0, \quad t > 0, \quad z = L. \qquad [5]$$

After the moisture front reaches the bottom of the sample, $\partial\theta/\partial t$ will tend to approach a constant value equal to v_o/L. A similar assumption was used by Passioura and Cowan (1968) in an analysis of radial flow of water to plant roots, and by Passioura (1977) in deducing soil water diffusivities from one-step outflow experiments. Equation [1] then reduces to:

$$\frac{\partial v}{\partial z} \approx -\frac{v_o}{L} = -\frac{d\bar{\theta}}{dt}, \qquad [6]$$

where $\bar{\theta}$ is the average water content of the column. At time t the average increment in water content $\overline{\Delta\theta}$ is given by:

$$\overline{\Delta\theta} = (v_o t)/L = [\textstyle\int_0^L (\theta - \theta_n)dz]/L, \qquad [7]$$

and $\bar{\theta}$ will tend to $\theta[z = L/2]$ whenever v_o and L are small.

The theory that follows demonstrates the effects of v_o, L, and $k(h)$ on the evolving water content profiles.

Integration of Eq. [6] using boundary condition Eq. [4] gives:

$$v = \left(1 - \frac{z}{L}\right) v_o. \qquad [8]$$

To calculate from Eq. [2] and Eq. [8] the evolution of pressure head profiles, we consider the class of soils with an exponential dependence of the hydraulic conductivity upon the pressure head:

$$k(h) = \beta \exp(\alpha h), \qquad [9]$$

where α and β are constants.

Introducing the matrix flux potential ϕ defined by

$$\phi = \textstyle\int_{-\infty}^{h} k(h)dh, \qquad [10]$$

and using Eq. [9] in Eq. [2] gives:

$$v = -\frac{\partial\phi}{\partial z} + \alpha\phi. \qquad [11]$$

Substituting Eq. [11] into Eq. [8] gives this linear differential equation:

$$\frac{\partial\phi}{\partial z} - \alpha\phi = \left(\frac{z}{L} - 1\right) v_o. \qquad [12]$$

The solution of Eq. [12] is:

$$\phi = \frac{v_o}{\alpha}\left(1 - \frac{z}{L} - \frac{1}{\alpha L}\right) + c \exp \alpha z. \qquad [13]$$

Setting $z = 0$ in Eq. [13] gives an expression for the integration constant c:

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Table 1—The physical properties of soils used in the experiment.

Soil	Clay	Silt	Fine sand	Coarse sand	Bulk density	Initial moisture content θ_n
	%				kg m^{-3}	cm^3/cm^3
Bungendore fine sand	1.5	0.9	73.0	24.6	1.58×10^3	0.050
Glebe loam	16.6	21.2	34.2	28.0	1.55×10^3	0.266
Brindabella silty clay loam	33.0	26.0	25.0	16.0	1.16×10^3	0.166

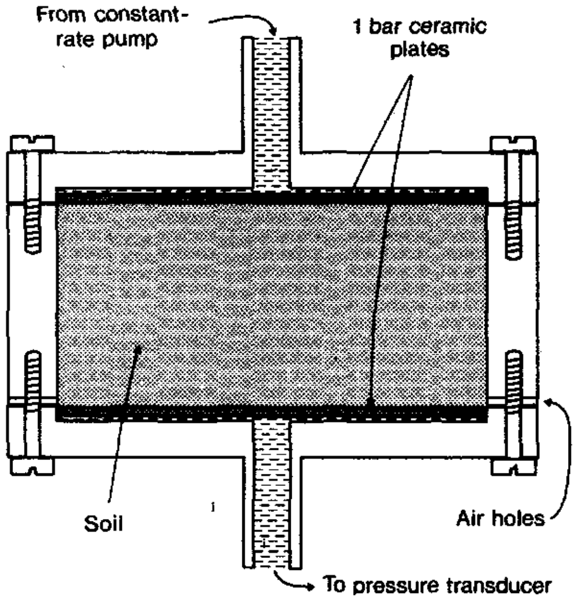

Fig. 1—A cross-section of the soil cell. Soil diameter = 25.4 mm; soil depth = 12.9 mm.

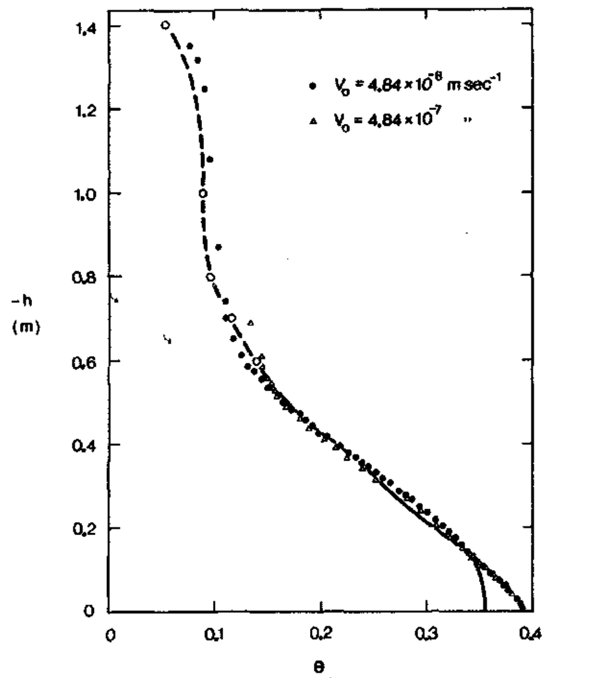

Fig. 2—The wetting experimental points $\theta(h)$ for fine sand obtained by imposing the fluxes indicated. The solid line is a $\theta(h)$ curve for a column (Smiles et al., 1981), and the open circles and dashed line are derived using the method of Haines (1930).

$$c = \frac{\beta}{\alpha}\exp(\alpha h_o) + \frac{v_o}{\alpha}\left(\frac{1}{\alpha L} - 1\right), \qquad [14]$$

where h_o is the pressure head at the soil surface. Introducing Eq. [14] into Eq. [13] and using Eq. [9] and Eq. [10] to solve for h gives:

$$h = \frac{1}{\alpha}\ln\left\{\frac{v_o}{\beta}\left[\left(1 - \frac{z}{L} - \frac{1}{\alpha L}\right) + \exp \alpha z\left(\frac{\beta}{v_o}\exp(\alpha h_o) + \frac{1}{\alpha L} - 1\right)\right]\right\}. \qquad [15]$$

Equation [15] describes pressure head profiles as a function of z, L, α, β, and h_o.

MATERIALS AND METHODS

Three soils were used in the experiment: a fine sand (Bungendore), an aggregated (<2-mm fraction) silty clay loam (Brindabella), and an undisturbed loam (Glebe). The relevant physical properties of the three soils are given in Table 1. The fine sand and silty clay loam were packed into the cell shown in Fig. 1. For the loam, undisturbed samples 25.4 mm in diameter were obtained from the field and trimmed to accurately fit the cell. A constant flux was imposed at the top of the samples through a 1-bar ceramic plate, using a "Unita" constant-rate syringe pump. The pressure head at the bottom of the sample was measured continuously using a tensiometer consisting of a 1-bar ceramic plate connected to a pressure transducer.

The sets of $\theta(h)$ data were obtained for the fine sand using fluxes of 4.84 × 10^{-8} and 4.84 × 10^{-7} m sec^{-1}. For the silty clay loam and loam, a single flux of 4.84 × 10^{-8} m sec^{-1} was used. Alternative wetting moisture characteristics were obtained for the three soils using thin samples on a Haines apparatus (Haines, 1930). A separate sample was used for each pressure increment. For the fine sand $\theta(h)$ data were available for columns at equilibrium with a water table for h = −0.5 m to 0 (Smiles et al., 1981). Data obtained during measurement of $k(\theta)$ relations (Perroux et al., 1981) provided a dynamic $\theta(h)$ relationship for the silty clay loam.

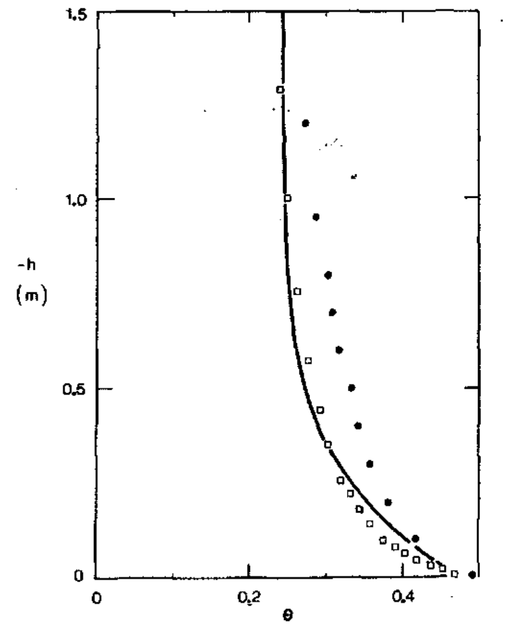

Fig. 3—Wetting $\theta(h)$ data for Brindabella silty clay loam (<2 mm). The squares are for the method of constant flux infiltration (v_o = 4.84 × 10^{-8} m sec^{-1}); the dots are for the method of Haines (1930); and the solid line is for a dynamic $\theta(h)$ relationship (Perroux et al., 1981).

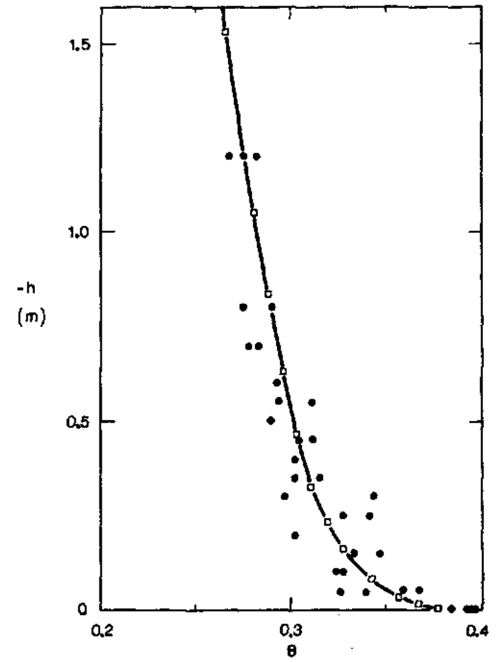

Fig. 4—The $\theta(h)$ data for Glebe loam (undisturbed samples). The squares and curve are for the method of constant flux infiltration ($v_o = 4.84 \times 10^{-8}$ m sec^{-1}); the dots are for the method of Haines (1930).

These data were obtained by constant flux infiltration into long, 50 mm in diameter columns; pressure heads were measured by a tensiometer–pressure transducer system and moisture contents by gamma ray attenuation.

RESULTS AND DISCUSSION

Figures 2, 3, and 4 show $\theta(h)$ relationships for the three soils using the constant flux method as well as static equilibrium methods. Also shown in Fig. 3 is the independently obtained dynamic $\theta(h)$ relation.

The fine sand data of Fig. 2 show good agreement,

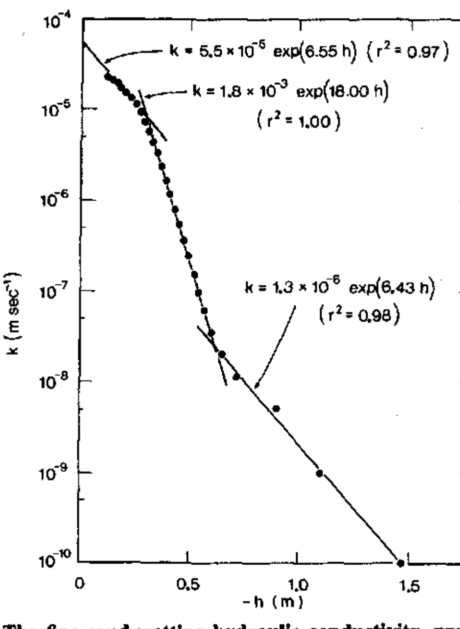

Fig. 5—The fine sand wetting hydraulic conductivity–pressure head relationships $k(h)$ and fitted exponential relations.

even at the higher flux, between the alternative methods except at high water content. It appears that when water is applied at a constant rate, less air is entrapped in the sand than when a column of sand is allowed to come to equilibrium with a water table.

In Fig. 3 the constant flux data agree well with the dynamic $\theta(h)$ relation for the silty clay loam but not with the static equilibrium data obtained from the Haines method. The disparity between moisture characteristics for aggregated soils, subjected to different pressure increments when wetted, has been demonstrated by Davidson et al. (1966), who attributed this to a difference in the water content distribution be-

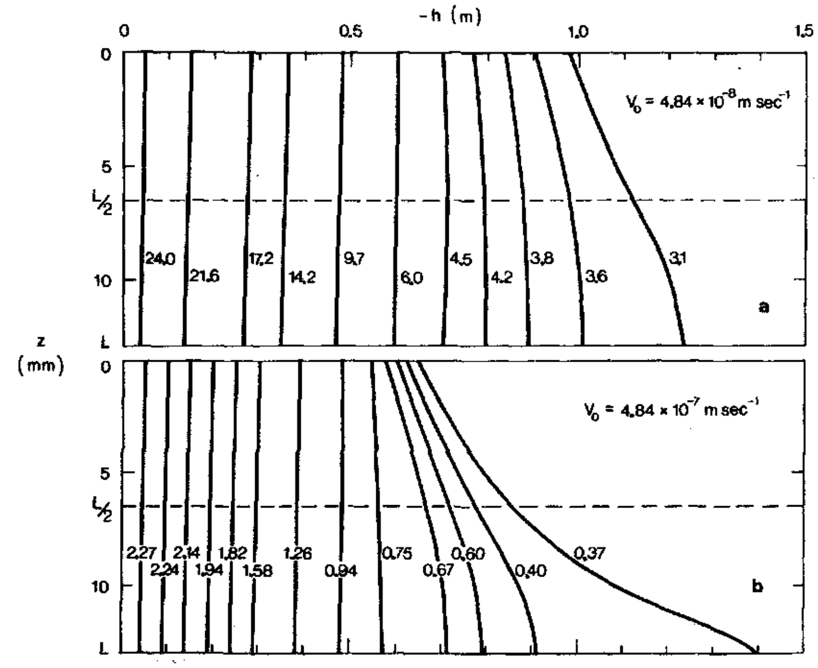

Fig. 6—Predicted pressure head profiles for the fine sand for imposed fluxes of (*a*) 4.84×10^{-8} m sec^{-1} and (*b*) 4.84×10^{-7} m sec^{-1}. The curves are labeled with the predicted time in hours for the profile to develop.

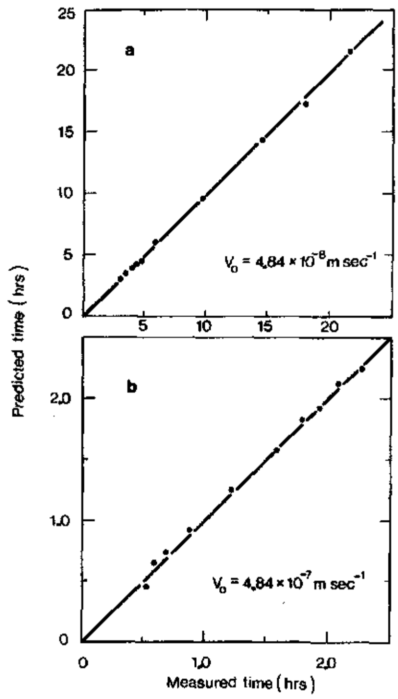

Fig. 7—Predicted and observed times for the soil at the bottom of the fine sand sample to reach pressure heads which correspond to the profiles shown in Fig. 6. The straight line represents exact correspondence.

tween the small pores within aggregates and the large pores between aggregates. The disparity here is, however, considerably greater than that reported by those authors.

The undisturbed samples of Glebe loam exhibit considerable heterogeneity at the scale of sampling used in this study (compare Haines data of Fig. 4). The $\theta(h)$ data obtained by the constant flux method fall centrally within the range of scatter.

Figure 5 shows a wetting $k(h)$ relationship for the fine sand obtained by combining the $k(\theta)$ relationship of Talsma (1974) and the $\theta(h)$ relationship of Fig. 2. A single straight line does not describe the relationship so it has been divided into three linear regions in the ranges of h: 0 to -0.3 m, -0.3 to -0.6 m, and -0.6 to -1.5 m. Lines have been fitted for these ranges as shown in the figure.

Figures 6a and 6b show the predicted evolution of pressure head profiles in the fine sand for fluxes of 4.84×10^{-8} m sec^{-1} and 4.84×10^{-7} m sec^{-1}. These profiles were calculated using Fig. 5 and Eq. [15]. The times at which they are realized were calculated by integrating the corresponding water content profiles and using Eq. [7]. At early times the difference between the measured pressure head at the bottom of the sample and that at the top of the sample is significant, particularly at the higher flux. The difference diminishes as the pressure head increases until it is negligible at -0.5 m for the higher flux and -0.7 m for the lower flux.

For the fine sand a comparison between the predicted and actual times taken for the soil at the bottom of the sample to reach a particular pressure head is shown in Fig. 7. The pressure heads for which the comparisons are made correspond to the profiles shown in Fig. 6a and 6b. The predicted times were calculated from Eq. [7] after integrating the water content profiles derived from the pressure head profiles of Fig. 6a and 6b, and the wetting $\theta(h)$ relationship of Fig. 2. The points lie near the 1 to 1 line indicating good agreement between actual and predicted times.

The analytical approach, used to predict the $h(z)$ profiles with respect to time and flux, is confirmed by comparison.

We reiterate, however, that the analysis merely confirms our insights, based on experience of water movement in soils, that the method should work. With no a priori information on the properties of a particular soil, the justification for the method lies in correspondence between sets of data obtained at substantially different low flow rates.

The method can be used for all nonswelling soils provided that the initial pressure head of the soil is not beyond the normal operating range of tensiometers (~0.7 m). Its use with undisturbed samples is limited by the need to shape thin samples to fit the cell and by the size of the cell in relation to the scale of soil heterogeneity.

Finally, we note that the analysis presented in this paper can be applied to other situations in which pressure head profiles develop in shallow soil profiles during constant-flux infiltration. For example, it may be used in the case of constant-flux infiltration into layered soils in which a fine soil overlies a coarser material. For that case, the time taken for the pressure head at the interface to rise to a value in excess of the water entry value of the coarser material could be calculated.

CONCLUSIONS

The method provides a simple and quick method for the measurement of the laboratory wetting curve of a soil sample. Accurate measurement can be obtained by using very low fluxes and/or correcting the measurements of the pressure head by use of Eq. [14]. The latter approach can be used only if reasonable estimates of α and β can be made.

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