A simple model of crop loss by weed competition from early observations on relative leaf area of the weeds

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Summary: Résumé: Zussamenfassung

A new simple empirical model for early pre-
diction of crop losses by weed competition was
introduced. This model relates yield loss to
relative leaf area of the weeds shortly after crop
emergence using the relative damage coefficient
q as the single model parameter. The model is
derived from the hyperbolic yield density
relationship and therefore accounts for the
effects of weed density. It is shown that the
model also accounts for the effect of different
relative times of weed emergence. A strong
advantage of the approach is that it can be used
when weeds emerge in separate flushes.

The regression model described experimental
data on sugar-beet – lambsquarters (*Beta
vulgaris* L. – *Chenopodium album* L.) and
maize–barnyard grass (*Zea mays* L. –
Echinochloa crus-galli L.) competition pre-
cisely. The model describes a single relationship
between crop yield loss and relative leaf area of
the weeds over a wide range of weed densities
and relative times of weed emergence. Possibili-
ties for scientific and practical application of the
model are discussed.

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*Un modèle simple d'estimation des pertes de
rendements dues à la compétition des adventices
basé sur des observations précoces de la surface
relative des feuilles des adventices.*

Un nouveau modèle empirique simple pour la
prévision précoce des pertes de récolte dues aux
adventices a été établi. Ce modèle relie les
pertes de rendement à la surface relative des
adventices peu après la levée de la culture en
utilisant le coefficient de dégat relatif q comme
seul paramètre du modèle. Le modèle provenait
de la relation hyperbolique entre rendement et
densité et de ce fait prend en compte les effets
de la densité des adventices. Ceci montre que le
modèle prend en compte également les différ-
entes époques de levée des adventices. Un
grand avantage de l'approche est que l'utilisa-
tion est possible quand les adventices lèvent en
vagues séparées.

Le modèle de régression a décrit avec préci-
sion les données expérimentales de la compéti-
tion des chénopodes (*Chenopodium album* L.)
vis à vis des betteraves (*Beta vulgaris* L.), et des
panics pied de coq (*Echinochloa crus galli* L.)
vis à vis du maïs (*Zea mays* L.). Le modèle
décrit une relation simple entre la perte de
rendement et la surface foliaire relative des
feuilles sur un grand nombre de densités
d'adventices et d'époques relatives de levée
d'adventices.

Les possibilités pour des applications scienti-
fiques et pratiques de modèle sont discutées.

*Ein einfaches Modell über Ertragsverluste durch
Unkrautkonkurrenz auf der Basis der relativen
Blattfläche der Unkräuter*

Es wurde ein einfaches Modell zur frühen
Vorhersage von Ertragsverlusten durch Un-
krautkonkurrenz entwickelt. Dieses Modell
stellt den Bezug von Ertragsverlusten zur

relativen Blattfläche der Unkräuter kurz nach
dem Auflaufen der Kulturpflanzen her, wobei
als einziger Modellparameter der Schadens-
Koeffizient q benutzt wird. Das Modell
wurde aus der hyperbolischen Ertrag/Dichte-
Beziehung entwickelt und trägt deshalb dem
Einfluß der Unkrautdichte Rechnung. Es wurde
gezeigt, daß das Modell auch die von verschiede-
nen Keimzeiten der Unkräuter ausgehenden
Wirkungen berücksichtigt, so daß es selbst bei
schubweisem Auflaufen von Unkräutern
benutzt werden kann. Das Regressionsmodell
beschrieb Versuchsergebnisse der Konkurrenz
zwischen der Zucker-Rübe (*Beta vulgaris* L.)
und Weißem Gänsefuß (*Chenopodium album*
L.) sowie Mais (*Zea mays* L.) und Hühnerhirse
(*Echinochloa crus-galli* L.) genau. Das Modell
beschreibt die Einzelbeziehung des Ertragsver-
lusts zur relativen Blattfläche der Unkräuter
über einen weiten Bereich der Unkrautdichte
und verschliedenen Keimzeiten der Unkräuter.
Die wissenschaftlichen und praktischen An-
wendungsmöglichkeiten des Modells werden
diskutiert.

Introduction

The success of weed management programmes
which are directed towards the minimization of
herbicide use, largely depends upon the ability
to predict the effects of weeds on crop yield.
Damage relationships that quantify yield losses
on the basis of early observations of weed infes-
tations are essential for any short- or long-term
economic analysis.

Many empirical models have been developed
to describe the responses of crops yield to one or
more parameters with which weed infestation
can be characterized. The most important
parameters appear to be weed density (Spitters,
1983; Cousens, 1985), and relative time of
emergence of the weeds with respect to the crop
(Håkansson, 1983; Cousens *et al.*, 1987).
Because both weed density and the period
between crop and weed emergence determine
the competitive relations between crop and
weeds (Cousens *et al.*, 1987; Kropff, 1988a, b),
precise prediction of yield loss on the basis of
early observations should be based on both
these factors. However, in practice weeds
often emerge in successive flushes, making it

impossible to use a descriptive model which
accounts for the effect of the relative time of
weed emergence.

Another approach to quantify crop weed
competition has been introduced by Spitters &
Aerts (1983). On the basis of existing eco-
physiological models of crop growth, they
developed a dynamic simulation model in
which competition between the crop and weeds
for light and water is simulated at the process
level. Such models are useful for obtaining
more insight into competition processes
through detailed analysis of experimental data.
After thorough validation these models can be
used to generate data sets for other situations,
because these models are valid for a wide
range of conditions. However, these models
require many parameter estimates for both the
crop and the weeds. The parameter values may
be difficult to derive and, moreover, the un-
certainties of the many parameter estimates
accumulate in the final prediction error. There-
fore too detailed approaches will not be
suitable for management applications. Eco-
physiological models can be helpful, however,
in identifying appropriate variables for use in
simple descriptive models.

Spitters & Aerts (1983) suggested that a
relationship between relative leaf area and yield
loss would be far more appropriate to predict
yield loss than a relationship based on weed
density. It was shown by Kropff (1988a) using
simulated data that a close relationship exists
between relative leaf area of the weeds and yield
loss over a wide range of weed densities and
relative times of weed emergence.

Spitters, Kropff & de Groot (1989) suggested
an approach based on the hyperbolic yield
density function, in which the plant densities of
each species are replaced by their LAI's (Leaf
Area Index in m^2_{leaf} m^{-2}_{ground}) monitored early
in the growing season. However, the approach
was not worked out in detail nor evaluated with
experimental data. In the present paper, an
empirical model will be mathematically derived
from the hyperbolic yield loss weed density
relationship and evaluated with experimental
data. The independent variable in the model is
the leaf area index of a weed species as a frac-
tion of total leaf area index of all species.
Special attention will be paid to the influence of
the moment of observation on the parameter
value.

Materials and methods

The model

The starting point in the derivation of the model is the response of crop yield to plant density, which can often be described by a rectangular hyperbola (e.g. Shinozaki & Kira, 1956; Holliday, 1960; de Wit, 1960):

$$Y_{cm} = \frac{N_c}{b_o + b_c N_c} \quad (1)$$

where Y_{cm} is the yield of the crop in monoculture in g m^{-2}, N_c is the plant density of the crop in numbers m^{-2}, and b_o and b_c are the model parameters. The effect of other species (here weeds) can be introduced in this equation in an additive way (Suehiro & Ogawa, 1980; 1981 and Spitters, 1983):

$$Y_{cw} = \frac{N_c}{b_o + b_c N_c + b_w N_w} \quad (2)$$

where Y_{cw} is the crop yield in a weedy situation, and N_w is the number of weed plants m^{-2}. The parameter b_c measures intraspecific competition between crop plants and the parameter b_w measures interspecific competition effects of the weeds on the crop.

A simple one parameter expression for yield loss (YL) as a function of the relative weed density (N_w/N_c) can then be derived from Equations 1 and 2 when crop density is constant:

$$YL = 1 - \frac{Y_{cw}}{Y_{cm}} = \frac{a \dfrac{N_w}{N_c}}{1 + a \dfrac{N_w}{N_c}} \quad (3)$$

where a characterizes the competitive effect of the weed on the crop:

$$a = \frac{b_w N_c}{b_o + b_c N_c} \quad (4)$$

Generally, the crop is grown at such densities that monoculture yield (Y_{cm}) approaches its maximum value, so that the parameter b_o (equation 1) can be neglected. The expression for the parameter a then approaches to:

$$a = \frac{b_w}{b_c} \quad (4a)$$

Although this relation fits closely to experimental data (Cousens, 1985; Spitters *et al.*, 1989), the value of the parameter a may vary greatly over years or locations, primarily as a result of differences in the period between crop and weed emergence and differences in growing conditions (Cousens *et al.*, 1987; Kropff, 1988a). In practice, weeds of the same species differ in size partly because weeds emerge often in flushes. Because the competitive strength of a species is strongly determined by its share in leaf area at the moment when the canopy closes and interplant competition starts (Spitters & Aerts, 1983; Kropff, 1988a), it is better to weigh the density with the average leaf area of the individual plants at the moment of observation. Weighing densities with the average leaf area per plant (LA) at the moment of observation results in the leaf area index (LAI) of the species:

$$LAI = N \times LA \text{ or } N = \frac{LAI}{LA} \quad (5)$$

Combining Equations 3 and 5 results in an expression for yield loss as a function of the ratio between the LAI of the weed and the LAI of the crop:

$$YL = 1 - \frac{Y_{cw}}{Y_{cm}} = (q\frac{LAI_w}{LAI_c})/(1 + q\frac{LAI_w}{LAI_c}) \quad (6)$$

when a new model parameter (q; the relative damage coefficient) is introduced:

$$q = a \frac{LA_c}{LA_w} \quad (7)$$

in which LA_c and LA_w are the average leaf areas per plant of the crop and the weed at the moment of observation. It is assumed here that the effect of different periods between crop and weed emergence on the parameter a can be compensated by multiplying a with the ratio of the average leaf area per plant, resulting in a parameter q (the relative damage coefficient) which accounts for the effect of weed density and the period between crop and weed emergence.

However, a parameter which may be easier to estimate than the ratio of the leaf area indices is the share in total leaf area of the weed species (L_w):

$$L_w = \frac{LAI_w}{LAI_c + LAI_w} \qquad (8)$$

The model in Equation 6 can be reparameterized to express yield loss of the crop as a function of the relative leaf area of the weed:

$$YL = \frac{qL_w}{1 + (q-1)L_w} \qquad (9)$$

Equation 9 can easily be extended in an additive way to allow for more weed species:

$$YL = \frac{\Sigma q_i L_{w,i}}{1 + \Sigma (q_i - 1) L_{w,i}} \qquad (9a)$$

When the crop is grown at such a density that monoculture yield reaches its maximum value and the crop and weeds have identical physiological and morphological characteristics, the relative damage coefficient q approaches to unity which results in a linear relation (the diagonal 1:1; Fig. 1), which directly follows from Equations 4a and 7. When a weed is a stronger competitor than the crop, the relative damage coefficient q will be larger than one and a convex curve is found above the diagonal line. When the crop is the strongest competitor, the relative damage coefficient q will be smaller than one and a concave curve is found under the diagonal line. When the crop is grown at low suboptimal densities and the crop and weeds have identical physiological and morphological characteristics, the relative damage coefficient q will be smaller than unity since the weeds can occupy space which cannot be occupied by the crop anyway (see Equations 4 and 7). The theoretical relations for different values of the relative damage coefficient q are shown in Fig. 1.

The competitive strength of a species is strongly determined by its share in leaf area at the moment when the canopy closes and interplant competition starts (Spitters & Aerts, 1983; Krupff, 1988a, b). To enable precise decision making in weed management, however, yield loss caused by the weeds has to be estimated as early as possible after crop emergence. Because the relative damage coefficient q depends upon
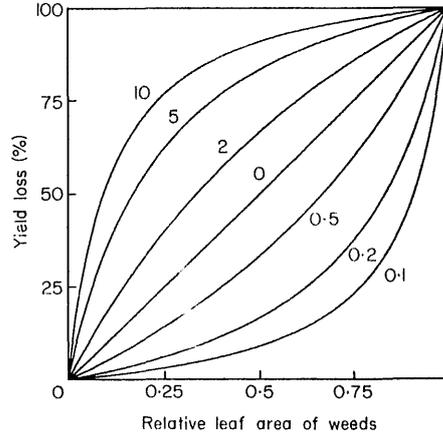

Fig. 1 Theoretical relations between yield loss and relative leaf area of the weeds according to Equation 9 at different values of the parameter q.

the ratio of the leaf area per plant of the crop and the weed (LA_c/LA_w), it is important to know how the relative leaf area of weeds changes in the period between crop emergence and the moment when the canopy closes. In the early growth phase, when the observations on weed infestation have to be made, the canopy is not yet closed and the crop and weed plants generally grow exponentially according to the function:

$$LA_t = LA_0 \times exp\,(RGRL\,t) \qquad (10)$$

in which LA_t represents the leaf area plant at time t, LA_0 the leaf area at the reference time 0 (the moment of observation for which the relative damage coefficient q has been determined from experimental data), $RGRL$ the relative growth rate of the leaf area ($°C^{-1}\,d^{-1}$) in that t is the time expressed in degree days (°C d). The relative growth rate of the leaf area $RGGL$ is only relevant in early growth phases when plants grow exponentially and can easily be determined by growth analysis of free growing plants.

From Equations 10 and 7 it can be derived that the change in time of the relative damage coefficient q in the period of exponential growth when the canopy is not closed equals:

$$q = q_0 \, exp\,(RGRL_c - RGRL_w)\,t \qquad (11)$$

where q_0 is the value of q when L_w is observed at $t = 0$ (the moment of observation for which the

relative damage coefficient q has been determined from experimental data) and t indicates the period between $t = 0$ and the moment of observation (in degree days) for which the relative damage coefficient q will characterize the effects. When the weeds and the crop have the same value of $RGRL$, the relative damage coefficient q will be equal for all different dates of observation. When q is determined for a given crop weed combination at a certain period after crop emergence, the value of the relative damage coefficient q at other days of observations in the early growth period when plants are not yet competing can be calculated using Equation 11 if the relative growth rates of leaf area, for each species are known.

Experiments

Sugar-beet-Chenopodium album L.

Field experiments with sugar-beets and *Chenopodium album* L. were conducted in 1984, 1985 and 1986 on a sandy loam soil at Droevendaal Experimental Station at Wageningen. The sugarbeets were grown at a density of 110 000 plants per ha at a spacing of 0·3 m both between the rows and within the row in 1984 and 1985 and at 0·5 m between the rows and 0·18 m within the row in 1986. In 1984 a natural population of *C. album* L. plants was thinned to 0, 5·5 or 11 plants per m². Both the weeds and the crop emerged on 27 April. In 1985 and 1986 the weeds were sewn by hand, after pretreatment with gibberellic acid and nitrate to stimulate germination, at a density of 5·5 plants per m² in 1985 and 10 plants per m² in 1986. In 1985 sugarbeets emerged on 9 May and the weeds on 19 May. In 1986 the crop emerged on 4 May and the weeds on 25 May and 3 June in two treatments only differing in the date of weed emergence. Other weeds were removed by

hand. Details on experimental design and procedure were given by Kropff (1988a, b).

Maize-Echinochloa crus-galli L.

Maize cv. LG11 was grown on a sandy soil at Wageningen in 1982 and 1983. In 1982 maize was grown in rows (0·75 m distance between rows and 0·12 m within rows) and in 1983 maize was grown at a spacing of 0·3 m between and within the rows (both years 110 000 plants per ha). In 1982 maize emerged at 15 May and in 1983 at 5 June. Plots were not irrigated. *Echinocloa crus-galli* L. densities were established by thinning natural populations to 0, 100, 200 and 300 plants per m² in 1982, whereas in 1983 naturally established densities were used. In 1982, the weeds emerged 5 days after the crop and in 1983 the weeds emerged 2 days before the crop. Further details on experimental design and methods were given by Kropff *et al.* (1984), Spitters (1984) and Spitters *et al.* (1989).

Results and discussion

Sugar-beet-Chenopodium album L.

Observed data. The results of the five sugar-beet-*C. album* experiments are summarized in Table 1. From these data it is obvious that a hyperbolic relationship does not exist between yield loss and weed density, as a result of the large differences in the relative times of weed emergence. However, the data of the sugar-beet-*C. album* L. experiments were precisely described by the yield loss-relative leaf area model presented in this paper, with relative leaf area determined 30 days after crop emergence (Fig. 2, Table 2). The value of the relative damage coefficient q is much larger than 1, indicating that *C. album* is a stronger competitor than sugar-beet (Table 1).

Table 1 Observed and simulated yield losses of sugar-beet crops due to *Chenopodium album* L. competition in different years at the same site and different densities and different periods between crop and weed emergence (Kropff, 1988a)

Year	Weed density (plants m⁻²)	Weed emergence (days after crop)	Observed yield loss (%)	Simulated yield loss (%)
1984	5·5	0	79 ± 6	77
1984	22	0	93 ± 1	88
1985	5·5	10	37 ± 6	30
1986	9·1	21	7 ± 1	15
1986	9·7	30	−6 ± 5	3

Simulated data. It was shown by Kropff (1988a, Table 1, last column) that simulated yield losses could be precisely stimulated by an eco-physiological model for crop weed competition from differences in weed density, dates of weed emergence and weather data. Analysis of the data with the eco-physiological model demonstrated that the large differences in yield loss between the experiments could be attributed mainly to differences in the period between crop and weed emergence. The close relationship in Fig. 2 confirms the theory that the yield loss–relative leaf area model accounts for both differences in weed density and periods between crop and weed emergence.
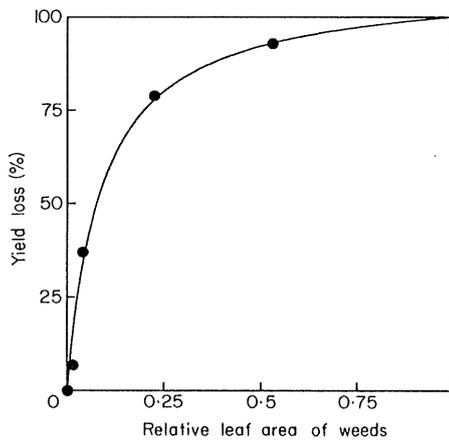

Fig. 2 Yield loss in sugar-beet related to relative leaf area of the weed *Chenopodium album* L., determined 30 days after sugar-beet emergence for experimental data from 5 experiments, conducted in 3 subsequent years (observed ●; fitted ——).

Because the eco-physiological model simulated yield loss in sugar-beet due to *C. album* competition precisely over a wide range of densities and relative times of weed emergence (Table 1, last column), the eco-physiological model was used to perform a number of theoretical experiments in which weed density and date of weed emergence were varied over a wide range. In these simulation experiments, it was assumed that water was in ample supply and average weather conditions for Wageningen were used. These results showed a close relationship between relative leaf area of the weeds and yield loss (Kropff, 1988a). The data from the theoretical experiments were also well described by the yield loss-relative leaf area model (Fig. 3a). The estimated value of the

Table 2 Estimated parameter values using the regression model (Equation 9) to fit observed and simulated data sets on competition between *Chenopodium album* L. (simulated with 3 different maximum heights: H = 120 cm, H = 60 cm and H = 30 cm) and sugar-beet (crop height was 60 cm), and competition between maize and *Echinochloa crus-galli* L. in 1982 and 1983

	q
Sugar-beet-*Chenopodium album* L., observed (Fig. 2)	12·10 ± 1·94
Sugar-beet-*Chenopodium album* L., simulated (Fig. 3)	
H120	9·62 ± 0·72
H60	1·22 ± 0·03
H30	0·06 ± 0·003
Maize-*Echinochloa crus-galli* L. 1982, observed (Fig. 4)	0·49 ± 0·05
Maize-*Echinochloa crus-galli* L. 1983, observed (Fig. 4)	2·30 ± 0·27

relative damage coefficient q was close to the value estimated from the experimental data (Table 2, H120), although average weather data were used in the simulation experiments.

To illustrate the importance of plant height in competitive relationships, the simulation runs were repeated, varying only the maximum height of the weed thereby creating new theoretical weed species. Normally the *C. album* plants grew twice as high as the sugar-beet crop (120 cm *vs* 60 cm). When the parameter value for maximum height of the weeds was reduced to 60 cm which is equal to the height of the crop, the competitive abilities of the species were almost equal, resulting in a value of the relative damage coefficient q near to 1 (Table 2, Fig. 3b). When the maximum height of the weed was restricted to half the value of maximum crop height (30 cm), the crop became far more competitive than the weed resulting in a drop in the value of the relative damage co-efficient q to 0·06 (Table 2, Fig. 3c). For all three sets of simulated experiments the relationship between relative leaf area and yield loss were well described by the regression model, indicating that the presented empirical model indeed allows a precise description of yield loss on the basis of early observations of the relative leaf area of the weeds over a wide range of weed densities and relative times of weed emergence.

Maize-Echinochloa crus-galli *L.*

The results of fitting the regression model to the data sets on maize-*E. crus-galli* competition are
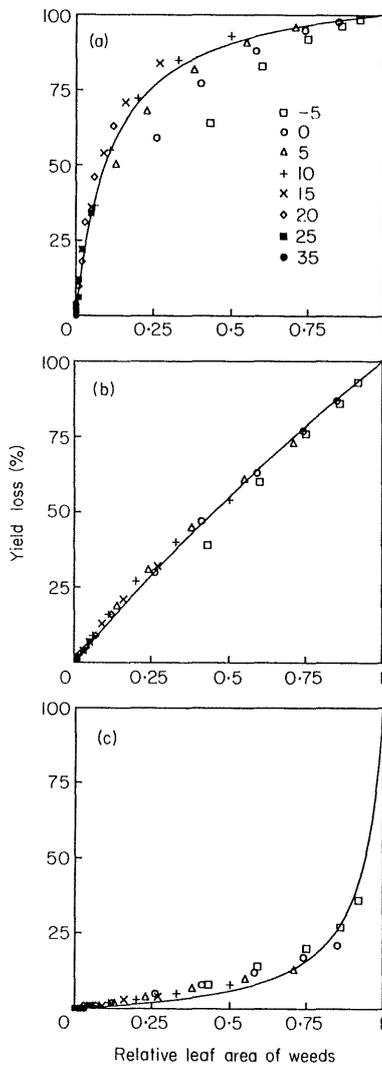
Fig. 3 Relation between simulated yield loss in sugar-beet and relative leaf area of *Chenopodium album* L. determined 30 days after sugar-beet emergence. *Chenopodium album* L. densities were 5·5, 11, 22, 44 and 88 plants m^{-2} at a wide range of periods between crop and weed emergence (see legend in (a)) for weeds with a maximum height of 120 cm (a), 60 cm (b) and 30 cm (c). Maximum height of the crop was 60 cm. Lines give the result of fitting the simulated data with a regression model (Equation 9).

shown in Fig. 4. The estimated parameter values are presented in Table 2. In these experiments only weed density was varied, although the relative time of weed emergence differed between the years. In 1982 the weeds emerged 5 days after the crop and in 1983 the weeds emerged 2 days before the crop. A reasonable

fit was found for both years, although the relationship clearly differed between the years, which resulted in different values of the relative damage coefficient q (Table 2). In 1982 the crop was the strongest competitor ($q < 1$) whereas the weed was the strongest competitor in 1983 ($q > 1$). The difference in the effect of the weeds between years was largely due to the extreme drought in 1983 which occurred around the period of stem elongation of the maize (Kropff *et al.*, 1984, Spitters, 1984). However, the yield loss–relative leaf area model shows less difference between the years than the hyperbolic yield density functions presented by Kropff *et al.* (1984) and Spitters *et al.* (1989), because the model presented in this paper accounts for differences in relative time of weed emergence. The following calculations show the advantage of the yield loss–relative leaf area model over the hyperbolic yield density equation. Expected yield loss at 100 *E. crus-galli* plants m^{-2} for 1983 was calculated using parameter estimates from the 1982 data for both regression models. A yield loss of 8% was estimated by the hyperbolic yield loss weed density function and 45% yield loss was estimated by the yield loss–relative leaf area model, whereas observed yield loss was 81%. This supports the interpretation made using the eco-physiological models that the stronger competitive effects in 1983 were only partly caused by the earlier emergence of the weeds with respect to the crop (Kropff *et al.*, 1984). These results demonstrate that application of the approach presented in this paper (or any other empirical model) to situations where large differences in water availability may occur should be based on experimental data from realistic but contrasting situations. This allows the prediction of the variation in yield loss which can be expected as a result of unpredictable differences in weather conditions.

The one parameter model presented in this paper provides a good description of crop yield loss, expressed in total above ground biomass, as a function of the relative leaf area of the weeds shortly after crop emergence. It was shown theoretically and by application of the model to experimental data that the model accounts for the effects of both weed density and the relative time of weed emergence. The model is superior to the extended hyperbolic yield density functions which account for differences in relative time of weed emergence, because
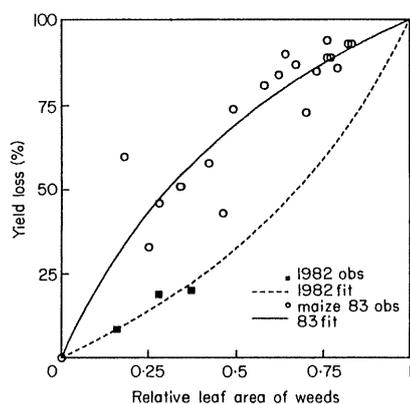
Fig. 4 Relation between yield loss in maize and relative leaf area of the weed *Echinochloa crus-galli* L., determined 20 days after maize emergence for experimental data from 1982 and 1983.

accurate estimation of the relative time of weed emergence is not required, which is not feasible in practice (Kropff, 1988a). Moreover, the yield loss–relative leaf area model also handles the effect of different emergence flushes of the weeds for the same reasons mentioned in relation to differences in relative times of weed emergence.

Practical application of predictive models for crop losses due to weed competition requires a prediction of yield losses as soon as possible after crop emergence, to allow timely application of weed control measures. However, the main problem for all types of approaches is the yield loss caused by weeds emerging after the date of observation. The reliability of all models increases with the length of the period between crop emergence and the date of observation of weed infestation. From critical period experiments, it is well known that weeds emerging after a crop and weed specific period (generally a few weeks) have minimal effects on crop yield. Since available control methods restrict the possibilities for post emergence control (especially in sugar-beet crops) it is important to quantify the minimum period between crop emergence and observation for reliable prediction of yield losses. Observations may start after crop emergence to decide whether weed control is needed, but the observations have to be continued until the end of this period.

For practical application of the model, values of the relative damage coefficient q in the

particular crop must be estimated (and its range when extreme environmental stress may occur) for each weed species. The relationship between the value of the relative damage coefficient q and the period between crop emergence and the date of observation can be estimated by daily observations in the field or by Equation 11, if the relative growth rates of the leaf area are known from simple growth analysis on free growing plants. When the crop-weed canopy is not yet closed, the relative leaf area will be strongly correlated to the relative leaf cover, which is relatively easy to determine. Simple and accurate methods to estimate relative leaf cover need to be developed.

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