AN EXAMPLE OF A RAINFALL-RUNOFF MODEL FOR DESIGN FLOOD COMPUTATION

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Stefan Ignar

Department of Hydraulic Structures
Warsaw Agricultural University
02-766 Warsaw
Poland

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1. INTRODUCTION

Mathematical models in hydrology are often used as a tool for flood analysis. Such an analysis is carried out to determine the magnitude of extreme flows with a low probability of occurrence, the so called design discharges.

The main problem of mathematical modelling of small agricultural watersheds is the lack of recorded data. It requires research to apply simple, conceptual rainfall-runoff models with only a few parameters. Parameters of these models can be determined from correlation formulae, topographic maps and published tables.

This report describes the practical application of a conceptual model developed by Wackermann for design flood evaluation with assumed probability of occurrence. The method assumes the equality of probabilities for design precipitation and discharge. The described alghorithm consists of four stages leading to evaluation:

a) total rainfall - P
b) effective rainfall - H
c) direct flow hydrograph - Qp
d) total flood flow hydrograph - Q

The first three of them will be described. The fourth stage consists of summation of two hydrographs: direct flow Qp and groundwater flow Qg. The values of Qg are relatively very small compared to Qp and it is possible to assume that direct flow hydrograph can be treated as a total flow hydrograph.

A practical application of the method was carried out for a small agricultural watershed (area = 6.5 km^2) in east Holland.

2. DESCRIPTION OF THE WATERSHED

The Hupselse Beek runs from east to west through a slightly undulating rural landscape in the eastern part of The Netherlands. This region of sandy soils is well above sea level. The catchment area is mainly covered with grass. The top of the underlying thick tertiory formation of marine clays is found of shallow depths in the east and slopes down to the west. These marine clays are covered with younger sand deposits. The thickness of this sand aquifer varies between 1 and 8 m from east to west. Consequently the transmissivity and the storage capacity of the soil are relatively small, the groundwater table in this region is shallow, about 50 cm below ground surface in winter whereas in summer time it may decline to about 1.30 m. Locally groundwater levels may rise to the surface during prolonged wet periods.

3. TOTAL RAINFALL

The described method utilizes as an input a design or critical rainfall histograph that imitates some severe future or historical event. If rainfall records are unavailable, design histographs are found from empirical formulae describing relationships between probability of occurrence, duration and intensity of the rain. For regions with rainfall records such a relationships can be developed for particular stations.
In order to determine the input histograph it is necessary to assume a probability of occurrence and to determine the duration of critical storm. The next step is to obtain storm intensity based on the selected probability and duration. Duration of input rainfall is usually equated to the time of concentration of the watershed. The time of concentration is assumed to be equal to flow time from the most remote point in the drainage area to the outlet of interest (Viessman et al., 1977). The Kirpich equation can be used for the time of concentration determination:

$$t_c = 0.0663 \cdot \left(\frac{L}{\sqrt{I}}\right)^{0.77} \qquad \ldots \quad [1]$$

where:

t_c = time of concentration (h)

L = the horizontal projection of the channel length from the most distant point to the basin outlet (km)

I = slope between the two points (-)

Because this formula gives only a rough estimation of t_c, it is necessary to find the critical rainfall duration by a trial method, computing the flood hydrographs for a few,usually longer, durations.

For the Hupselse Beek watershed the relation between probability, duration and intensity of rainfall was assumed as for station De Bilt (Buishand, Velds, 1980).

Developed relations for an assumed 1% probability lead to the intensity-duration curve shown in Fig. 1.

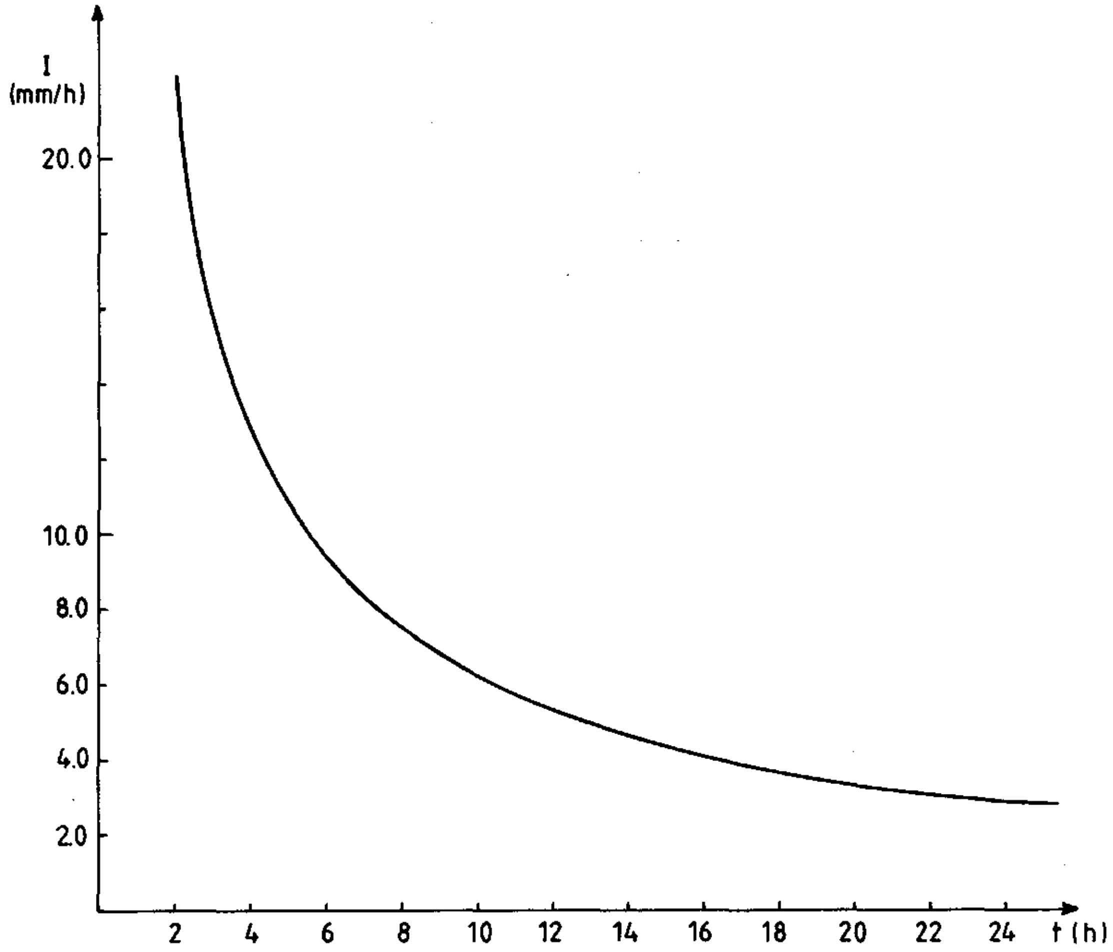

Fig. 1. Intensity - duration curve for station De Bilt. (p = 1%)

Time of concentration from Kirpich formula is 2.75 h. Duration used in the calculations were: 2, 4, 6, 8, 12, 14 and 16 hours. Rainfall time distribution was uniform.

4. EFFECTIVE RAINFALL

Effective rainfall is a part of total rainfall remaining after withdrawing of losses consisting of infiltration , evapotranspiration, interception and depression storage. This rainfall is transformed by the surface watershed into direct runoff.

Among the many methods used in engineering hydrology for effective rainfall determination, the SCS (Soil Conservation Service) Curve Number method is one of the most often used.

According to this method, the volume of effective rainfall is subjected to the CN (Curve Number) parameter depending on soil type, land use, soil conservation practices and antecedent moisture conditions. This parameter is related to the maximum retention, S in mm:

$$S = 25.4 \cdot \left(\frac{1000}{CN} - 10\right) \qquad \ldots [2]$$

and effective rainfall after time $t_i = i \cdot \Delta t$ can be calculated from the formula:

$$H_{ti} = \sum_{j=1}^{i} \Delta Hj = \begin{cases} 0 & \text{for } P_{ti} - 0.2 \cdot S \leq 0 \\ \dfrac{(P_{ti} - 0.2 \cdot S)^2}{P_{ti} + 0.8 \cdot S} & \text{for } P_{ti} - 0.2 \cdot S > 0 \end{cases} \qquad \ldots [3]$$

where:

H_{ti}= effective rainfall in time from t_0 to t_i (mm)

S = maximum potential retention of the watershed, i.e. difference between total rainfall and direct runoff after a long time (mm)

CN = method parameter (-)

P_{ti}= total rainfall in time from t_0 to t_i (mm)

ΔH_j= effective rainfall in j-time interval (mm)

ΔP_j= total rainfall in j-time interval (mm)

Using this formula it is possible to determine the effective rainfall in subsequent time intervals. The value of the CN parameter can be evaluated from tables developed by SCS. In this method soils are classified as A, B, C or D according to the following criteria:

A. (Low runoff potential) Soils having high infiltration rates even in thoroughly wetted and consisting chiefly of deep well to excessively drained sands and gravels. They have a high rate of water transmission.

B. Soils having moderate infiltration rates if thoroughly wetted and consisting chiefly of moderately deep to deep, moderately well to well-drained soils with moderately fine to moderately coarse textures. They have a moderate rate of water transmission.

C. Soils having slow infiltration rates if thoroughly wetted and consisting chiefly of soil with a layer that impedes the downward movement of water, or soils with moderately fine to fine texture. They have a slow rate of water transmission.

D. (High runoff potential) Soils having very slow infiltration rates if thoroughly wetted and consisting chiefly of clay soils with a high swelling potential, soils with a daypan or clay layer at or near the surface, and shallow soils over nearly impervious material. They have a very slow rate of water transmission.

A CN value is extracted from Table 1. A composite CN for a watershed having more than one land use, treatment or soil type can be found by weighting each curve number according to its area. The curve numbers in Table 1 are applicable to average antecedent moisture conditions.

Table 1 Runoff Curve Numbers for hydrologic soil-cover complexes (Antecedent moisture condition II, and $I_a=0.2S$)

Cover			Hydrologic Soil Group			
Land use or cover	Treatment or Practice	Hydrologic Condition	A	B	C	D
Fallow	Straight row	--	77	86	91	94
Row crops	Straight row	Poor	72	81	88	91
	Straight row	Good	67	78	85	89
	Contoured	Poor	70	79	84	88
	Contoured	Good	65	75	82	86
	Contoured and terraced	Poor	66	74	80	82
	Contoured and terraced	Good	62	71	78	81
Small grain	Straight row	Poor	65	76	84	88
		Good	63	75	83	87
	Contoured	Poor	63	74	82	85
		Good	61	73	81	84
	Contoured and terraced	Poor	61	72	79	82
		Good	59	70	78	81
Close-seeded legumes	Straight row	Poor	66	77	85	89
or rotation meadow	Straight row	Good	58	72	81	85
	Contoured	Poor	64	75	83	85
	Contoured	Good	55	69	78	83
	Contoured and terraced	Poor	63	73	80	83
	Contoured and terraced	Good	51	67	76	80
Pasture or range		Poor	68	79	86	89
		Fair	49	69	79	84
		Good	39	61	74	80
	Contoured	Poor	47	67	81	88
	Contoured	Fair	25	59	75	83
	Contoured	Good	6	35	70	79
Meadow		Good	30	58	71	78
Woods		Poor	45	66	77	83
		Fair	36	60	73	79
		Good	25	55	70	77
Farmsteads		--	59	74	82	86
Roads (dirt)		--	72	82	87	89
(hard surface)		--	74	84	90	92

Other antecedent moisture conditions (AMC) are:

AMC I : A condition of watershed soils where the soils are dry but not wilting point, and when satisfactory plowing or cultivation takes place.

AMC II : The average case for annual floods.

AMC III: When heavy rainfall or light rainfall and low temperatures have occurred during the 5 days previous to the given storm.

Table 2 gives total 5-day antecedent rainfall for different AMC. Conversion of the curve numbers to moisture categories I or III is given in Table 3.

Table 2 Classification of Antecedent Moisture Conditions.

Condition	5-day antecedent rainfall, mm	
	Dormant season	Growing season
I	up to 13	less than 35
II	13-28	35 to 53
III	over 28	over 53

Table 3 Curve Numbers (CN) for wet (AMC III) and dry (AMC I) Antecedent Moisture Conditions corresponding to an average Antecedent Moisture Condition.

CN for AMC II	Correspondig CN's	
	AMC I	AMC III
100	100	100
95	87	98
90	78	96
85	70	94
80	63	91
75	57	88
70	51	85
65	45	82
60	40	78
55	35	74
50	31	70
45	26	65
40	22	60
35	18	55
30	15	50
25	12	43
20	9	37
15	6	30
10	4	22
5	2	13

For a gauged watershed the CN value can be evaluated from measured rainfall and runoff by iterative methods (Banasik, Ignar, 1983).
For the Hupselse Beek watershed the average CN value calculated using 10 flood events was 91.8.
CN value determined for the watershed applying the hydrologic soil-cover complex procedure with the use of the data about type of soil and land use was much lower and was equal to 73.6.

5. RAINFALL RUNOFF TRANSFORMATION

Among the many rainfall-runoff models which have been developed for flood flow evaluation, practical use is constrained to simple models with few, easy to determine parameters. One such model is the Wackermann (1981) model of two parallel cascades of linear reservoirs consisting of two reservoirs (Fig. 2). It is a special case of the more general Diskin model. Parameters of these model (i.e. K1, K2-retention coefficients for first and second cascade, and β-dividing coefficient for input effective rainfall) can be evaluated from formulae developed by Thiele (1981) from data recorded in over 90 watersheds from West Germany:

$$K1 = 0.7308 \cdot \left(\frac{L}{\sqrt{I}}\right)^{0.2175} \qquad \ldots [5]$$

$$K2 = 2.0246 \cdot \left(\frac{L}{\sqrt{I}}\right)^{0.2814} \qquad \ldots [6]$$

$$\beta = 2.0188 \cdot \left(\frac{L}{\sqrt{I}}\right)^{-0.5078} \qquad \ldots [7]$$

where:

L,I = like in formula 1

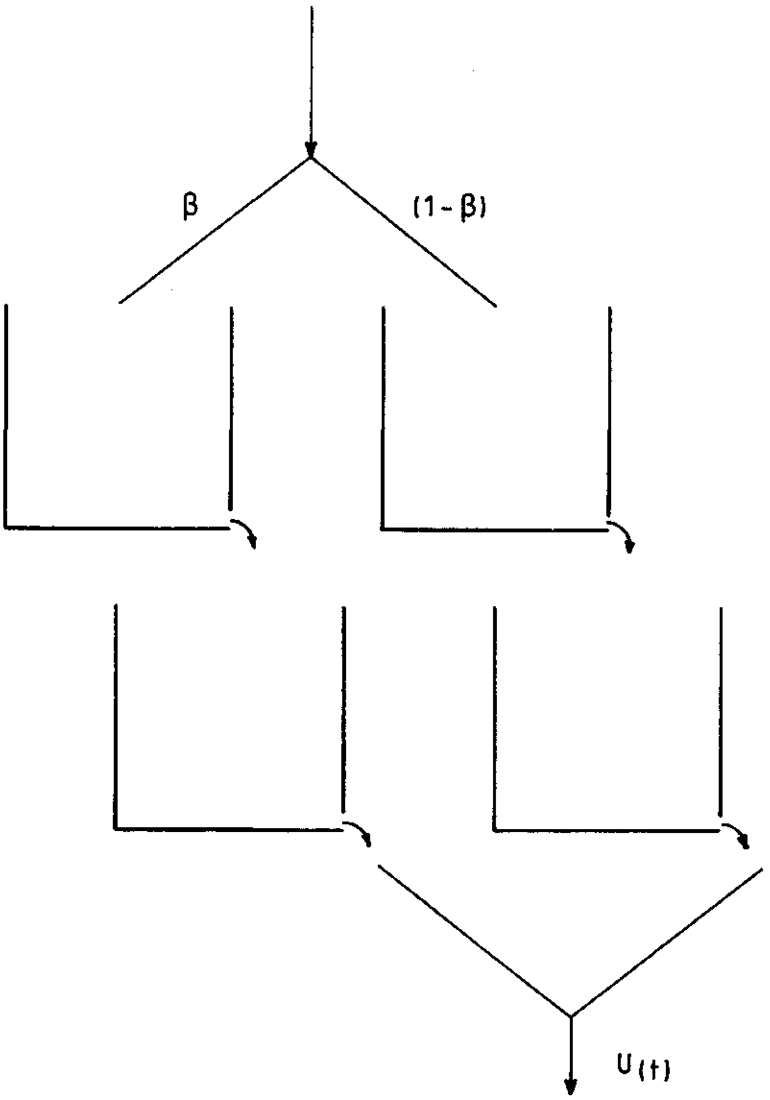

Fig. 2. Conceptual rainfall - runoff model by Wackermann.

Instantaneous Unit Hydrograph (IUH) ordinates caused by unit of effective rainfall in time → 0 can be determined from equation:

$$u_t = \beta \cdot u_{1(t)} + (1-\beta) \cdot u_{2(t)} \qquad \ldots \quad [8]$$

$$t = 1, 2 \ldots m$$

where:

$u_{1(t)}$, $u_{2(t)}$ - ordinates of IUH for first and second cascade, respectively (1/h), calculated from the relationship:

$$u_{j(t)} = \frac{t}{K_j^2} \cdot e^{-\frac{t}{K_j}} \qquad \ldots \quad [9]$$

where:

j = 1,2 - for first and second cascade

K_j = retention coefficient for j cascade

t = time from the start of IUH

Ordinates of the unit hydrograph caused by 1 mm of effective rainfall with Δt duration for watershed area F and used for rainfall-runoff transformation are calculated from the equation:

$$h_t = \frac{F}{3.6} \cdot \bar{u}_t = \frac{F}{7.2} \cdot (u_t + u_{t-1}) \qquad i=1,2 \ldots m \qquad \ldots [10]$$

where:

h_t = unit hydrograph ordinates ($m^3/s \cdot mm$)

u_t = like in equation 8

F = area of the watershed (km^2)

$\bar{u}_t$ = $1/2 \cdot (u_t + u_{t-1})$ in (1/h)

1/3.6 = unit coefficient

Calculation of direct flow from effective rainfall histograph and unit hydrograph is shown in Fig. 3. It can be written in general form:

$$Q_{p(i)} = \sum_{j=1}^{\min(i,n)} h_k \cdot \Delta H_j \qquad k=i-j+1,\ i=1,2 \ldots m+n-1 \qquad \ldots [11]$$

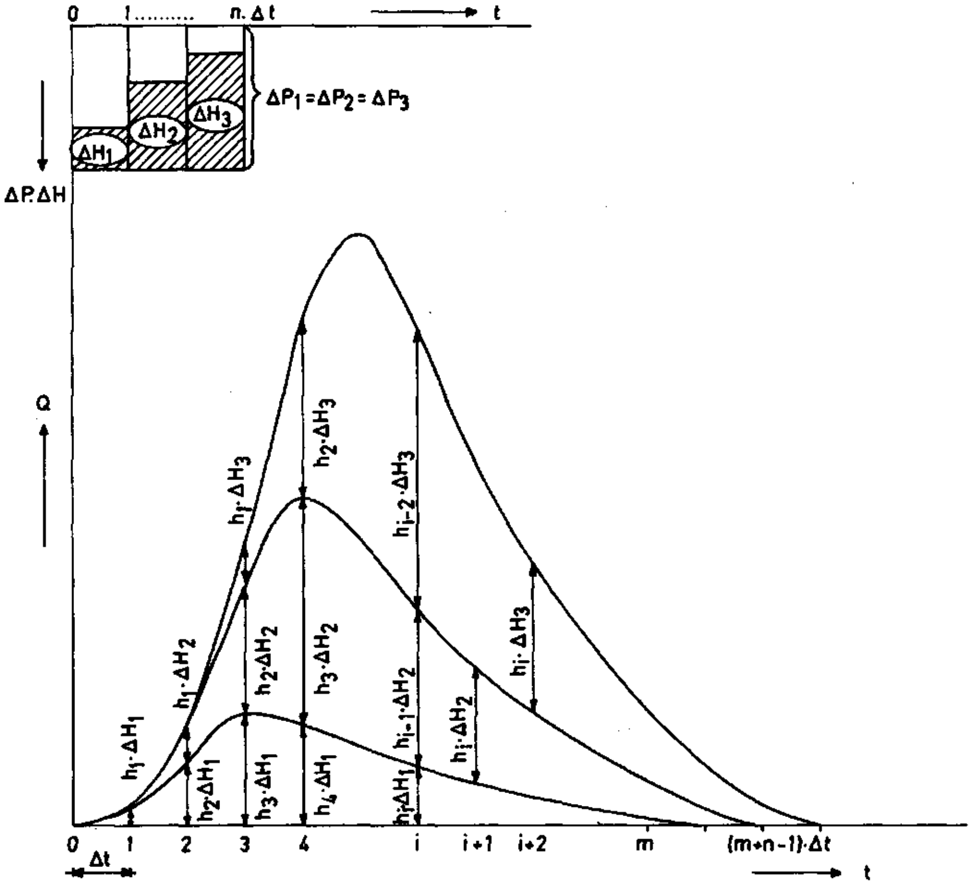

Fig. 3. Direct flow calculation.

6. WACKERMANN MODEL PARAMETERS EVALUATION

Parameters of the Wackermann model for the Hupselse Beek watershed were calculated in two ways. First they were obtained from formulae 5 to 7, applying L=4 km and I=1 0/00. Calculated values were $\beta=.173$, K1=2.094, and K2=7.904.

The second method was based on the optimalization method. The procedure based on the direct search technique with constraints proposed by Rosenbrock was adopted (Kuester, Mize, 1973) for the automatic calibration of the model parameters. This method is the most commonly used in hydrologic research works. The sum of the squares of the differences between simulated and observed direct flow ordinates was chosen as an objective function among many forms described in the literature about model calibration:

$$F = \sum_{i=1}^{N} (Q_{O(i)} - Q_{c(i)})^2 \qquad \ldots \quad [12]$$

where:

F - objective function

$Q_{O(i)}$ = the i-th value of the observed flow (m^3/s)

$Q_{C(i)}$ = the i-th value of the computed flow (m^3/s)

N - numbers of values in the flow series

The computer program which contains described methods was developed in the Department of Hydraulic Structures, at the Warsaw Agricultural University. Ten rainfall-runoff events were taken from the recorded data for the optimization calculations. The results are shown in Table 4. The recorded events were divided into two sets (1 to 6 and 7 to 10) in order to make it possible to verify the model parameters with the set of independent data (events 7 to 10). So the optimized parameters were calculated as an average for all events and for 1 to 6 events.

Table 4 Optimized Wackermann model parameters

No	Date	β	K1	K2
1	22-08-77	.295	3.073	7.312
2	11-12-77	.144	1.664	5.781
3	20-03-78	.159	6.053	4.090
4	02-02-80	.230	1.222	3.589
5	17-12-80	.333	2.620	6.528
6	09-01-81	.323	.999	4.862
7	14-01-81	.341	1.130	4.958
8	31-12-81	.357	5.124	2.429
9	06-06-82	.499	3.984	2.199
10	21-03-83	.444	7.344	2.428
	Average 1:6	.226	2.616	5.339
	Average 1:10	.270	3.132	4.521
	Thiele formulae	.173	2.094	7.904

Parameter values from the Thiele formulae are shown for comparison. In order to compare the effect of different methods of parameter evaluation, three instantaneous unit hydrographs are shown in Figure 4 for three calculated sets of parameters.

Next, the verification was done using parameters computed in a different way, for four flood events (7 to 10). The criterion proposed by Delleur, Sarma and Rao (1973) was used for comparison of observed and simulated hydrographs. It was the so-called special correlation coefficient in the form:

$$R_S = \left[\frac{2 \sum_{i=1}^{N} Q_{O(i)} \cdot Q_{C(i)} - \sum_{i=1}^{N} (Q_{C(i)})^2}{\sum_{i=1}^{N} (Q_{O(i)})^2} \right]^{1/2} \qquad \ldots [13]$$

where:

$Q_{O(i)}$, $Q_{C(i)}$, N - like in form 12

The authors of the criterion determined five intervals making it possible to evaluate the agreement between the obseved hydrographs and the one computed by the model, by using five grades from excellent (denoted by 5) to poor (denoted by 1). These intervals are:

$0.99 \leq R_s < 1.0$	excellent = 5
$0.95 \leq R_s < 0.99$	very good = 4
$0.90 \leq R_s < 0.95$	good = 3
$0.85 \leq R_s < 0.90$	fair = 2
$0.00 \leq R_s < 0.85$	poor = 1

The results of the verification are given in Table 5.

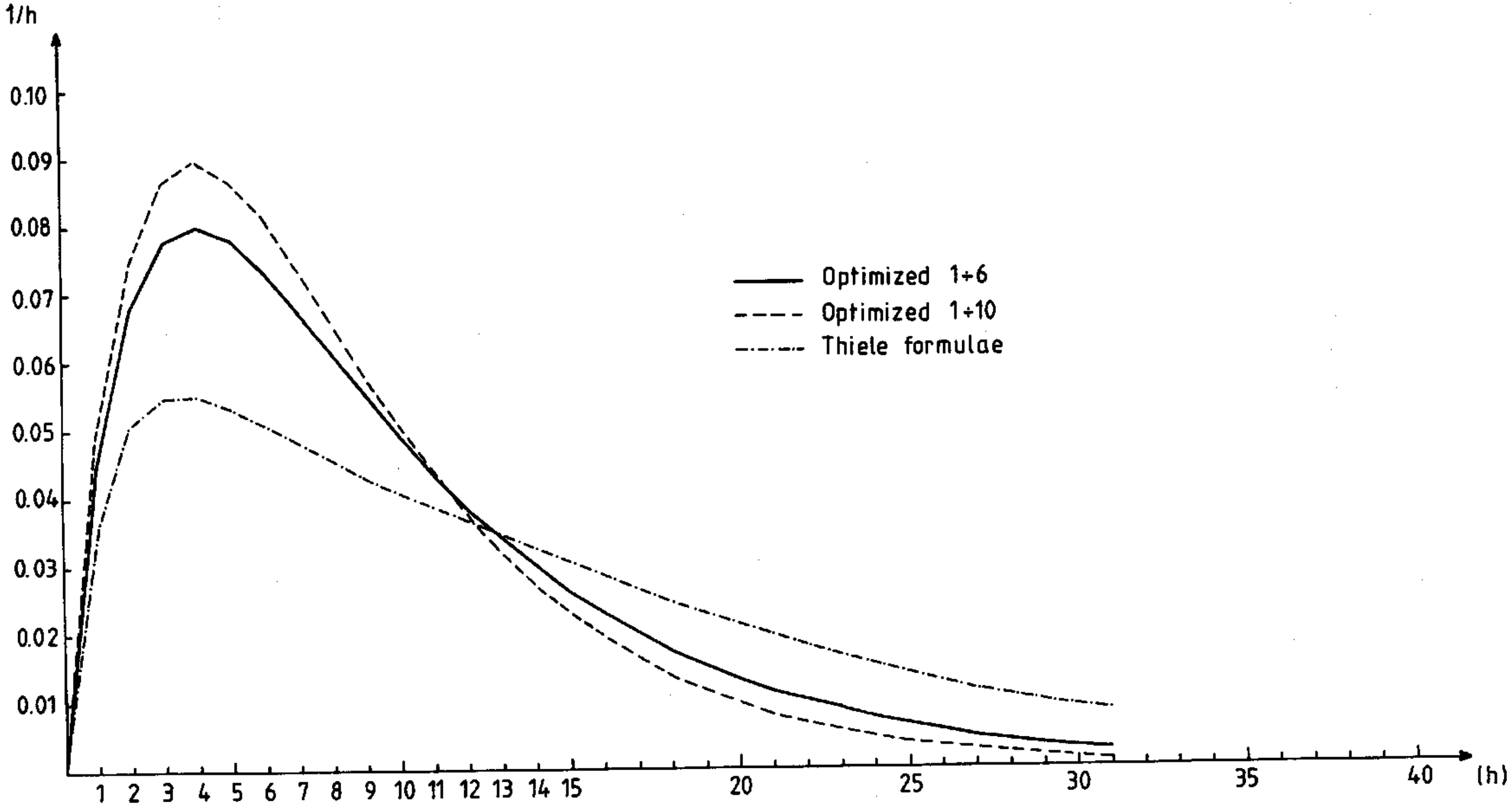

Fig. 4. Instantaneous unit hydrographs for three sets of parameters.

Table 5 Results of Wackermann model verification.

No	Optimization 1:10		Optimization 1:6		Thiele formulae	
	RS	grade	RS	grade	RS	grade
7	.963	4	.955	4	.896	2
8	.973	4	.956	4	.876	2
9	.952	4	.935	3	.863	2
10	.978	4	.976	4	.927	3

7. DESIGN FLOOD SIMULATION

The described rainfall-runoff model was used for design flood simulation for the Hupselse Beek watershed. The CN parameter, assessed as the average of 10 observed events was equal to 91.8. Eight total rainfall durations were applied to check their influence on flood magnitude. There were 2, 4, 6, 8, 10, 12, 14 and 16 hours,time of concentration evaluated from Kirpich formula was 2.75 hours). Total rainfall intensities were taken from the relationship shown in Figure 1. The simulated hydrographs are presented in Figure 5 for CN equal to 91.8 and in Figure 6 for CN=73.6. Rainfall intensities and amounts together with peak flows are shown in Table 6.

Table 6 Simulated design floods.

No	TP [h]	Total rainfall intensity [mm/h]	Total rainfall amount [mm]	Peak flow [m^3/s]	
				CN=73.6	CN=91.8
1	2	22.2	44.4	.94	4.10
2	4	13.1	52.4	1.47	5.13
3	6	9.3	55.8	1.74	5.48
4	8	7.4	59.2	1.97	5.70
5	10	6.2	62.0	2.15	5.76
6	12	5.3	63.6	2.21	5.66
7	14	4.6	64.4	2.18	5.42
8	16	4.1	65.6	2.19	5.20

In order to evaluate the influence of CN value on peak flows additional simulations were conducted for CN=60, 70 and 80 and for rainfall durations up to 20 hours. The relationships between flood peaks and rainfall duration for different CN are presented in Figure 7.

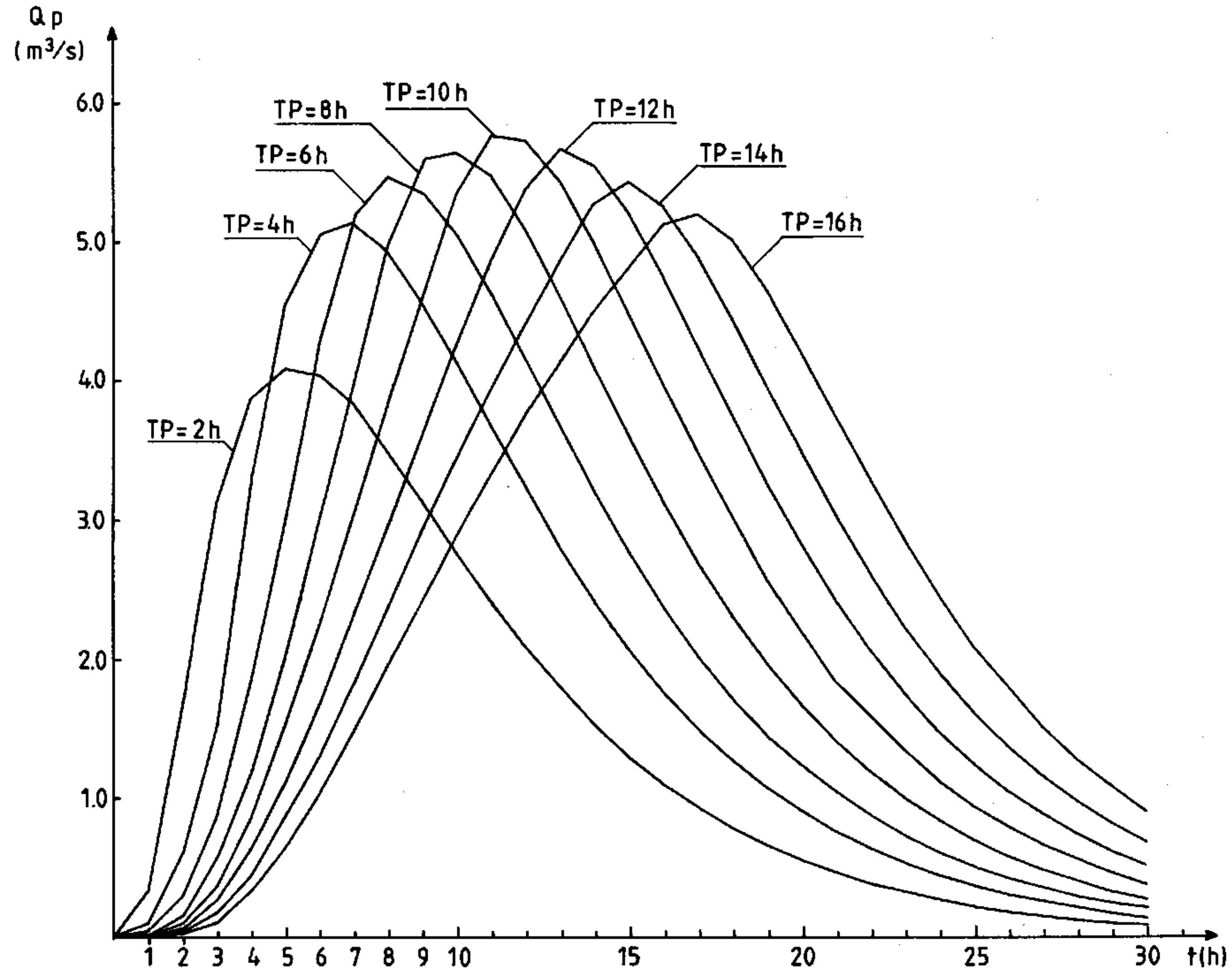

Fig. 5. Direct runoff hydrographs for CN = 91.8

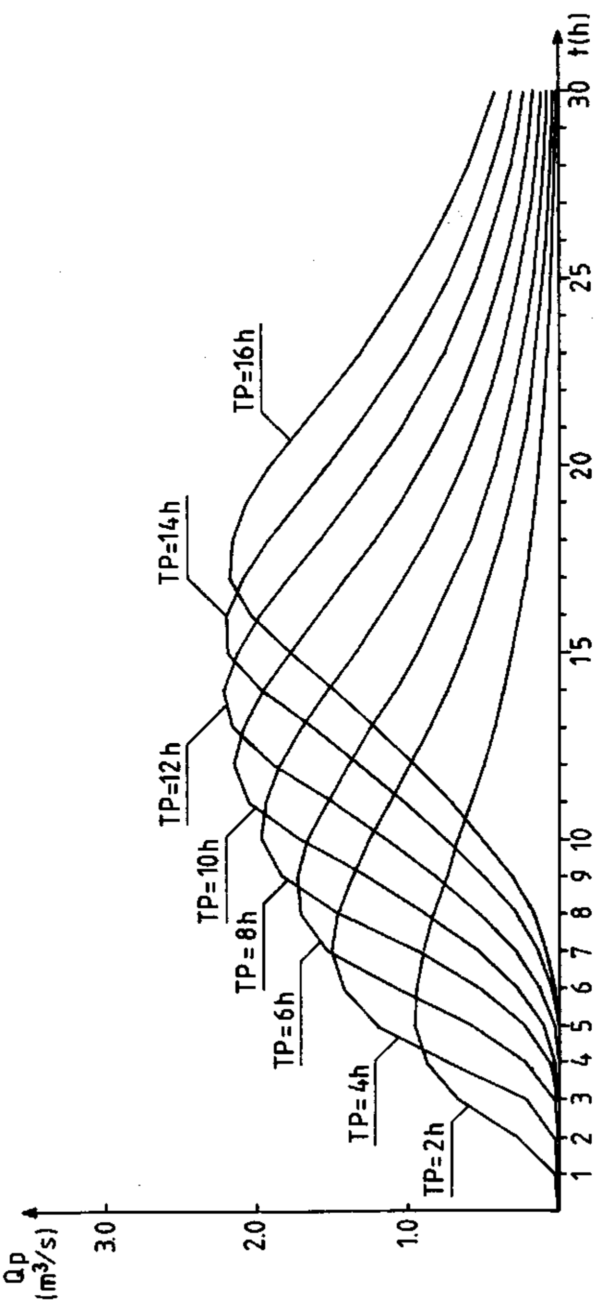

Fig. 6. Direct runoff hydrographs for CN = 73.6

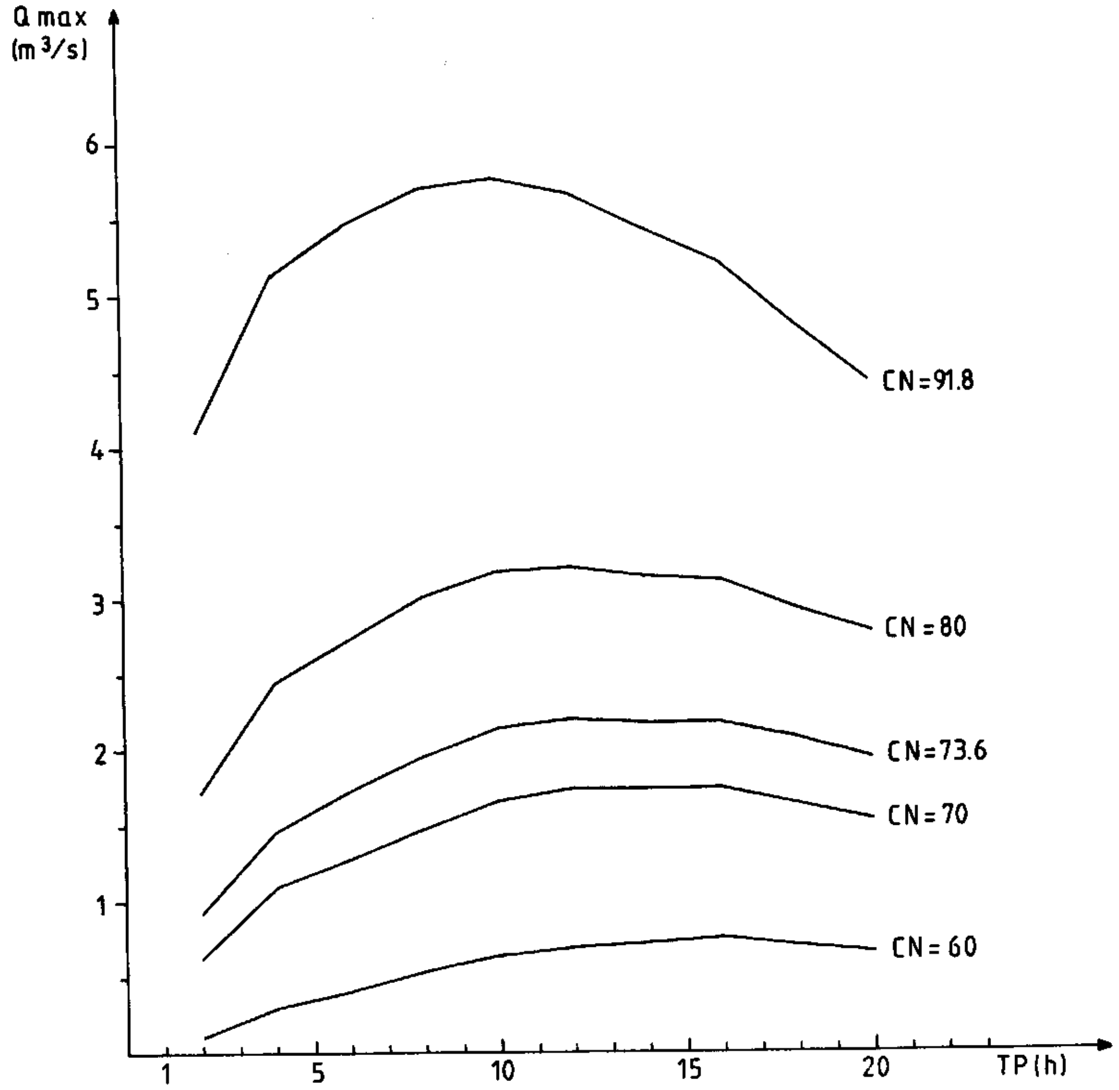

Fig. 7. Relationships between flood peaks Q max and rainfall duration TP.

8. RESULTS AND DISCUSSIONS

Calibration of the Wackermann model has shown discrepancies among parameter values obtained in two different ways. As is shown in Table 4, the optimized set of parameters is scattered, with average values of .270, 3.132 and 4.521 for β, K1 and K2 (for all events). Parameter values computed from Thiele formulae are respectively: .173, 2.094 and 7.904. In order to evaluate the influence of different parameter values on Instantaneous Unit Hydrograph, three hydrographs were generated from these parameters and are shown in Figure 4. It can be seen that all three hydrographs peak at four hours, but peak values differ significantly (44% for Thiele formulae parameters, and 11% for parameters optimized for 6, compared to 10 events). This difference is probably caused by the very small slope of the main river, compared to watershed in West Germany, for which the Thiele fomulae were determined.

Model verification results with the use of four events (no. 7 to 10) are shown in Table 5. It can be seen that for both optimized set of parameters verification gave very good results (with one event graded as good). for the Thiele method parameters the results were much worse (three grades fair and one good).

The average CN parameter value of ten events (calculated from observed rainfall-runoff data) was 91.8. The value determined by the hydrologic soil cover complex procedure (as developed by SCS) includes data on type of soil and land use and was much lower (73.6). Such differences are described in the literature (Bales, Berson, 1982). If the soil cover complex procedure is used for ungauged watersheds, direct flow for observed rainfall events many be underestimated. However, when the probability of flooding is low, CN values obtained in this way can be used. This method has the advantage of being simple and though it can be used for ungauged watersheds, but it can produce results in which less confidence may be placed. An additional explanation for such discrepancy between CN values obtained by the two methods is shown in Table 4. Almost all events were recorded in the wet, winter period. So AMC III conditions were more likely to occur.

Design flood hydrographs for CN=91.8 and 73.6 with 8 different rainfall durations are shown in the Figures 5 and 6. Critical rainfall duration for CN=91.8 was 10h and for CN=73.6 12h and peak flows were 5.76 m^3/s and 2.21 m^3/s respectively. Flood hydrographs for CN=73.6 are close to the expected for the Hupselse Beek watershed. Figure 7 shows that the critical rainfall duration gets rapidly longer with decreasing values of the CN parameter. It was a few times longer than the time of concentration calculated from Kirpich formulae. This confirms the idea that the critical rainfall durations has to be found by running simulation models for different rainfall durations. Analysis of the relationships in Figure 7 also confirmed a dependence of flood peak flows on CN parameter values. (Peak flows increase with increasing CN values).

Presented results show that the described Wackermann model can be used for design flood hydrograph computation; however the user should be aware of a possible lower accuracy when using this model for ungauged basins, when applying Thiele formulae.

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