FROM ONE-STEP TO MULTI-STEP
Determination of soil hydraulic functions by outflow experiments

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1. INTRODUCTION

Unsaturated flow in the upper soil layers plays a key role in the hydrological cycle. Many engineering problems encountered in agriculture, hydrology, meteorology and environmental protection can only be solved with thorough knowledge of the water flow in the vadose zone. In general this flow is described by Richard's equation and the soil hydraulic functions, $\theta(h)$ and $k(h)$. Currently a multitude of laboratory and field methods exist to determine these highly nonlinear functions (Klute, 1986; Dirksen, 1990). Most of these methods require restrictive initial and boundary conditions, making the measurements time consuming and expensive.
Recently a new approach became feasible when computer facilities, flow simulation models and optimization algorithms improved. This approach combines measurements, concepts of unsaturated flow and numerical computer techniques to determine in a transient flow process the unknown soil properties. For this so called inverse approach an experiment is set up with prescribed but arbitrary initial and boundary conditions. The problem is solved with an appropriate numerical model and by parameterization of the soil hydraulic functions. The unknown parameters in these functions are estimated by minimizing deviations between observed and model-predicted output.
Only a few experiments on parameter estimation in the field are published. Dane and Hruska (1983) studied gravity drainage from a clay loam soil. Initial water content profiles and profiles after 7 and 25 days of drainage were measured with a neutron probe. Tensiometers were installed at a depth of 0.9 m. Dane and Hruska obtained good agreement between the predicted and independently obtained $\theta(h)$ relation, but overestimated $k(h)$ curve by one order of magnitude when measured K_s values were used. This was attributed to macropore flow occurring during ponded infiltration, which was used to estimate K_s. Kool et al. (1987a) applied parameter estimation on drainage from a 3 m diameter by 6 m deep lysimeter, which experiment is described in detail by Abeele (1984). The lysimeter was filled with silty sand and instrumented with neutron probe access tubes and tensiometers. After saturation the lysimeter was allowed to drain for 100 days with a covered soil surface. They estimated 4 parameters in the Mualem - van Genuchten model (1980), using measured water contents and the pressure head at 0.4 m depth in the object function. Both $\theta(h)$ and $k(h)$ showed close agreement with independent data.
Although in general laboratory measurements on $\theta(h)$ and $k(h)$ appear to deviate from field measurements, the advantage of the laboratory is the wider range on which the curves can be determined, the shorter duration of the experiment and the possibility to manipulate the initial and boundary conditions. Zachmann et al. (1981) investigated which measurements ($\theta(z)$, $h(z)$ or $Q_0(t)$) at a hypothetical drainage test contained the best information to determine the soil hydraulic functions. $Q_0(t)$ resulted in the best approximation, which is encouraging since outflow is easy to collect in the laboratory. A later study (Zachmann et al., 1982) shows the importance of selecting correct parametric forms for $\theta(h)$ and $k(h)$. When incorrect expressions are used, it may still be possible to obtain an acceptable fit of simulated outflow but the estimated hydraulic properties may be in error. Kool et al. (1985) investigated the One-step outflow method. They performed numerical experiments on two hypothetical soils to investigate the sensitivity to error and the uniqueness of the solution. Their results indicate that an accurate solution of the parameter identification problem may be obtained when (i) input data include the final cumulative outflow and cumulative outflow volumes corresponding to at least half of the final outflow; (ii) final cumulative outflow corresponds to a sufficient large fraction (e.g. > 0.5) of the total

water between saturated and residual water contents; (iii) experimental error in outflow measurements is low; and (iv) initial parameter estimates are reasonably close to their true values. Parker et al. (1985) applied the One-step outflow method to four soils of distinct texture. They used equilibrium $\theta(h)$ data to estimate $k(h)$ and predict the outflow process. From this they suggest that the ability of the model to describe a flow process is improved when $\theta(h)$ and $k(h)$ are simultaneously optimized on the outflow data. This will distribute errors due to model inaccuracies to both $\theta(h)$ and $k(h)$ in a manner which optimizes the ability of the model to describe the entire transient flow process. Optimization on cumulative outflow with time, supplemented with the equilibrium outflow and the measured water content at h = -150 m, yielded satisfactory results when estimated $\theta(h)$ and $k(h)$ were compared with equilibrium $\theta(h)$ data and with the results of analytical analysis according to Passioura (1976). Tamari (1988) investigated experiments on evaporating soil samples in the laboratory (as described in Chapter 5). He compared the instantaneous profile method with nonlinear parameter estimation adapted from Kool et al. (1987). Tamari concluded that in case the fitting is performed on simulated evaporation the parameter estimation method is unique, using only tensiometer data and two points of the retention curve (h = -3.2 resp. -15800 cm). But in case experimentally determined evaporation data are used the parameter estimation fails to give an unique solution, and additional data should be provided.

The purpose of this report is to extend the error analysis on the One-step outflow method, to report on experimental data of the One-step outflow method in comparison with other methods and to give suggestions on the performance of outflow experiments. It gives an example of the promising features of parameter estimation techniques in determination of soil hydraulic functions.

2. METHODS AND MATERIALS

An undisturbed soil sample is placed in a Tempe pressure cell on top of a ceramic plate (Fig. 1). The sample is saturated from below until equilibrium in the burette is reached. The outflow experiment starts by increasing pneumatic pressure at top of the sample. This induces unsaturated flow in the soil sample, while the ceramic plate stays saturated. The cumulative outflow of the water ($Q_o(t)$) is recorded in the burette.

Three stages can be distinguished in the outflow as illustrated in Fig 2. In the first stage (I), the flow rate is determined by resistance to flow in the ceramic plate, so that $Q_o(t)$ is proportional to time t. After some time the flow rate decreases as the soil itself starts to limit the flow rate. This results in a second stage (II), during which the column behaves as though semi-infinite, and $Q_o(t)$ is linearly related to $\sqrt{t}$. Eventually the boundary condition at the top manifests itself, the initial concentration at the top begins to decrease rapidly, and $Q_o(t)$ ceases to be linear with respect to $\sqrt{t}$. This is the third stage (III) (Passioura, 1976).

Originally, Gardner (1956,1962) derived an analytical method to determine diffusivity D by assuming $D(\theta)$ constant over the length of the soil core. Although Doering (1965) found good agreement with other methods, Gupta et al. (1974) demonstrated that for some cases Gardner's original method may be in error by as much as a factor of three and they modified the method. Their method uses only the latter stage of the outflow, when ceramic plate impedances are negligible. Passioura (1976) simplified the computations of Gupta, by assuming $\delta\theta/\delta t$ to be effectively constant throughout the core at any given time. The calculated values should be accurate within about 15%, provided D increases monotonically with θ over the range of interest. Valianzas et al. (1988) combined the simplicity of Passioura's method with some intuitive ideas borrowed from Gupta et al.(1974). They derived a direct method that even in case of extreme nonlinearities in the ln $D(\theta)$ relationship and in cases where the $D(\theta)$ relationship is not monotonous, gives reliable results. Recently Valianzas et al. (1990) extended the analysis from determination of $D(\theta)$ to simultaneous determination of $\theta(h)$ and $k(h)$ by assuming Brooks and Corey moisture retention curves and $(k(h)/K_s)^c$-type conductivity functions.

In this report a numerical simulation model is used, as it is more flexible in initial and boundary conditions and allows error analysis.

Combination of Darcy's law and the principle of mass conservation leads for unsaturated flow in a rigid porous medium to Richards' equation :

$$\delta\theta/\delta t = \delta[k(h)(\delta h/\delta x-1)]/\delta x \qquad (1)$$

The combined system of soil and porous plate has the following initial and boundary conditions :

$h = h_o(x)$	$t = 0$	$0 \le x \le L$	(2a)
$\delta h/\delta x = 1$	$t > 0$	$x = 0$	(2b)
$h = h_L - h_a$	$t > 0$	$x = L$	(2c)

where $x = 0$ is the top of the soil core, L is the height of sample plus ceramic plate, h_L is the initial water potential below the ceramic plate, and h_a is the applied pneumatic pressure.

The flow can be simulated when the soil hydraulic functions are known. As the purpose of the experiment is to determine these properties, the hydraulic functions are described by analytical functions of which the unknown parameters are optimized. Flexible and common used analytical functions are those introduced by van Genuchten and Mualem (van Genuchten, 1980; Mualem, 1976; van Genuchten and Nielsen, 1985) :

$$S_e = (\theta - \theta_r)/(\theta_s - \theta_r) \tag{3}$$

$$h(S_e) = [(S_e^{-1/m} - 1)^{1/n}]/\alpha \tag{4}$$

$$k(S_e) = K_s \, S_e^{l} \, [1 - (1 - S_e^{1/m})^m]^2 \tag{5}$$

where $m = 1 - 1/n$

where α, n and l are empirical parameters, θ_r and θ_s are respectively the residual and saturated water content and K_s is the saturated hydraulic conductivity. In this report the Mualem - van Genuchten model (MVG) is used.

The numerical solution of (1) and (2) is obtained by a modified version of the Galerkin finite element model of van Genuchten (1978).

To simulate the flow process parameters of the hydraulic functions have to be assumed. One criterium for the correctness of the MVG model may be an accurate simulation of the outflow process (observed $Q_o(t_i)$ versus calculated $Q_c(t_i, \mathbf{b})$). Another criterium is the matching between MVG model and independent data of $\theta(h)$ or $k(h)$. When, in addition, weighting factors are included to account for the differences in magnitude and accuracy of the measurements, the following objective function may be derived:

$$\begin{aligned} O(\mathbf{b}) &= \sum_{N1} [w_i(Q_o(t_i) - Q_c(t_i, \mathbf{b}))]^2 \\ &+ \sum_{N2} [W_1 v_i(\theta(h_i) - \hat{\theta}(\mathbf{b}, h_i)]^2 \\ &+ \sum_{N3} [u_i(k(h_i) - \hat{k}(\mathbf{b}, h_i)]^2 \end{aligned} \tag{6}$$

with w_i, v_i and u_i weighting coefficients for individual measurements, $\hat{\theta}$ and $\hat{k}$ are the computed data according to the MVG model, N1, N2 and N3 are resp. the number of data of $Q_o(t_i)$, $\theta(h_i)$ and $k(h_i)$, and W_1 is a pre-weight factor of the retention data :

$$W_1 = \left(\sum_{N1} Q_o(t_i) \,/\, N1\right) \,/\, \left(\sum_{N2} \theta(h_i) \,/\, N2\right) \tag{7}$$

The used objective function does not contain a pre-weighting for $k(h)$; this should be included in u_i.

For a given parameter set the derivatives of $O(\mathbf{b})$ to fitting parameters are calculated. Next, applying Marquardt's maximum neighbourhood method, a parameter set is determined which decreases $O(\mathbf{b})$ effectively. This is repeated until $O(\mathbf{b})$ is minimal, which corresponds to the optimum parameter set.

The simulation of the outflow and the optimization of the MVG parameters are performed by the program MULSTP which is based on ONESTP (Kool, 1985). The modifications to ONESTP are mentioned in Appendix B. The main are :

- The exponent l in Mualem's derivation of k is no longer fixed to 0.5, but has been made variable and may be optimized.

- Not only outflow after one step-increase of pneumatic pressure but also after more steps can be simulated and optimized.

- It is optional to fix $\theta(h)$ parameters and optimize four parameters of $k(h)$.

3. SENSITIVITY ANALYSIS

The cumulative outflow $Q_o(t)$ was calculated for different sets of parameter values of the MVG model. Subsequent optimization on $Q_o(t)$ showed that the optimization routine is able to find the right parameters in case of error-free outflow and reasonable initial estimates of the parameters.

However, the outflow of a real sample may be influenced by :

Measurement errors :

- reading errors
- leakage of water

Deviating initial or boundary conditions :

- deviating initial pressure
- error impedance ceramic plate
- layered sample

Inadequate model concept :

- flow equation
- parametric model

A sensitivity analysis was performed on these influences involving a loamy sand and a clay loam. The parameters of these soils are listed in Table 1 as soil 1 and 2. In Fig. 3 the soil hydraulic functions of these soils are depicted.

Table 1. Parameters of soils used in sensitivity analysis.

soil texture		1 loamy sand	2 clay loam	3 sandy loam	4 clay loam
α	(cm^{-1})	0.025	0.042	0.010	0.0067
n	(-)	2.000	1.125	2.000	1.395
θ_r	(-)	0.080	0.000	0.170	0.240
θ_s	(-)	0.300	0.420	0.470	0.450
K_s	(cm/hr)	1.00	2.50	3.13	0.00248
l	(-)	0.50	-3.706	0.50	0.50

Evaluation of differences between soil hydraulic functions is possible by comparing parameters or figures. RMS (Root Mean Square) expresses the difference between soil hydraulic functions quantitatively in the following way :

For $\theta(h)$: $\mathrm{RMS} = \{ 1/N \sum_{N} (\theta_1(h_i) - \theta_2(h_i))^2 \}^{1/2}$ (8)

For $k(h)$: $\mathrm{RMS} = \{ 1/M \sum_{M} (\log(k_1(h_i)) - \log(k_2(h_i)))^2 \}^{1/2}$ (9)

In this report the range of comparison is $0 < \mathrm{pF} < 4.0$ with $N = M = 25$.

3.1 READING ERRORS

On the calculated exact cumulative outflow $Q_o(t_i)$ a random error was added :

$$Q_p(t_i) = Q_o(t_i) + 2P\,(R-0.5) \qquad (10)$$

where $Q_p(t_i)$ (L^3) is the outflow inclusive reading error, P (L^3) is the maximum absolute error and R is a random number between 0 and 1. Table 2 shows RMS for $P = 0.4\ cm^3$. Kool et al. (1985) calculated the effect of random error increasing with Q. They showed that the effect is small at the 2% error level and quite dramatic at the 15% error level. In practice errors in reading from the burette are $\leq 0.1\ cm^3$, which affects the accuracy only to a small extent.

Table 2. Difference in optimized curves caused by errors in input data.

	RMS			
	loamy sand		clay loam	
	θ	$\log k$	θ	$\log k$
$P = 0.4\ cm^3$	0.0205	0.3375	0.0190	0.1262
$L = 0.1\ cm^3/hr$	0.0327	1.4993	0.0378	0.3046
$h_o(x) = -30.0$ cm	0.0367	1.2755	0.0092	0.2574
$K_{plate} * 1/3$	0.0143	0.8739	0.0188	0.1239

3.2 LEAKAGE OF WATER

When leakage occurs, the error is systematic and therefore more serious than random reading errors. In formula :

$$Q_p(t_i) = Q_o(t_i) - L\,t_i \qquad (11)$$

with L = rate of leakage (L^3/T). Even a small rate of leakage ($L = 0.1\ cm^3/hr$) causes a high RMS (Table 2).

3.3 DEVIATING INITIAL PRESSURE

Partly saturation of the sample at the start of the experiment affects the outflow by both the volume occupied by the bubbles and compression of some of the air bubbles when the pneumatic pressure is increased. A generally accepted hypothesis for air entrapment is that the advance of water through preferred pores or pore sequences on some occasions results in the sealing by water of all paths through which gas would need to escape for the complete wetting of another pore or group of pores. This hypothesis suggests that air entrapment is favoured by a material containing a wide range of pore sizes, since this would result in a wide range of interfacial velocities in the pores. Peck (1969) describes the entrapment, stability and persistence of air bubbles in soil water. He calculated the time needed for air bubbles to get in equilibrium with a sudden change in water potential (0 - 3 bar) to be of the order 10^4 sec. He suggests that the slow growth of entrapped bubbles may account for the unexpected slow release of water observed in some outflow experiments.

The effect of partly saturation on $Q_o(t)$ after an increase of pneumatic pressure is difficult to simulate as the whole flow process is involved. We approached the error in parameter estimation by simulating the outflow with $h_o(0) = -30$ cm and using this outflow to optimize $\theta(h)$ and $k(h)$ under the assumption $h_o(0) = 0$. Table 2 shows the comparatively large effect on RMS on the optimized curves. This effect may be prevented by starting the experiment at some suction.

3.4 ERROR IMPEDANCE CERAMIC PLATE

Before the One-step experiment starts, the resistance of the ceramic plate is measured independently. During the One-step experiment the outflow is determined mainly by the plate as long as

$$d_{plate}/K_{plate} \gg d_{soil}/k(h)_{soil} \tag{12}$$

with d_{plate} and d_{soil} the respective heights (L). Therefore, the first observations of the outflow process contain more information of the plate than of the soil. In general however, the effect of errors in the input resistance of the ceramic plate is not large. If the parameters are optimized with K_{plate} one-third of the value for which the outflow was calculated, the RMS is still low as can be seen in Table 2 again.

3.5 LAYERING

Intuitively someone may expect, when pneumatic pressure is applied at top of a soil sample, the soil water to be driven out from above. In contrast, after air is able to reach the ceramic plate, the soil near the ceramic plate looses its water quickly in order to develop a downward suction gradient. Because of the lower water content, it has a relatively high resistance compared with the rest of the sample. In case of layered samples this may lead to results that are not representative for the whole sample.

As a demonstration the outflow was calculated for samples consisting of two soils of which the layer-thickness was varied. Figure 4A and 4B shows the results for loamy sand above clay loam and the opposite. The parameters of the separate soils are given in Table 1 (soil 1 and 2). A small clay loam layer near the ceramic plate results in an outflow which corresponds more or less to the ratio volume loamy sand / volume clay loam (Fig. 4A). This is also the case when increasing the thickness of the clay loam. In case a small loamy sand layer occurs underneath a clay loam sample, the outflow is larger during

the first period of the experiment (Fig. 4B). After some time, however, the loamy sand impedes the flow by its low hydraulic conductivity. Then, the outflow is even less than in case of a homogeneous clay loam sample. The outflow is out of the expected range and optimizations will certainly display large deviations. We can not say, however, that in general only material of coarser texture near the ceramic plate may result in unreliable optimizations. This depends on the shape of both $\theta(h)$ and $k(h)$. The simulations with layered samples were repeated for a sandy loam and a clay loam (soil 3 and 4 in Table 1). In this case sandy loam near the plate gives outflow corresponding to the portion of each soil (Fig. 4D). However, a layer of clay loam as small as 0.49 cm near the plate, greatly affects the outflow (Fig. 4C) and by that the estimated functions.
This illustrates the large influence that the lowest part of the sample has on the optimization. In practice the texture in a small and selected sample will not deviate as assumed in the previous calculations. But disturbances at the bottom of the sample or bad contact between soil and ceramic plate may have a similar detrimental effect on outflow analysis. In case the soil surface near the ceramic plate is unequal, it should be smoothened carefully with material of the same texture.

3.6 FLOW EQUATION

Richard's equation is based on a combination of Darcy's law and conservation of mass. It has proven its value for macroscopic modelling in the unsaturated zone. However, it is questionable whether it describes accurately the flow in a small sample on which a high pneumatic pressure is applied :

- On the microscopic scale their exists no continuum of soil, water and air, as supposed in Darcy's law.
- The fluxes that occur in the sample during the experiment are much larger than those under natural circumstances.
- Although theoretically suction under the ceramic plate and pneumatic pressure at the top of the sample have the same effect, the difference in absolute gas pressure might cause deviations. For instance, entrapped air will expand when suction is applied but shrink in case of gas pressure increase.
- The large transition in 'texture' between soil and ceramic plate might cause deviations from Darcian flow

To get an impression we simulated the outflow of samples of four soils with $\theta(h)$ and $k(h)$ as determined by other laboratory methods. Bias due to the parametric model of the soil hydraulic functions was circumvented by fitting the MVG parameters of $\theta(h)$ and $k(h)$ independently on the data (α and n for $k(h)$ are different from those of $\theta(h)$). The soils and independent methods are further described in Chapter 5. The four soils exhibit quite different flow characteristics as shown in the simulated outflow (Fig. 12).
Outflow of several samples of each soil ($n > 13$) was also measured. Fig. 13 shows the simulated and measured outflow for the four soils. Although the band width of outflow data is quite large, the measured outflow clearly reflects the shape of the simulated outflow. An exception is the sandy soil. Here the outflow during the first four hours is larger than predicted by the model.

An accurate flow equation is essential for reliable parameter estimation. Although the previous simulations support the ability of the Richards' equation to describe the unsaturated flow induced by a large pneumatic pressure increment, it is still difficult to quantify exactly its effect on parameter estimation. The flow conditions during the One-step outflow experiment, particularly the fluxes, deviate to a large extent from the natural flow conditions. A small inaccuracy of the applied flow equation might have a pronounced effect on the estimation of $\theta(h)$ and $k(h)$. It should be noted, however, that this also applies to analytical methods, used to estimate $D(\theta)$ or both soil hydraulic functions from outflow data (Gardner, 1956,1962; Gupta et al., 1974; Passioura, 1976; Valianzas et al., 1988,1990).

3.7 PARAMETRIC MODEL

The MVG model contains 6 parameters with more or less a physical meaning. The effects of α, n, K_s and l on the soil hydraulic functions are shown in Fig 5. The parameter α corresponds roughly to the inverse of h (cm) at the inflection point ($\delta\theta/\delta h$ max.), while n determines the gradient $\delta\theta/\delta h$. The effect of α and n on $k(h)$ is similar to that on $\theta(h)$. The parameters K_s and l affect only $k(h)$. K_s moves $k(h)$ up- and downward, while l changes the slope $\delta k/\delta h$. The parameters θr and θs determine the range of watercontent. Concerning the retention curve α, n, θ_r and θ_s have a clear physical meaning (α corresponds to the main air-entry value, while n is a measure for the width of the pore-size distribution). With the theory of Mualem $k(h)$ can be predicted from $\theta(h)$. In that case K_s equals the saturated hydraulic conductivity and l accounts for the correlation between pores and the flow path tortuosity. Van Genuchten and Nielsen (1985) applied the retention model to a large data set of both disturbed and undisturbed field soils. Without the restriction $m = 1-1/n$ they achieved a satisfying description of the observed data with the retention model. Our experience with fitting Eq. 4 under the restriction $m = 1-1/n$ is that a close agreement can be achieved between data points and fitted curve. In case K_s and l are kept variable, the prediction of the $k(h)$ function from $\theta(h)$ is not too rigid. K_s acts as a vertical scale parameter while l influences the slope $\delta k/\delta h$ (Fig. 5).
Russo (1988) paid attention to the effect of the chosen model for the soil hydraulic functions on the determination of these functions by parameter estimation. Russo compared three models : Mualem - van Genuchten (1980), Brooks and Corey (1964), and Gardner (1958). The models of Brooks and Corey and of Gardner were extended such that $k(h)$ is predicted from $\theta(h)$ with the Mualem (1976) theory. Russo found in case of outflow measurements of a silt loam soil the MVG model to be the most accurate and the most consistent with the data. Also the Akaike Information Criterion selected the MVG model as the most appropriate. Prerequisite is a variable parameter l in the MVG model (Russo, 1988).

We assume the MVG model to be flexible enough when the parameters α, n, K_s, l and one of the watercontents (either θ_r or θ_s) are kept variable.

4. NONUNIQUENESS AND INSTABILITY OF THE PARAMETER VALUES

In general nonuniqueness is caused by high correlation between parameters or by lack of sensitivity of outflow data to a certain parameter. Excessive sensitivity, on the other hand, may lead to instability, which means that slight variations in data produce disproportionally large changes in the solution. These problems are closely connected to the parameterset that is optimized. In many cases it is not feasible to invert for all parameters simultaneously. A more realistic approach is to design the experiment in such a way that direct information on the least sensitive parameters is obtained, enabling either their elimination from the inverse problem, or providing a good starting value for these parameters and subsequently constraining them to remain close to this value (Kool et al., 1988).

We checked the sensitivity of the outflow to each parameter for the loamy sand and clay loam. The outflow $Q_{ref}(t)$ was calculated for the original parameters. Next the parameters were increased one by one with 10 % (see Table 3) and the outflow $Q_c(t)$ was calculated. Figure 6A and 6C shows the relative difference $(Q_c(t)-Q_{ref})/Q_{ref}$ during the outflow. The effects of θ_s and θ_r are exactly opposite. This means that only one of these parameters can be estimated from the outflow experiment. The parameters α, n, K_s, and l show the largest relative influence for $t < 4$ hr. Note however that in the objective function absolute differences are used (Eq. 6). The outflow is least sensitive to K_s and l.
Also the effects of a wrong estimate of K_{plate} and $h_o(0)$ were calculated (Fig. 6B and 6D). The K_{plate} affects the outflow only during the first stage of the experiment. The effect of $h_o(0)$ is large compared to the effects of the parameters. This is in accordance with the RMS calculated for deviating K_{plate} and $h_o(0)$ (Table 2).

Table 3. The effect of the MVG parameters, K_{plate}, and $h_o(0)$ on the outflow.

		loamy sand		clay loam	
		original	changed	original	changed
α	(cm^{-1})	0.025	0.0275	0.042	0.046
n	(-)	2.00	2.20	1.125	1.237
θ_r	(-)	0.08	0.088	0.00	0.04
θ_s	(-)	0.30	0.33	0.42	0.46
K_s	(cm/hr)	1.00	1.10	2.50	2.75
l	(-)	0.50	0.55	-3.706	-3.33
$h_o(0)$	(cm)	-20.0	0.00	-20.0	0.00
K_{plate}	(cm/hr)	0.0050	0.0080	0.0050	0.0080

To examine the effect of the number and combinations of parameters which are optimized, a set of outflow data was analyzed using the parameter sets mentioned in Table 4. The outflow data belonged to 13 samples of a loess soil which is further described in chapter 5 (soil 1, Table 6). In every set θ_r was fixed. It is of no use to keep both θ_r and θ_s variable, for the outflow is determined by the difference between these two. Set 1 has the largest flexibility, optimizing 5 parameters. In set 2 θ_s is fixed on the measured value. Set 3 differs from set 2 in removing the boundaries of l. In set 4 only α, n and K_s are optimized. The effect of fixation of K_s was examined in set 5.

All these combinations were able to reproduce measured outflow correctly for the individual samples. In Fig. 7 the resulting mean curves of the 13 samples are shown. Comparison with other methods (Chapter 5) showed set 1 to be a good estimation. Optimization 3 (no boundaries on l) is deviating. Apparently this gives too much freedom to the optimization. The mean curves of the 4 other sets look quite similar, but differ in individual curves. To investigate the effect of these differences, the individual curves were used to simulate the waterbalance during a growing season of grass with the program SWATRE (Feddes, 1978; Hopmans, 1987) and to calculate functional criteria which characterize soil hydraulic functions for certain flow conditions (Wösten et al., 1986). The optimization of 3, 4 and 5 showed significant deviations with set 1. Set 2 was quite similar to set 1. For a full account of the results see Verhoef (1990).

Table 4. Different sets of fitting parameters used in optimization of Verhoef (1990)

set	α	n	θ_r	θ_s	K_s	l
1	var	<var>	fix	<var>	var	<var>
2	var	<var>	fix	fix	var	<var>
3	var	<var>	fix	fix	var	var
4	var	<var>	fix	fix	var	fix
5	var	<var>	fix	<var>	fix	<var>

<var> means variable within boundaries :

$1.1 < n$
$-0.5 < l < 1.5$
$\theta s - 0.02 < \theta s < \theta s + 0.02$

Set 1 seems to be the most suitable set. Both α and n are needed to describe $h(S_e)$ accurately. Either θ_r or θ_s are needed to determine the range of the retention curve. By keeping K_s and l (partly) variable, $k(h)$ is not too rigidly predicted from $\theta(h)$. The saturated hydraulic conductivity is often determined by the soil structure, while the unsaturated hydraulic conductivity is determined by the soil texture. Near saturation a small change in suction (some cm's) may change k over one or more orders of magnitude. Since in the outflow experiment the soil hydraulic functions are determined over the range $1.5 < \mathrm{pF} < 3$, it implies that by optimizing K_s it looses its physical meaning and becomes a fitting parameter, determined only by the soil texture.
If not stated otherwise, the optimizations of both the one-step and multi-step outflow experiments in this report are performed with set 1.

Non-uniqueness may also be caused by local minima in the response surface of the objective function. We used the Marquardt's maximum neighbourhood method, which is fast but sensitive to local minima. Realistic initial values of the parameters are necessary to achieve the right solution. By performing the optimization with different initial values the absolute minimum can be determined. Other methods, like the Simplex method (Nelder and Mead, 1965), are less sensitive to local minima but require substantially more computer time.

5. COMPARATIVE STUDY

The Dutch Standardization Institute (NNI) is collecting information on methods for measuring the *k(h)* function. To get insight in the performance of different methods, three Dutch institutes were invited to apply commonly used methods on undisturbed samples of the same soil. Table 5 gives an overview of the institutes and methods involved.
Criteria for selection of the soils were homogeneity and texture. The four soils selected are listed in Table 6. Information on bulkdensity, organic matter and texture is mentioned in Appendix A.
In this chapter soil 1-4 will be named loess, sand, silt loam and loam, respectively.

Table 5. Methods and institutes involved in the comparative study.

Method	Institute
Needle	Landscape and environmental research group, University of Amsterdam
Crust and Evaporation	Winand Staring Centre for integrated land, soil and water resources, Wageningen
One-step outflow	Department of hydrology, soil physics and hydraulics, Agricultural university Wageningen

Table 6. Investigated soils in comparative study.

n	classification NL	FAO	depth cm	place
1	zandige leem	silt loam (loess)	30-50	Groesbeek
2	leem arm zand	sand	70-90	Kootwijk
3	lichte klei	silt loam	35-55	Wageningen
4	lichte klei	loam	60-70	Zeeland

5.1 NEEDLE METHOD (SPRINKLER INFILTROMETER METHOD)

This method may be applied for $0 < h < -100$. A sample with a diameter of 15 cm and a height of 25 cm is placed on a sandbox (Fig. 8). By adjusting the waterlevel in the sandbox different suctions in the sample can be achieved. With a pulse pump a small flux in the sample is induced. After equilibrium flow is reached, the hydraulic tension in the sample is measured with 4 tensiometers on 4-5 cm distance. Accurate measurements are possible if at a certain flux such a suction is applied that gravity flow is reached. Then according to Darcy's law *k* is equal to the flux. In case the gradient $\delta h/\delta x$ deviates from 0 (non-gravitational flow), the calculation of *k(h)* is ambiguous and a mean *k(h)* is calculated for each layer between tensiometers. More details can be found in Freyer and de Lange (1990) and Klute (1986).

5.2 CRUST METHOD

This method, applied to samples of a height of 20 cm and 20 cm in diameter, is restricted to the wet range ($h > -40$ cm). A crust is made of a mixture of sand, cebar powder and water. The ratio between these three materials determines the crust conductivity which has to be less than the saturated conductivity of the soil. In this way a negative pressure head is induced underneath the crust. Above the crust a small layer of water will be maintained until equilibrium flow is reached. With a tensiometer 5 cm below the crust h is measured (Fig. 9). By assuming gravitational flow in the sample, $k(h)$ can be calculated. Repeating measurements can be made at higher suctions by putting a new crust on top of the old crust. In case h underneath the crust falls below -10 to -15 cm, the sample should be placed on a column of coarse sand to ensure the right suction at the lower boundary of the sample (Fig. 9) (Stolte, 1990).

5.3 EVAPORATION METHOD

In soil samples of 8 cm height and 10.3 cm in diameter tensiometers are installed at four depths (Fig. 10). After saturation the samples are placed on weighing equipment. Next, water is allowed to evaporate from the top under normal laboratory conditions. During the experiment the soil water potential and the total weight of the sample are measured. The experiments end when the suction at the upper tensiometer exceeds the air entry value of the cup (approx. 800 mbar). For heavy clay this takes 1 to 2 days, for coarse sand it may take 2 to 3 weeks.
With an efficient iterative method suggested by Wind the water retention curve is calculated from the measurements. When the retention curve is known it is possible to calculate the water contents in the soil profile during the experiment. Further the flux is zero at the lower boundary, while the flux at the upper boundary follows from the decrease in weight. Next the fluxes between the soil compartments are calculated. The gradients in hydraulic head follow directly from the tensiometer readings. Finally, the unsaturated conductivities can be calculated straightforward with Darcy's law (Veerman, 1989).

5.4 RESULTS OF THE REFERENCE METHODS

Per location each method was applied on 2 samples. The results are depicted in Fig. 11. There is a clear difference in ranges covered by the 3 methods. Error analysis per method showed that the crust method may underestimate k with a factor 2, resulting from uncertainty of the h gradient (Stolte, 1990). For the needle method, those measurements that could be in error due to a large gradient of $\delta k/\delta x$ or a small gradient of dh/dx (< 0.2), were rejected (Freyer and de Lange, 1990). Concerning the evaporation method, numerical analysis of Tamari (1988) indicated that the mean of the estimated $k(h)$ values corresponds with the real values. Only in the wet range, at small gradients of h, errors increase. Also an inaccurate description of $\theta(h)$ by a polynome will influence the $k(h)$ estimation.
The correspondence between the crust method and needle method is quite good. The transition between the needle and evaporation method is satisfactory at the loess and loam soil. However, for the sand and silt loam soil the transition is problematic. This is probably due to the small gradients of the evaporation method in the overlapping range.
In Fig. 11 also a fitted curve based on the MVG model is drawn. The parameters

α, n, K_s, and l are optimized independent of the retention curve. The corresponding parameters are listed in Table 7.

Table 7. Parameters MVG model for $k(h)$ fitted on combined data of crust, sprinkler infiltrometer and evaporation method.

soil	α (cm^{-1})	n (-)	K_s (cm/hr)	l (-)
loess	0.0130	1.1489	3.4630	-0.0011
sand	0.0470	4.7121	8.0367	-1.0563
silt loam	0.3647	2.2926	0.8260	-2.0067
loam	0.0430	1.3420	1.0035	0.0001

5.5 RESULTS OF THE ONE-STEP OUTFLOW METHOD

The resulting $k(h)$ curves of the crust, needle and evaporation method, as mentioned in Table 7, were used to evaluate different methods of optimizing One-step outflow data. Table 8 shows for 5 methods the supplied information and the parameters that were estimated. Optimization 1 uses only the outflow data in the objective function. In optimization 2 and 3 $\theta(h)$ data are added to the objective function (Eq. 6). The difference between these optimizations is the weight given to the $\theta(h)$ data. The $\theta(h)$ data were not determined on the same samples that were used in the outflow experiments. They were determined on at least 6 different samples of each soil (Freyer and de Lange, 1990). In opt. 4 and 5 the four parameters of the $\theta(h)$ function were fixed on their optimum values according to the $\theta(h)$ data and the outflow data were used only to estimate the $k(h)$ function. In opt. 4 K_s and l of this function were optimized using α and n of $\theta(h)$, while in opt. 5 all parameters of the $k(h)$ function (α,n,K_s and l) were optimized. In this case the concept of the MVG model, in which $k(h)$ is predicted from $\theta(h)$, was left to create more freedom to describe the $k(h)$ function.

Table 8. Different methods of estimating $k(h)$ from One-step outflow data.

number optimization	data objective function	fixed	optimized
1	$Q(t)$	-	$\alpha,n,\theta_s,K_s,l^{(1)}$
2	$Q(t) + \theta(h)$	-	$\alpha,n,\theta_s,K_s,l^{(1)}$
3	$Q(t) + \theta(h)^{(2)}$	-	$\alpha,n,\theta_s,K_s,l^{(1)}$
4	$Q(t)$	$\theta(h)$	K_s,l
5	$Q(t)$	$\theta(h)$	$\alpha^{(3)},n^{(3)},K_s,l$

(1) Boundaries as set 1 in Table 4
(2) Extra weight
(3) These parameters are independent from those of the retention function

For each soil the outflow was measured from 13-17 samples of 100 cm^3. A question is how to derive 'representative' soil hydraulic functions for the four soils, given the variation shown by the samples. As the relation between the soil hydraulic functions and the flux in unsaturated flow is nonlinear, their exist no mean $\theta(h)$ and $k(h)$ that predict the mean flux for any flow condition. For instance in a dry year the mean evapotranspiration will be predicted by other effective or representative soil hydraulic functions than in a wet year. The same applies to outflow experiments. This is not the place to elaborate on the determination of 'representative' soil hydraulic functions. The approach used in this report is :

- optimization of the soil hydraulic functions for each sample
- determination of representative $\theta(h)$ and $k(h)$ by calculating the arithmetic mean of θ and the geometric mean of k as function of h.

The effect of this method of averaging on the simulated outflow is shown in Fig 13. It shows the outflow data for each soil, together with the simulation of the outflow, using the independent determined mean $\theta(h)$ and the mean $k(h)$ of an optimization (opt. 5). It shows that the mean $\theta(h)$ and $k(h)$, determined in the mentioned way, predict the average $Q_o(t)$ with reasonable accuracy. This implies that the reverse approach, in which first the outflow curves of all the samples of a certain soil are averaged and next with this mean outflow curve and the mean $\theta(h)$ the parameters of $k(h)$ are optimized, will lead to the same $k(h)$ function.

The mean curves, resulting from the optimizations for each sample, are drawn in Fig. 14 together with the fitted curves (Fig. 11) through the reference methods. Although in the figures pF ranges from 0.0 to 4.0 the comparison with data of the crust, needle and evaporation method is only valid between $1.0 < pF < 2.5$. To give an impression of the band width around the mean curves, in Fig. 15 the 20 %, 50 %, and 80 % values of k, calculated at a certain h, are depicted (opt. 5).

A more quantitative comparison between the different optimizations and the curves of the reference methods is given by the RMS (Eq. 9). The RMS is calculated on the range $1.0 < pF < 2.5$. Table 9 gives a summary of the calculations. The RMS is not only calculated with respect to the curve of the reference methods, but also with respect to the geometric mean curve of the particular soil and optimization. The first gives an impression of the deviation with the crust, needle and evaporation method, while the second is a measure of the spread of the optimized curves for a particular soil and optimization.

Table 9 and Fig. 14 show that optimization on $Q(t)$ only is not sufficient. Especially for the sand and silt loam soil the difference with the reference methods is large, more than 2 orders. This improves when $\theta(h)$ data are added. The effect is even more favourable when extra weight is given to the $\theta(h)$ data. The air-entry value for the sand soil is quite well estimated and the overestimation of $k(h)$ of the silt loam is less (average 1.26 order) than without extra weight. The $\theta(h)$ data are needed to get unique solutions. Fig. 16 shows two sets of soil hydraulic functions resulting from optimization on the same sample (using method 1 and 3 resp.). They are able to predict the outflow equally well. How is this possible with such deviating $\theta(h)$ and $k(h)$? An important part of the mechanism is shown in the same figure. After a certain time t the outflow is known from the data, which determines the mean θ in the sample unambiguously. This results for both simulations in a different h, but almost the same k ! The effect on $\delta h/\delta z$ is minor as $\delta h/\delta z$ increases rapidly near the ceramic plate. This means that the fluxes ($v = k\ \delta h/\delta z$) and $Q_c(t)$ are the same and that $k(\theta)$ is more or less unique defined. This is also the case in Fig. 16.
Fig. 17 shows the large differences in pressure distribution calculated for this outflow experiment. By adding $\theta(h)$ to the objective function, the

retention curve is more or less fixed to its position and this type of nonuniqueness is prevented.

Opt. 4 results in large deviations although the results are better than opt. 1. Overall opt. 3 and opt. 5 shows the least difference with the reference soils. It supports that the MVG model is accurate enough to describe the soil hydraulic functions. The spread of the optimized curves per soil has not improved for opt. 5 in comparison with opt. 3. This is attributed to the fact that one mean $\theta(h)$ curve was used. Then, by fixation of $\theta(h)$, the variation that exists in both the $\theta(h)$ and $k(h)$ between samples is forced into the optimized $k(h)$.

Table 9. Root Mean Square Deviation (RMS) for $k(h)$ using different methods of optimizing One-step outflow data.

n	reference	loess	sand	silt loam	loam
1	independent(1)	0.795	2.697	2.477	0.540
	mean(2)	0.508	0.788	1.324	0.409
2	independent	0.770	1.274	2.684	0.606
	mean	0.537	0.818	1.159	0.548
3	independent	0.962	1.286	1.262	0.438
	mean	0.669	0.819	0.977	0.293
4	independent	0.799	1.959	1.608	0.568
	mean	0.637	1.077	1.214	0.484
5	independent	0.797	1.403	0.751	0.590
	mean	0.677	1.099	0.652	0.508

(1) Fitted conductivity function through needle, crust and evaporation method

(2) Geometric mean curve per soil and optimization

6. FROM ONE-STEP TO MULTI-STEP

As shown in Chapter 5, the outflow after one pressure increase contains not enough information to estimate θ(h) and $k(h)$ simultaneously. To achieve uniqueness, independent data of either θ(h) or $k(h)$ must be added.
Another approach is to increase the pneumatic pressure step by step in stead of applying one large pressure increase. It is quite complicated to analyze this outflow analytically, but numerically it is not a problem. Some of the earlier mentioned disadvantages of the One-step variant of the outflow method are than circumvented.

First the high fluxes at the start of the experiment. It is questionable whether this flow is described accurately enough with Darcian flow. The fluxes occurring after a pressure increase of 1 bar on a saturated sample are some orders of magnitude larger than those in field situations. If 6 steps are applied and as a criterium the outflow of a sample is equally divided over each pressure increase, the first step is approx. 30-40 cm (depending on the soil texture). This means that the fluxes during the first period are decreased with a factor 25 to 30 !

A second advantage is that the outflow is determined by the whole sample instead of a small layer at the bottom. Fig. 18A shows the pressure distribution in a sample, simulated for a clay loam (Table 1, soil 2), and which is common for an One-step experiment. The soil water potential decreases sharply near the plate to accommodate the cumulative flux in the relatively dry bottom layer. As a consequence, the upper parts of the sample hardly affect $Q_0(t)$. In case the pneumatic pressure is gradually increased, the differences in the sample of k and hence $\delta h/\delta z$ are smaller (Fig. 18B). A larger part of the sample affects the outflow. The outflow data contain more information on the mean soil hydraulic functions of the sample.

The problems with uniqueness, that can appear in optimizing $Q_0(t)$ for One-step without additional $\theta(h)$ data, do not occur with Multi-step if the periods between each pressure increases are large enough to force $h(x)$ to reach more or less equilibrium with suction in the ceramic plate. As Fig. 18C shows, intervals of a day between pressure increments fulfils this requirement.

Also from a practical point of view Multi-step is attractive. All the measurements are performed in the same equipment and a set of samples can be examined within two weeks.

Disadvantages of Multi-step are the larger amount of readings to be made, the diffusion of gas through the ceramic plate (this should be removed), and the increase of computation time.

6.1 EXPERIMENT

Samples from three soil layers in Sint Oedenrode (province Noord Brabant) were collected. Earlier One-step outflow measurements on these soils with only $Q_0(t)$ data in the objective function resulted in almost similar estimates of θ(h) and $k(h)$ curves for both the sand and the loamy sand.

To investigate the effect of increasing pressure step by step, the following data were measured of the samples mentioned in Table 10 :

- $Q_o(t)$ data, increasing pneumatic pressure in 6 days to 1 bar
- $Q_o(t)$ data, increasing pneumatic pressure in 6 hours to 1 bar
- $Q_o(t)$ data, applying 1 bar in one step

Table 10. Soil horizons from Sint Oedenrode.

	classification		depth	layer
n	NL	FAO	cm	
2,3,4,7,9,10	lemig fijn zand	loamy sand	50-60	1
11,12,13,14,15	leemarm fijn zand	sand	10-15	2
16,18,19	leemarm fijn zand	sand	50-60	3

6.2 RESULTS

The optimized soil hydraulic functions per sample for the three different outflow experiments are drawn in Fig. 19. In this figure is also depicted the mean θ in the sample after one day of equilibration derived from the outflow experiment with daily increase. These data are an upper boundary for θ at a certain h. It shows an overestimation of θ at higher suctions by One-step. Pressure increases per hour improve the estimation of θ but are still too high. For the most samples daily pressure increase seems to give realistic optimizations. As expected the difference in $k(h)$ is quite large, while $k(\theta)$ is more unique defined. For example at sample 2 θ equals 0.04 - 0.12 for pF = 2.5. At this pF the differences in $k(h)$ are of an order of 2.5, while $k(\theta)$ varies less than 1 order (θ = 0.08).

In Fig. 20 the retention curves of the samples are grouped per horizon and compared with the mean of the more or less equilibrated $\theta(h)$ data after one day. The variability per horizon is less and the correspondence to the $\theta(h)$ data better for daily pressure increases in comparison with pressure increases per hour or one large pressure increment.

Another experiment is intended in which the analysis of an outflow experiment with daily pressure increase is compared with independent $\theta(h)$ and $k(h)$ data.

7. CONCLUSIONS

Outflow experiments in the laboratory in combination with a numerical model and nonlinear parameter estimation techniques are attractive. They are flexible in initial and boundary conditions, the measurements are quick, cheap and simple and $\theta(h)$ and $k(h)$ are fit simultaneously over a wide range. The flexibility of the method is tempered by the requirement that the parameter estimation problem is sufficiently well-posed to give unique and stable solutions.

Sensitivity analysis on One-step outflow experiments showed that reading errors and ceramic plate impedance do not disturb the optimization. Leakage of water has detrimental effects, but can technically be solved. Inaccuracies due to entrapped air at the start of the experiment may be circumvented by starting the experiment at some suction. The MVG model for describing the soil hydraulic functions seems to be flexible enough. Still two uncertainties remain. First, deviations in texture or disturbances of the soil near the ceramic plate may influence the optimization to a large extent. Second, the flow conditions during the flow experiment, particularly the fluxes, deviate in such extent from the natural flow conditions, that the validity of the estimated $\theta(h)$ and $k(h)$ for natural flow conditions still remains uncertain.

The performance of the One-step outflow procedure for estimating $k(h)$ for different input data has been compared with measurements by the needle, crust and evaporation method. Fitting on $Q(t)$ data exclusively appeared to be insufficient. Either independent $\theta(h)$ data or fixation of the retention function were necessary to achieve unique solutions. In that case the estimated $k(h)$ agrees well with the measurements by the other methods.

Outflow experiments in which the pressure is increased in several steps (Multi-step) meet the two deficiencies displayed by One-step. A larger part of the sample influences effectively the outflow, and, by that, decreases the high sensitivity of the outflow to the soil condition near the ceramic plate. Also the soil water fluxes are less high, giving more confidence in application of the estimated soil hydraulic functions under natural conditions.

In numerical simulations the Multi-step approach showed to give unique results, using only $Q(t)$ data as input. Also the first series of laboratory experiments are encouraging.

Recommended research topics for the future are :

- The influence of the numerical schematisation of the transition soil/ceramic plate on the simulated outflow
- Reduction of the plate impedance to get more information on the wet part of the soil hydraulic functions
- The time at which measurements should be made in outflow experiments.
- Derivation and use of confidence limits around the estimated parameters.
- Other laboratory or field methods that provide well-posed problems for nonlinear parameter estimation of the soil hydraulic functions.

COMMON USED SYMBOLS

α	empirical shape factor MVG model	L^2
b	trial vector of parameter values	-
D	soil water diffusivity	L^2T^{-1}
h	soil water potential	L
h_a	applied pneumatic pressure	L
h_L	water potential at $x = L$ and $t = 0$	L
$h_0(0)$	water potential at $x = 0$ and $t = 0$	L
k	unsaturated hydraulic conductivity	LT^{-1}
K_s	saturated hydraulic conductivity	LT^{-1}
L	height of sample plus ceramic plate	L
l	pore connectivity factor, MVG model	-
MVG	Mualem - van Genuchten	-
n	empirical shape factor MVG model	-
$Q_o(t)$	measured cumulative outflow	L^3
$Q_c(t)$	calculated cumulative outflow	L^3
RMS	root-mean-square of difference, Eq. 8 and 9	-
t	time (at start experiment $t = 0$)	T
θ	volumetric water content	-
θ_r	residual volumetric water content	-
θ_s	saturated volumetric water content	-
x	place in sample (top of sample $x = 0$)	L

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APPENDIX A

Particle distribution, bulk density and organic matter content of soils used in comparative study (Chapter 5).

Soil	1	2	3	4
Texture	silt loam	sand	silt loam	loam
Bulk density (g/cm3)	1.55	1.43	1.49	1.61
Organic matter (% weight)	0.95	0.27	2.09	1.38
Particle distribution :				
2000 -1000 μm	0.21	1.50	0.38	0.00
1000 - 500 μm	1.04	7.62	2.74	0.02
500 - 250 μm	4.96	31.89	9.73	0.12
250 - 125 μm	6.40	48.58	3.99	3.63
125 - 63 μm	4.16	9.18	3.60	42.07
> 63 μm	16.78	98.77	20.45	45.84
< 63 μm	83.22	1.23	79.55	54.16
< 32 μm	40.07	0.85	70.92	34.87
< 16 μm	18.58	0.79	54.22	27.45
< 8 μm	13.84	0.69	41.11	24.47
< 4 μm	12.03	0.69	32.58	22.11
< 2 μm	10.71	0.69	26.84	27.45

APPENDIX B

MODIFICATIONS TO THE PROGRAM ONESTP

Kool et al. (1985) developed the program ONESTP, a nonlinear parameter estimation program for evaluating soil hydraulic properties from One-step outflow experiments. The program was adapted to be able to perform multi-step outflow experiments, to make the parametric model more flexible and to facilitate testing. In the source listing the added or changed lines are written in small characters. The main alterations are :

1) The exponent l in Mualem's derivation of k is no longer fixed to 0.5, but has been made variable and may be optimized.

2) It is possible to fix $\theta(h)$ and optimize α, n, K_s and l for the conductivity curve only. This provides extra flexibility to $k(h)$.

3) The pneumatic pressure may be increased step by step.

4) At selected times the distributions of h and θ as function of x are printed in file DISTP.DAT.

5) The maximum number of observations is enlarged to 100.

6) In case the optimization is performed for 3 different initial parameter sets, the RMS is calculated between 1-2 and 1-3.

7) A summary of the optimization results is printed in file EXTRACT.DAT.

In the original program some bugs were traced :

- The array PIN(I) has a global function. It should be put in a COMMON block.

- When the optimization is restarted with different initial parameter values the array P(I) should be reset to its original values and the array E(I) used in the steepest descent calculation, should be reset to zero.

- In subroutine STAGE1, line 17, the counter should be I in stead of L.

VARIABLES INPUTFILE PROGRAM MULSTP

```
          10        20        30        40        50        60
1 NCASE  NPR NOUT KHAL NRES
2      DESCRIPTION OF THE SIMULATION OR OPTIMIZATION
3               SAMPL
4
5    NN  LNS      DNUL     DUMMY      AIRP      EPS1      EPS2
6        SLL       PLL      DIAM      CPLT NOBB MDAT MODE  MIT
7
8       B(1)      B(2)      B(3)      B(4)      B(5)      B(6)
9   INDEX(1)  INDEX(2)  INDEX(3)  INDEX(4)  INDEX(5)  INDEX(6)
10   BMIN(1)   BMIN(2)   BMIN(3)   BMIN(4)   BMIN(5)   BMIN(6)
11   BMAX(1)   BMAX(2)   BMAX(3)   BMAX(4)   BMAX(5)   BMAX(6)
--        AT        RT      WCRT      WCST
--        ZX
--     HO(I)     FO(I) ITYP     WT(I)           (NOBB times)
--    TPRESS    PRESSU                          (-AIRP times)
--    TPRINT                                    ( NPR times)

          10        20        30        40        50        60
```

VARIABLE	PLACE	DESCRIPTION
NCASE	1-5	Number of cases considered
NPR	6-10	Number of times on which $h(z)$ and $\theta(z)$ are printed in DISTP.DAT
NOUT	11-15	Observations : 0 - printing in outputfile 1 - no printing in outputfile
KHALL	16-20	Option to fix the retention curve and optimize α, n, K_s and/or l of the conductivity function 0 - simultaneous optimization of $\theta(h)$ and $k(h)$ 1 - $\theta(h)$ is fixed, $k(h)$ is optimized
NRES	21-25	1 = Print the simulated data in OBSERV.DAT
SAMPL	16-20	Code for sample
NN	1-5	Number of nodes
LNS	6-10	Number of nodes in soil
DNUL	11-20	Initial time step
DUMMY	21-30	No significance
AIRP	31-40	Positive : One step method --> pneumatic head Negative : Multi step method --> number of pressure steps
EPS1	41-50	Temporal weighting coefficient
EPS2	51-60	Iteration weighting coefficient
SLL	1-10	Length of soil core
PLL	11-20	Thickness of porous plate
DIAM	21-30	Diameter of soil core
CPLT	31-40	Conductivity of porous plate
NOBB	41-45	Number of observations
MDAT	46-50	Mode for observation data : 1 = transient flow data only 2 - last $Q(t)$ entry represents equilibrium outflow
MODE	51-55	Mode for type of calculation : 0 - flow equation is solved for initial parameter values 1 = optimization process continues until parameter values converge or the number of iterations reaches MIT; all intermediate parameter values are printed 2 - as MODE=1 except parameter values are only printed at the end of every iteration 3 = as MODE=2 but $\theta(h)$ and $k(h)$ according to final parameter values are also printed With negative sign --> the program continues with two other sets of intial parameters
MIT	55-60	Maximum number of iterations in optimization routine
B(1)	1-10	Initial value of α
B(2)	11-20	Initial value of n
B(3)	21-30	Initial value of θ_r
B(4)	31-40	Initial value of θ_s
B(5)	41-50	Initial value of K_s
B(6)	51-60	Initial value of l
INDEX(I)		Index indicating fixed or loose parameters : 0 - parameter B(I) is known and is kept constant

		1 = parameter B(I) is not known and will be optimized
BMIN(I)		Minimum permissible parameter values
BMAX(I)		Maximum permissible parameter values
AT	1-10	Fixed α of $\theta(h)$ (This line only if KHALL=1)
NT	11-20	Fixed n of $\theta(h)$
WCRT	21-30	Fixed θ_r of $\theta(h)$
WCST	31-40	Fixed θ_s of $\theta(h)$
ZX	1-10	Initial pressure head at the center of the sample
HO(I)	1-10	Time (ITYPE=0) or pressure head (ITYPE=1 or 2)
FO(I)	11-20	Outflow (ITYPE=0), moisture content (ITYPE=1) or unsaturated conductivity (ITYPE=2)
ITYPE(I)	21-25	Data type : 0 = $Q(t)$ 1 = $\theta(h)$ 2 = $k(h)$
WT(I)	25-35	Array containing weights assigned to every observation
TPRESS	1-10	Time on which pressure is increased
PRESSU	11-20	Applied pneumatic pressure
TPRINT	free	Times on which h and θ against depth are printed in DISTP.DAT

EXAMPLE INPUT & OUTPUT FILE FOR PROGRAM MULSTP

The next inputfile contains two outflow experiments. The first experiment is derived from Kool and Parker. The optimization is performed on outflow data, supplemented by data on $\theta(h)$ and $k(h)$ and the equilibrium outflow. In the second experiment the pressure is increased in 6 steps from 30 to 1000 cm. The optimization is performed on outflow data, without additional data. The corresponding outputfiles (total and summary) are also shown. Other features, that are not included in the example but that might be useful, are :

- 3 optimizations on the same dataset, starting with different initial parameters to check uniqueness
- give different weights to the observations (especially when both outflow and retention data are used)
- summary of optimization results in file EXTRACT.DAT
- on selected times $h(z)$ and $\theta(z)$ can be calculated
- the retention function can be fixed, while of $k(h)$ the parameters α, n, K_s and l are optimized.

INPUTFILE :

```
  2    0    1    0    0
  SANDY LOAM  ONE-STEP  INCLUDING  O(H) AND K(H) DATA
  MONSTER     SLOAM

 29    25    1.E-05     2.57    1000.0       1.0       1.0
    4.0       0.57       5.4     0.005   12    2    2   10

  0.0150    2.2500    0.1700     0.470     3.125       1.5
       1         1         0         0         1         1
               1.1                                     0.0
              10.0     0.350                           5.0
 -2.0
 0.017    3.707
 0.033    6.953
 0.083   13.087
 0.167   136
 0.333   18.572
 0.750   20.689
 1.500   22.070
 3.000   23.104
 6.0     23.849
999.99   24.575
-15000.0  0.172       1
  -10.0  2.50         2
 LOAMY SAND   MULTI-STEP DATA ONLY
 MONSTER      SINT1

 46   41 0.100E-03     0.00      -6.0        1.0       1.0
   5.10       0.57      5.00   0.00490   40    1    2   15

  0.0216     1.540     0.000     0.304     2.210     -0.52
       1         1         0         1         1         1
  0.0000     1.100     0.000     0.284     0.000     -2.52
```

0.0000	10.000	0.000	0.324	0.000	1.48
-30.00					
0.067	0.35				
0.233	1.10				
0.483	1.95				
0.983	3.20				
1.483	4.25				
2.483	5.65				
3.483	6.60				
4.483	7.35				
5.483	7.85				
7.133	8.55				
22.700	9.85				
22.833	10.30				
23.000	11.40				
23.250	12.85				
23.750	15.00				
24.250	16.55				
25.250	18.05				
26.300	18.65				
28.217	19.05				
47.033	19.25				
47.167	19.55				
47.333	19.75				
47.583	19.85				
48.233	20.10				
48.700	20.20				
49.950	20.45				
50.883	20.55				
52.300	20.75				
55.183	20.95				
72.983	21.50				
143.517	21.95				
143.717	22.45				
144.000	22.55				
144.850	22.55				
146.550	22.65				
148.733	22.70				
151.117	22.75				
167.183	23.00				
167.400	23.15				
167.750	23.20				
0.00	60.00				
22.77	100.00				
47.10	150.00				
143.63	240.00				
167.23	480.00				
167.80	1000.00				

OUTPUTFILE :

```
************************************************************
*                                                          *
*   SANDY LOAM  ONE-STEP  INCLUDING  O(H) AND K(H) DATA    *
*                                                          *
*                     MONSTER SLOAM                        *
*                                                          *
************************************************************
```

PROGRAM PARAMETERS

```
NUMBER OF NODES.....................(NN)................ 29
  NODE AT SOIL-PLATE BOUNDARY.........(LNS)............... 25
INITIAL TIME STEP...................(DNUL)............  .10E-04
PNEUMATIC PRESSURE..................(AIRP)............1000.000
TEMPORAL WEIGHTING COEFF............(EPS1)............   1.00
ITERATION WEIGHTING COEFF...........(EPS2)............   1.00
MAX. ITERATIONS.....................(MIT)............... 10
DATA MODE...........................(MDATA).............  2
NO. OF OBSERVATIONS.................(NOBB).............. 12
```

SOIL AND PLATE PROPERTIES

```
SOIL COLUMN LENGTH..................(SLL).............   4.000
COLUMN DIAMETER.....................(DIAM)............   5.400
THICKNESS OF PLATE..................(PLL).............    .570
PLATE CONDUCTIVITY..................(CONDS(2))......  .5000E-02
SATURATED MOISTURE CONTENT..........(WCS).............    .470
RESIDUAL MOISTURE CONTENT...........(WCR).............    .170
FIRST COEFFICIENT...................(ALPHA)...........    .015
SECOND COEFFICIENT..................(N)...............   2.250
SATURATED CONDUCTIVITY SOIL.........(CONDS(1)).....   .3125E+01
EXPONENT L MUALEM-GENUCHTEN.........(EXPL)............   1.500
```

INITIAL PRESSURE AT CENTER SAMPLE :   -2.0

ITERATION NO	SSQ	ALPHA	N	CONDS	EXPL
0	.1762D+01	.0150	2.2500	3.1250	1.5000
1	.2954D+00	.0066	2.1584	3.4481	1.0398
2	.8433D-01	.0077	2.1286	2.8206	.7732
3	.4884D-01	.0092	2.0200	2.9530	.5444
4	.1500D-01	.0097	2.0111	3.0099	.5054
5	.3717D-02	.0099	2.0034	3.0842	.5001
6	.3428D-02	.0099	2.0040	3.0845	.5008

MASS BALANCE ERROR IN FE SOLUTION DURING FINAL RUN WAS   .9076 %

RSQUARE FOR REGRESSION OF PREDICTED VS OBSERVED =1.00000

CORRELATION MATRIX

	1	2	3	4
1	1.0000			
2	-.9685	1.0000		
3	.9000	-.8834	1.0000	
4	-.9372	.9575	-.7457	1.0000

NON-LINEAR LEAST-SQUARES ANALYSIS: FINAL RESULTS

			95% CONFIDENCE LIMITS	
VARIABLE	VALUE	S.E.COEFF.	LOWER	UPPER
ALPHA	.00990	.0001	.0098	.0100
N	2.00398	.0026	1.9980	2.0100
CONDS	3.08451	.0100	3.0614	3.1076
EXPL	.50084	.0049	.4896	.5121

--------OBSERVED & FITTED OUTFLOW --------

NO	TIME (HR)	OBS	FITTED	RESI-DUAL
1	.017	3.707	3.704	.003
2	.033	6.953	6.947	.006
3	.083	13.087	13.081	.006
4	.167			.002
5	.333	18.572	18.572	-.000
6	.750	20.689	20.691	-.002
7	1.500	22.070	22.072	-.002
8	3.000	23.104	23.106	-.002
9	6.000	23.849	23.850	-.001
10	999.990	24.575	24.574	.001
11	-15000.000	.172	.172	.000
12	-10.000	2.500	2.505	-.005

```
****************************************************************
*                                                              *
*   LOAMY SAND   MULTI-STEP DATA ONLY                          *
*                      MONSTER SINT1                           *
*                                                              *
****************************************************************
```

PROGRAM PARAMETERS

NUMBER OF NODES.....................(NN)................ 46
NODE AT SOIL-PLATE BOUNDARY.........(LNS)............... 41
INITIAL TIME STEP...................(DNUL)............ .10E-03
PNEUMATIC PRESSURE..................(AIRP)............ -6.000
TEMPORAL WEIGHTING COEFF............(EPS1)............ 1.00
ITERATION WEIGHTING COEFF...........(EPS2)............ 1.00
MAX. ITERATIONS.....................(MIT)................ 15
DATA MODE...........................(MDATA)............. 1

NO. OF OBSERVATIONS................(NOBB)............. 40

SOIL AND PLATE PROPERTIES

SOIL COLUMN LENGTH.................	(SLL)............	5.100
COLUMN DIAMETER.....................	(DIAM)...........	5.000
THICKNESS OF PLATE..................	(PLL)............	.570
PLATE CONDUCTIVITY..................	(CONDS(2))......	.4900E-02
SATURATED MOISTURE CONTENT..........	(WCS)............	.304
RESIDUAL MOISTURE CONTENT...........	(WCR)............	.000
FIRST COEFFICIENT...................	(ALPHA)..........	.022
SECOND COEFFICIENT..................	(N)..............	1.540
SATURATED CONDUCTIVITY SOIL.........	(CONDS(1)).....	.2210E+01
EXPONENT L MUALEM-GENUCHTEN.........	(EXPL)...........	-.520

INITIAL PRESSURE AT CENTER SAMPLE : -30.0

6 STEPS IN PNEUMATIC PRESSURE :

TIME	PRESSURE
.00	60.0
22.77	100.0
47.10	150.0
143.63	240.0
167.23	480.0
167.80	1000.0

ITERATION NO	SSQ	ALPHA	N	WCS	CONDS	EXPL
0	.8133D+01	.0216	1.5400	.3040	2.2100	-.5200
1	.5491D+01	.0405	2.3962	.2840	.2866	-2.5200
2	.3407D+01	.0344	2.3863	.3040	.1433	-2.5200
3	.2324D+01	.0278	2.4023	.3240	.0185	-2.5200
4	.1403D+01	.0275	2.4731	.3240	.0353	-2.5200
5	.1003D+01	.0240	2.7971	.2938	.0316	-2.5200
6	.8527D+00	.0216	3.0475	.2840	.0304	-2.5200
7	.8192D+00	.0217	3.0651	.2840	.0336	-2.5200
8	.7762D+00	.0222	3.0632	.2840	.0350	-2.5200
9	.7660D+00	.0223	3.0606	.2840	.0359	-2.5200
10	.7626D+00	.0223	3.0579	.2840	.0365	-2.5200
11	.7603D+00	.0223	3.0553	.2840	.0371	-2.5200
12	.7584D+00	.0223	3.0528	.2840	.0375	-2.5200
13	.7567D+00	.0223	3.0505	.2840	.0379	-2.5200
14	.7550D+00	.0223	3.0483	.2840	.0383	-2.5200
15	.7537D+00	.0222	3.0462	.2840	.0387	-2.5200

MASS BALANCE ERROR IN FE SOLUTION DURING FINAL RUN WAS .7498 %

RSQUARE FOR REGRESSION OF PREDICTED VS OBSERVED = .98934

CORRELATION MATRIX

	1	2	3	4	5
1	1.0000				
2	-.9197	1.0000			
3	.9620	-.9761	1.0000		
4	-.2697	.1820	-.1820	1.0000	
5	-.4946	.6018	-.4999	.6813	1.0000

NON-LINEAR LEAST-SQUARES ANALYSIS: FINAL RESULTS

VARIABLE	VALUE	S.E.COEFF.	95% CONFIDENCE LIMITS LOWER	UPPER
ALPHA	.02225	.0021	.0180	.0264
N	3.04620	.2887	2.4601	3.6323
WCS	.28400	.0195	.2443	.3237
CONDS	.03867	.0115	.0154	.0619
EXPL	-2.52000	.4457	-3.4250	-1.6150

--------OBSERVED & FITTED OUTFLOW --------

NO	TIME (HR)	OBS	FITTED	RESI-DUAL
1	.067	.350	.313	.037
2	.233	1.100	.922	.178
3	.483	1.950	1.677	.273
4	.983	3.200	2.907	.293
5	1.483	4.250	3.915	.335
6	2.483	5.650	5.531	.119
7	3.483	6.600	6.787	-.187
8	4.483	7.350	7.780	-.430
9	5.483	7.850	8.566	-.716
10	7.133	8.550	9.522	-.972
11	22.700	9.850	11.462	-1.612
12	22.833	10.300	11.815	-1.515
13	23.000	11.400	12.480	-1.080
14	23.250	12.850	13.223	-.373
15	23.750	15.000	14.336	.664
16	24.250	16.550	15.193	1.357
17	25.250	18.050	16.453	1.597
18	26.300	18.650	17.335	1.315
19	28.217	19.050	18.210	.840
20	47.033	19.250	18.788	.462
21	47.167	19.550	19.126	.424
22	47.333	19.750	19.596	.154
23	47.583	19.850	20.066	-.216
24	48.233	20.100	20.828	-.728
25	48.700	20.200	21.144	-.944
26	49.950	20.450	21.516	-1.066
27	50.883	20.550	21.607	-1.057
28	52.300	20.750	21.651	-.901
29	55.183	20.950	21.663	-.713
30	72.983	21.500	21.664	-.164
31	143.517	21.950	21.664	.286
32	143.717	22.450	22.093	.357
33	144.000	22.550	22.637	-.087
34	144.850	22.550	23.078	-.528

35	146.550	22.650	23.126	-.476
36	148.733	22.700	23.127	-.427
37	151.117	22.750	23.127	-.377
38	167.183	23.000	23.127	-.127
39	167.400	23.150	23.719	-.569
40	167.750	23.200	23.824	-.624

LIST OF FIGURES

Figure 12 Predicted outflow for the four soils with $k(h)$ according to crust, needle and evaporation method (Table 7) and $\theta(h)$ as mean of six samples.

□ loess + sand ◇ silt loam △ loam

Figure 13. Comparison between measured outflow (□) and simulated outflow (————), based on soil hydraulic functions measured by a combination of crust, needle and evaporation method (par. MVG see Table 7). Also the simulated outflow for the mean soil hydraulic functions of optimization 5 is shown (—·—).

Figure 14. Mean curves (n > 13) for four soils, using different data in the object function. The numbers 1-5 refer to Table 8. The fitted line through the crust, needle and evaporation method is called REF.

Figure 15. The band width of the estimated $k(h)$ functions for each soil. REF stands for the fitted line through the crust, needle and evaporation method. The distribution is determined per h. GEM equals the arithmetic mean of log k, while 50% corresponds to the median. 20% means that one-fifth of the lines at a certain h have a lower k.

Figure 16. Two sets of soil hydraulic functions resulting from optimization (method 1 and 3 resp., Table 8) of the same outflow of a loess sample.

Figure 17. $h(z)$ simulated during an One-step outflow experiment of samples with soil hydraulic functions according to Fig. 16. The top of the sample is at depth = 0.0 cm, while the transition soil - ceramic plate occurs at depth = 5.1 cm.

Times (hr) at which $h(z)$ is shown :

□ 0.2 + 0.5 ◇ 1.1 △ 3.0 × 8.0 ▽ 24.0

Figure 18. The effect of one large or five smaller pressure increments on $h(z)$. (A) shows $h(z)$ for One-step. (B) and (C) apply to Multi-step with pressure increments at t = 0, 24, 48, 96, and 120 hour. (B) represents $h(z)$ one hour after a pressure increment, (C) represents $h(z)$ one hour before a pressure increment.

A	□ 0.12	+ 0.60	◇ 1.60	△ 3.20	× 7.00	▽ 28.00
B	□ 25.0	+ 49.0	◇ 73.0	△ 97.0	× 121.0	
C	□ 23.0	+ 47.0	◇ 71.0	△ 95.0	× 119.0	▽ 139.0

Figure 19. Optimized soil hydraulic functions, using $Q(t)$ data in the objectfunction, in comparison with $\theta(h)$ data derived from outflow experiments with daily pressure increase. Three different time schedules were applied on the same sample :

◇ At once   △ Per hour   + Per day

Figure 20. Estimated $\theta(h)$ functions in comparison with $\theta(h)$ data obtained after one day of equilibration. Different time schedules for pressure increments are used on the same samples. *At once* shows the result of the One-step outflow experiment. In case of Multi-step, 6 steps were applied, each giving approx. the same amount of outflow. *Per hour* and *Per day* refer to the time between the pressure increments.

Mean of $\theta(h)$ data from daily pressure increments.

□ + ◇ △ × ▽ Optimized $\theta(h)$ for the different samples. The numbers refer to Table 10.

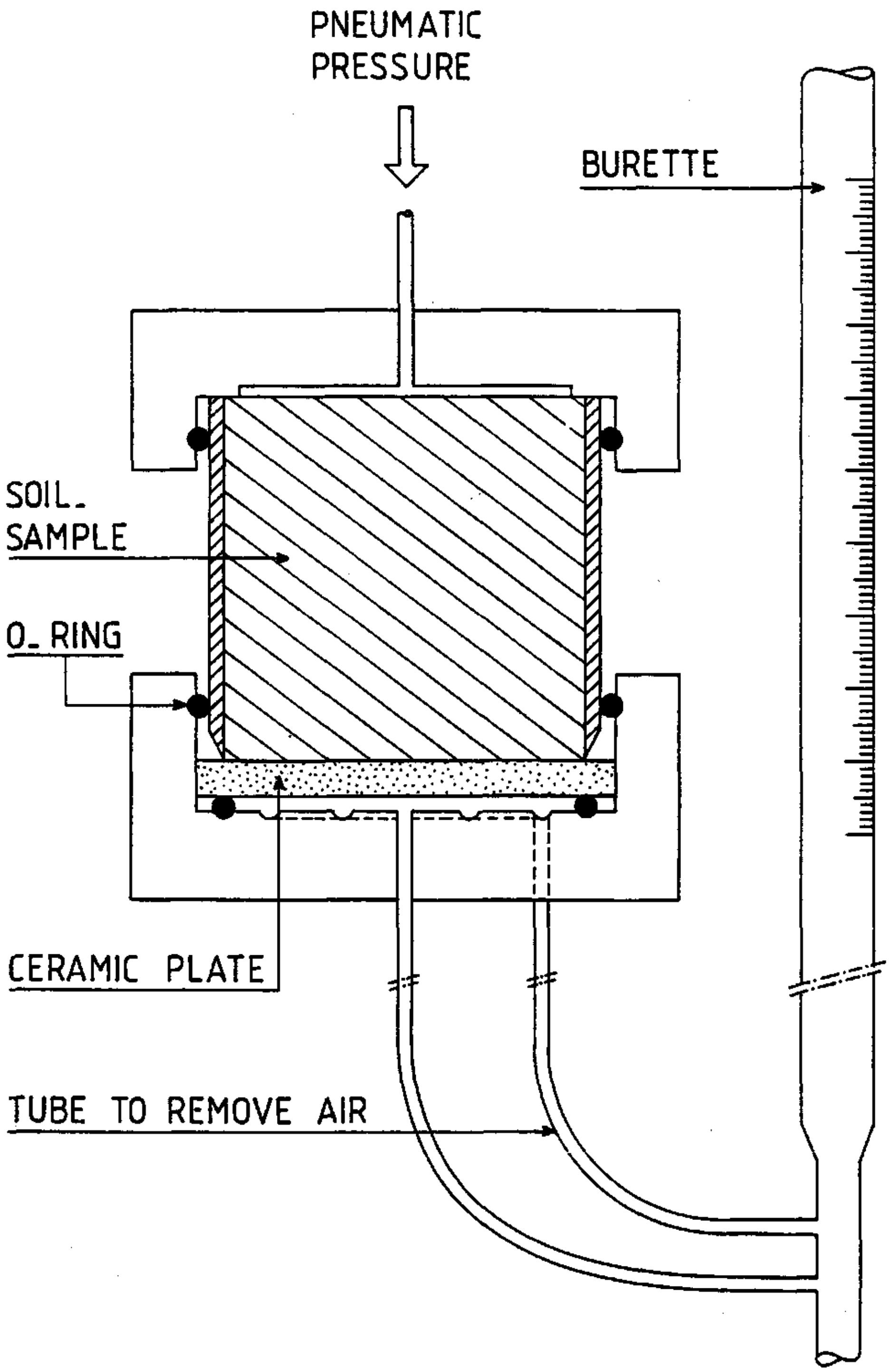


Figure 1. Cross section of Tempe-pressure cell.

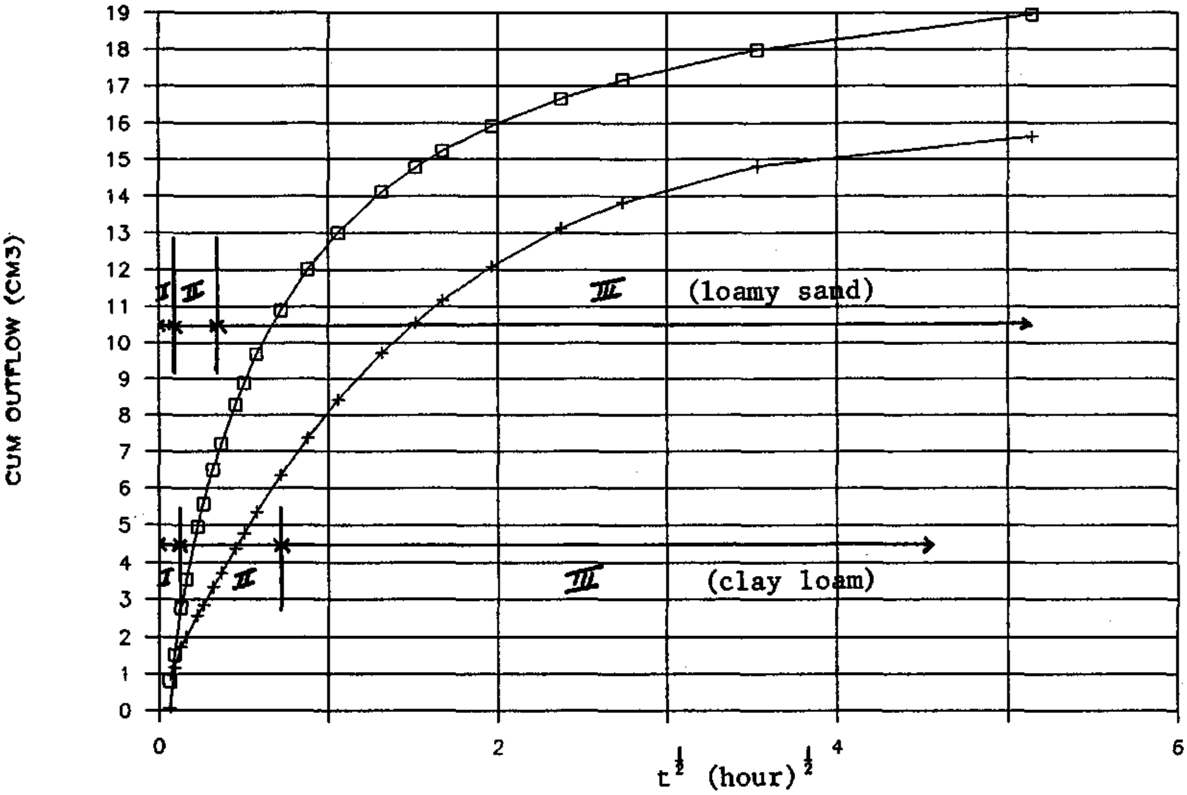


Figure 2. Stages in outflow (after Passioura, 1976)
□ Loamy sand
+ Clay loam

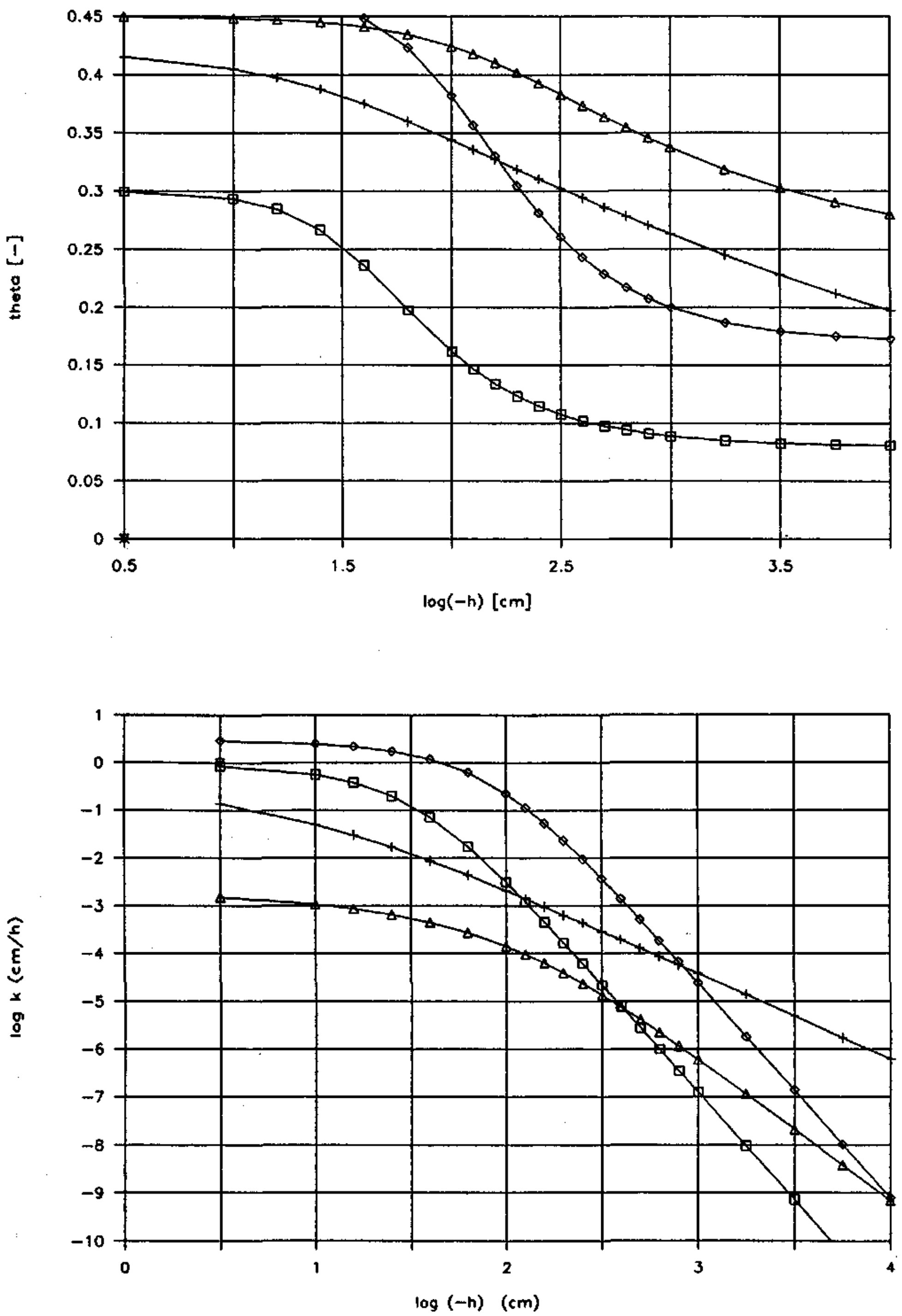
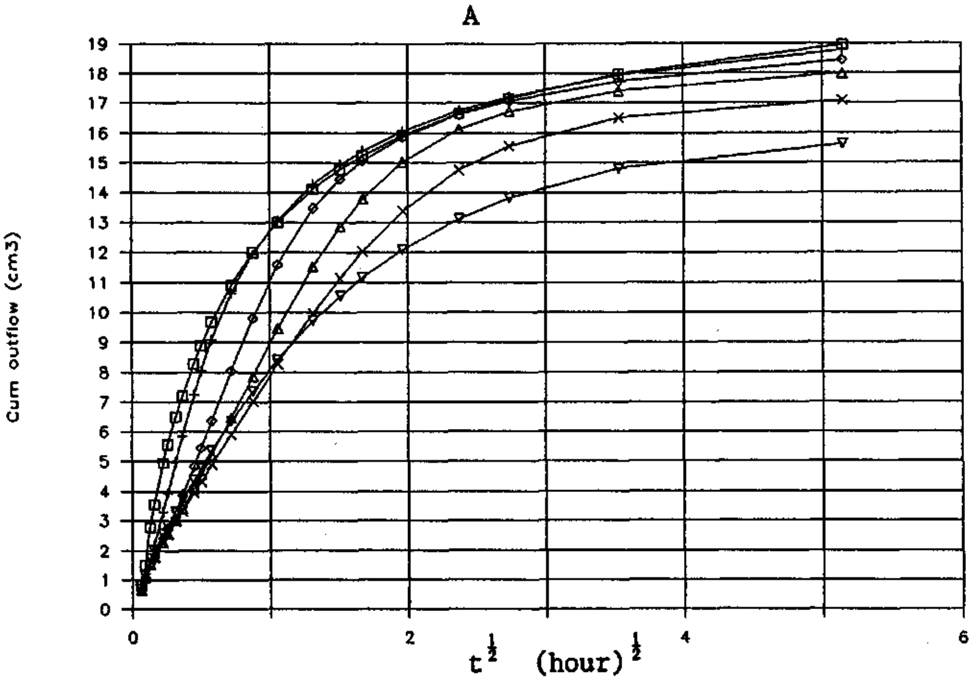


Figure 3. Soil hydraulic functions for soils used in the sensitivity analysis. The MVG parameters are mentioned in Table 1.

□ 1. Loamy sand (Wösten, 1987)
+ 2. Clay loam (Wösten, 1987)
◇ 3. Sandy loam (Kool et al., 1985)
△ 4. Clay loam (Kool et al., 1985)



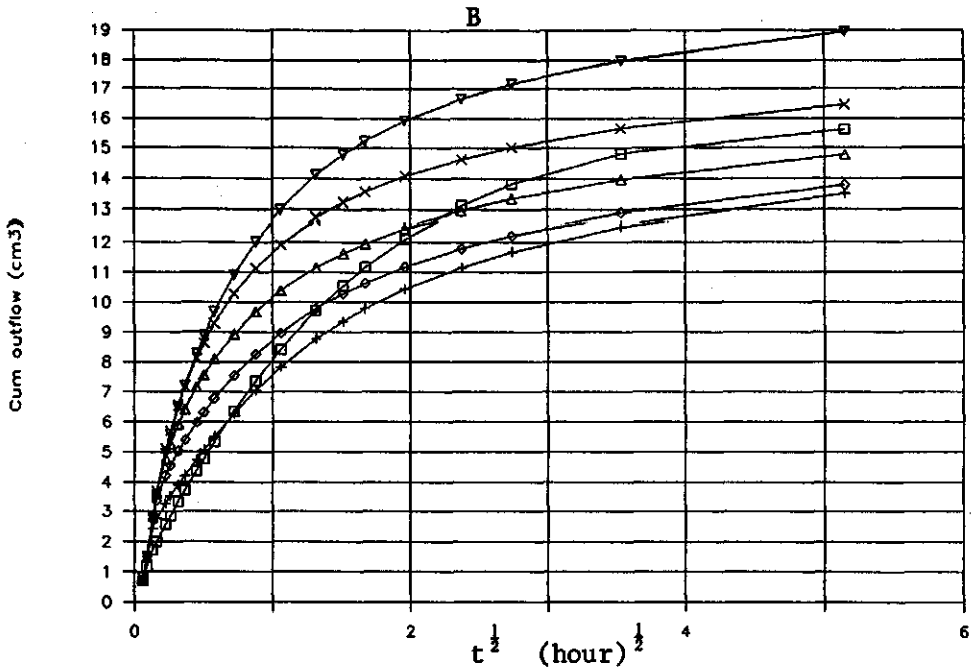


Figure 4. Outflow of non-homogeneous soil samples. The MVG parameters are mentioned in Table 1.

A. Top loamy sand (soil 1), bottom clay loam (soil 2).
B. Top clay loam (soil 2), bottom loamy sand (soil 1).
C. Top sandy loam (soil 3), bottom clay loam (soil 4).
D. Top clay loam (soil 4), bottom sandy loam (soil 3).

Thickness (cm) bottom layer (total height = 5.10 cm) :
□ =0.00 +=0.49 ◇=1.11 △=1.97 ×=3.07 ▽=5.10

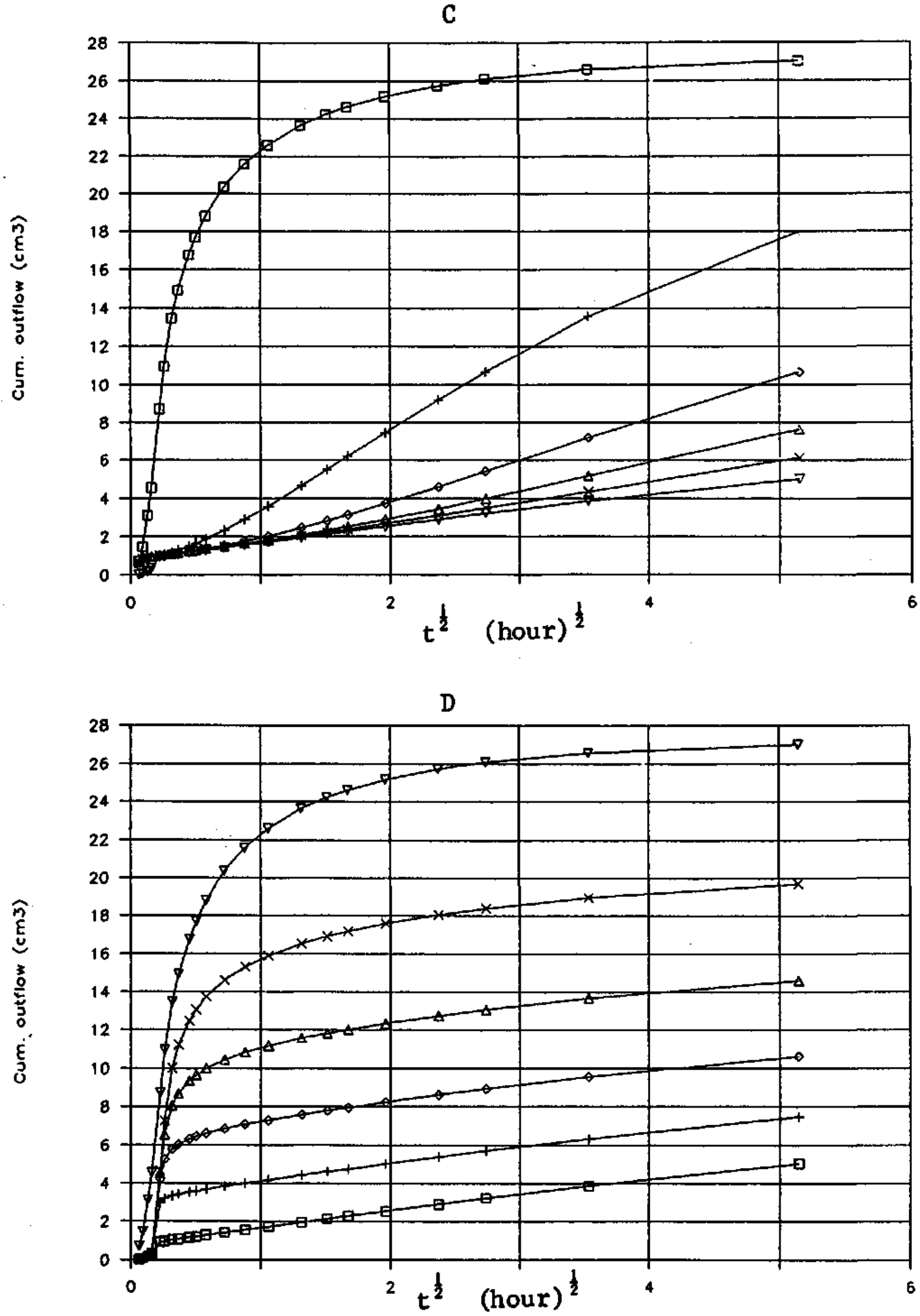
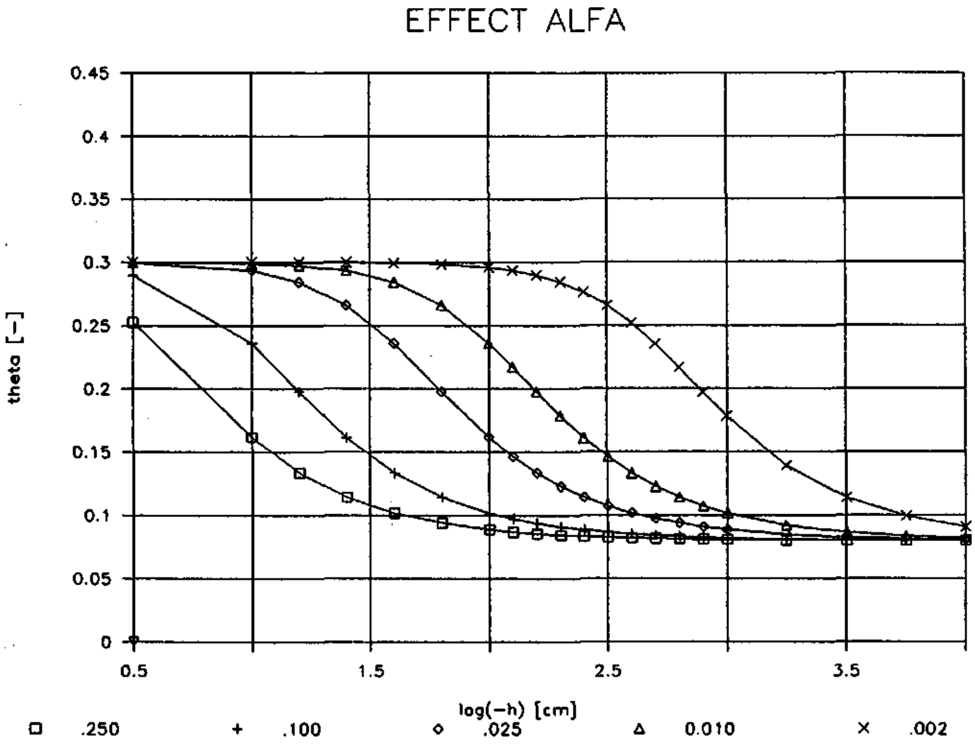


Figure 4. Outflow of non-homogeneous soil samples. The MVG parameters are mentioned in Table 1.

A. Top loamy sand (soil 1), bottom clay loam (soil 2).
B. Top clay loam (soil 2), bottom loamy sand (soil 1).
C. Top sandy loam (soil 3), bottom clay loam (soil 4).
D. Top clay loam (soil 4), bottom sandy loam (soil 3).

Thickness (cm) bottom layer (total height = 5.10 cm) :
□ =0.00 + =0.49 ◇ =1.11 △ =1.97 × =3.07 ▽ =5.10



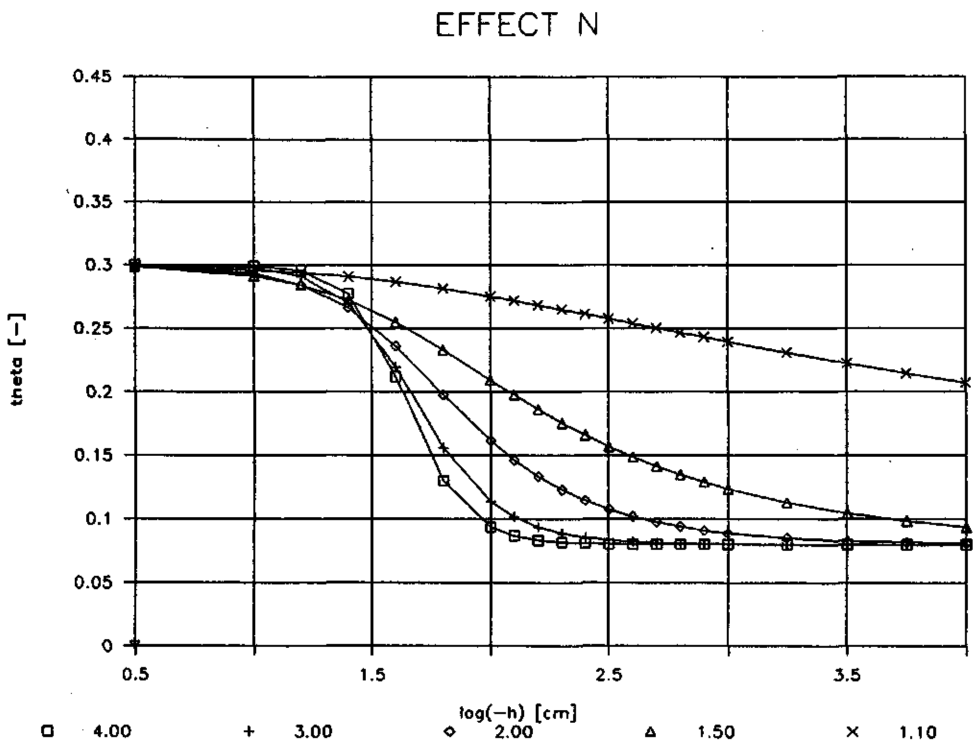
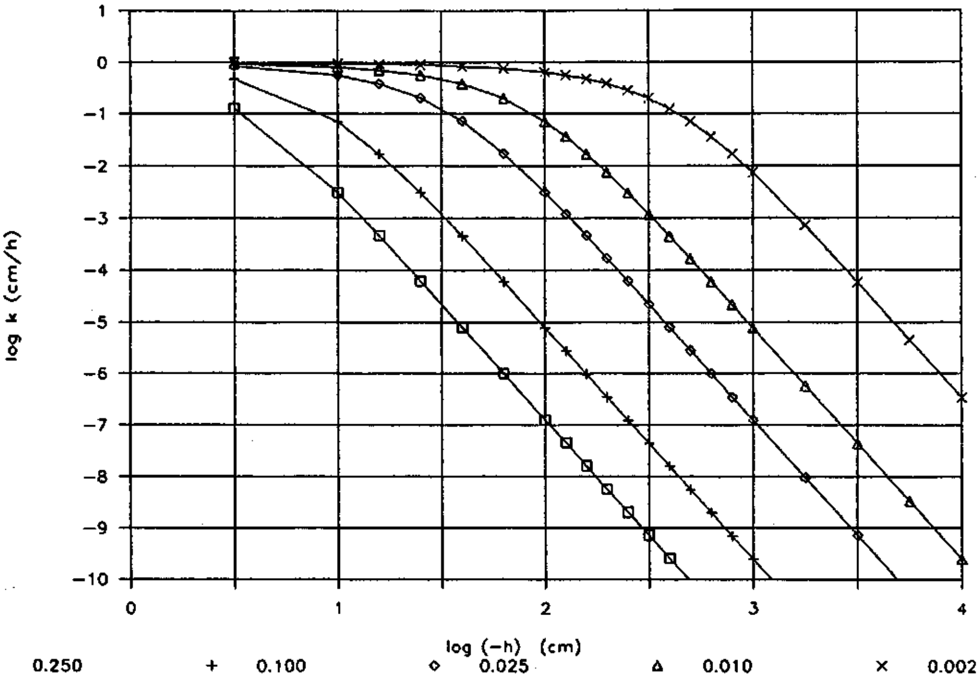
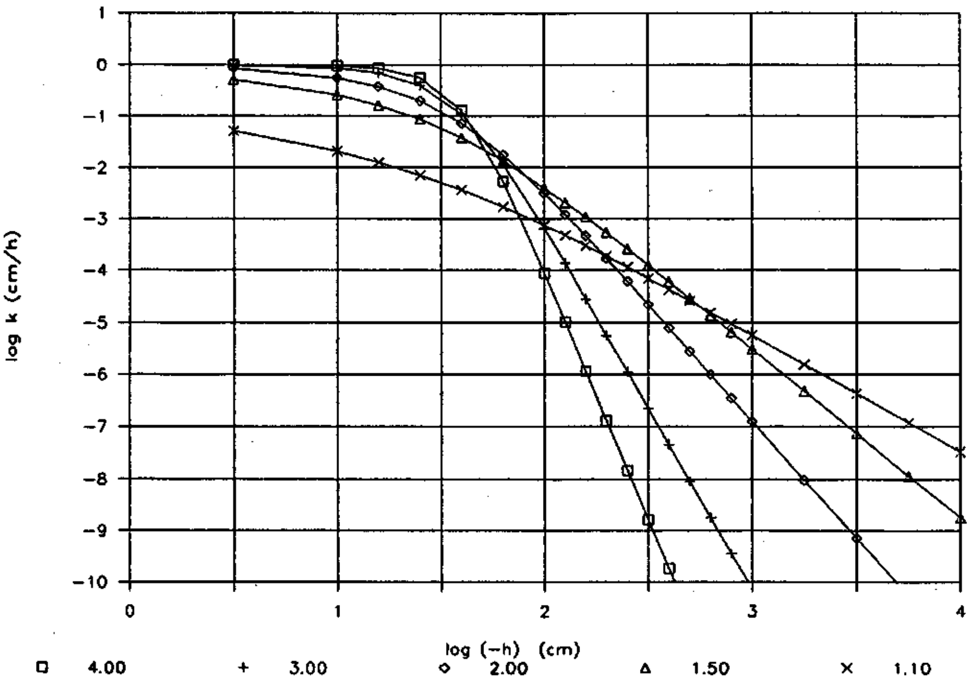


Figure 5. The effect of the MVG parameters α, n, K_s and l on the description of soil hydraulic functions. Function of departure is soil 1 (Table 1). The parameters are one by one changed to the values mentioned under the figures.

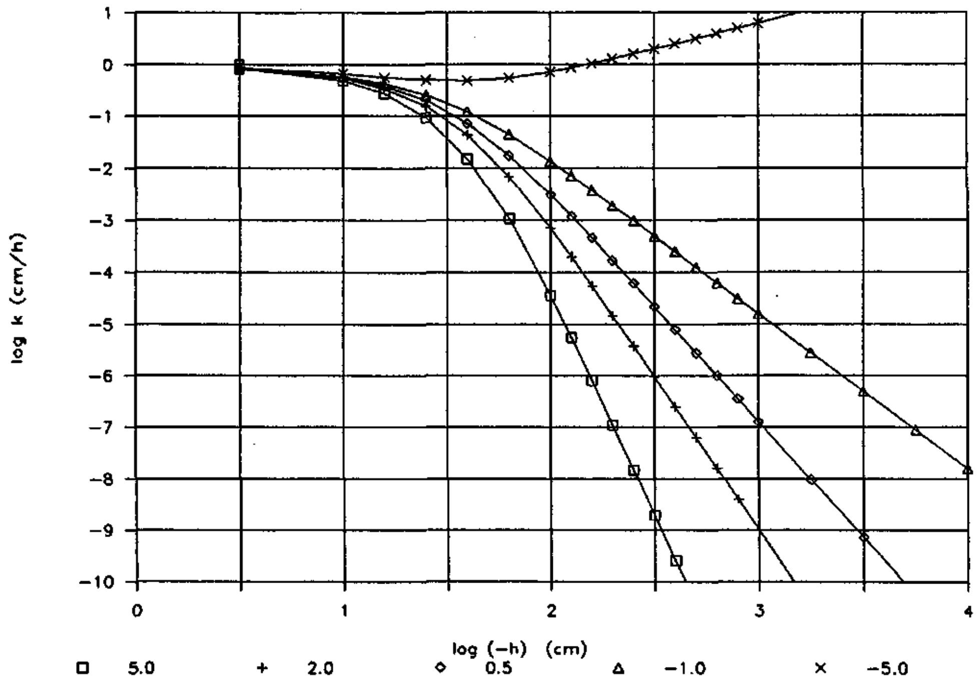
EFFECT ALFA



EFFECT N



EFFECT L



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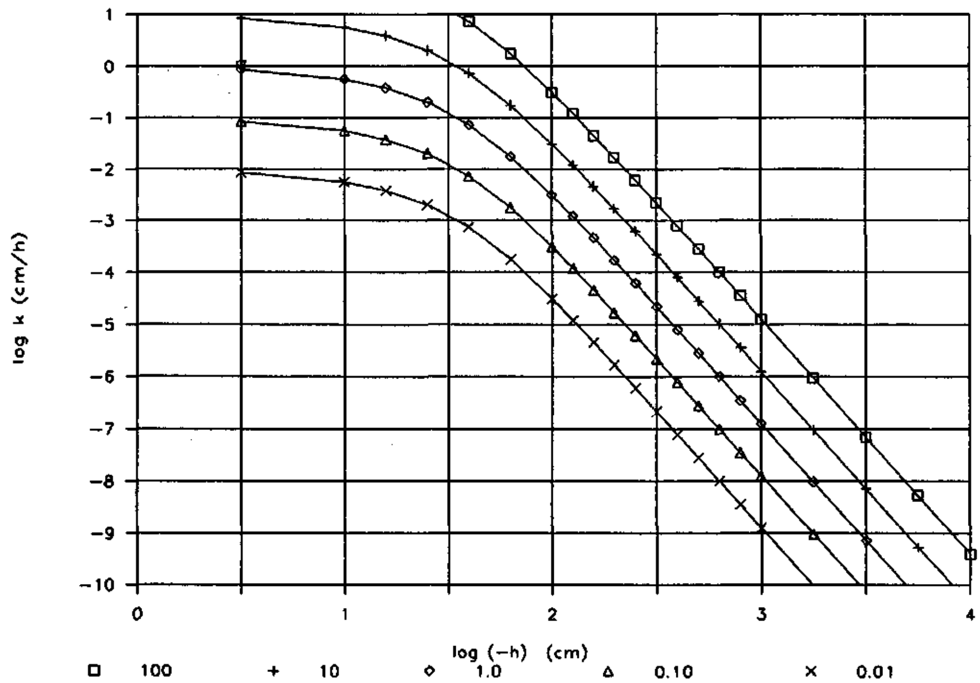
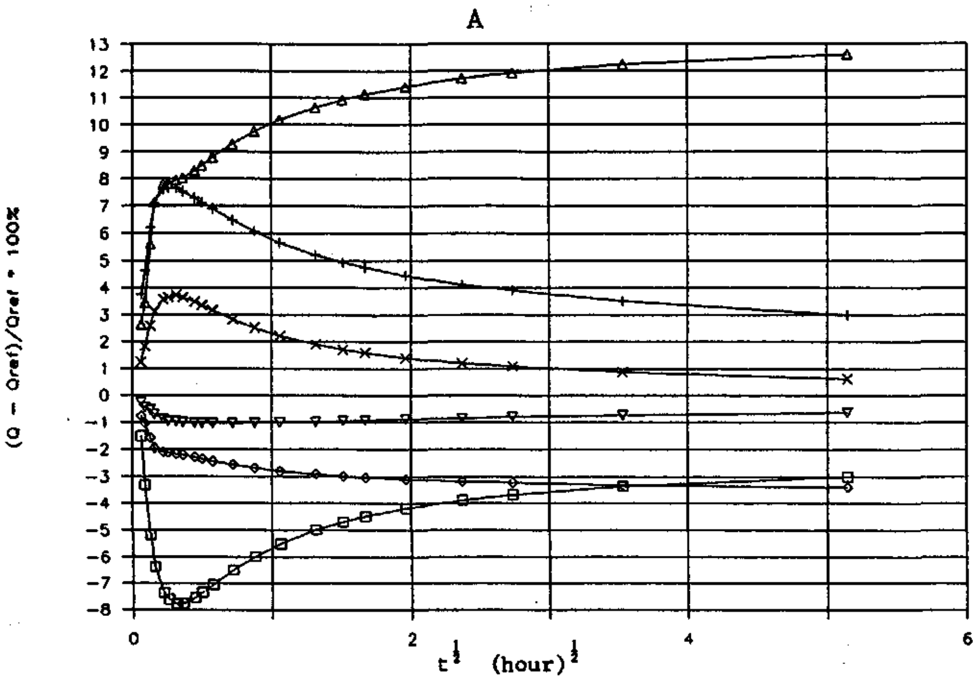




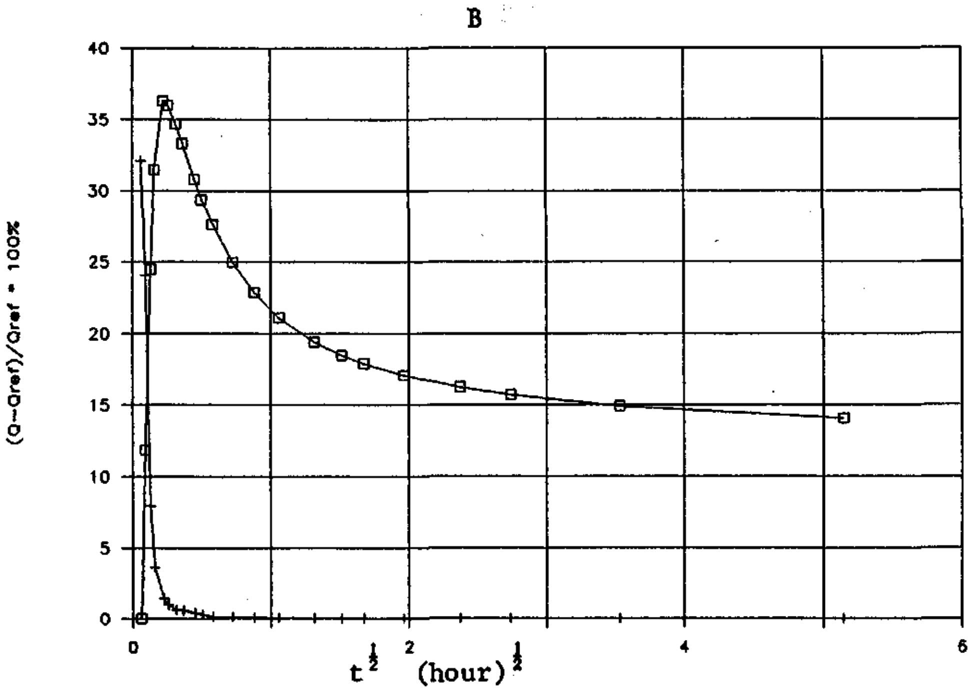
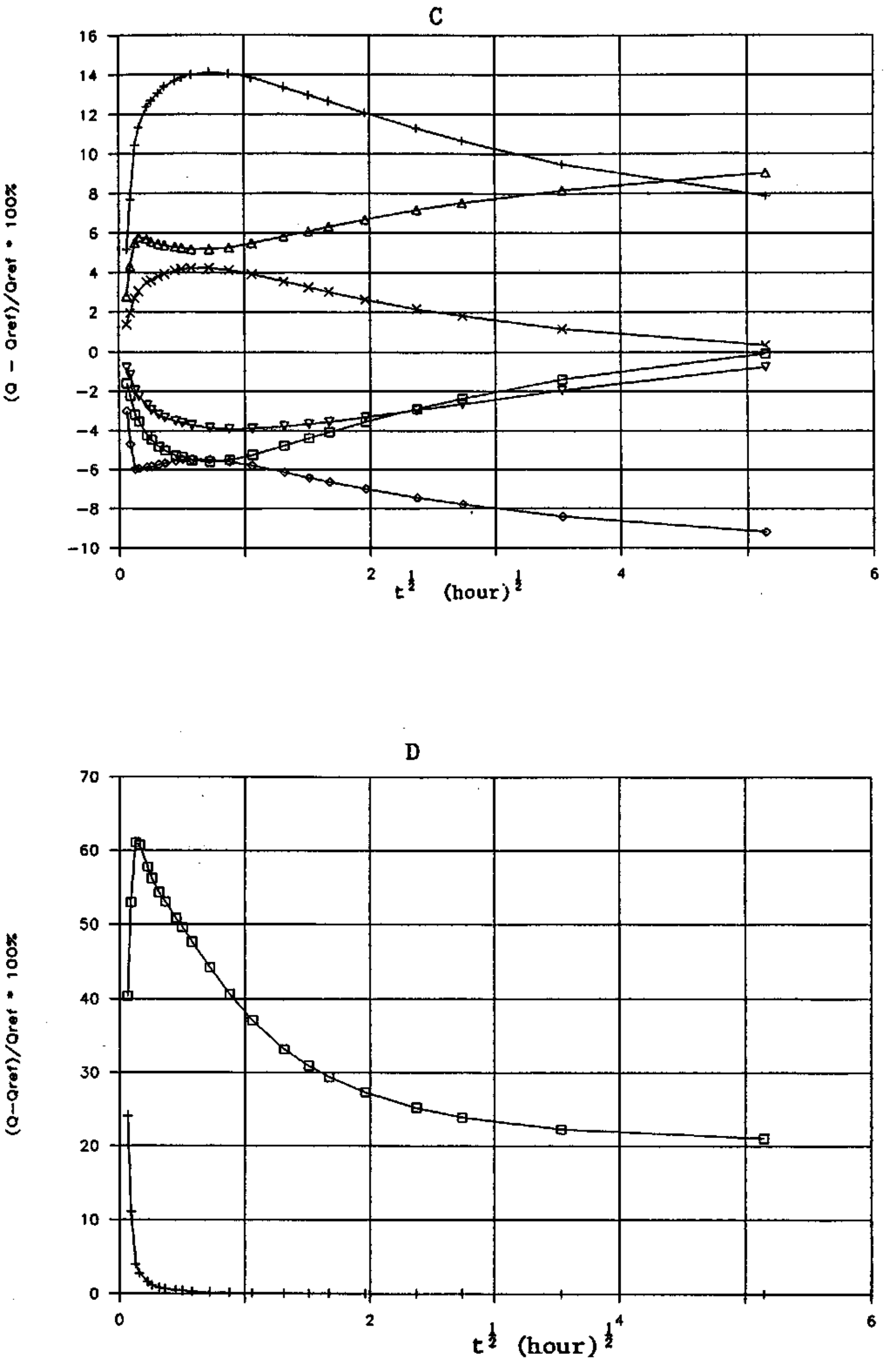


Figure 6. The sensitivity of the outflow to the parameters in the MVG model, and to $h_0(0)$ and K_{plate}. The parameters are one by one changed to the values mentioned in Table 3. Qref equals the outflow of the original parameters.

A,B : Loamy sand C,D : Clay loam

□ - α + - n ◇ - θ_r △ - θ_s × - K_s ▽ - l

□ - $h_0(0)$ + - K_{plate}



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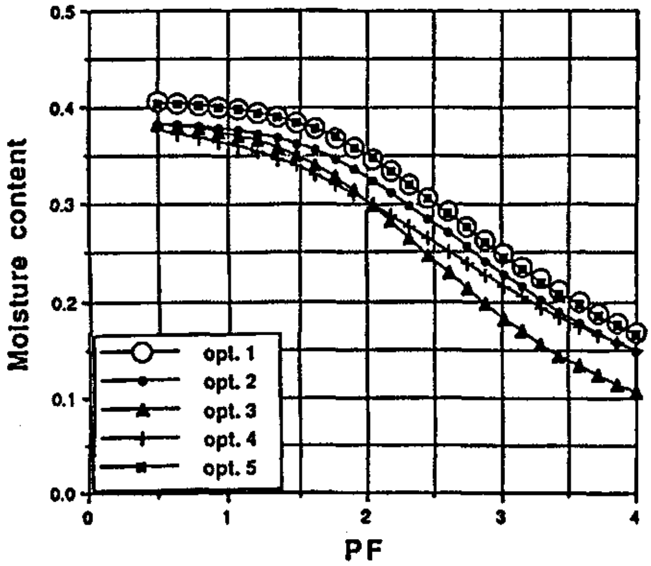


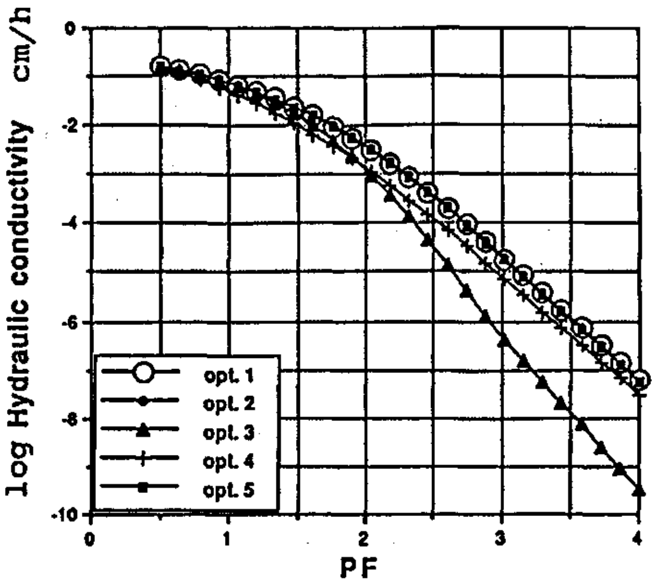


Figure 7. Arithmetic mean $\theta(h)$ and geometric mean $k(h)$ of optimized curves obtained for 13 samples of the Groesbeek soil. Opt. 1 to 5 refer to the five parameter sets mentioned in Table 4.

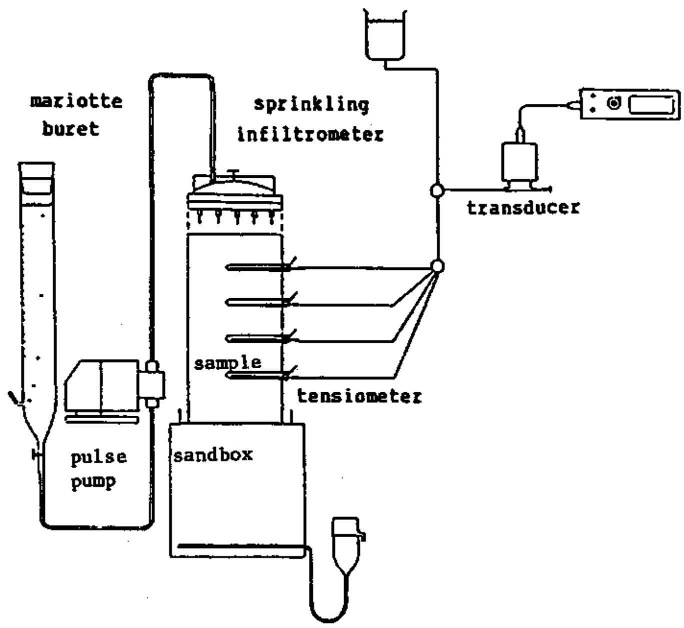


Figure 8. System to measure $k(h)$ according to the needle method.

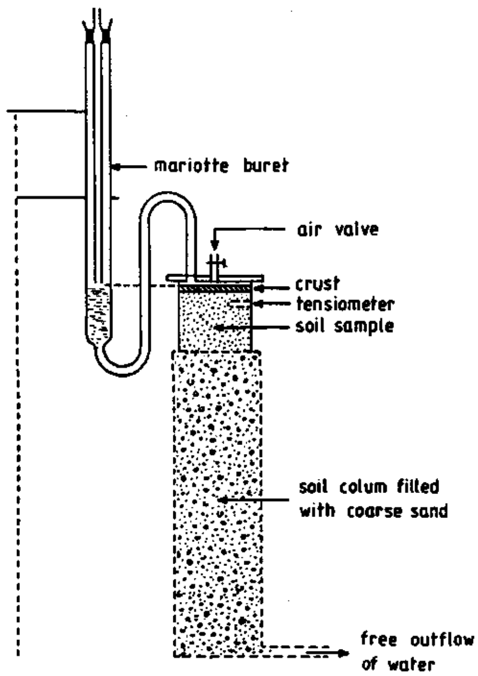


Figure 9. Arrangement to measure $k(h)$ according to the crust method.

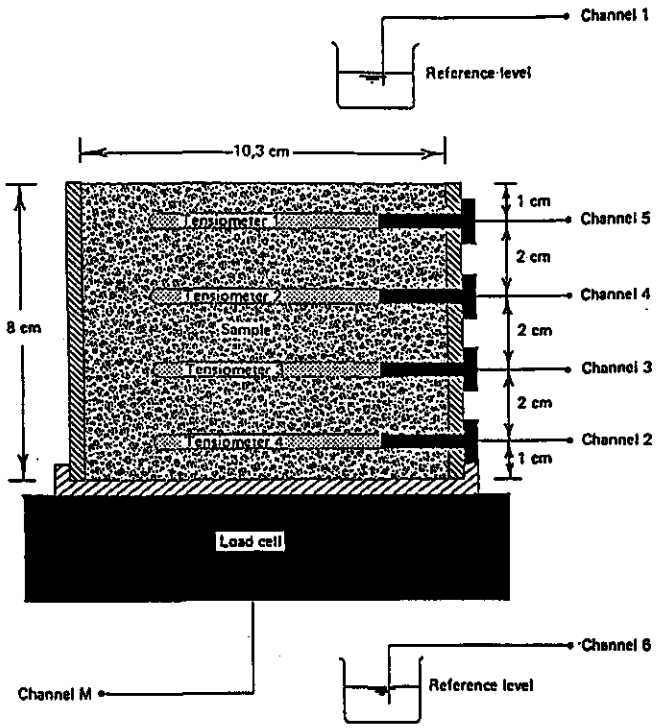


Figure 10. Set-up for the evaporation method.

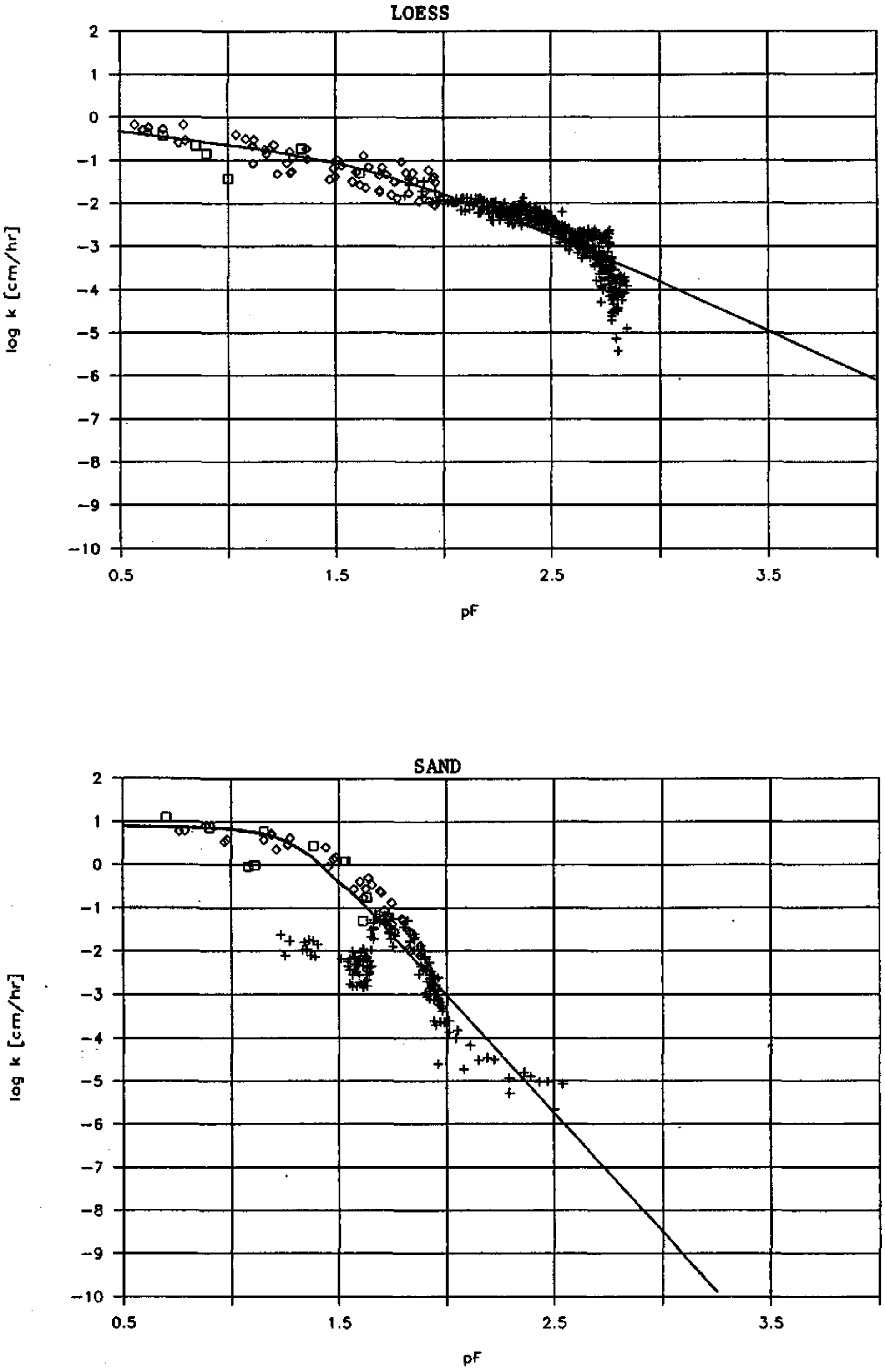
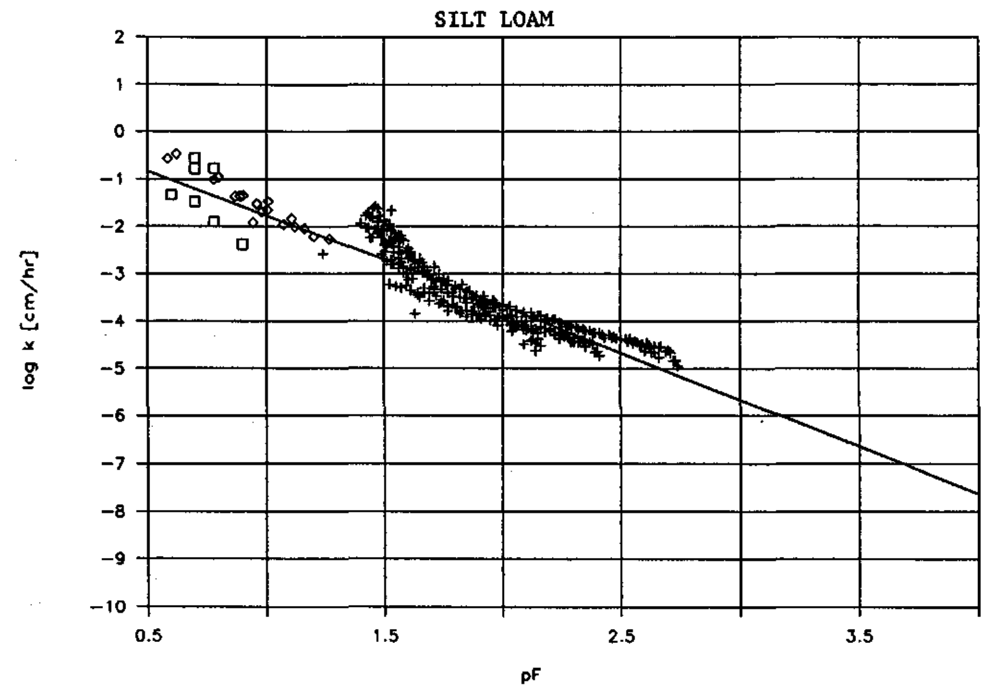
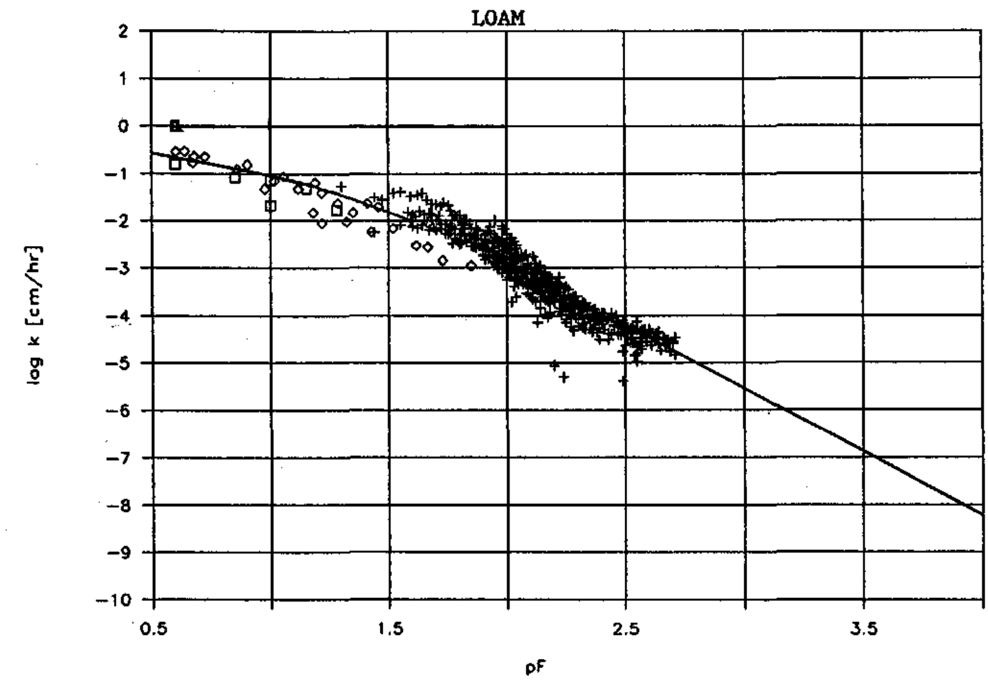


Figure 11. The results of the crust, needle and evaporation method for four soils. Also the fitted lines are drawn. The MVG parameters of these lines are mentioned in Table 7.

□ Crust   + Evap   ◇ Needle





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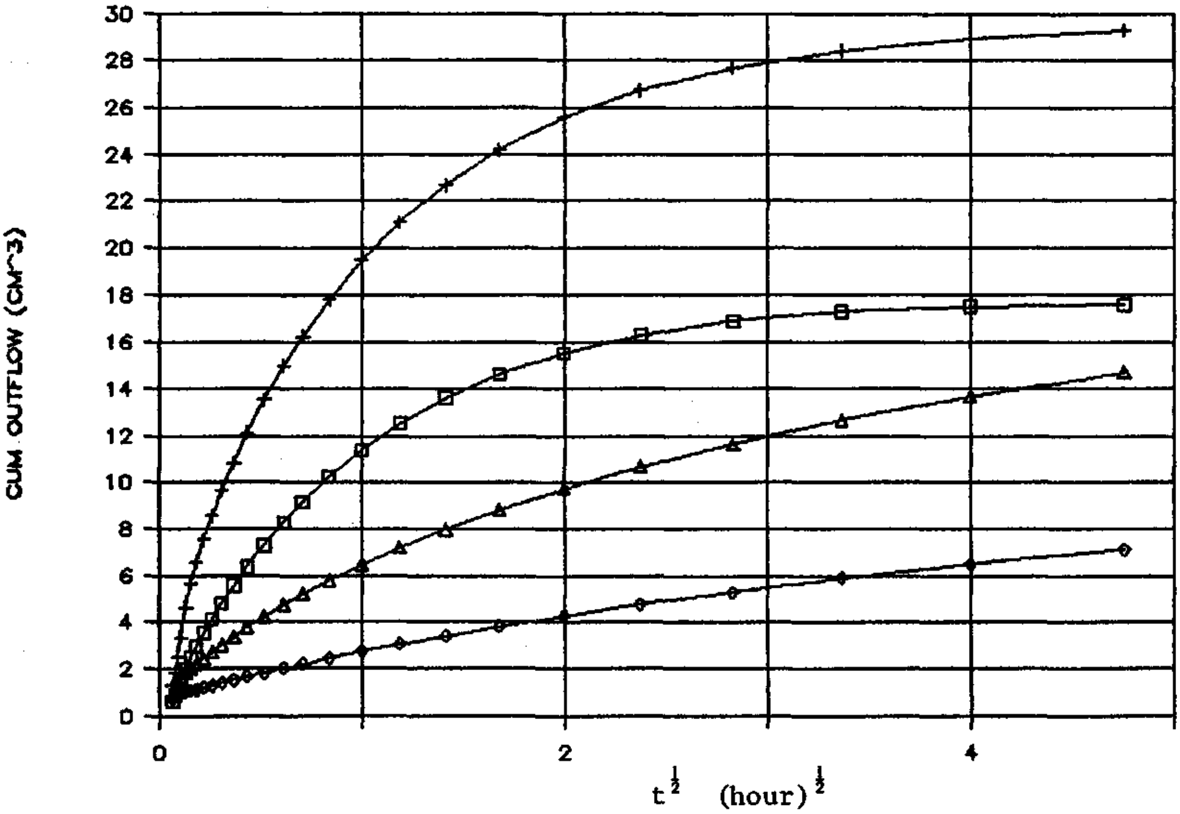


Figure 12 Predicted outflow for the four soils with $k(h)$ according to crust, needle and evaporation method (Table 7) and $\theta(h)$ as mean of six samples.

□ loess + sand ◇ silt loam △ loam

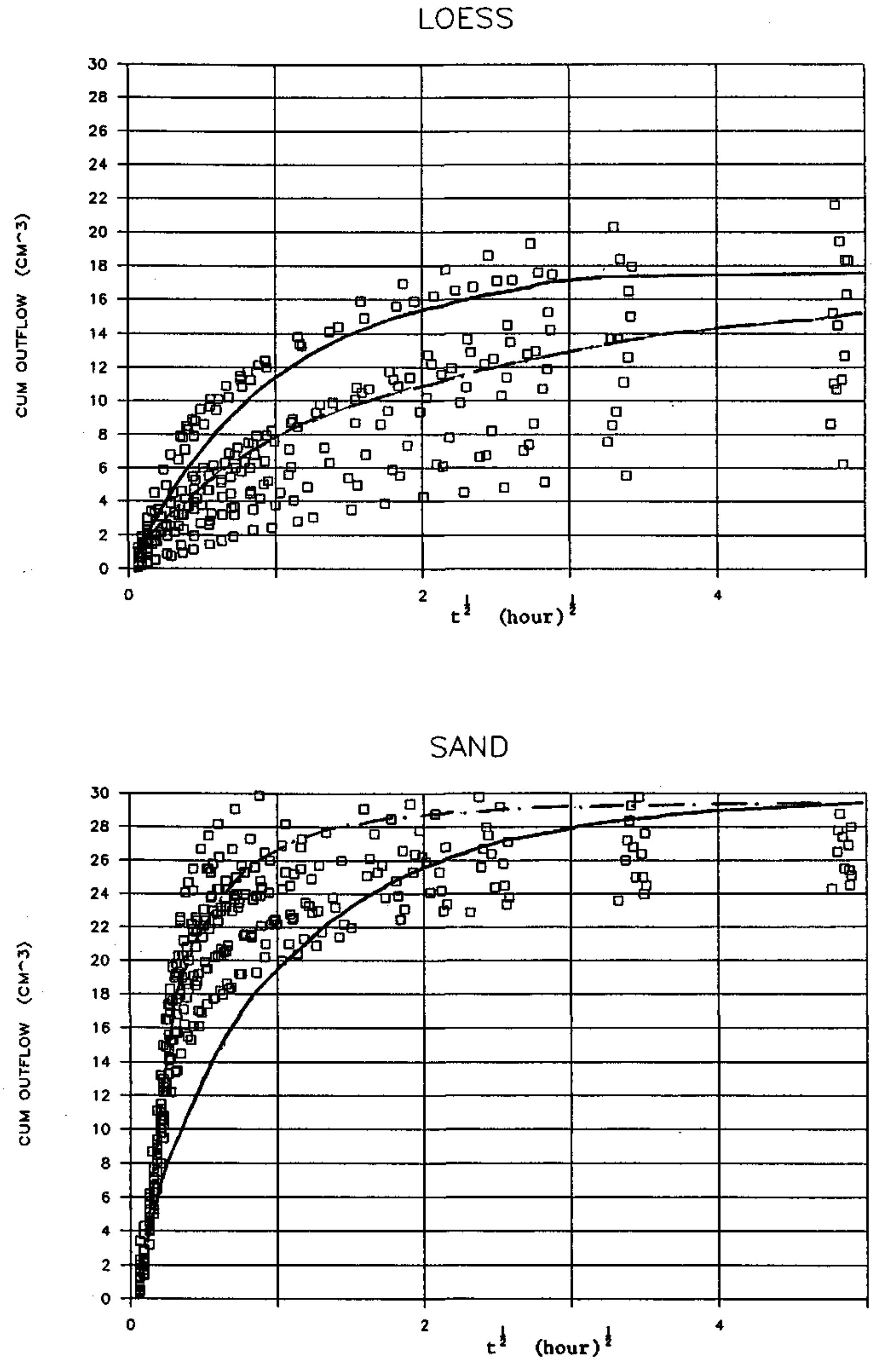
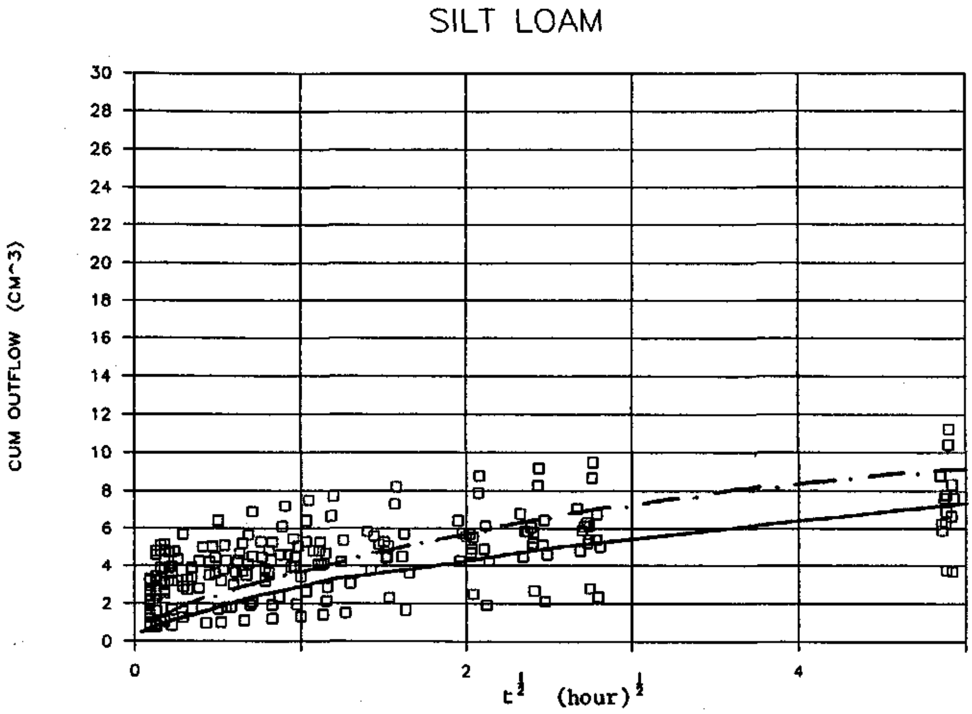
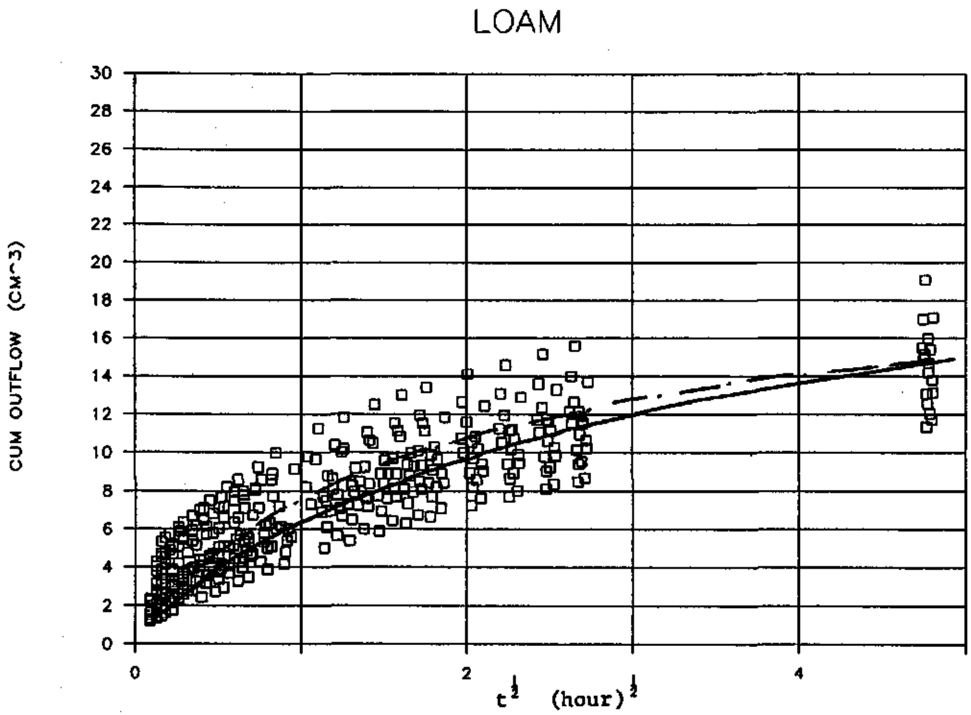


Figure 13. Comparison between measured outflow (□) and simulated outflow (———), based on soil hydraulic functions measured by a combination of crust, needle and evaporation method (par. MVG see Table 7). Also the simulated outflow for the mean soil hydraulic functions of optimization 5 is shown (—·—).





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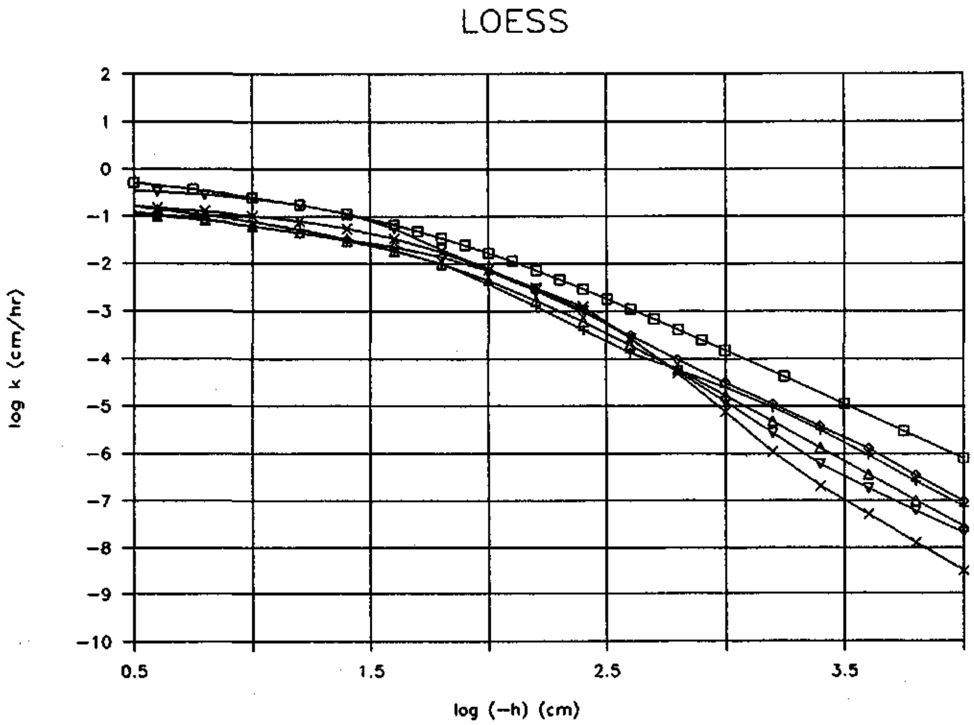


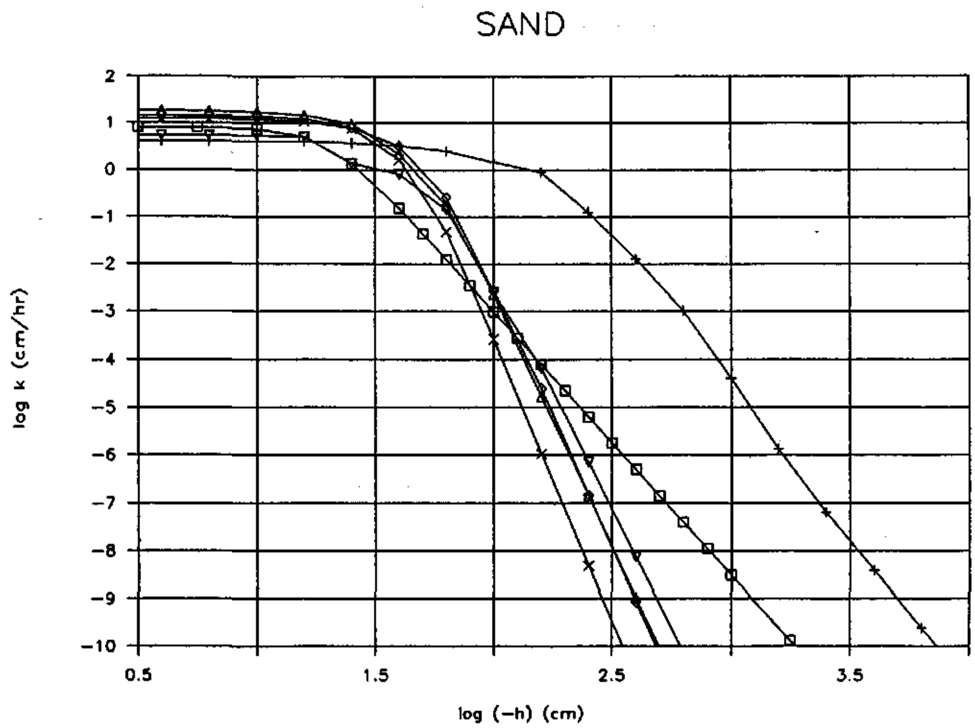
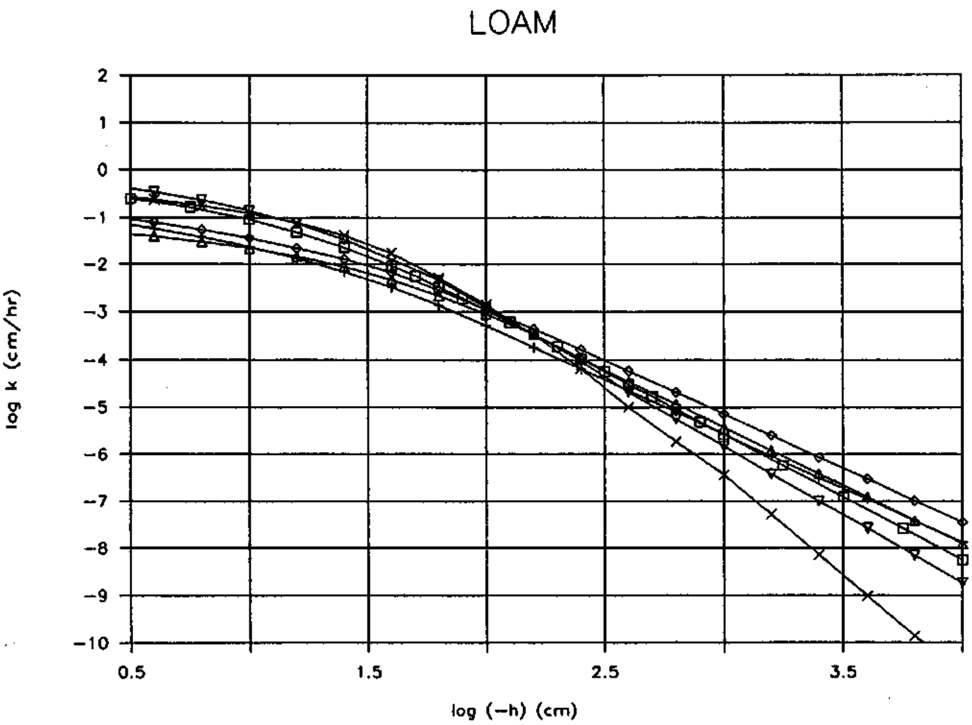


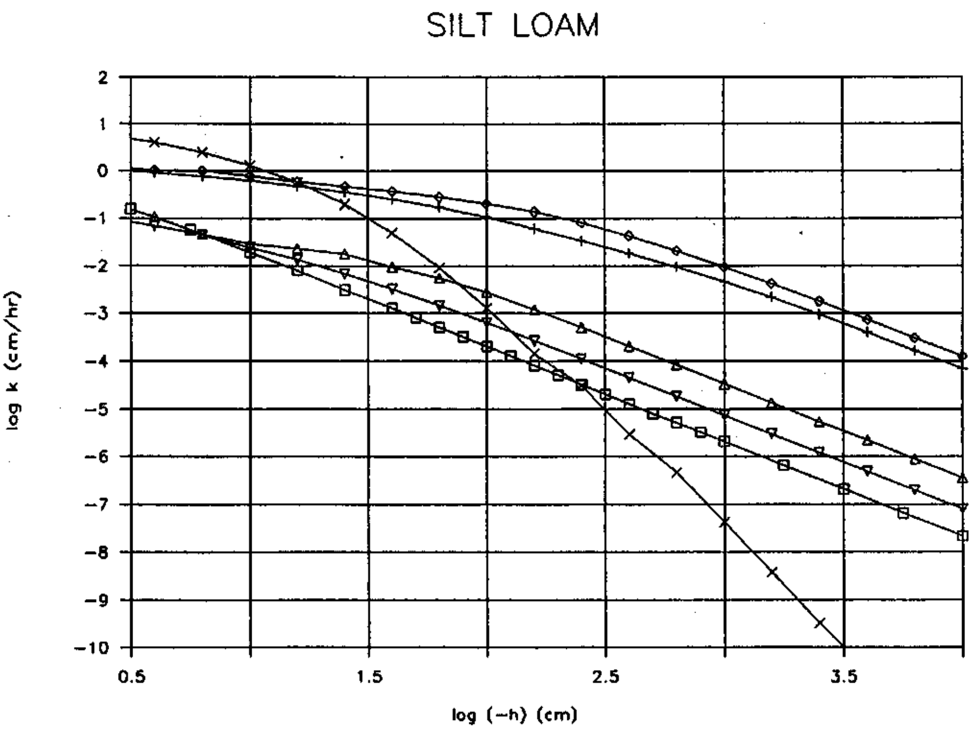
Figure 14. Mean curves (n > 13) for four soils, using different data in the object function.

□ The fitted line through the crust, needle and evaporation method.

Optimization (Table 8) :

+ 1 ◇ 2 △ 3 × 4 ▽ 5





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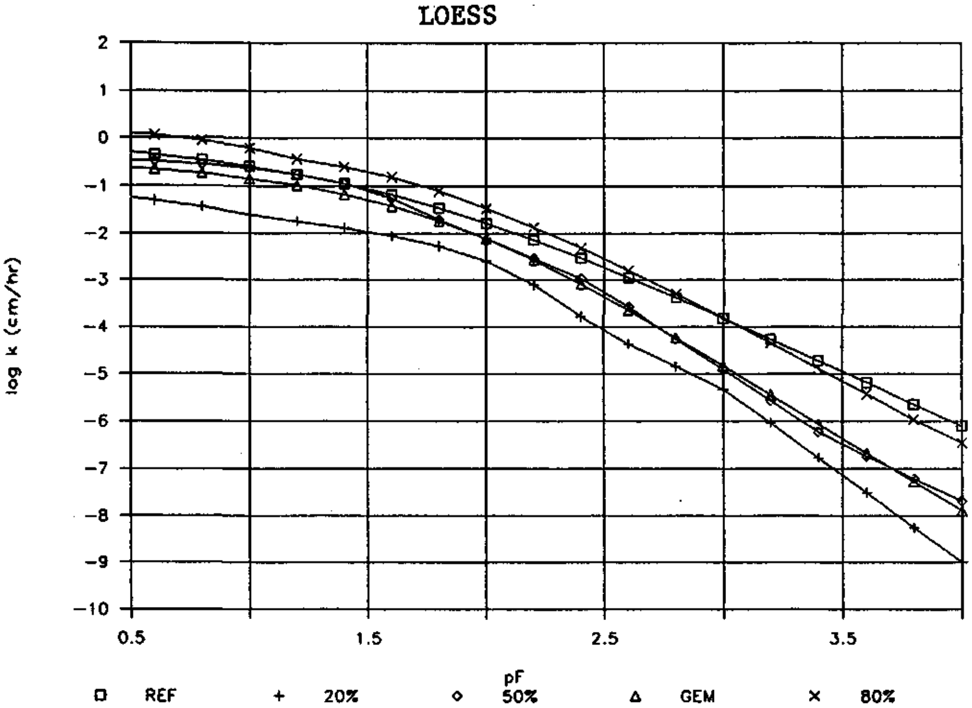


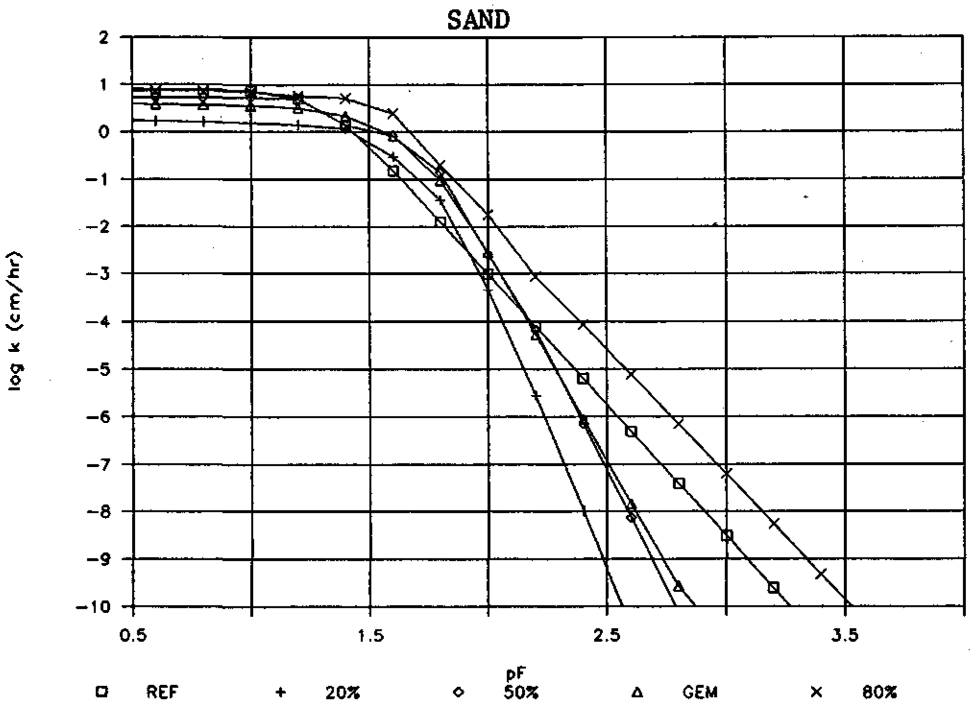
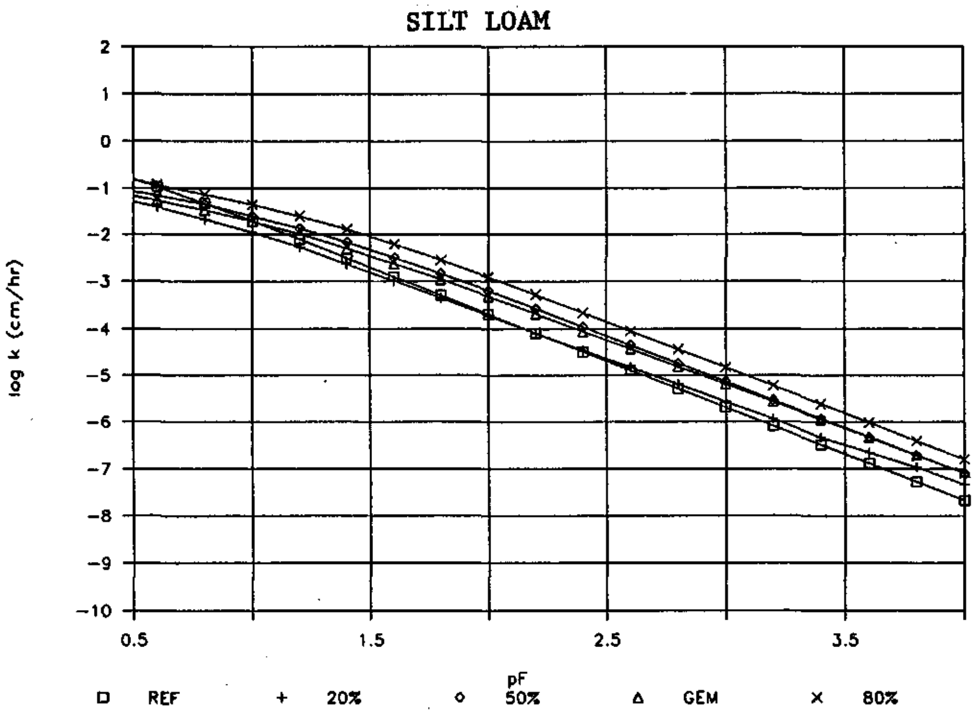
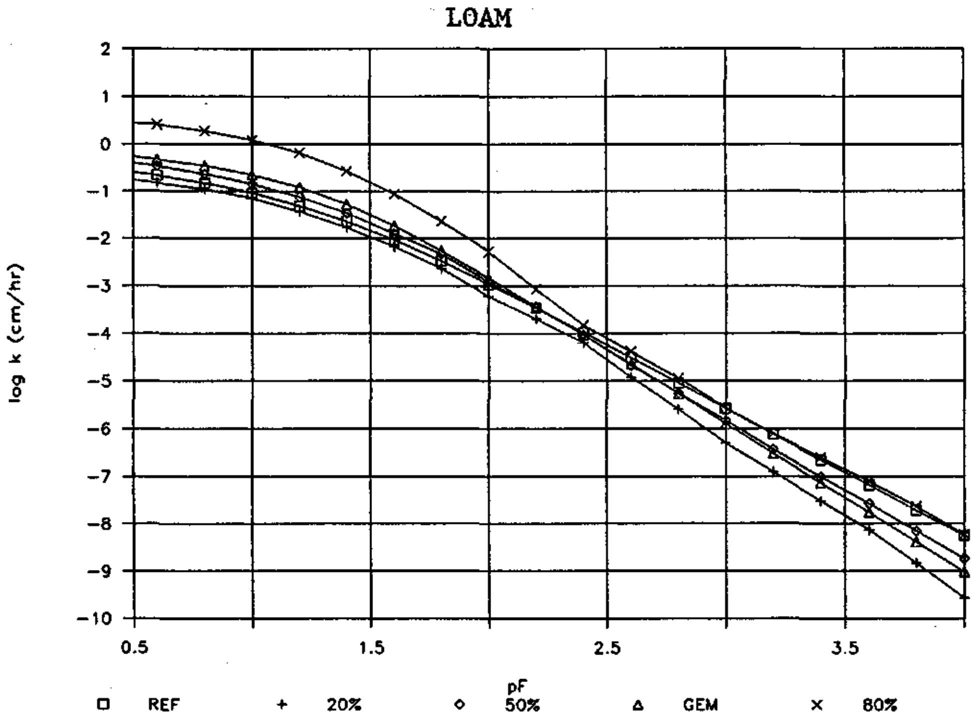


Figure 15. The band width of the estimated $k(h)$ functions for each soil. REF stands for the fitted line through the crust, needle and evaporation method. The distribution is determined per h. GEM equals the arithmetic mean of log k, while 50% corresponds to the median. 20% means that one-fifth of the lines at a certain h have a lower k.





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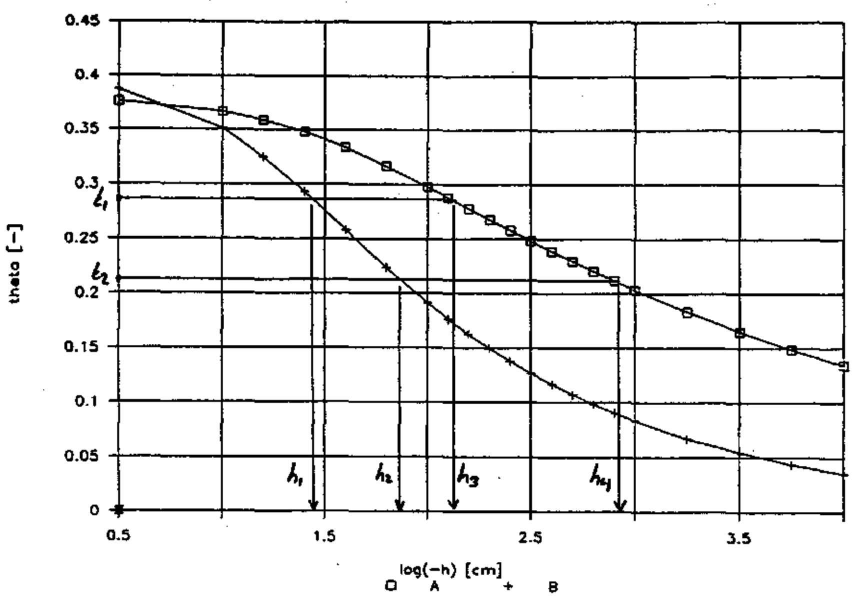
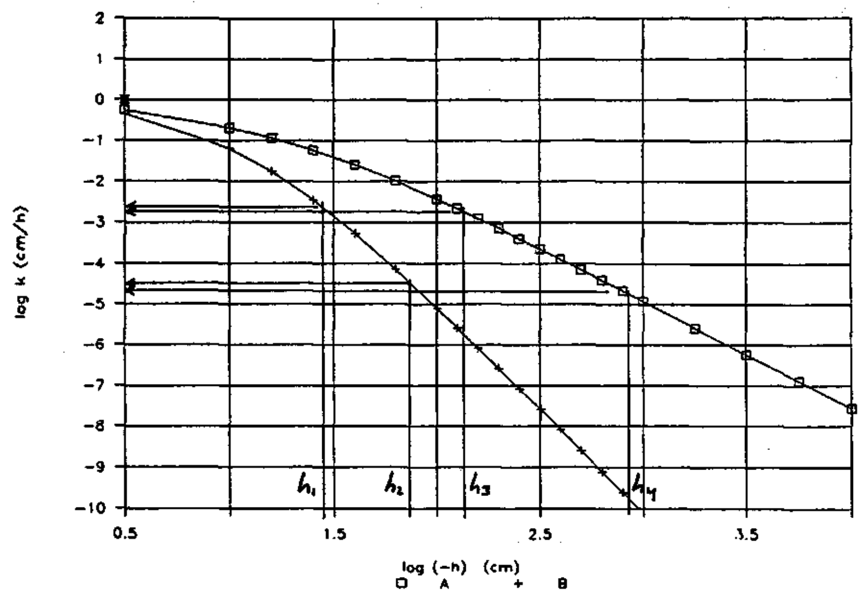




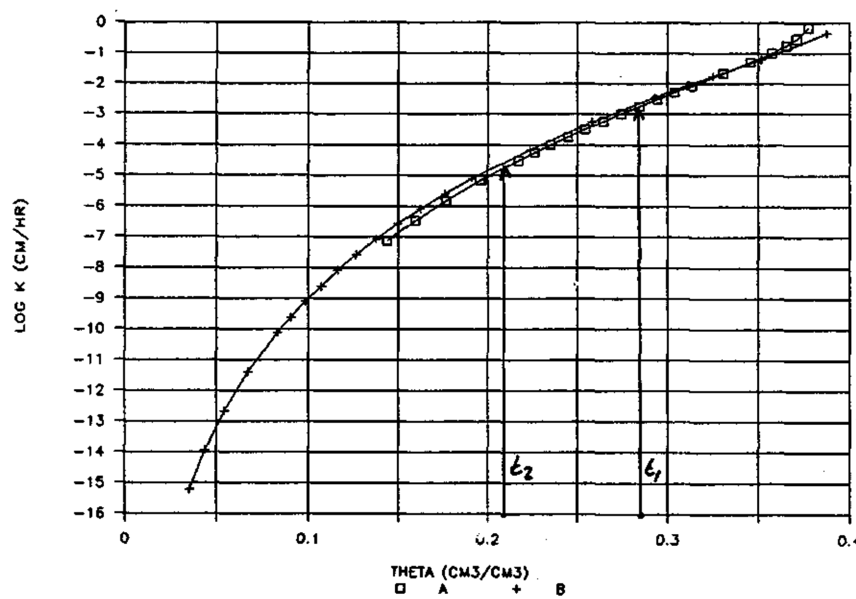
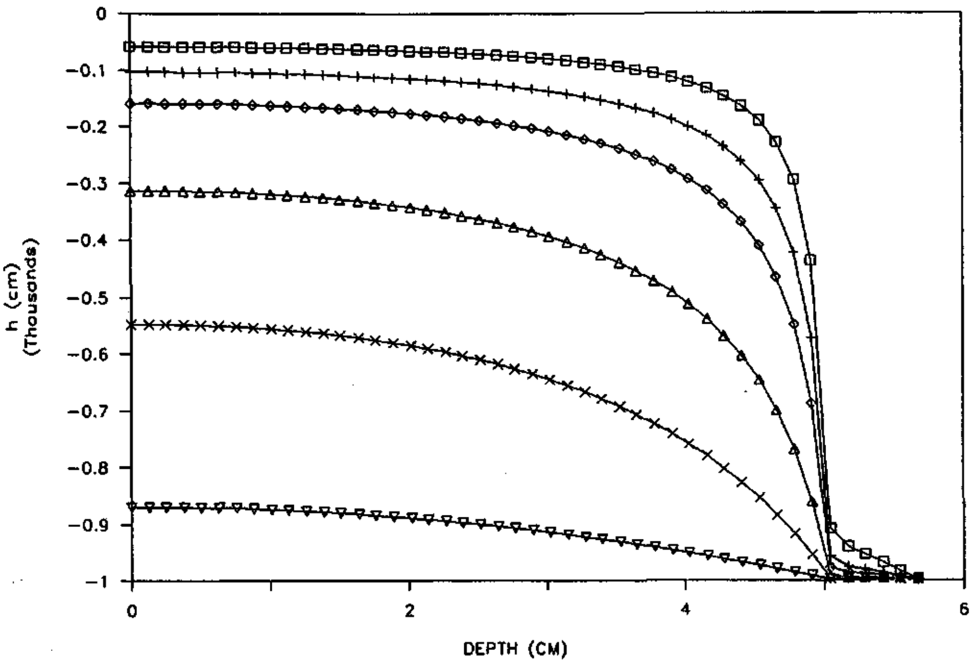


Figure 16. Two sets of soil hydraulic functions resulting from optimization (method 1 and 3 resp., Table 8) of the same outflow of a loess sample.



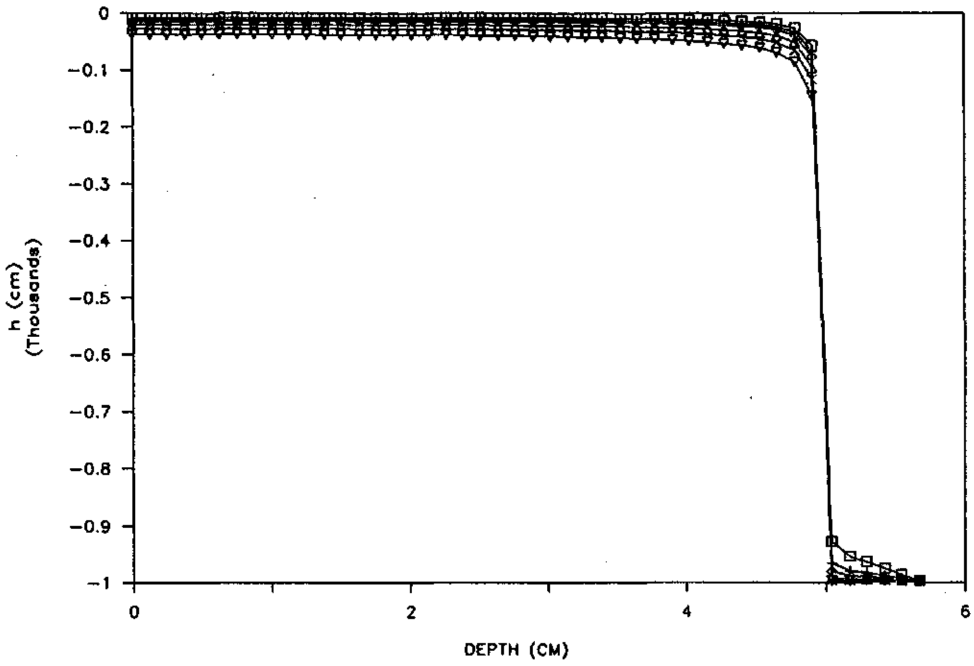
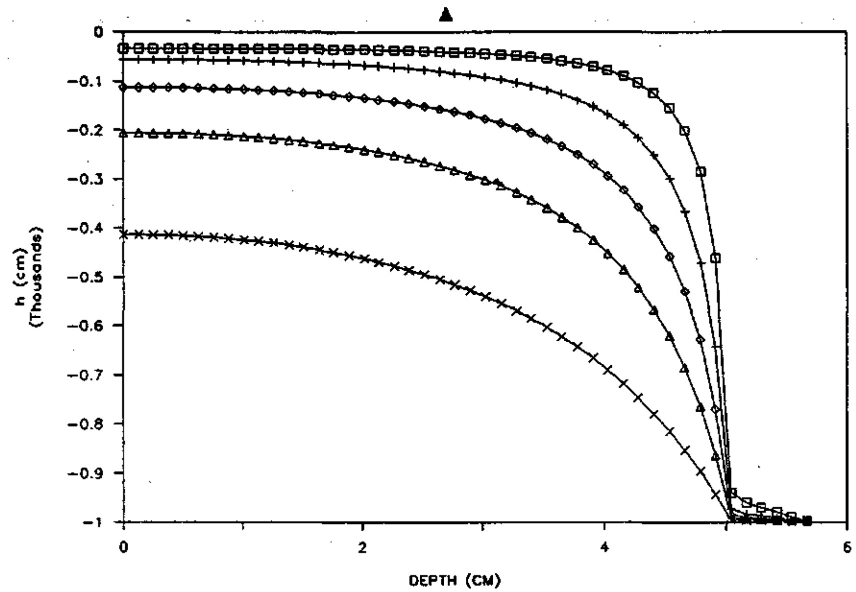
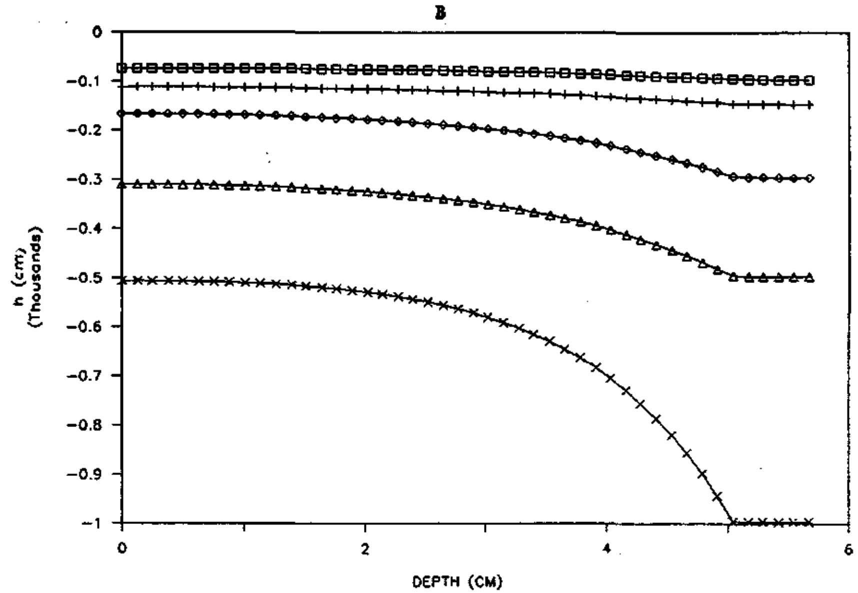


Figure 17. $h(z)$ simulated during an One-step outflow experiment of samples with soil hydraulic functions according to Fig. 16. The top of the sample is at depth = 0.0 cm, while the transition soil - ceramic plate occurs at depth = 5.1 cm.

Times (hr) at which $h(z)$ is shown :

□ 0.2 + 0.5 ◇ 1.1 △ 3.0 × 8.0 ▽ 24.0





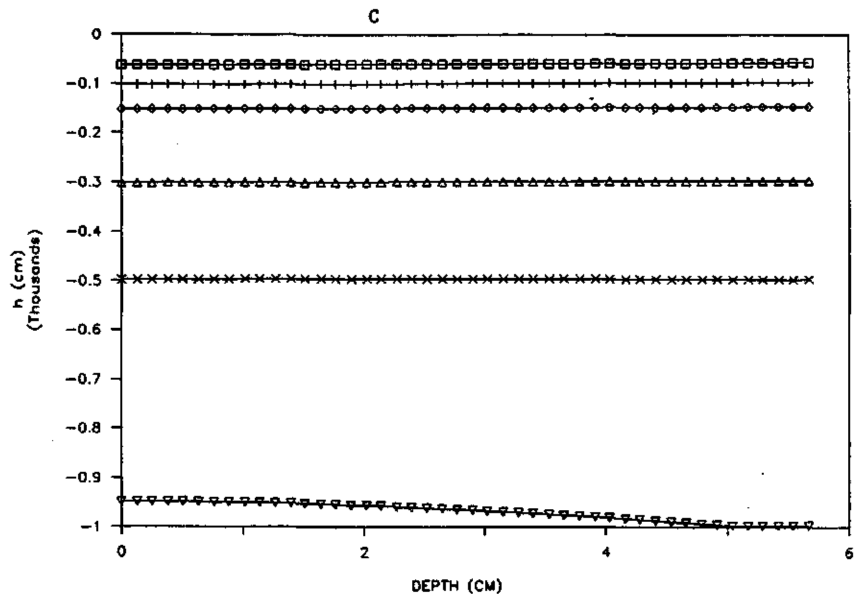
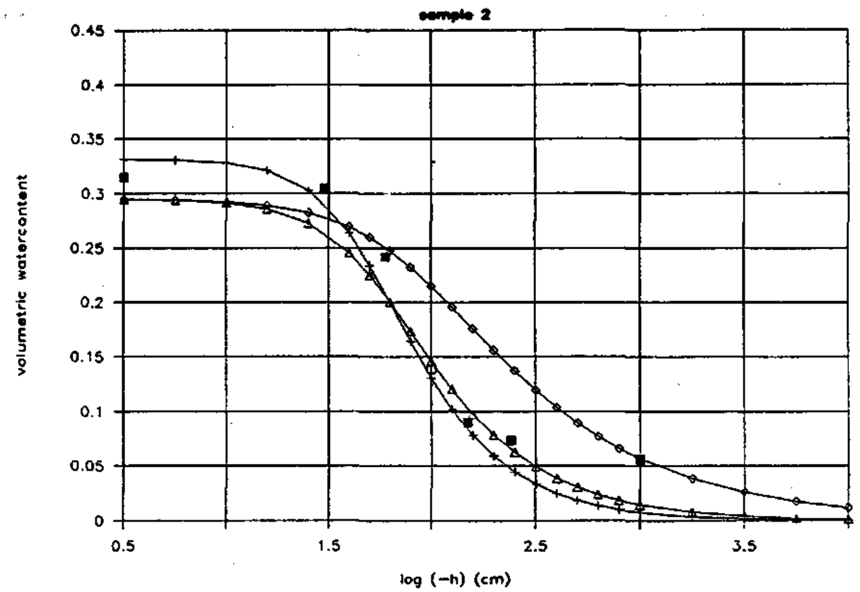
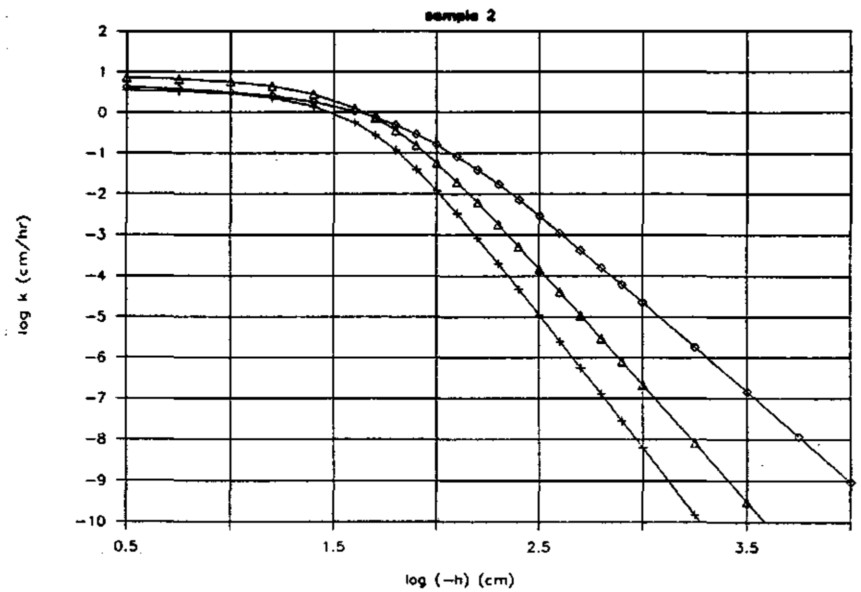


Figure 18. The effect of one large or five smaller pressure increments on $h(z)$. (A) shows $h(z)$ for One-step. (B) and (C) apply to Multi-step with pressure increments at t = 0, 24, 48, 96, and 120 hour. (B) represents $h(z)$ one hour after a pressure increment, (C) represents $h(z)$ one hour before a pressure increment.

A	□ 0.12	+ 0.60	◇ 1.60	△ 3.20	× 7.00	▽ 28.00
B	□ 25.0	+ 49.0	◇ 73.0	△ 97.0	× 121.0	
C	□ 23.0	+ 47.0	◇ 71.0	△ 95.0	× 119.0	▽ 139.0





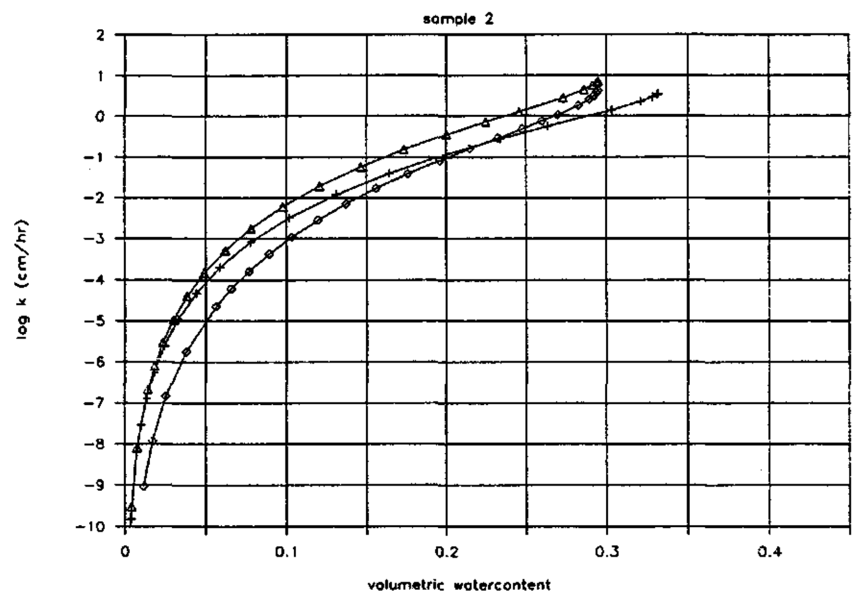
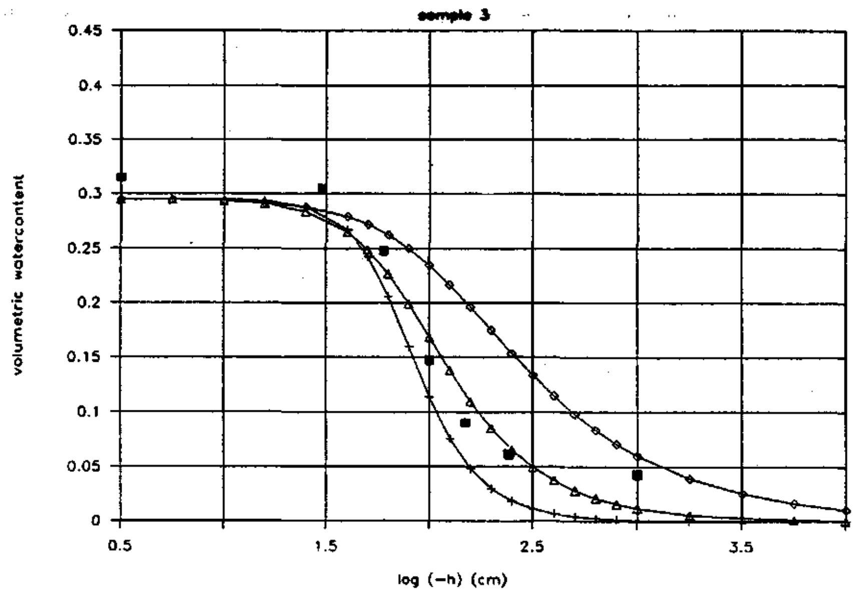
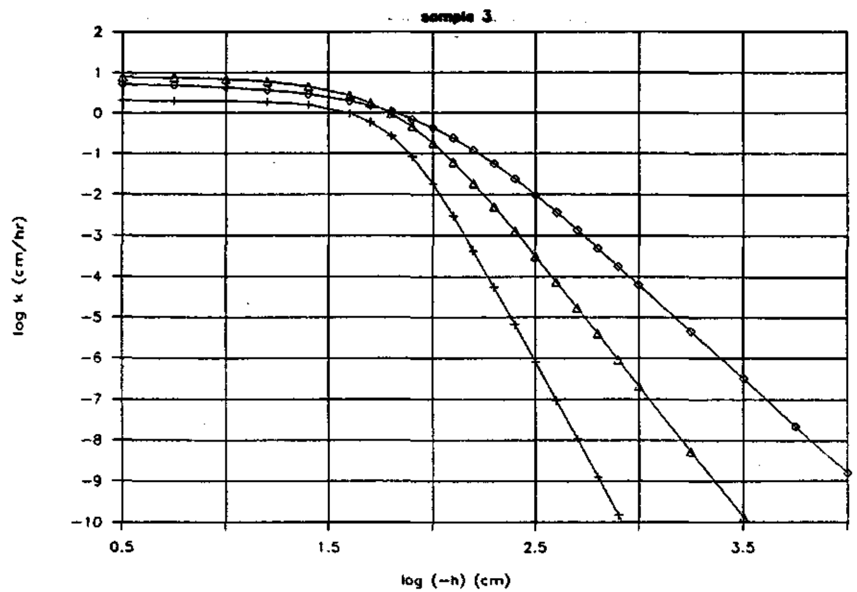
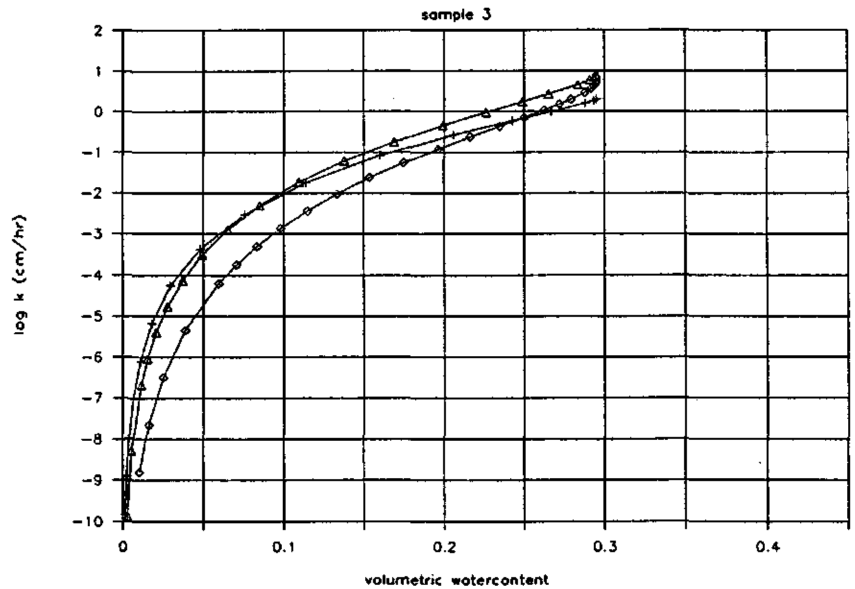


Figure 19. Optimized soil hydraulic functions, using $Q(t)$ data in the objectfunction, in comparison with $\theta(h)$ data (■) derived from outflow experiments with daily pressure increase. Three different time schedules were applied on the same sample :

◇ At once   △ Per hour   + Per day







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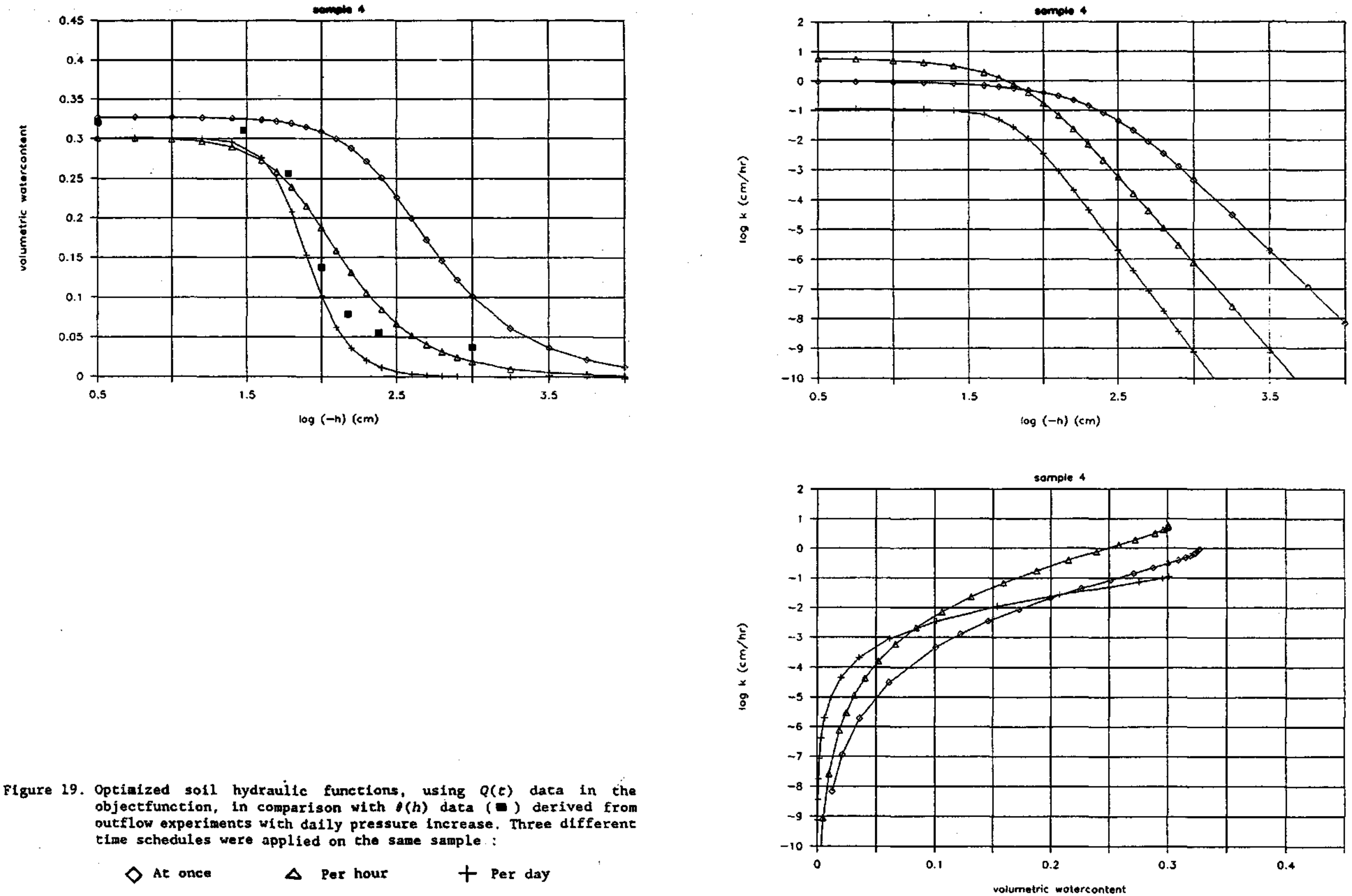

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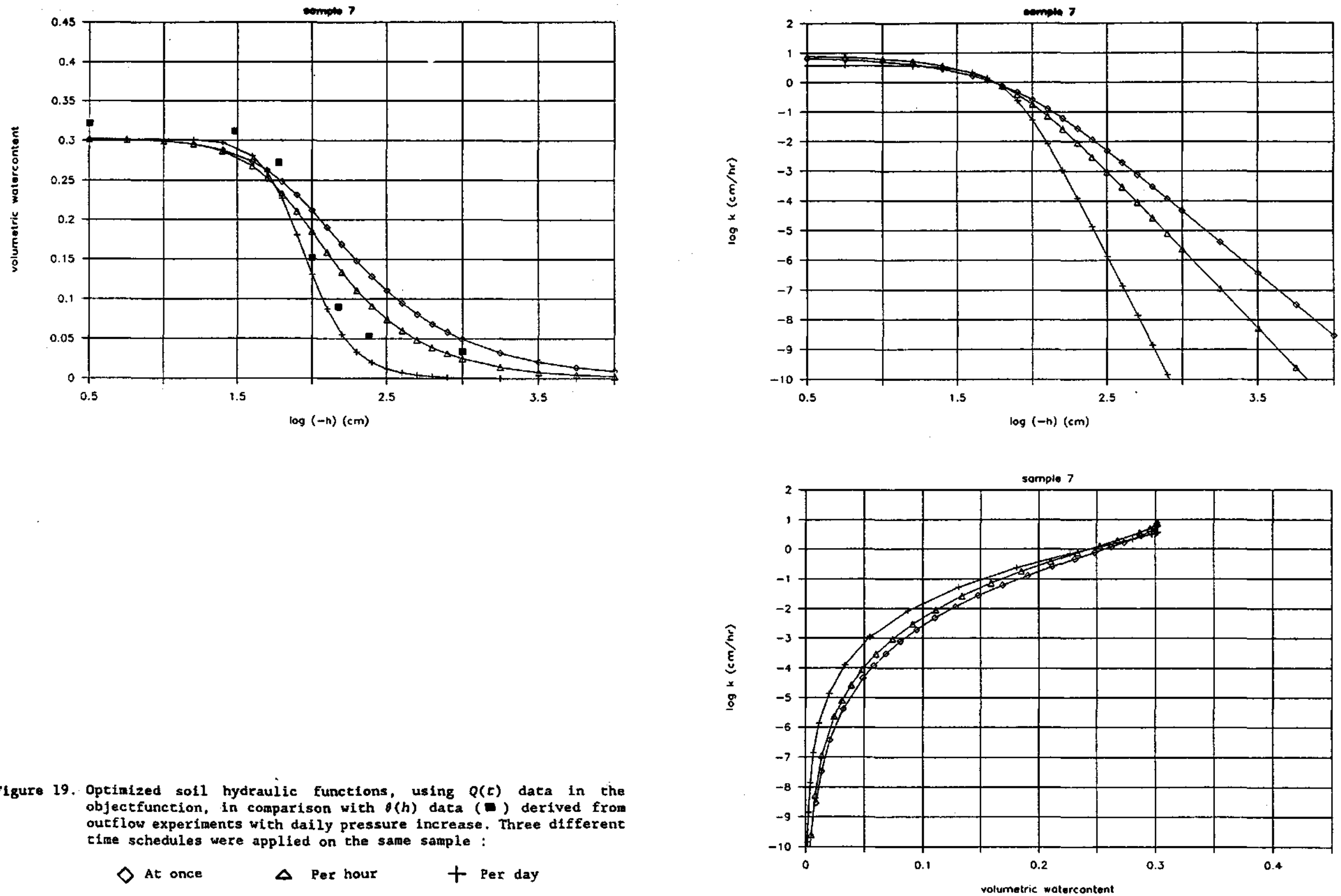

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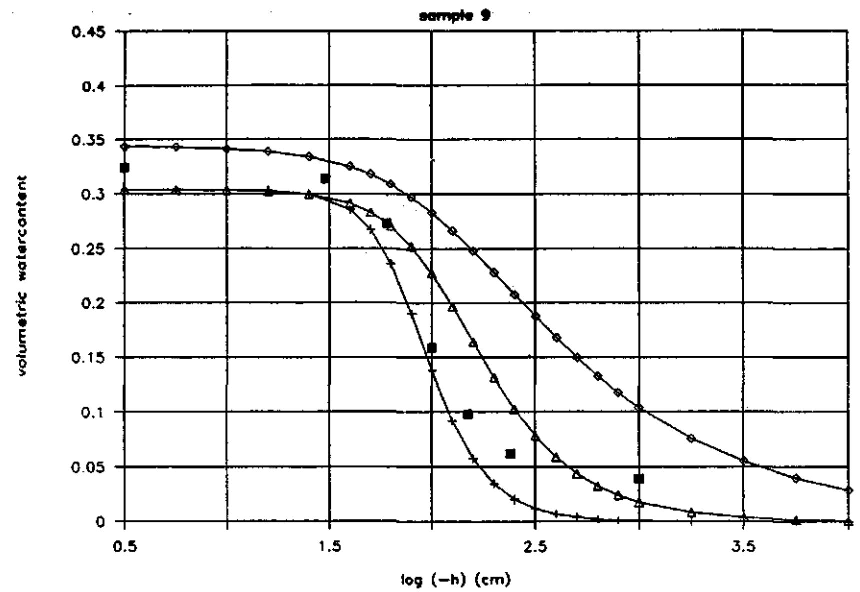

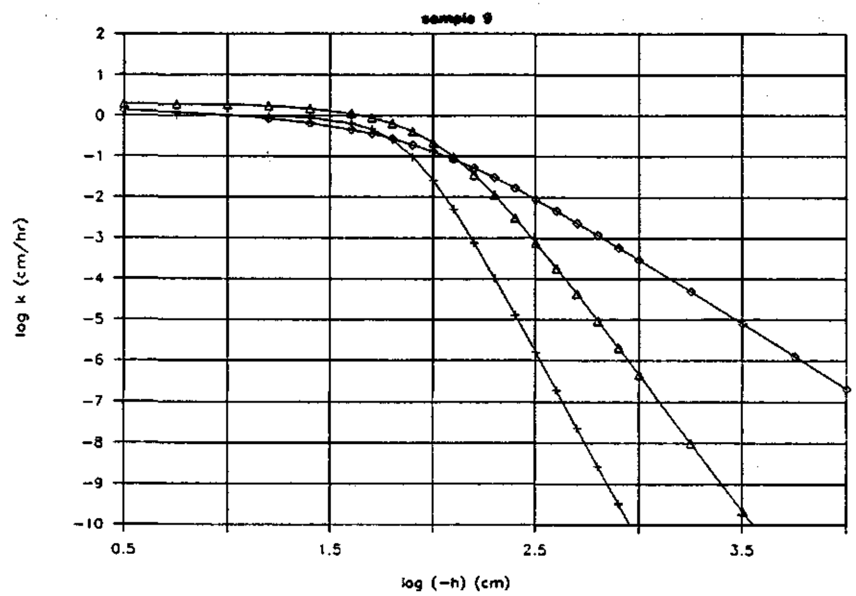

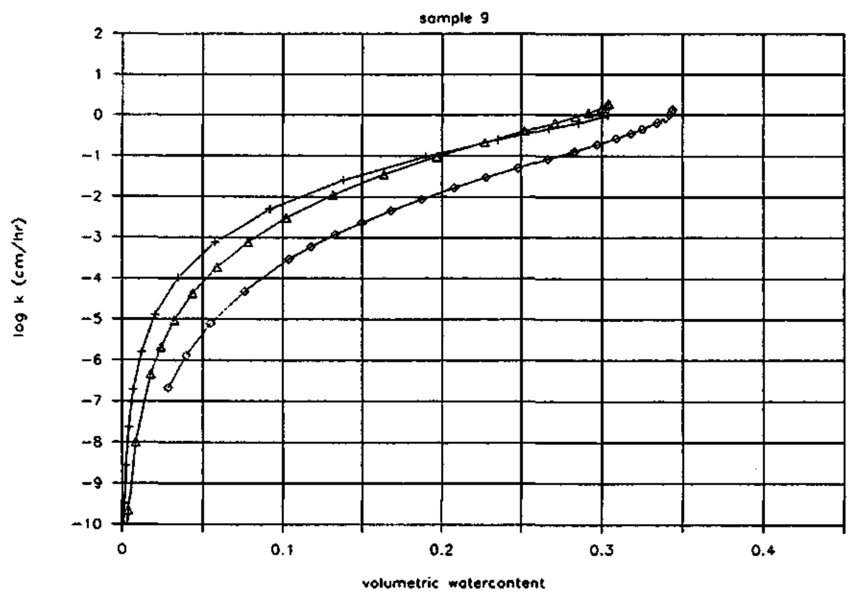

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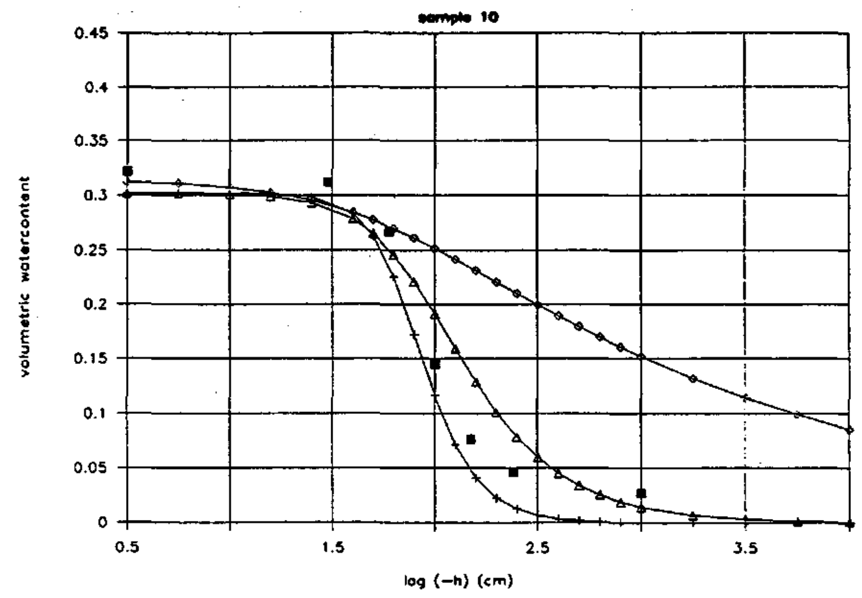

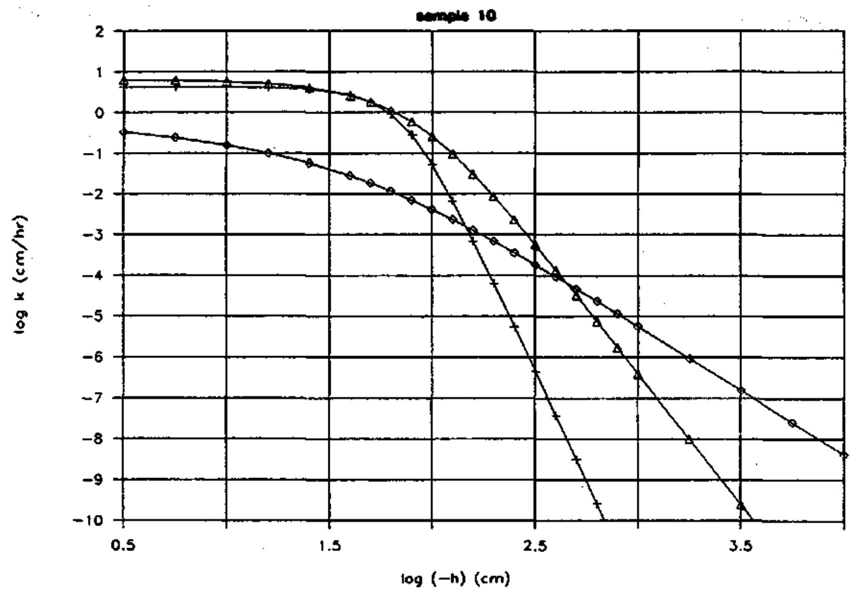

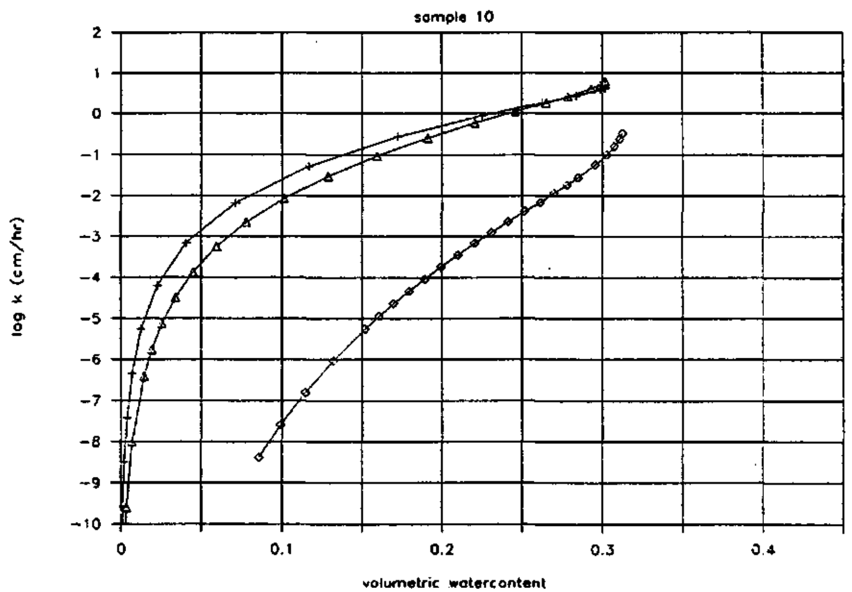

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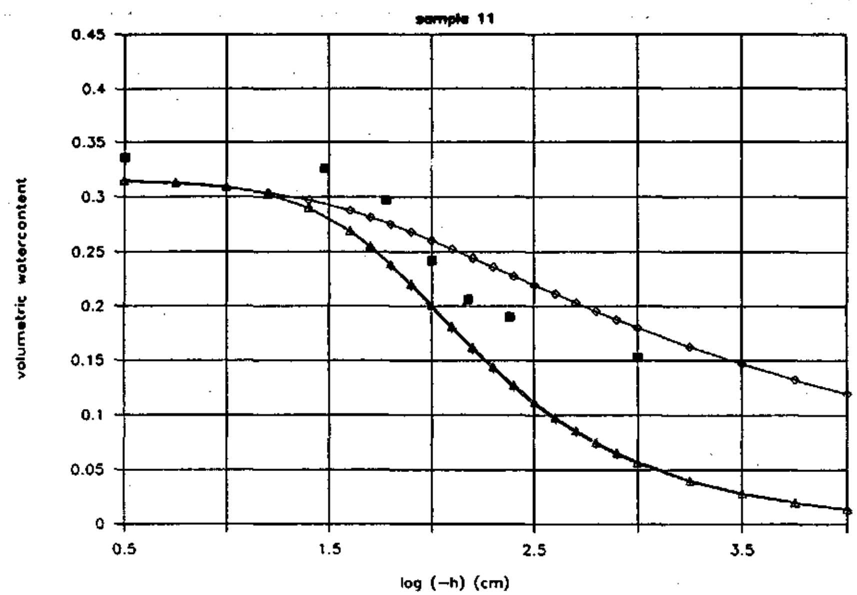

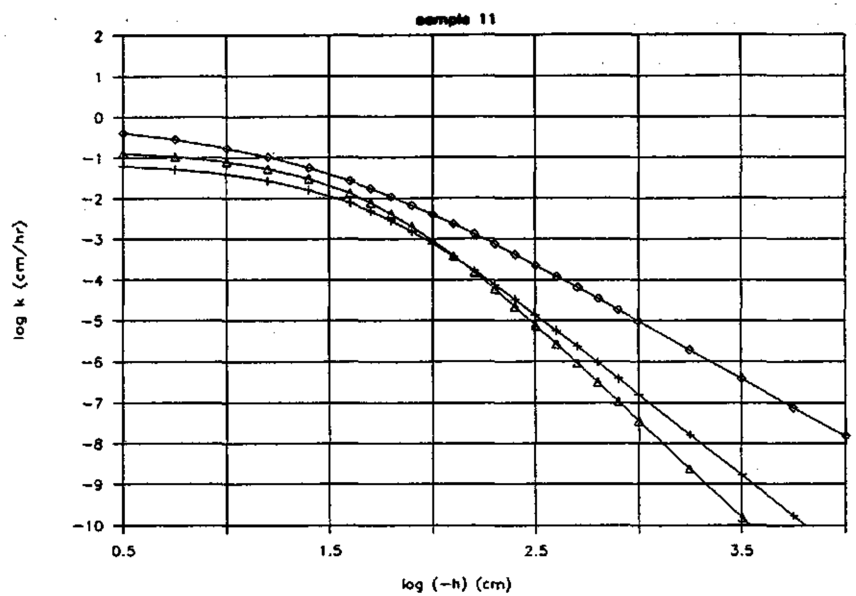

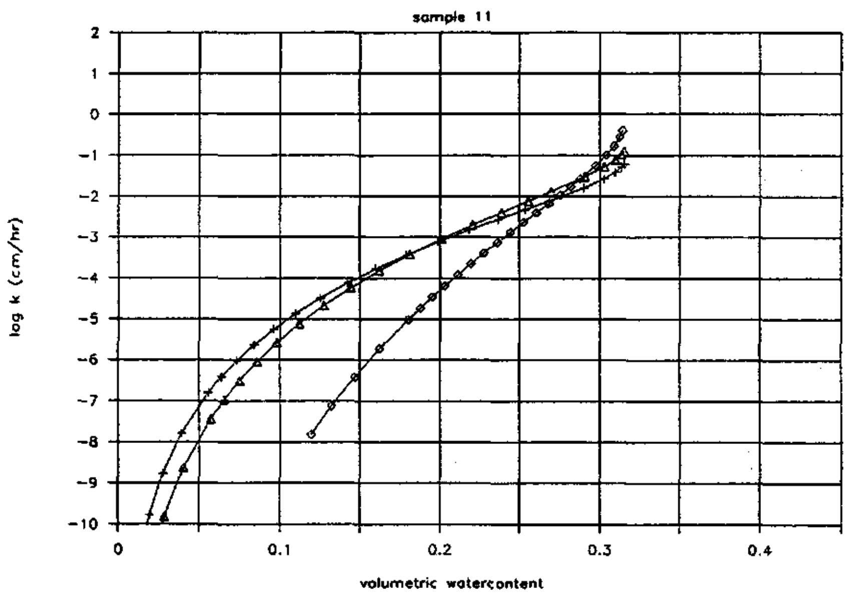

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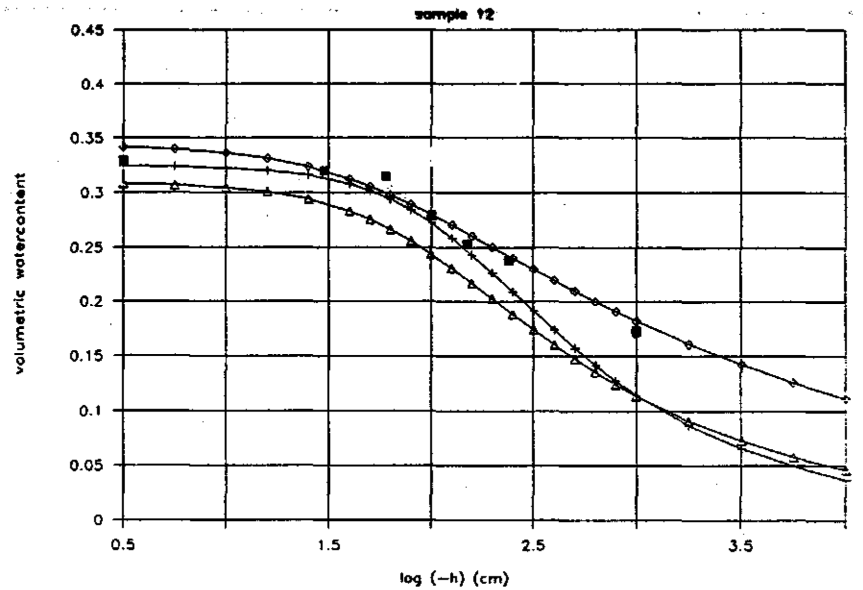

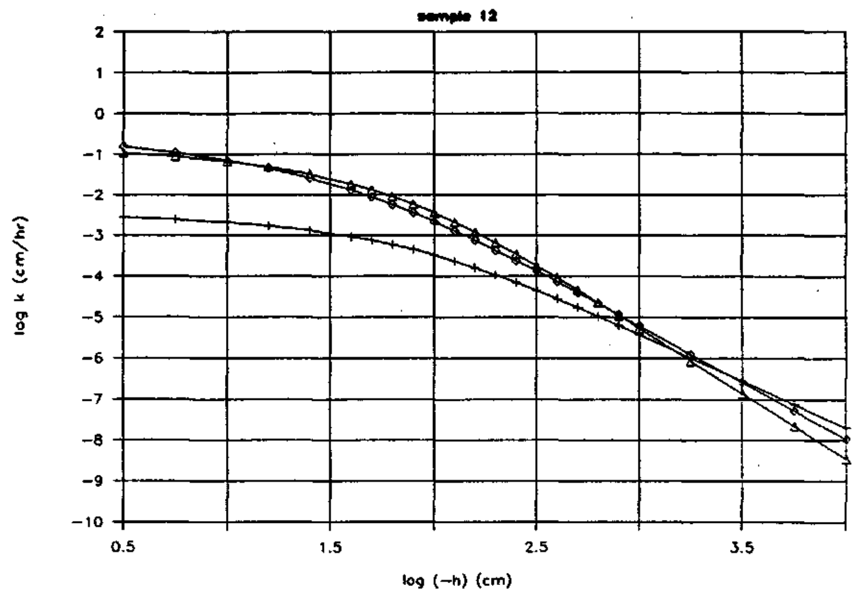

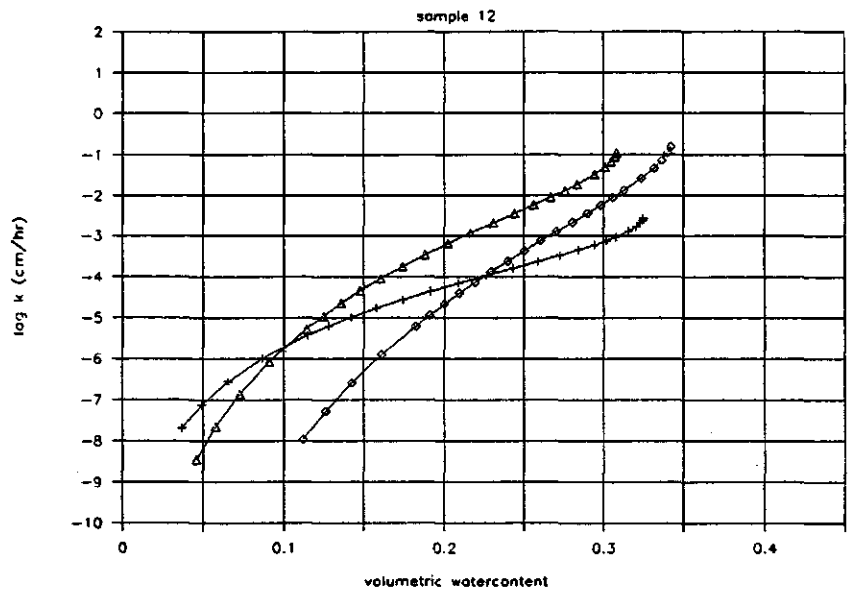

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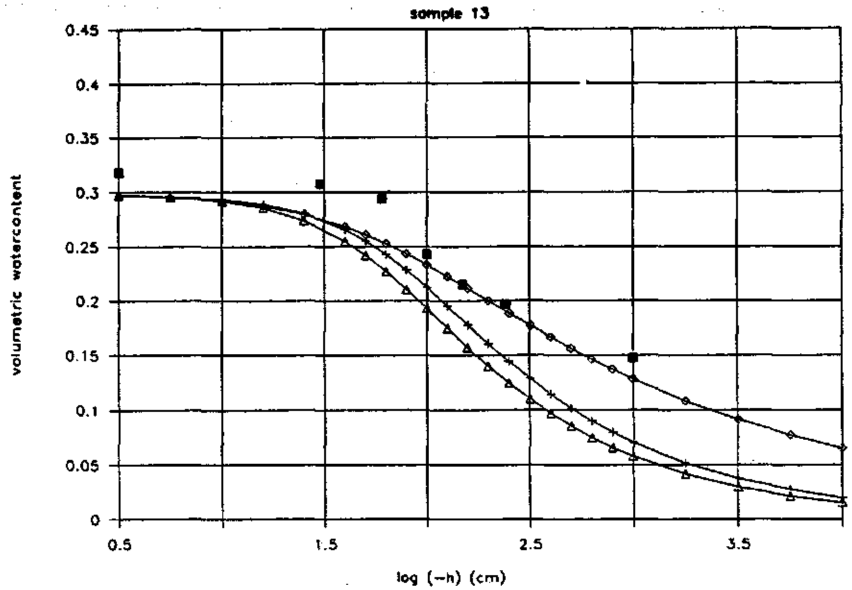

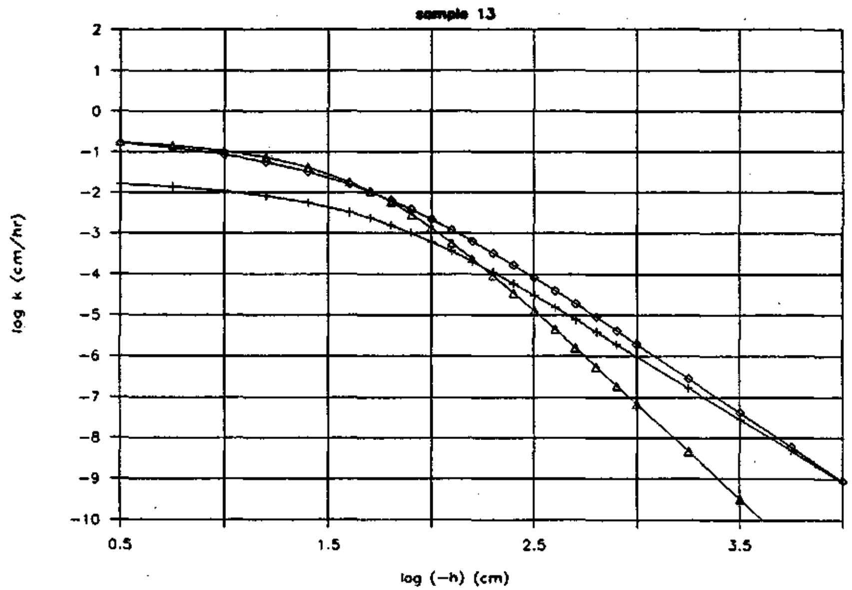

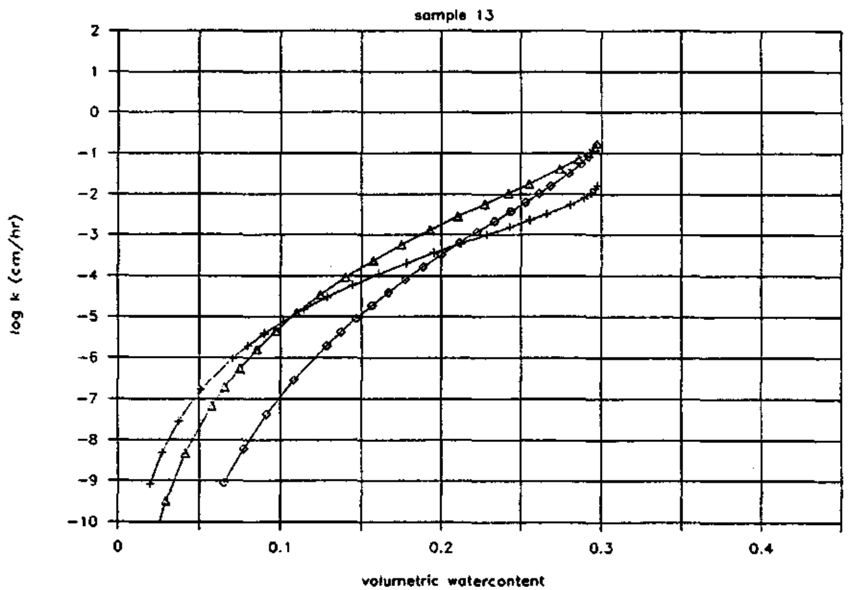

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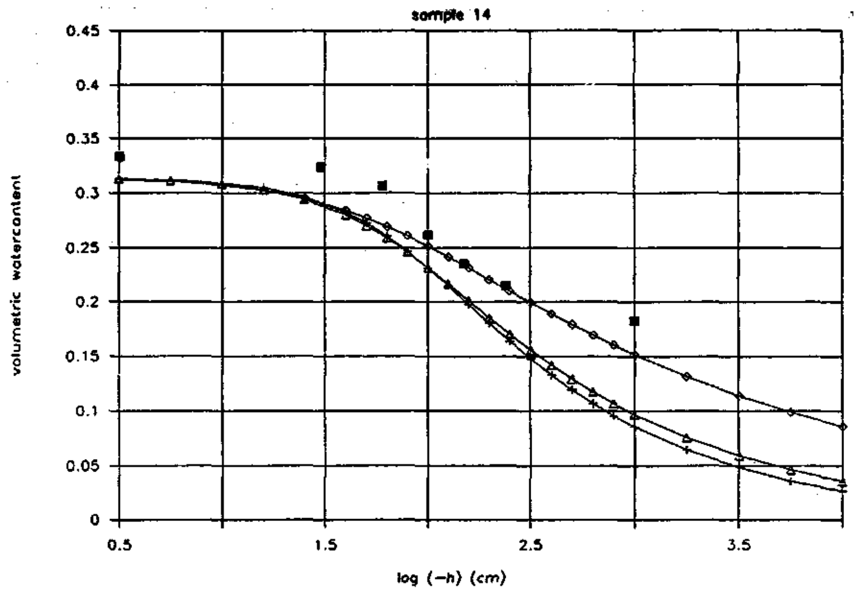


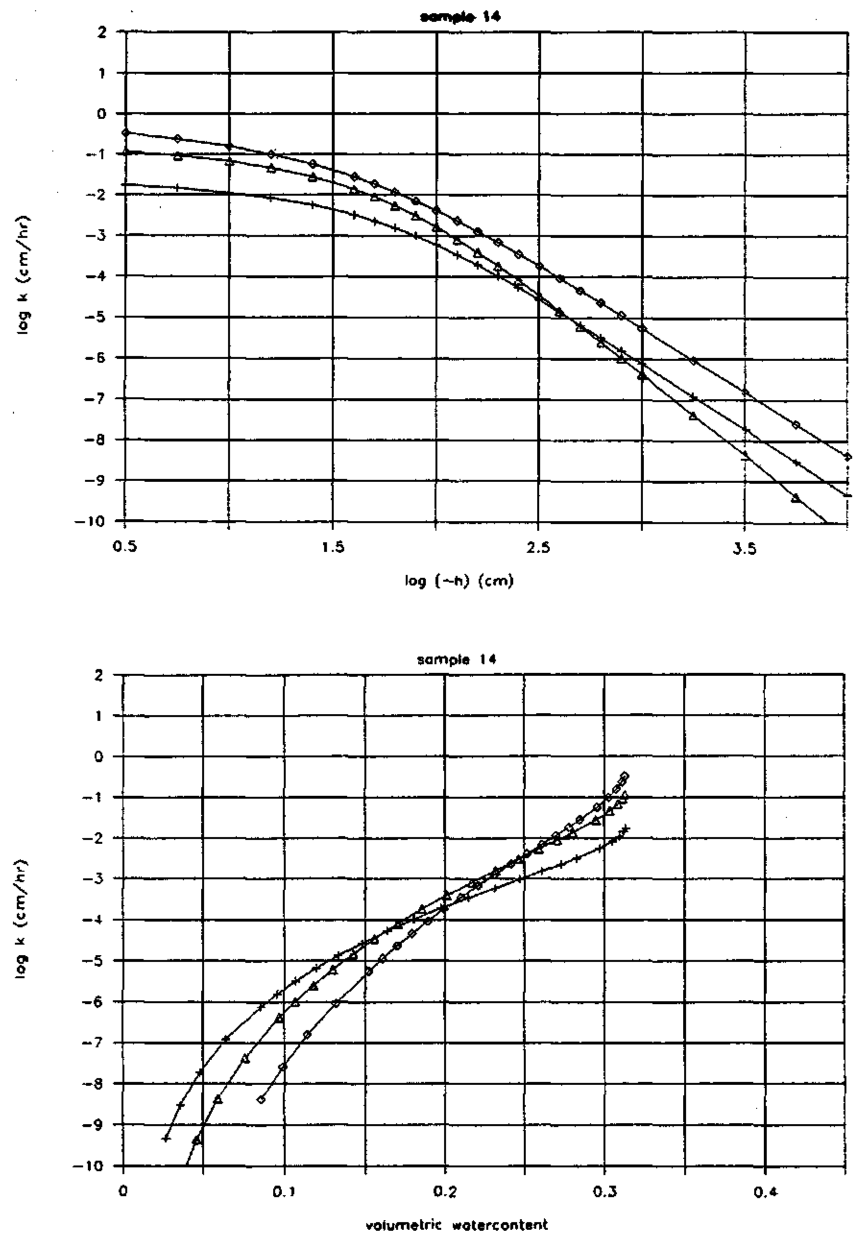
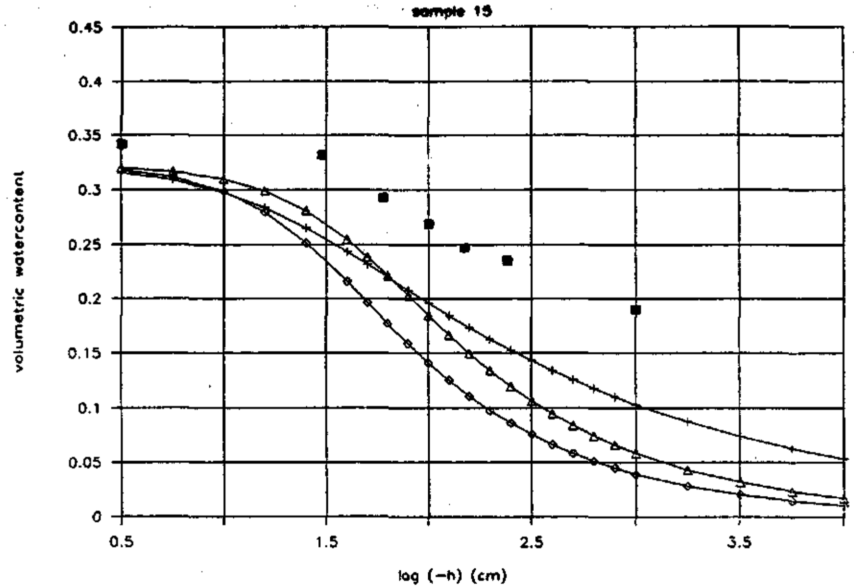
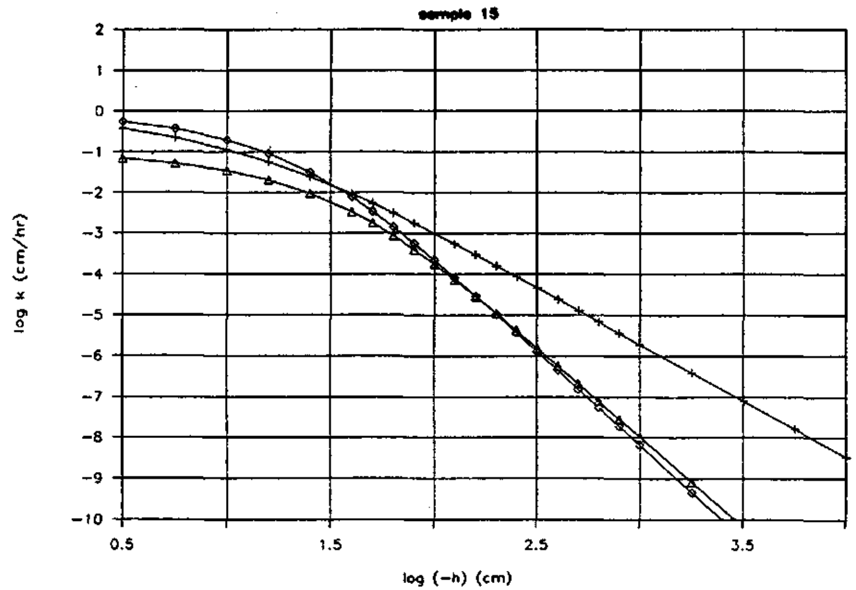
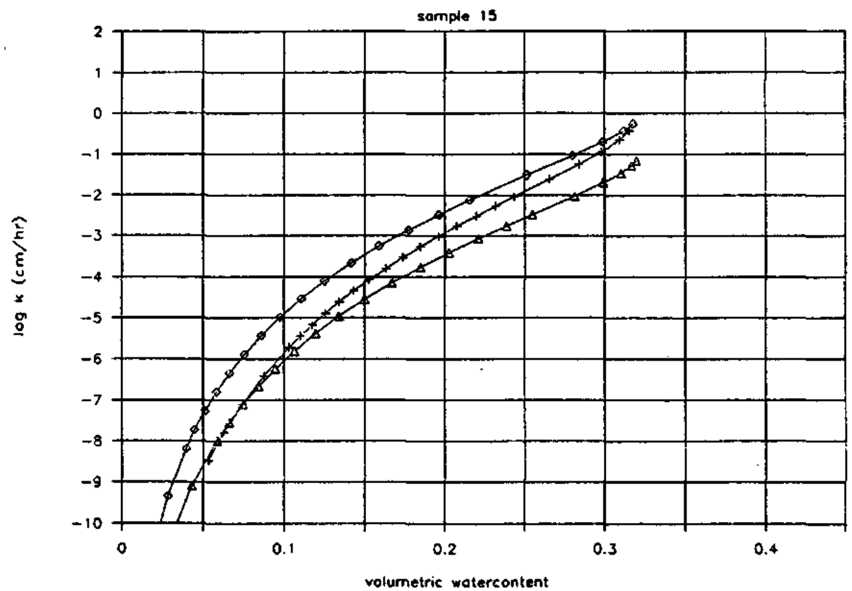


Figure 19. Optimized soil hydraulic functions, using $Q(t)$ data in the objectfunction, in comparison with $\theta(h)$ data (■) derived from outflow experiments with daily pressure increase. Three different time schedules were applied on the same sample :

◇ At once △ Per hour + Per day







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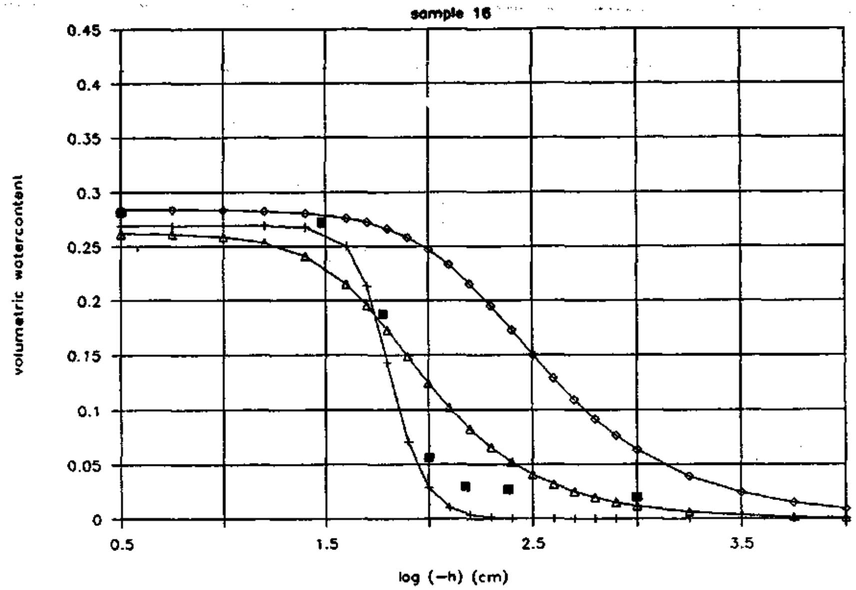

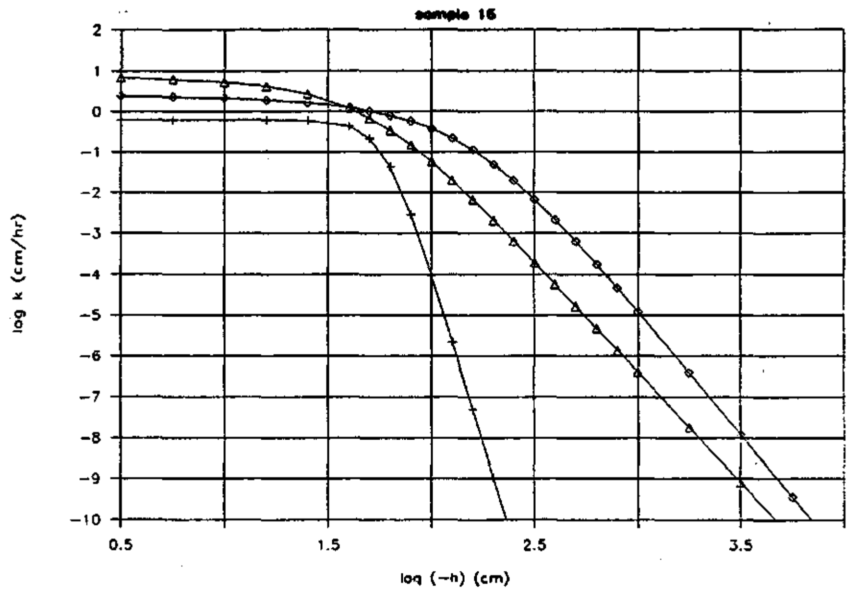

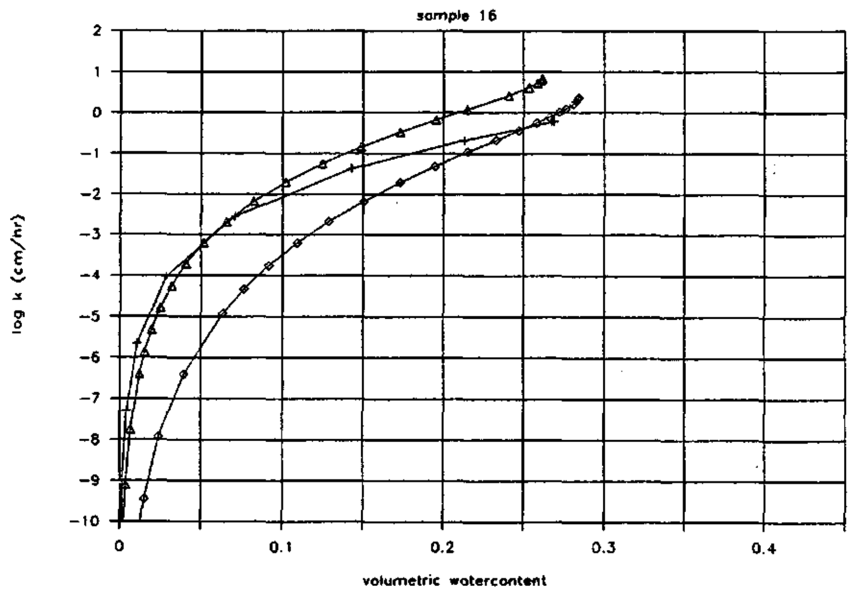

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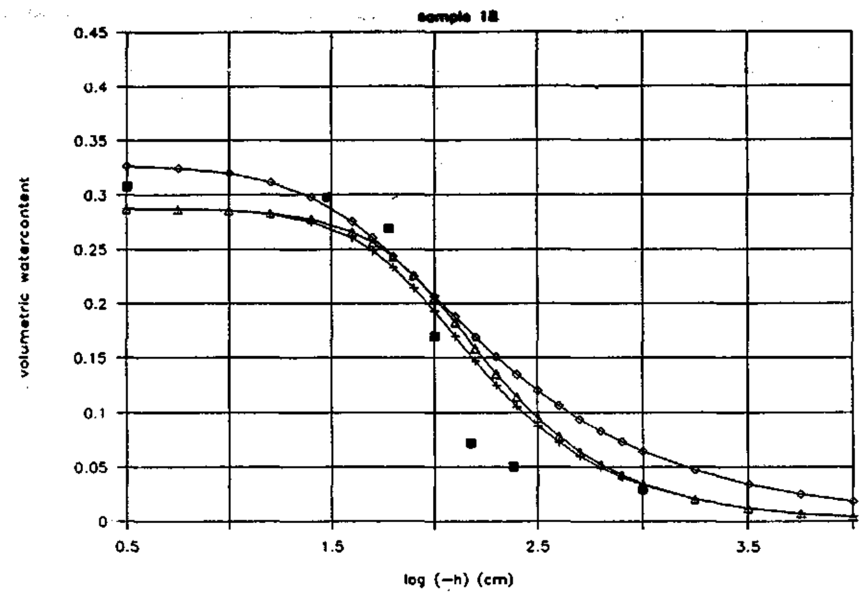

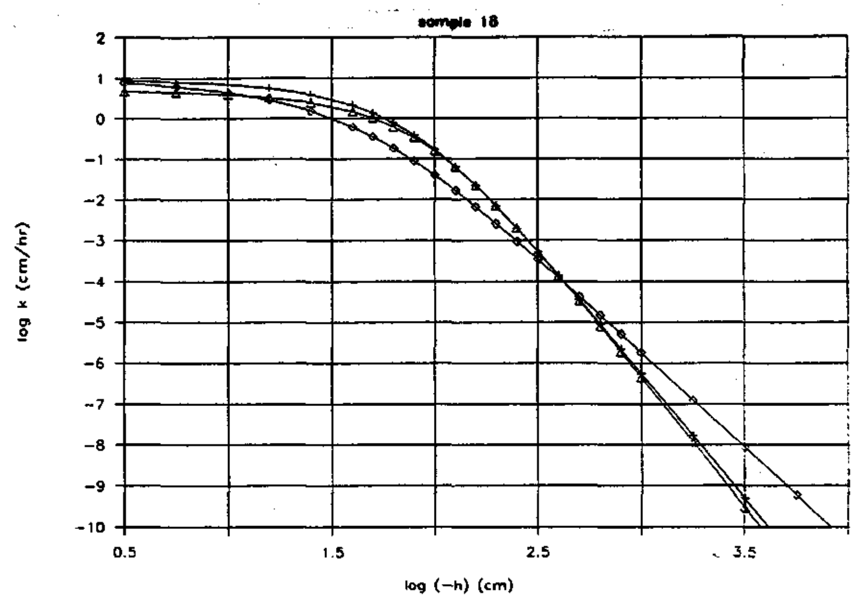

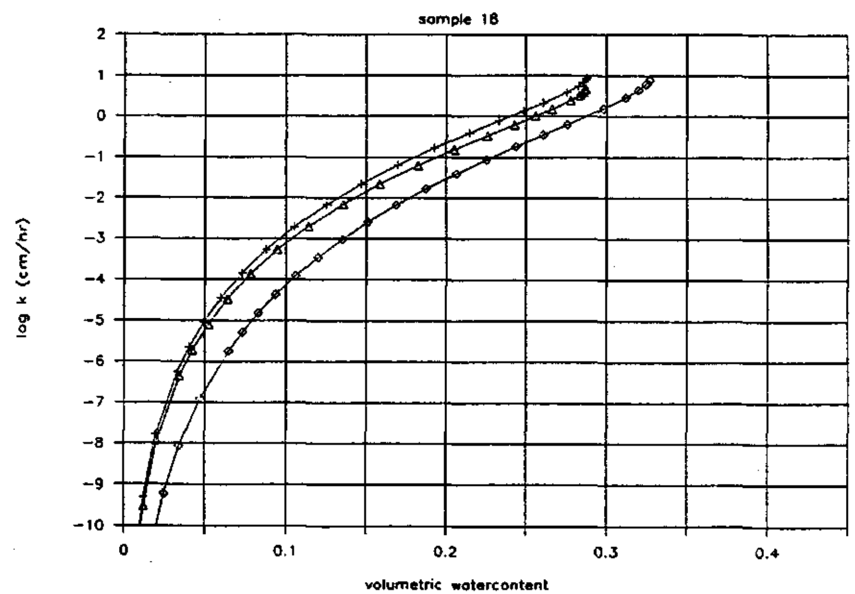

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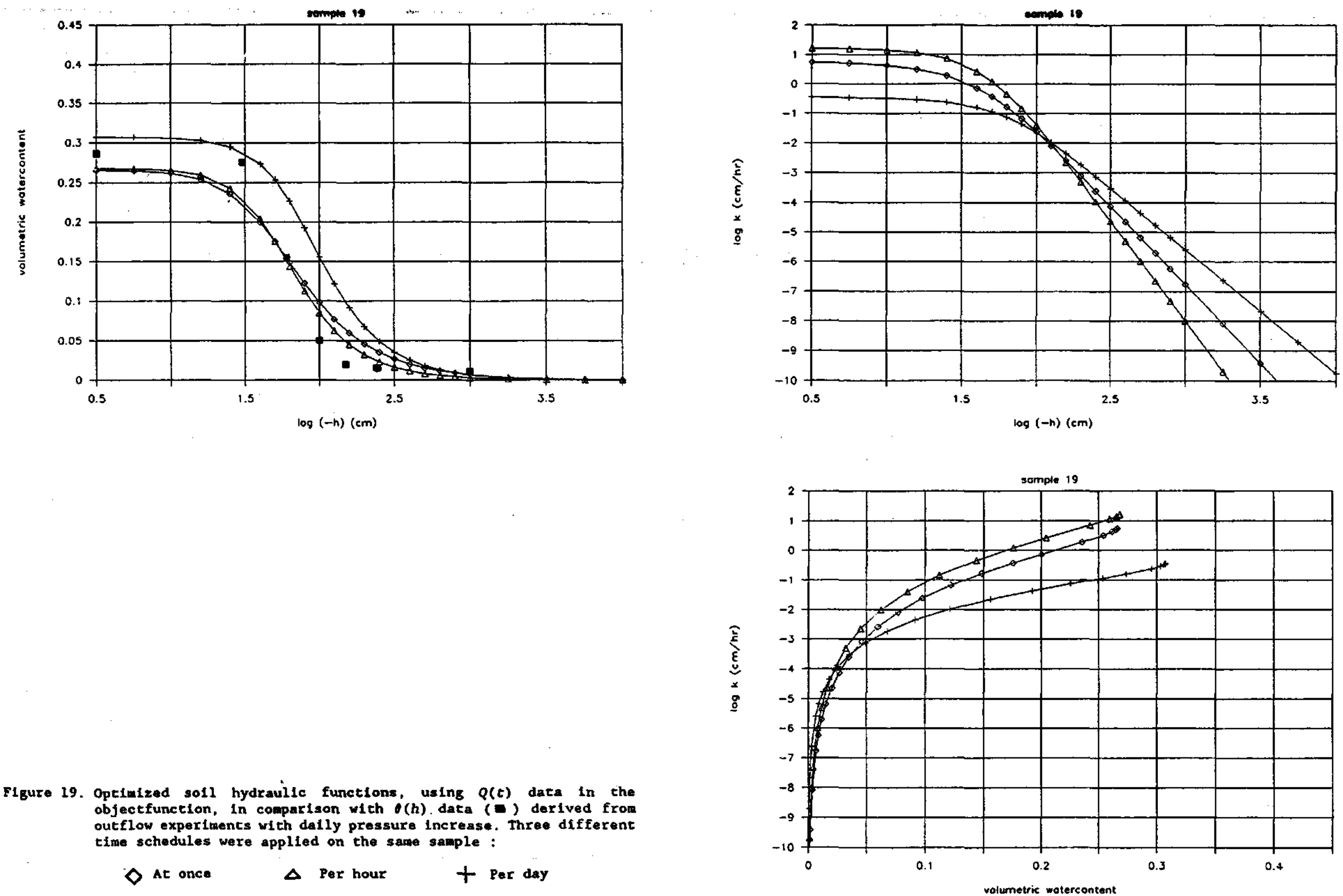

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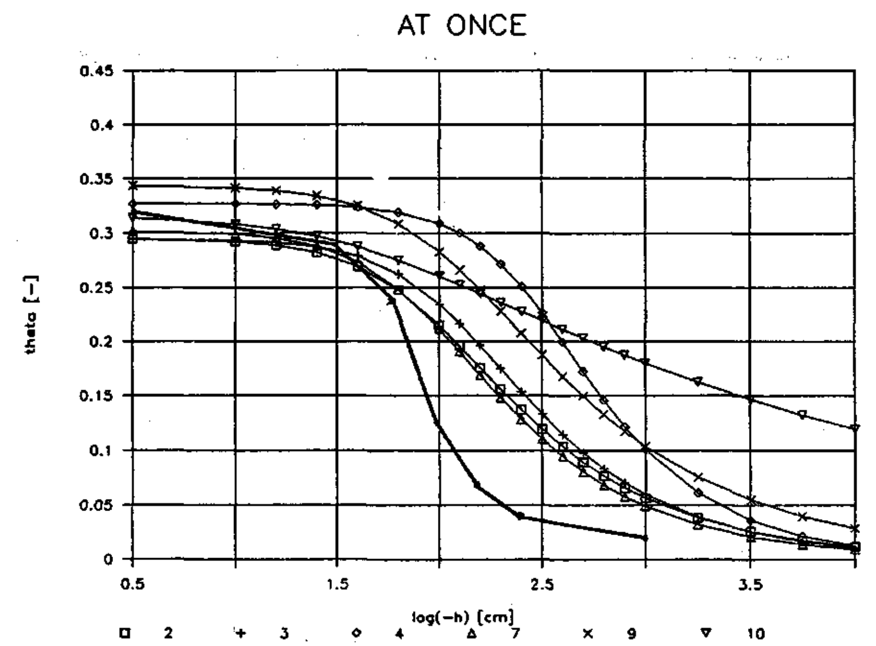
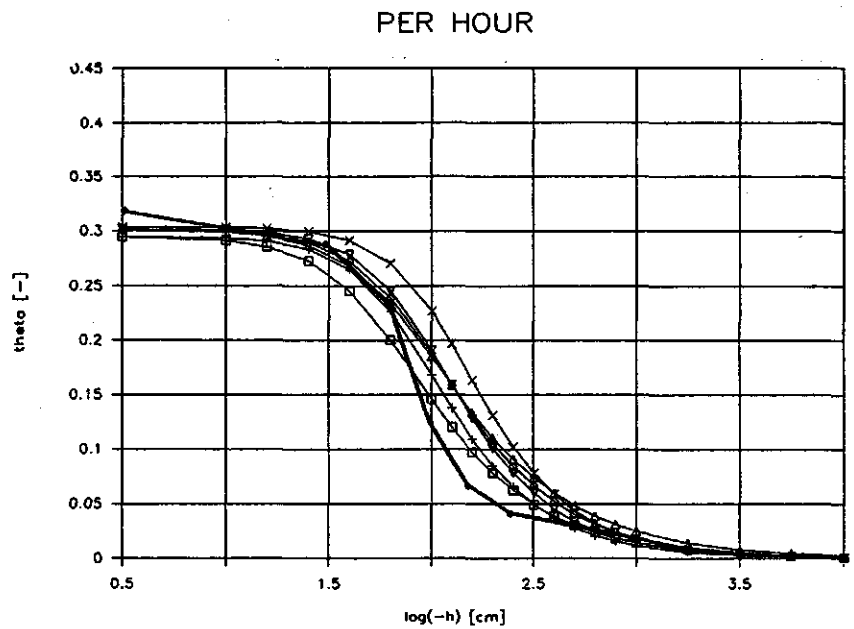




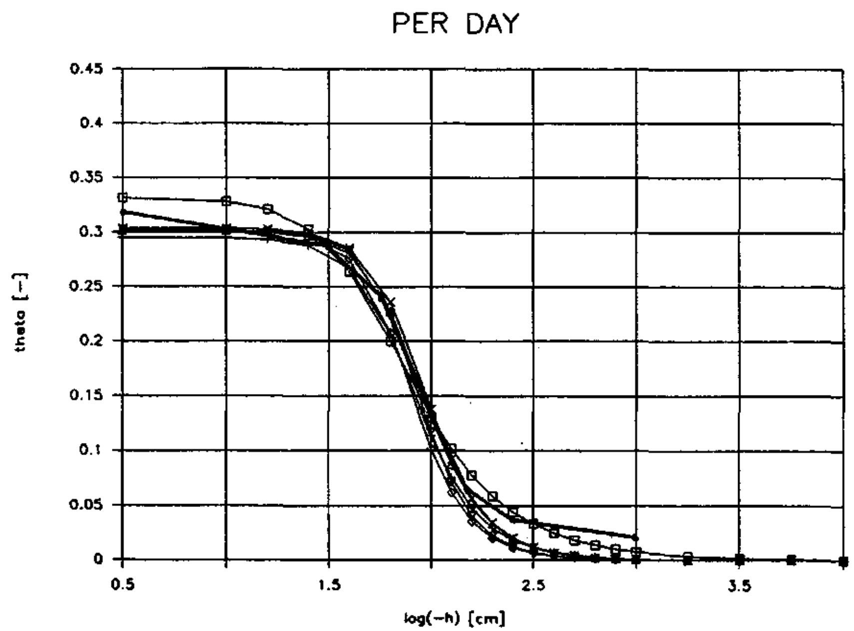
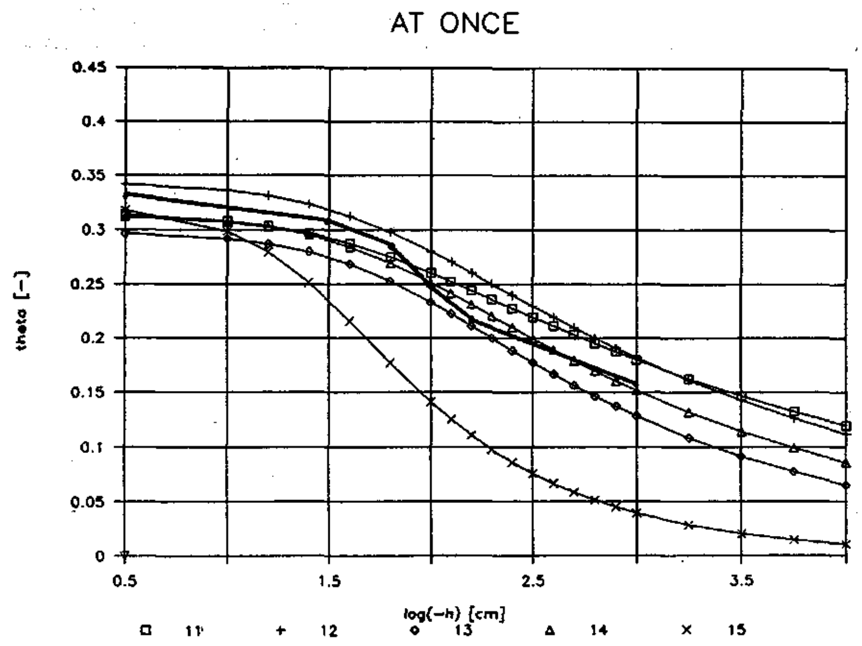
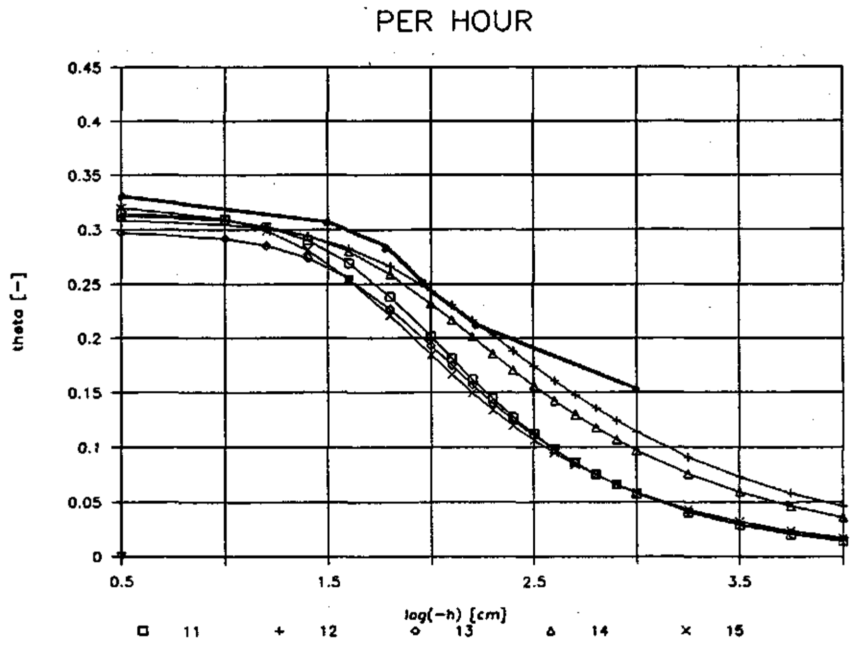


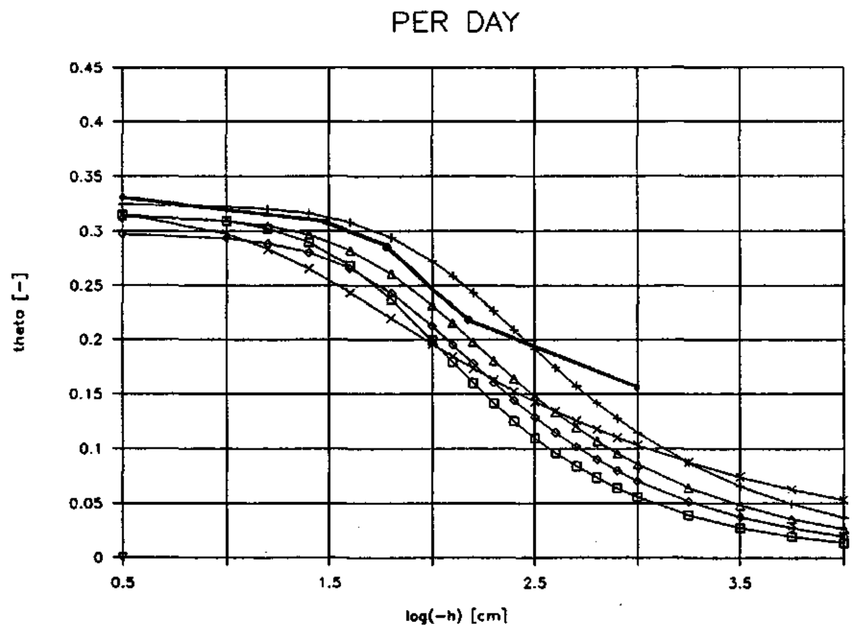
Figure 20. Estimated $\theta(h)$ functions in comparison with $\theta(h)$ data obtained after one day of equilibration. Different time schedules for pressure increments are used on the same samples. *At once* shows the result of the One-step outflow experiment. In case of Multi-step, 6 steps were applied, each giving approx. the same amount of outflow. *Per hour* and *Per day* refer to the time between the pressure increments.

—◆—◆—◆— Mean of $\theta(h)$ data from daily pressure increments.

□ + ◇ △ × ▽ Optimized $\theta(h)$ for the different samples. The numbers refer to Table 10.







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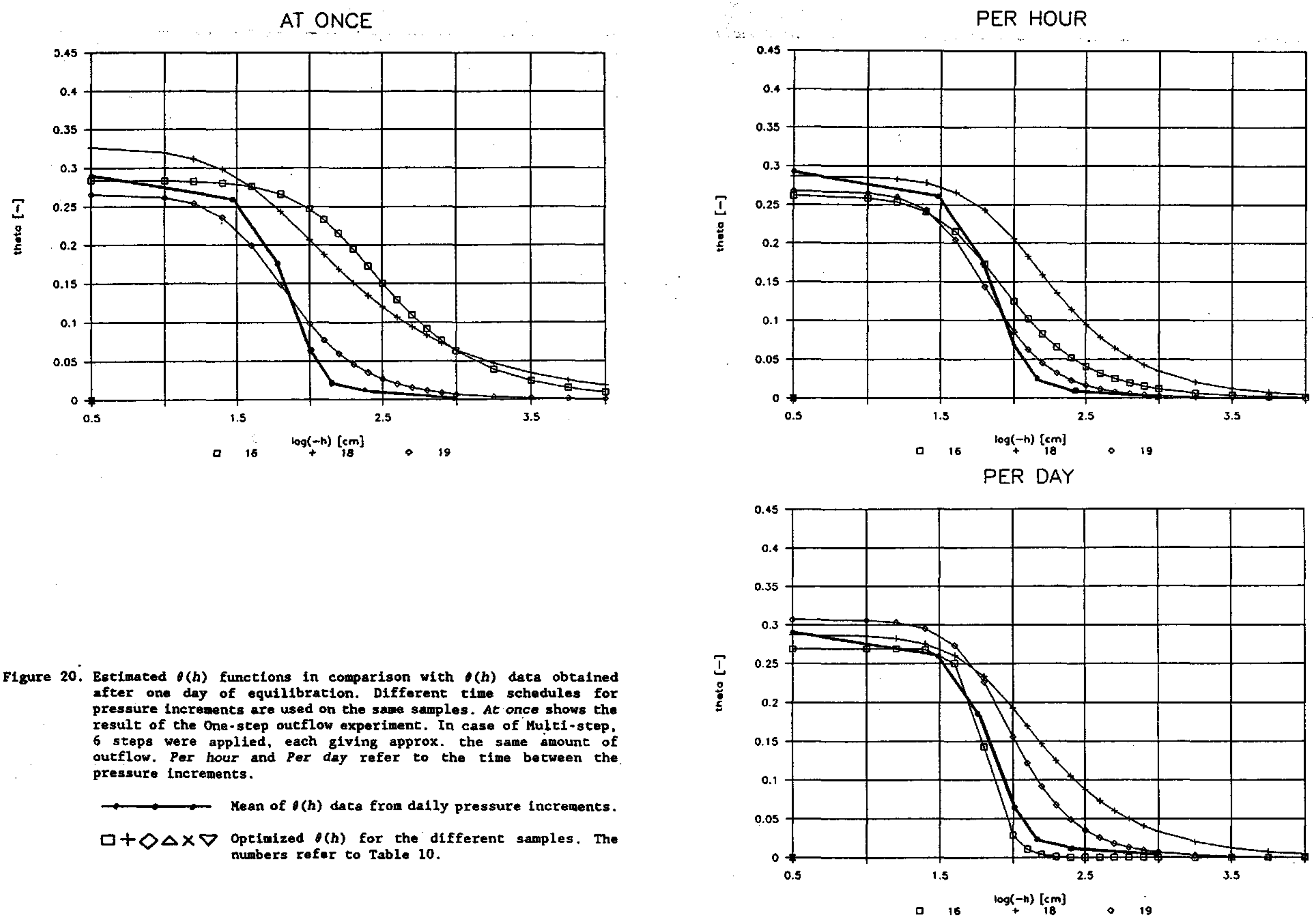
—◆—◆—◆— Mean of $\theta(h)$ data from daily pressure increments.

□ + ◇ △ × ▽ Optimized $\theta(h)$ for the different samples. The numbers refer to Table 10.



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Mean of $\theta(h)$ data from daily pressure increments.

□ + ◇ △ × ▽ Optimized $\theta(h)$ for the different samples. The numbers refer to Table 10.