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CONVECTIVE TRANSPORT OF SOLUTES BY STEADY FLOWS
II. SPECIFIC FLOW PROBLEMS

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ABSTRACT

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A comprehensive theory describing convective transport of solutes was presented in Part I. In this paper the general theory is applied to specific flow problems. The relatively simple problem of leaching to drains and ditches induced by an input distributed uniformly over the surface is discussed in detail. It is shown that if the ratio of the halfspacing between the drains or ditches and the depth to the impermeable layer is larger than about five, then the system approximates an apparently well-mixed system, i.e., then the transit time density distribution is approximately exponential. The general theory is also used to evaluate the literature on many other problems.

INTRODUCTION

The transfer function approach to the transport of solutes, that was introduced in Part I (Raats, 1978) was inspired by analogous treatments of certain industrial processes by Danckwerts (1953) and of tracers in hydrological systems by Eriksson (1961, 1971). These authors treated their respective systems largely as black boxes, paying not much attention to the internal dynamics. Typically, they regarded functions analogous to the transit time density distribution function, $dq/d\tau$, as probability density distributions. The stream tube model developed in I provides a simple, physical interpretation of all concepts. It also allows for an explicit treatment of the initial distribution of a solute over the region, of the distribution of fluxes of water and solutes over the input surface, of desaturation, and of anisotropy of the soil.

In this paper I discuss applications of the theory to specific flow systems. The leaching to drains and ditches will be the primary interest.

The idea to develop a comprehensive theory of convective transport of solutes based upon flow patterns originated during a series of seminars given jointly with W.R. Gardner at the University of Wisconsin in the fall of 1972.

Specifically, it was the demonstration by Gardner of the simple dependence of transit time upon water content, hydraulic conductivity, distance, and head difference for the one-dimensional case (cf. the finite difference equation (1.22) of Part I) that induced me to study the multidimensional problem. A year later I presented a brief outline of the "material approach" at the Annual Meeting of the American Society of Agronomy. Most of the details were worked out while I was on the staff of the U.S. Salinity Laboratory at Riverside, Calif. There I showed that following parcels of solute is also useful in describing the transport of solutes across the rootzone (Raats, 1975, 1977). By focussing upon the spatially distributed uptake of water and the flow pattern, one can formulate a detailed description of the leaching process. Of course, the simple, integral concept of a leaching fraction (U.S. Salinity Laboratory Staff, 1954) fits naturally in the new framework. At Riverside, J. Letey and J. van Schilfgaarde provided stimuli to look into the fate of periodic inputs (see Section 4 of Part I and Section 7 of this paper, see also Jury, 1975 a and b) and anisotropic media (see Section 5 of Part I), respectively. After joining the Institute for Soil Fertility at Haren (Gr.), The Netherlands, I learned from Th.J. Ferrari that the convective transport of solutes to drains and ditches was discussed at that Institute in December 1972 by L.F. Ernst from the Institute of Land and Water Management Research at Wageningen, The Netherlands. Unfortunately, Ernst's contribution is available only as an internal note of his Institute, written in Dutch (Ernst, 1973). I thank Ernst for allowing me to include the outline of his work in Section 8.2.

6. PISTON FLOW SYSTEMS

If the inflow and outflow surfaces are parallel planes, concentric cylinders, or concentric spheres, then along every streamline the transit time will be equal to the turnover time. Such systems will be called *piston flow systems*. On the basis of Eq. (2.2) and Darcy's law, Muskat (1934, 1946) calculated successive isochrones for linear, two-dimensional radial, and spherical displacement. In these problems, the forms of the isochrones are obvious and only the associated residence time needs to be determined. For a piston flow system, water introduced at time t_0 will appear in the output at time $t_0 + \bar{\tau}$, where $\bar{\tau}$ is the turn-over time defined earlier. For any flow system, the amount of water, η, that was applied to the actual system after time t_0, and has already left the system at time $t_0 + \bar{\tau}$ is equal to the amount of water that was present at time t_0, and has not yet left the system at time $t_0 + \bar{\tau}$ (equality of areas bij and jek in Fig.2 of I):

$$\eta = \int_0^{\bar{\tau}} q \, \mathrm{d}\tau = \int_{\bar{\tau}}^{\infty} (1 - q) \mathrm{d}\tau. \tag{6.1}$$

Following Danckwerts (1953), I will call this quantity η the hold-up of the flow system. The hold-up is a measure of the deviation of the actual system from a piston flow system.

7. APPARENTLY WELL-MIXED SYSTEMS

7.1 Analysis

Consider the flow in a saturated soil with porosity θ in the rectangular region shown in Fig.4, with the y-axis and the line $y = Y$ being a bounding

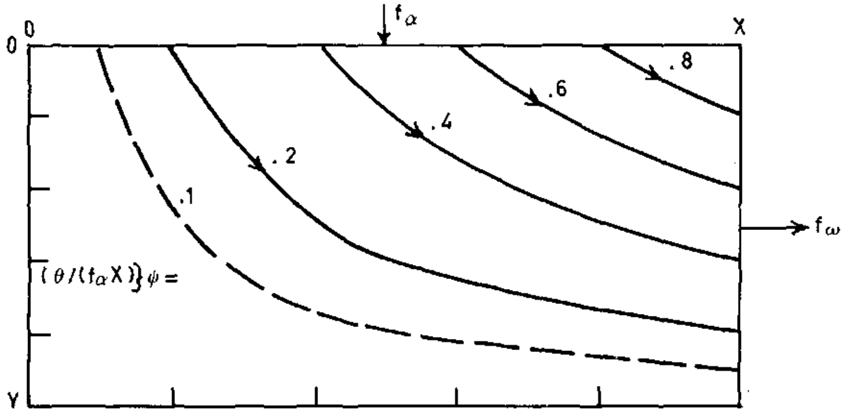

Fig.4. Plane flow pattern with uniform input and uniform output.

streamline, a uniform input f_α along the surface $y = 0$, and a uniform output $f_\omega = f_\alpha X/Y$ along the surface $x = X$. This flow is a good approximation for steady flow to ditches or tile drains, if the water table is flat and the distance $2X$ between the ditches or tile drains is large relative to the depth, Y, of the impermeable layer. The turnover time, $\bar{\tau}$, for the water in this region is equal to the volume of water in the system, θXY, divided by the rate of input, $f_\alpha X$:

$$\bar{\tau} = \frac{\theta Y}{f_\alpha}. \tag{7.1}$$

The total potential function, φ, and the stream function, ψ, given by

$$\varphi = (2\bar{\tau})^{-1}\{(Y-y)^2 - x^2\}, \tag{7.2}$$

and

$$\psi = \bar{\tau}^{-1}\, x\,(Y-y), \tag{7.3}$$

satisfy the boundary conditions. The flow pattern is shown in Fig.4. The streamlines are rectangular hyperbolas that have the y-axis and the line $y = Y$ as asymptotes. The equipotential lines are also hyperbolas for which the y-axis and the line $y = Y$ are axes of symmetry.

The components of the velocity vector are given by

$$v_x = \frac{\partial x}{\partial t}\Big|_X = -\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y} = x/\bar{\tau}, \tag{7.4}$$

$$v_y = \frac{\partial y}{\partial t}\Big|_X = -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x} = (Y-y)/\bar{\tau}. \tag{7.5}$$

Eqs. (7.4) and (7.5) describe a very simple variation of the components of the velocity over the flow region. The vertical component of the velocity v_y, is uniform over any horizontal cross section and varies linearly from a value $f_\alpha/\theta = y/\bar{\tau}$ at the input surface located at $y = 0$ to zero at the impermeable base located at $y = Y$. The horizontal component of the velocity, v_x, is uniform over any vertical cross section and varies linearly from the plane of symmetry located at $x = 0$ to $(f_\alpha X)/(\theta Y) = x/\bar{\tau}$, at the output surface located at $x = X$. Integration of (7.4) and (7.5) gives

$$x = x_0 \exp(t - t_0)/\bar{\tau}, \tag{7.6}$$

$$y = Y - (Y - y_0) \exp -(t - t_0)/\bar{\tau}, \tag{7.7}$$

where x and y are the coordinates of a particle at time t whose coordinates at time t_0 were x_0 and y_0. Eq.(7.6) shows that, as a result of the uniformity of the horizontal component of the velocity over any vertical cross section, material surfaces that are vertical will remain so in the course of time. Similarly, Eq.(7.7) shows that material surfaces that are horizontal will remain so in the course of time. In other words, all horizontal planes form a family of isochrones with all parcels in each horizontal plane having the same residence time, τ_α, and all vertical planes form a family of isochrones with all parcels in each vertical plane having the same residual transit time, τ_ω (Fig.5).

The transit time, τ, for a parcel introduced into the system at $x = x_0$ is found by setting $x = X$ and $t - t_0 = \tau$ in (7.6).

$$\tau = \bar{\tau} \ln X/x_0 \tag{7.8}$$

Since parcels introduced near $x = X$ are the first to appear in the output, the cumulative transit time distribution function, q, is given by

$$q = 1 - x_0/X = 1 - \exp -\tau/\bar{\tau}. \tag{7.9}$$

Differentiation of (7.9) with respect to τ gives the transit time density distribution, $dq/d\tau$:

$$dq/d\tau = \bar{\tau}^{-1} \exp \tau/\bar{\tau}. \tag{7.10}$$

Eq.(7.10) is identical to the well-known transit time density distribution of a well-mixed system of volume θXY and rate of input $f_\alpha X$. Of course, being based on piston displacement, the present model is as far removed from a well-mixed model as is possible. The transit-time density distribution is

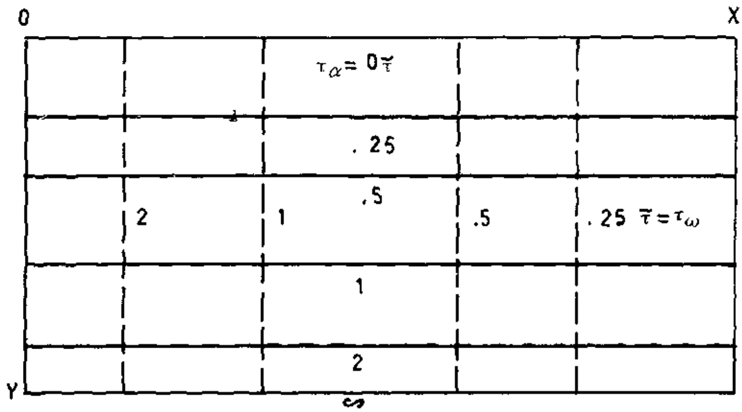

Fig.5. Isochrones with equal residence time (full lines) and with equal residual transit time (broken lines).

entirely dictated by the flow pattern given by Eq.(7.3). To assess the deviation of an actual system from a well-mixed system Danckwerts introduced the segregation, S, defined by

$$S = \int_0^T |q_{\text{actual system}} - q_{\text{well-mixed system}}| \, d\tau. \tag{7.11}$$

In the present context, these deviations are the result of transit time density distributions associated with particular geometries and boundary conditions; Danckwerts considered many other factors influencing the distributions.

Introducing (7.9) into (3.1), (3.2), and (3.4) and (3.5) shows that the bypass, B, the displacement, D, the overall leaching efficiency, E_a, and the marginal leaching efficiency, E_m, are given by

$$B = F\tau - F\bar{\tau}\{1 - \exp(-\tau/\bar{\tau})\}, \tag{7.12}$$

$$D = F\bar{\tau}\{1 - \exp(-\tau/\bar{\tau})\}, \tag{7.13}$$

$$E_a = \{1 - \exp(-\tau/\bar{\tau})\}\bar{\tau}/\tau, \tag{7.14}$$

$$E_m = \exp(-\tau/\bar{\tau}). \tag{7.15}$$

Consider a uniform, periodic input of solute dissolved in the water, i.e., an input whose input density distribution $r[q]$ is unity and whose time dependence is described by (4.15). The response to such an input by a system with a transit-time density distribution given by (7.10) is given by (4.16) with the amplitude a and the phase shift b given by

$$a = a_0(1 + \Omega^2\bar{\tau}^2)^{-1/2}, \tag{7.16}$$

$$b = \tan^{-1}\Omega\bar{\tau}. \tag{7.17}$$

Plots of a/a_0 and b, as functions of $\Omega\bar{\tau}$, are shown in Fig.6. If $\theta = 0.4$, $Y = 10$

m, and $q = 0.5$ m/year then $\bar{\tau} = 8$ year. If the input of the water is distributed uniformly over the year, then for annual variations of the input of salinity, the amplitude of the output is about one-eighth of that of the input, and the phase shift is 82.9°, or nearly 3 months.

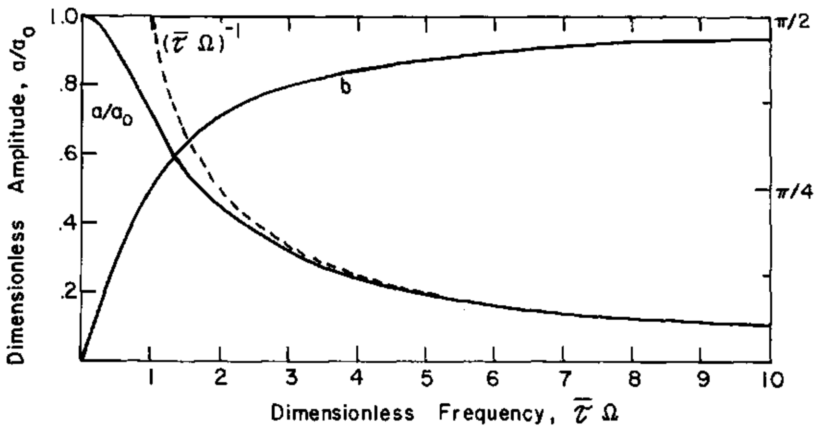

Fig.6. Response to periodic input of solute in plane flow with uniform input and uniform output.

The main conclusions of the above analysis of shallow system leaching to drains and ditches induced by an input distributed uniformly over the surface are that (1) all horizontal planes form a family of isochrones with all parcels in each horizontal plane having the same residence time, τ_α, (2) all vertical planes form a family of isochrones with all parcels in each vertical plane having the same residual transit time, τ_ω, and (3) the transit time density distribution is the same as that of a well-mixed system, i.e., it is exponential. The latter conclusion is also reached on the basis of a Dupuit-type of analysis (Raats, 1977). The key assumption is that at any distance from the drainage facility the horizontal component of the velocity is uniform over any vertical cross section and is given by:

$$v = (\theta A)^{-1} \int_0^x f_\alpha \, \mathrm{d}x, \tag{7.18}$$

where A is the cross-sectional area at x, and f_α is the rate of input per unit distance in the x-direction. Introducing (7.18) into (1.5) and setting $t - t_0 = \tau$ gives the general expression for the transit time from $x = x_0$ to $x = X$:

$$\tau = \int_{x_0}^{X} \theta A \left\{ \int_0^x f_\alpha \, \mathrm{d}x \right\}^{-1} \mathrm{d}x. \tag{7.19}$$

The transit time depends on the distribution of θ and A between x_0 and X and on the distribution of f_α over the entire interval 0 to X. If A is equal to a constant depth Y times unit thickness and θ and f_α are constants then (7.19) reduces to (7.8) from which in turn follow Eqs (7.9) to (7.17).

7.2 Evaluation of earlier work

Eriksson (1958) speculated that transport of uniform inputs of salt and water draining through a soil mass whose hydraulic conductivity decreases with depth would lead to an exponential distribution of arrival times. This model was used later without further justification in theoretical discussions of input/output relationships (Eriksson, 1961, 1971; Nir, 1964; Bolin and Rodhe, 1973; Nir and Lewis, 1975), and in attempts to fit experimental data (Eriksson, 1963; Peck and Hurle, 1973; Peck, 1973). The analysis in Section 7.1 provides a more explicit basis for Eriksson's speculation. Ericksson (1961, 1971) gave a detailed discussion of the frequency response of the well-mixed system, including the results (7.16) and (7.17) above.

The problem discussed in detail in Section 7.1 was also considered by Eldor and Dagan (1972). They gave expressions for the potential and stream functions, the velocity modulus, the location at successive times of a salinity front introduced at the soil surface, and an expression for the flushing efficiency. Unfortunately, what they identified as the cumulative inflow is only 50% of the cumulative inflow. The factors 1/2 and 2 in their Eqs. (60) and (62) compensate for this. The values of t in their Figs. 9a and 9b, and of $V^w(t)/V^w_{\text{total}}$ in Fig. 9c should be multiplied by two. These errors are perhaps the reason for their not recognizing that the response of the system is equivalent to that of a perfectly mixed system.

Eldor and Dagan (1972) have shown that diffusion and dispersion have only a very minor influence on the transit of a pulse of salts. This is not surprising since diffusion and dispersion will only have a very small influence on the distribution of the solute in the horizontal direction. Of course, if in Eq.(11) the period Ω^{-1} is much smaller than the turnover time $\bar{\tau}$, the response of the system will depend more on diffusion and dispersion.

Ernst (1973) arrived at the logarithmic Eq. (7.8) for the transit times by assuming a parabolic water table and introducing the resulting gradient of the total head into (1.17) of paper I. This is equivalent to the reduction, indicated earlier, of (7.19) to (7.8) on the basis of A being equal to a constant depth Y times unit thickness and of θ and f_α being constants. Ernst used complex variables to determine the plane flow pattern shown in Fig.2 and to show that the associated travel times are given by (7.8). He did not explicitly state any of the implications of Eq.(7.8) expressed in Eqs. (7.9) to (7.17).

In the analysis leading to the logarithmic Eq.(7.8) both the input and the output were assumed to be uniformly distributed over the input and output surfaces, respectively. Whereas such an input may result from rainfall or sprinkler irrigation, the output into ditches or tile drains will generally result in a very nonuniform distribution of the output over the surface $x = X$. Ernst (1973) proposed two different approximations that can be used to account for convergence to the drain. The first approximation is based on the assumpflow from a line source at constant total head to a point sink, and a plane flow from a point source to a point sink. For each of these problems, an explicit expression for $|\nabla H|^{-2}$ in terms of the total head, H, and the plane stream function, ψ, can be derived and (1.18) can be used to calculate the distribution of the travel times. The plane flow from a line source to a point sink represents flow of ponded water into a single drain or to a single well along a river. Muskat showed that for this problem the breakthrough time $\tau_{\min}$ is $2\pi\theta h^2/(3F)$, where h is the depth of the drain or the distance from the well to the river, and that the corresponding displacement D at breakthrough is $2\pi h^2/3$, i.e., that the leaching extends on the average to a distance of $\pi h/3$ on either side of the drain. This clearly indicates that for wide spacings the leaching process is not efficient.

The isochrones for plane flow from a point source to a point sink, calculated by Muskat, may be regarded as an example of a transformation of τ_α to $\tau_\omega = \tau - \tau_\alpha$, where the τ are the transit times between the source and the midplane of the source/sink pair. Bear and Jacobs (1965) calculated isochrones for plane flow to and from a well in a uniform flow field. They showed that in this case the coordinate in the direction of the uniform flow could be expressed in terms of the coordinate perpendicular to the flow and the stream function, so that (1.14) could be used to calculate the travel time distribution. An interesting feature of this problem is that the isochrones corresponding to large values of τ_α, respectively τ_ω, approach the divide between the uniform flow and the flow from the source, respectively into the sink. For a point source, a similar technique cannot be used, and Bear and Jacobs had to resort to a numerical solution.

Luthin et al. (1969) were the first to calculate travel times for an agricultural drainage system. They in essence used (1.22) to determine the distribution of travel times for flow of ponded water to drains. They used flow nets for certain drain radii, drain spacings, and locations of the drains relative to the distance between the soil surface and an impermeable layer, as calculated from an analytical solution given by Kirkham (1957). They presented their results in the form of isochrones of equal residence time τ_α. The agreement with observations on an interface between water with 10 000 ppm NaCl and ponded tap water in large sand tanks was only fair. The observed concentrations in the effluent were generally larger than the calculated concentrations. The discrepancy is possibly due to the large difference of the densities of the two fluids.

Ortiz and Luthin (1970) calculated travel time distributions along a streamline in an anisotropic soil directly from an expression for the magnitude of the velocity:

$$v = -(k_s/\theta)\, \partial H/\partial s, \qquad (8.1)$$

where k_s is the reciprocal resistivity scalar defined by

$$k_s = (\tau \cdot \mathrm{R}\tau)^{-1}, \qquad (8.2)$$

where **R** is the resistivity tensor. Introducing (8.1) into (1.5) gives

$$t - t_0 = -\theta \int_{s_0}^{s} (k_s\, \partial H/\partial s)^{-1}\, ds. \tag{8.3}$$

Ortiz and Luthin used a finite difference version of (8.3) in their study of movement of salts to tile drains in anisotropic soils with water ponded on the surface. The calculations are elaborate since even in homogeneous media, the reciprocal resistivity scalar, k_s, depends at any point along a streamline upon the direction of the streamline. The method outlined in Section 5 of I is much simpler.

In Fig.8, the results of Luthin et al. (1969) and of Ortiz and Luthin (1970) are summarized as q vs. $\tau/\bar{\tau}$ curves for seven different combinations of half spacing/depth ratios, X/Y, and drain depth/depth ratios, D/Y. The exponential distribution that was discussed earlier is also shown. The results show that narrow spacings and large drain depths result in large breakthrough times. The X/Y given for curves 4 to 7 are those for the equivalent isotropic systems.

Miyamoto and Warrick (1974 a) derived an analytical solution for flow of

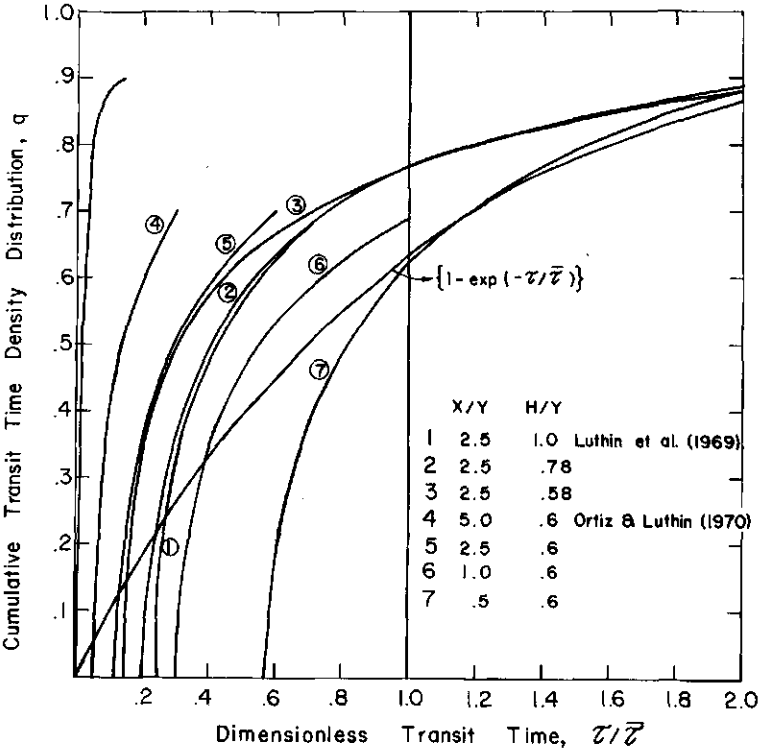

Fig.8. Transit time density distribution for ponded leaching and seven different geometries.

water ponded over the entire surface or over part of the surface to drains spaced equally at a certain depth in soils without an impermeable layer. They used a complex variable method to determine $|\nabla H|^{-1}$ as a function of the total head, H, and the plane stream function, ψ. The distribution of the travel times was calculated by introducing (1.18) into (2.5) and integrating numerically. The calculated isochrones, τ_α, agreed quite well with observations in a small tank filled with fine sand (Miyamoto and Warrick, 1974 a, Fig. 5). For five combinations of drain depth to drain spacing and of cover width to drain spacing, they also calculated the displacement D and the differential or marginal leaching efficiency $(1 - q)$. The latter agreed reasonably with the ratio of the measured concentration in the outflow to the initial concentration in the soil solution.

Miyamoto and Warrick (1974 b) did a similar analysis for water-filled ditches. For displacement into ditches, the isochrones advance much faster near the drains than in the region midway between them and a shallow impermeable sublayer accentuates this trend. With a wide, impermeable spoil bank near the ditch, the isochrones become nearly horizontal. For displacement by water moving from the ditches into a field, Miyamoto and Warrick found good agreement between calculated and observed isochrones.

Miyamoto et al. (1974 a), Warrick and Miyamoto (1974), and Miyamoto et al. (1974 b) used similar techniques to evaluate sorption patterns around pipes used for land disposal of waste gases. In that context, $\tau_{\min}$ is the time it takes for the gas front to reach the soil surface. To characterize any system, Miyamoto et al. introduced the ratio of the displacement at breakthrough, $F\tau_{\min}$, to the total volume of the system, $F\bar{\tau}$, as a uniformity parameter. This ratio is of course equal to $\tau_{\min}/\bar{\tau}$.

Zaslavsky and Levkovitch (1974) used a graphical technique, in essence based on (1.12), to determine isochrones for flow of ponded water to drains in soils with (like Luthin et al., 1969) and without an impermeable layer and for different depths and spacings of the drains. They used graphical analysis, electrical analogs, and Hele—Shaw models to obtain the flow patterns. The Hele—Shaw model also permitted direct observation of the isochrones. They defined quantities corresponding to the integral or overall leaching efficiency, E_a, the differential or marginal efficiency, $(1 - q)$, the breakthrough time, $\tau_{\min}$, and the displacement at breakthrough, $F\tau_{\min}$, and derived relationships between times in prototypes and models. They used the Hele—Shaw model to determine isochrones for flows with water ponded over only part of the surface. They also speculated about the potential of controlling the input distribution with sprinklers.

9. CONCLUDING COMMENTS

In view of the wide interest in convective transport of solutes, calculation of τ_α-isochrones and the function $q[\tau]$ should be a routine part of solving any steady flow problem. The theory presented in this paper shows the various ways such information can be used to describe leaching processes and effluent concentrations. The results can be easily extended to solutes experiencing adsorption and/or decay. Unsteady flows in saturated media can, in principle, be treated as successive steady flows. Elsewhere, I have treated convective transport of solutes in the presence of uptake of water by plant roots, assuming that all salts are excluded by the plant roots (Raats, 1975). Of great practical interest are the use of τ_α-isochrones and of the function $q[\tau]$ to summarize the effect of changing the geometry of the flow region and/or the boundary conditions, and of the input density distribution, $r[q]$, to compare alternative management practices for amendments.

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