NOTA 1021 januari 1978

Instituut voor Cultuurtechniek en Waterhuishouding
Wageningen

AN ELECTRONIC ANALOG FOR UNSATURATED FLOW AND ACCUMULATION OF MOISTURE IN SOILS

G.P. Wind* and A.N. Mazee**

* Institute of Land and Water Management Research (ICW), Wageningen, Netherlands

** Technical and Physical Engineering Research Service (TFDL), Wageningen, Netherlands

I N H O U D

ABSTRACT

An electronic analog model of the unsaturated zone was developed. The similarity between the integrated flow equation (5) and Ohm's law is the base of the model. The main difference between the two equations is compensated for by amplifiers. The model simulates one day in 2 seconds. There are ten normal layers, each with adjustable magnitude. Moreover there is a top layer in which infiltration, ponding and run-off are simulated, and a drain layer with adjustable drain-intensity. The normal layers are containing an adjustable resistor for the connection with an other layer and a function for the K (θ) relation. There are two transition layers which have to be placed at the boundary between two layers of different soil properties.

In saturated conditions the model is acting incorrectly. This causes a calculated thickness of the saturated layers of a too small magnitude, which can be compensated for by using an equivalent drain intensity, lower than the real one.

This model can be used to simulate the effect of natural rainfall and evaporation on moisture content at every depth. Soil physical properties and drainage conditions can be adjusted. Homogeneous as well as layered soil can be represented by the model. It can especially be applied to investigate drainage requirements of soils.

Some examples are given to show the applicability of the model. A short technical description is given at the end.

1. INTRODUCTION

Non-steady unsaturated flow and accumulation of moisture in soils can be described by models. This has to be done if one wants to forecast effects of drainage, tillage or soil improvement on the behaviour of soil moisture under natural, quickly changing weather conditions.

Numerical simulation models are the most versatile. They have few restrictions, except computer costs; these are prohibitive for long time series. Application of WIND and VAN DOORNE's (1975) numerical model costed about 1 US $ per calculated day. Very low in operation costs is the hydraulic analog of WIND (1972). However, this is a slow model with a small flexibility, because change of soil properties is laborious.

This paper describes a model which combines the advantages of both models mentioned and lacks the disadvantages of both. It is quick, flexible and has very low operation costs. This electronic model simulates the processes of infiltration, run-off, unsaturated flow and accumulation of moisture and drainage outflow. In a number of soil layers (mostly 10 are used, but more is possible) the soil properties k (ψ) and $\psi(\theta)$ of each layer can be adjusted. Drainage can be simulated in the model by one or more drain layers in which the drainage intensity can be chosen. Rainfall and evaporation input are given on paper tape. The time scale is one day per 2 seconds, so θ and k output can be read on line recorders and magnetic tape.

2. NOTATION AND SIGN-CONVENTIONS USED

A drain intensity (day^{-1})
a dimensionless factor $a = e^{\alpha\Delta z}$
E electric potential (Volt)
i electric current (Ampere)
k unsaturated conductivity ($cm.day^{-1}$)
k_0 conductivity at moisture pressure 0
k_d conductivity at drain depth
k_s conductivity at soil surface
r electric resistance (Ohm)
V vertical flux ($cm.\ day^{-1}$); upward is positive
V_d drain outflow ($cm.\ day^{-1}$); negative
V_{ro} run-off ($cm.\ day^{-1}$); negative if present
z height above soil surface (cm); negative below surface
z_d drain depth (cm)
z_g depth of groundwater table (cm)
Δz absolute value of distance between the centers of two layers (cm)
α exponent used in RIJTEMA's relationship between ψ and k (cm^{-1})
ϕ total potential expressed as hydraulic head (cm)
ψ capillary potential, moisture pressure (cm) in unsaturated zone negative
ψ_d moisture pressure at drain depth
ψ_s moisture pressure at soil surface
θ moisture content (volume fraction)
θ'_{ro} maximum depth of water ponded on soil surface (cm)
θ_s depth of water ponded on soil surface (cm)

3. GENERAL PRINCIPLE OF THE MODEL

In unsaturated soil the conductivity k in Darcy's law:

$$V = -k\frac{d\phi}{dz} \qquad (1)$$

is a function of soil moisture pressure ψ. In the wetter part of the moisture range RIJTEMA's (1965) expression can be used for this function:

$$k = k_0 \; e^{\alpha\psi} \tag{2}$$

The total potential ϕ is composed of the soil moisture suction ψ and the potential due to gravity z:

$$\phi = \psi + z \tag{3}$$

Combination of eq. (1), (2) and (3) gives

$$V \; e^{\alpha z} \; dz = - \; k_0 \; e^{\alpha\phi} d\psi \tag{4}$$

Integration of eq. (4) from z_1 to z_2 and from ϕ_1 to ϕ_2 under the assumption that V is constant over z between z_1 and z_2 gives a simple formula:

$$V = \frac{\frac{k_2}{a} - k_1}{\frac{a - 1}{a}} \tag{5}$$

In this function (5) the factor a represents

$$a = e^{\alpha\Delta z} \tag{6}$$

The numerator of (5) is therefore independent of k. There is some resemblance between eq. (5) and Ohm's law (7)

$$i = \frac{E_2 - E_1}{r} \tag{7}$$

If one represents the flow of moisture V by the flow of electricity i, the numerator $\frac{a-1}{a}$ by an electrical resistance r and the conductivity k_1 by an electric potential E_1 than the electric potential E_2 represents the conductivity $\frac{k_2}{a}$.

In this way an electric analog of unsaturated flow can easily be built (fig. 1). In this model the electric potential does not

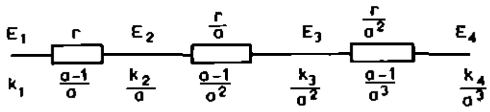

Fig. 1. Outline of an electric analog for unsaturated vertical flow; steady state model

represent moisture pressure or total potential, but unsaturated conductivity.

To make this model fit for non-steady processes, in each junction a capacitor has to be installed. This is done by a simple condensator, the capacity of it representing the moisture content θ. With a function-generator and an amplifier the corresponding value of $\frac{k}{a}$ is presented at the upper-side of the layer and the value of k at the lower side. This amplification in each junction has the advantage that all resistors can have the same value, in contrast with fig. 1. Moreover also the relation between E and k is now the same for all depths. Fig. 2 gives the outline of one layer of the model. A more precise description is given in a next chapter. In every layer the values of k and θ can be read without affecting the working of the model.

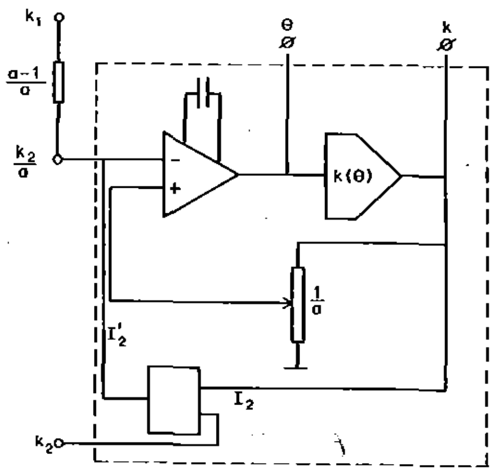

Fig. 2. Outline of the wiring diagram of one layer of the model

The soil properties are given in each layer by the value of a and the k (θ) relation. This curve is represented by three straight lines of which are given $\frac{dk}{d\theta}$ and the maximum value of θ. Fig. 3 gives an example of such a curve, fig. 4 shows the whole model and fig. 5 is a photograph of one layer.

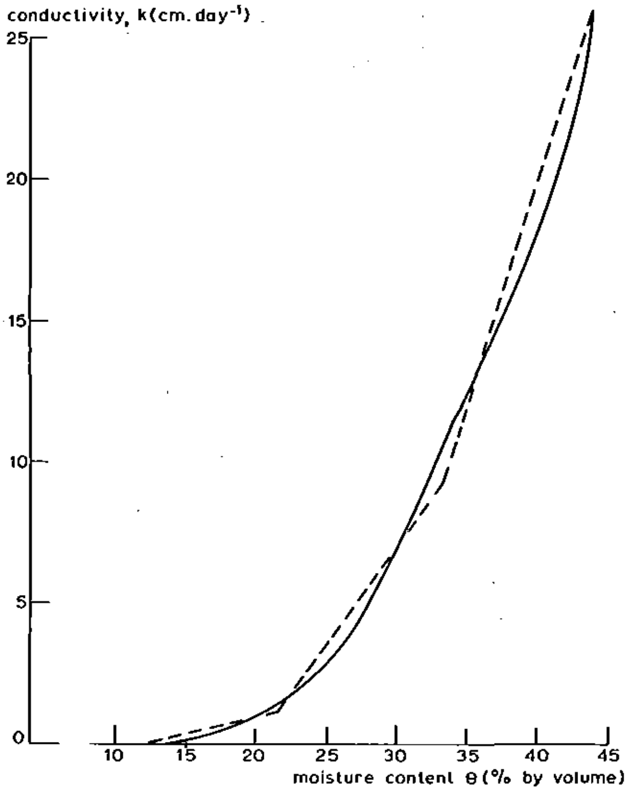

Fig. 3. Example of a k(θ) relationship and its realization in the model as three straight lines

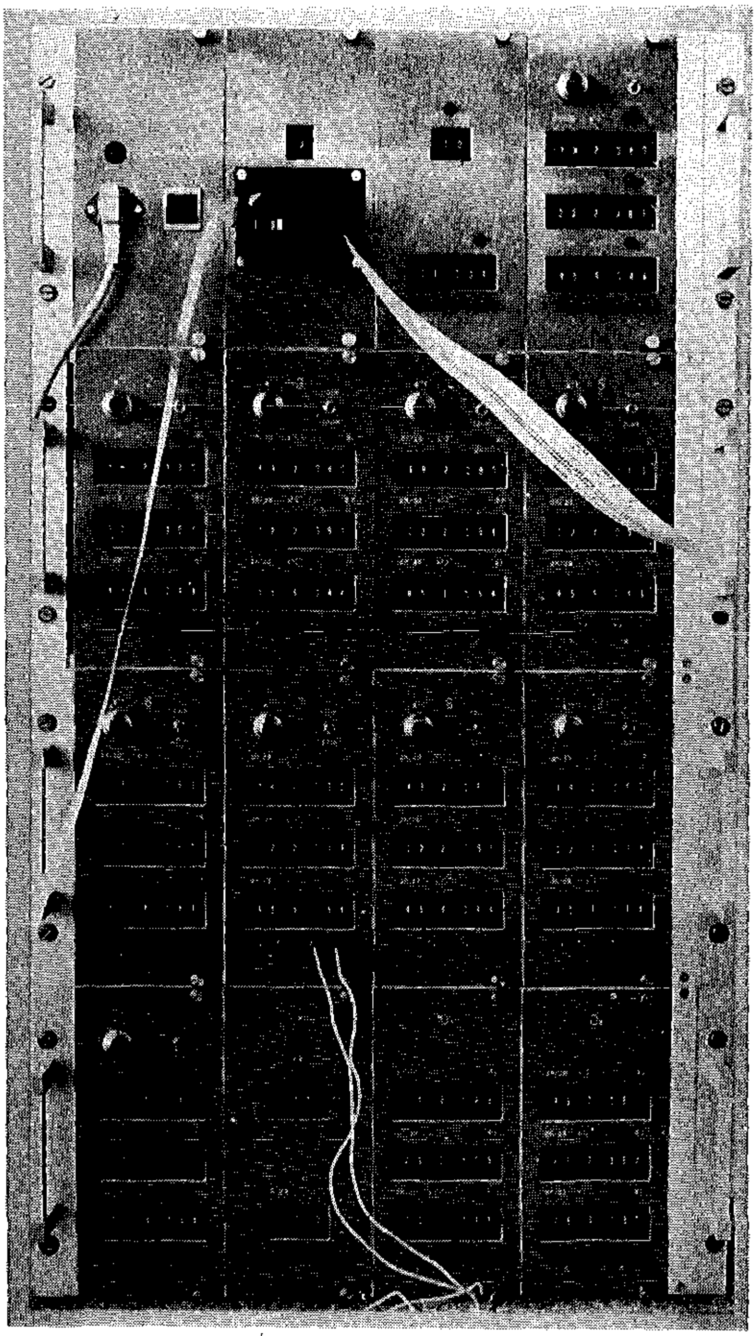

Fig. 4. Photograph of the electronic analog

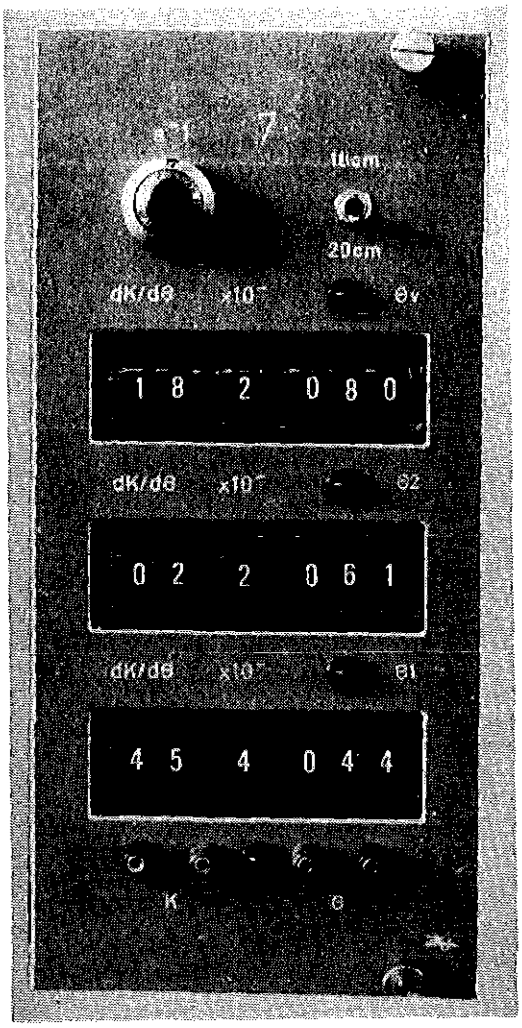

Fig. 5. Photograph of one normal layer

4. SCALES

In order to enable the use of line recorders and tape-writers for the model's output the time scale has been chosen at 2 seconds is 1 day. A velocity of 1 cm day^{-1} is represented by 10 μA and a conductivity of 1 $cm.day^{-1}$ by 0.333 Volt. From these 3 scales it can be calculated that 1 cm moisture is represented by 20 μ Coulomb and that the connecting adjustable resistors have a resistance of about 10^5 Ω.

The model-output, i.e. moisture content and conductivity of each layer, run-off and drain-outflow, are given in Volts. For flow velocities and conductivities, 1 Volt represents 3 $cm.day^{-1}$; for moisture contents 1 Volt stands for 10%.

5. RAIN AND EVAPORATION

Rain and evaporation are fed into the model's top layer with a paper-tape reader. The tape is read 5 times per day (= 2 seconds); the ASCII-code is used. In off-position of the tape-reader, a constant rainfall rate variable between 0 and 9 $mm.day^{-1}$ is applied. It is possible to change some scales so that more or fewer readings than 5 per day can be realized. For the normal scales the rain can be chosen in steps of 0.2 mm up to a maximum of 30 mm. Evaporation is mostly applied in the middle of the day, in steps of 0.1 mm up to 3.0 mm. If more readings are used higher evaporations are possible.

6. TOP LAYER

The top layer is representing soil surface. It receives electrical currents from the tape-reader representing rain and evaporation. The top layer is connected to the first layer with a resistor as is also the case for normal layers. In the top layer a value of a 0.5 instead of a has to be adjusted because the distance Δz is half the layers magnitude.

If rain rates are exceeding infiltration rates water is ponded upon the surface. In the model electricity has to be stored at the top layers' capacitor. The value of ψ_s indicating the moisture tension at soil surface equals the amount of moisture θ_s. So the conductivity is:

$$k_s = k_0 \, e^{\alpha\theta_s} \qquad (8)$$

It should be noted that k_s is exceeding k_0 now. The same will occur in saturated conditions in the sub soil which will be discussed later. Because the values of θ_s are small eq. (8) can be transformed into (9):

$$k_s = k_0 + \alpha k_0 \theta_s \tag{9}$$

This equation is brought into the top layer by a very low capacitive steady condensor working between zero and k_0 and a variable capacitor working from k_0 upward. The values of α and k_0 have to be adjusted in the top layer. A third value to be adjusted in this layer is that of the maximum ponding depth, θ_{ro} can be adjusted in steps of 0 1 cm to a maximum of 2.9 cm. The values of k_s and run-off rate V_{ro} can be read.

Not yet present is a device which reduces evaporation rates in dependence of the first layer's moisture condition.

7. DRAIN LAYER

The drain out-flow V_d (taken negative) is assumed to be dependent on the (positive) moisture potential at drain depth ψ_d as:

$$V_d = - A\psi_d \tag{10}$$

where A is the drainage intensity (day^{-1}) and ψ_d the potential at drain depth, expressed as hydraulic head.

As a value of ψ is not present in the model, but only that of k_d, equation (11) derived from (10) is applied in the drain layer of the model:

$$V_d = - \frac{A}{\alpha} \ln \frac{k_d}{k_0} \tag{11}$$

The values $\frac{A}{\alpha}$ and k_0 have to be adjusted. The drain outflow V_d can be read.

8. TRANSITION LAYER

At the boundary between two layers with different soil properties the values k and θ of upper and lower layer are different. The value of ψ is not present in the model. So the transition to a layer with different soil properties is a problem which has to be solved. The solution that has been chosen is to make transition layers without capacity and resistance but with the same function generator as normal layers.

So the θ-value of the transition layer is the same as that of the upper layer. In the transition layer a k(θ) relation has to be adjusted so that a k-value is generated which fits the soil properties of the lower layer. This k(θ) relation is composed of the ψ(θ) relation of the upper layer and the k(ψ) relation of the lower layer, see fig. 6.

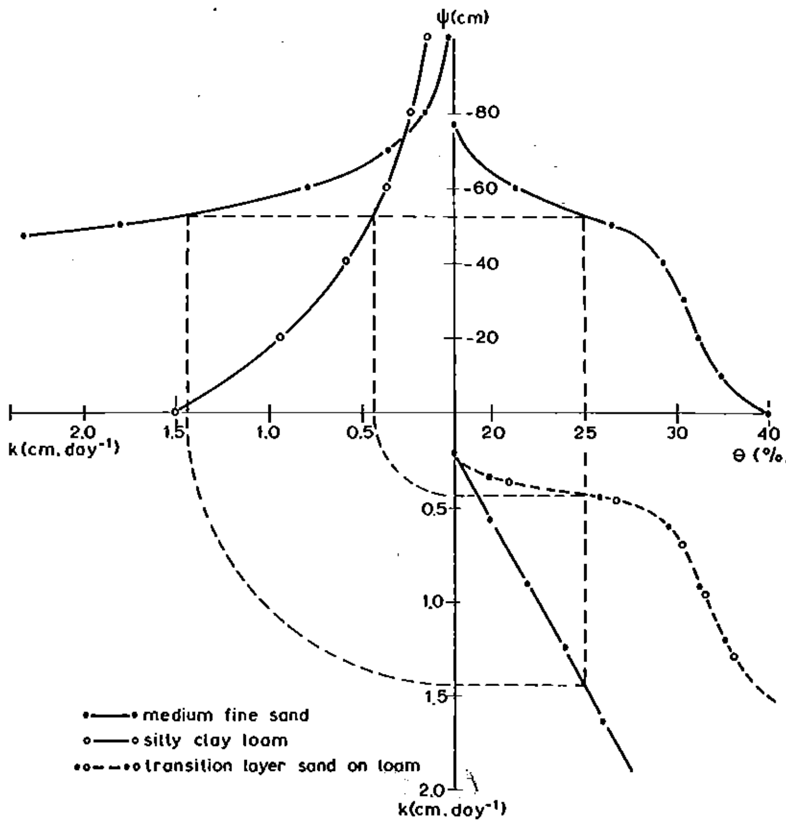

Fig. 6. The k(θ) relation of a transition layer has to be composed of ψ(θ) of the upper layer and the k(ψ) of the lower one

For unsaturated flow this solution is correct and works well. But if saturated conditions are occurring in the lower layer, the transition layer does not function well. Therefore another solution is considered in which the upper layer k, is directly translated into a k-value fitting the lower layer.

9. SATURATED CONDITIONS

The main problem of the model is that it is based on an unsaturated flow equation. In many studies of the unsaturated zone, drainage plays a role and near drain depth normally saturated conditions occur. In saturated conditions ψ is positive; the model is acting as an unsaturated model and calculates k-values exceeding k_0. Instead of equation (12) to be used for saturated flow:

$$V = k_0 \left(\frac{\psi 2 - \psi 1}{z_2 - z_1} + 1\right) \tag{12}$$

eq. (5) is used. Flow velocities calculated with (5) and (12) can differ considerably. But the model continues application of (5) whether the soil is saturated or not. For example if $\psi_1 = \psi_2 = + 20$ cm, $\Delta z = 10$ cm and $\alpha = 0.1$ cm^{-1} the flow velocities calculated with saturated eq. (12) and unsaturated eq. (5) are $- k_0$ and $- 7.4\ k_0$ respectively.

This is a large difference but calculations and comparison of flow velocities does not have much sense. Although Darcy's law is suggesting that gradient and conductivity are the cause and velocity their effect mostly the reverse is true. Flow velocities are controlled by rain and evaporation, being the causes; the moisture conditions are their effect.

If the thickness of the saturated layer above drain level is calculated correctly, the model functions well. By the use of the unsaturated eq. (5), however, a too low magnitude of its thickness is found.

According to saturated eq. (12) and drainage function (10) the correct magnitude of the saturated layer is:

$$(z_g - z_d)_{\text{correct}} = \frac{-k_0 V}{A (k_0 + V)} \tag{13}$$

By using the unsaturated eq. (5) combined with drainage function (10) the model finds:

$$(z_g - z_d)_{\text{model}} = -\frac{1}{\alpha} \ln \frac{e^{\frac{-\alpha V}{A}} + V/k0}{1 + V/k0} \tag{14}$$

The differences between the values of $z_g - z_d$ calculated with these equations are dependent on $\frac{\alpha V}{A}$ and $\frac{V}{k0}$. The higher α, A and V , and the lower k_0, the larger is the difference. The most important factor is $\frac{V}{k_0}$.

The error can be counterbalanced by adjusting a value $A^x < A$ in the model. It can be calculated from eq. (13) and (14) equaling these two. Therefore a certain value of V has to be chosen, normally near the highest value to be expected. Then for lower flow velocities the thickness of the saturated layer will come out somewhat too high. However, this is automatically counterbalanced by a feedback in the system, for a too high water table requires a certain amount of water. Fig. 7 shows that, provided that a good A^x is chosen, moisture content and drain outflow can be calculated well with the model.

10. SOME EXAMPLES

In fig. 7 the moisture content of top 10 cm and the drain outflow are shown, calculated with the numerical model FLOW of WIND and VAN DOORNE (1975) and with the electronic analog. The drain intensity used here was A = 0.014 day^{-1}; the adapted value calculated with eq. (13) and (14) for a saturated zone of 100 cm was A^x = 0.0082 day^{-1} for the electric model. If that was used, the differences between the two calculations were insignificant.

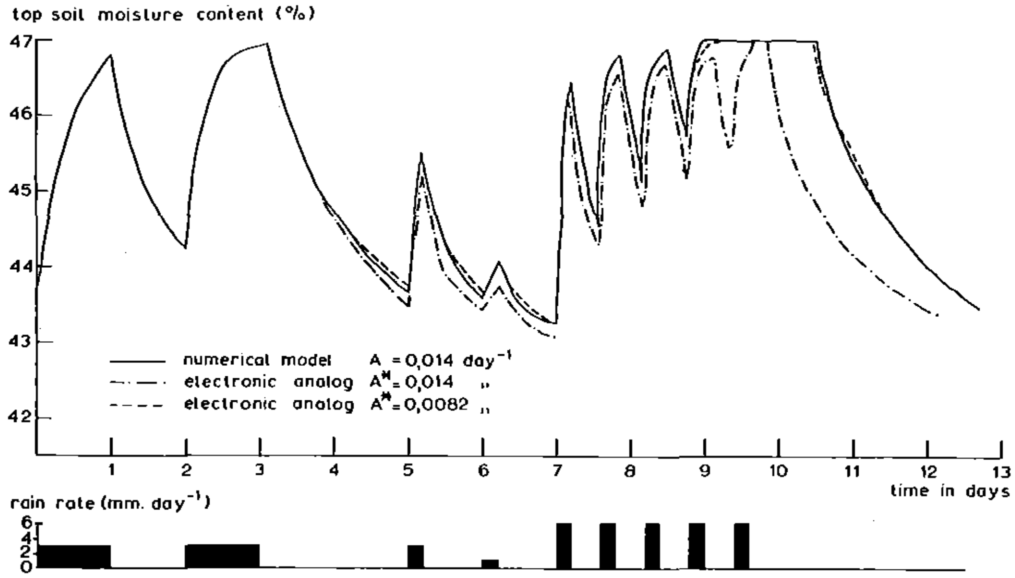

Fig. 7. Moisture content of top soil and drain outflow calculated with a numerical model and with the electronic analog

In the example of fig. 7 the three k(θ) line segments (e.g. the example in fig. 3) used in the model were practically the same as the k(θ) function used in the computer. Mostly there will be some differences and these will cause differences between the results of the two calculations. In fig. 8 this is shown for a homogeneous soil with k_0 = 2 cm.day^{-1} and α = 0,03 cm^{-1}. The k(θ) relationship and the three straight line pieces used are given in the inset. The drain depth is 100 cm, the intensity A = 1 in the electric model and infinitely large in the numerical calculation. Some differences in the moisture content at 5 and 35 cm depth can be seen, but they are smaller than 0.5% moisture.

In the very wet autumn of 1974 farmers in the Netherlands had a severe problem in harvesting potatoes, sugar beets and onions. The moisture suction in the top soil should be about 90 cm or drier to enable harvest on loam soils. Fig. 9 shows how the moisture suction of top soil varied in a part of this autumn in dependence of weather and drainspacing. The soil dit not become dry enough for harvesting operations, regardless the drain intensity. The lines of fig. 9 were

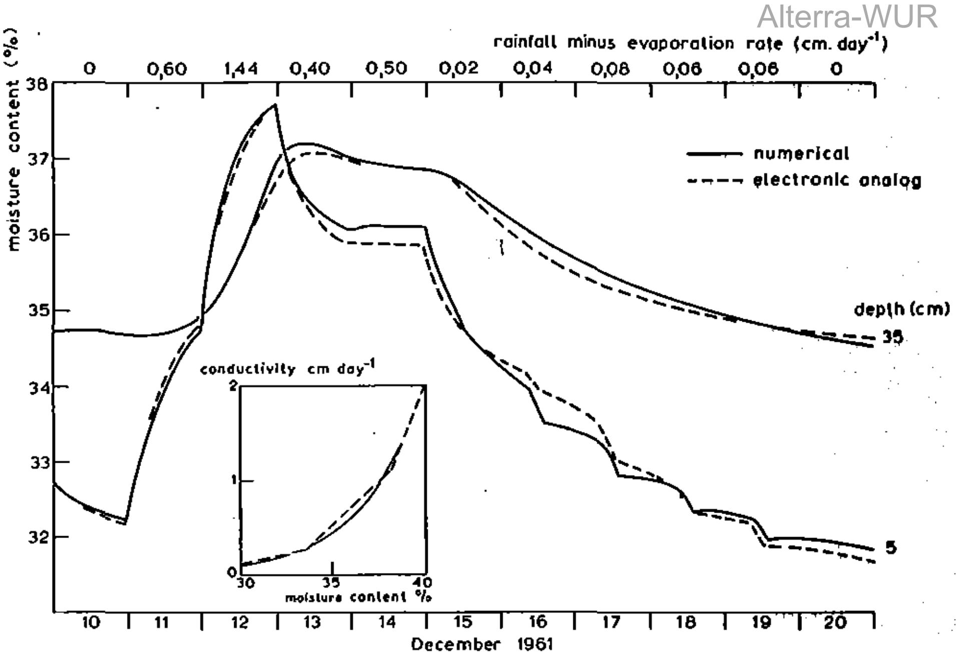

Fig. 8. Comparison of numerical and analog calculation of moisture content. Inset: k(θ) relationship and line segments used

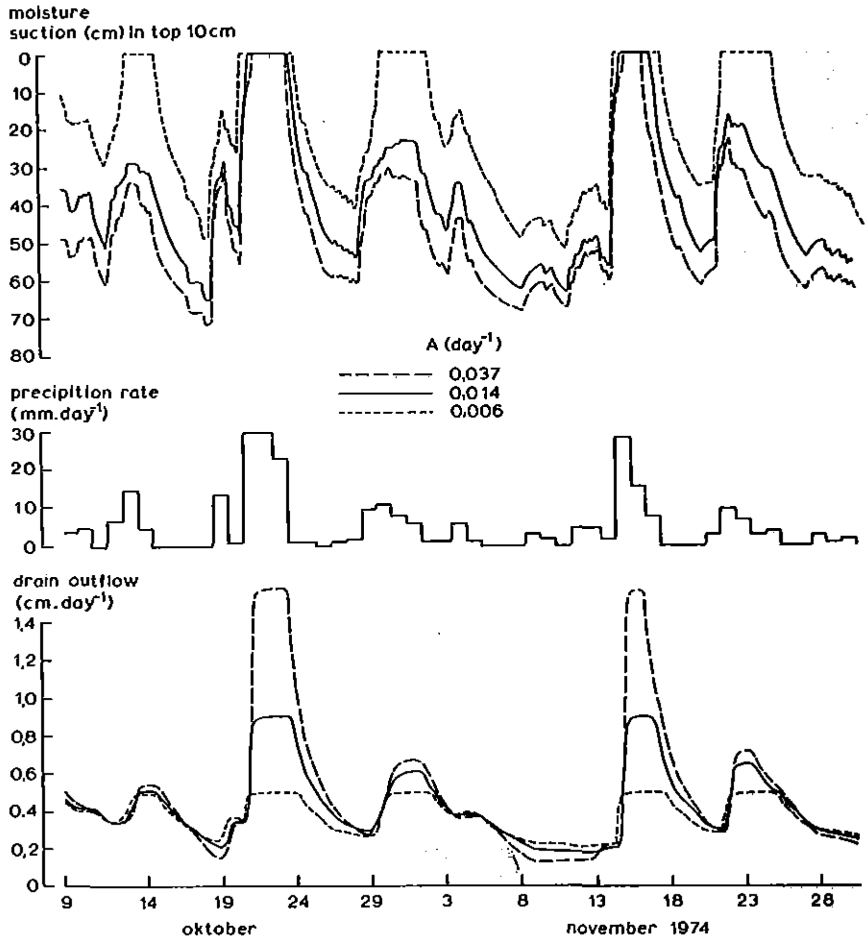

Fig. 9. Effect of drain-spacing on moisture suction of a top soil in the wet autumn of 1974

produced by the electric analog in about 2 minutes for a loam soil with k_0 = 2.8 cm.day^{-1} and α = 0.035 cm^{-1} and a drain depth of 100 cm. In the drain outflow graph one sees that the best drainage has a larger outflow than a poor drainage in wet periods. During dry periods the reverse can be true, e.g. October 19. Total discharge of the three drain intensities are not equal because considerable surface run-off occurred. Run-off in the analog was set to occur when more than 0.3 cm water was on the soil surface. If a larger amount had been chosen differences between the three drain spacings would have been larger, although the soil for all spacings should have been wetter at any time.

An example of the use of a transition layer is given in fig. 10. For the wet autumn of 1974 are compared the behaviour of a loam soil, a sandy soil and a soil consisting of 40 cm loam-on-sand. For the sandy soil the properties k_0 = 10 cm.day^{-1} and α = 0 05 cm^{-1} were used; the loam is the same as in fig. 9. The soils were taken to be drained nearly infinitely well (A = 0.17 day^{-1}, at a depth of 100 cm; about 12 times the Netherlands drainage design criterion).

In wet periods the loam soil is the wettest and the sandy soil the driest; the loam-on-sand soil then takes an intermediate position. In relatively dry periods the latter soil is mostly the driest; then the sandy soil is the wettest one. Striking is the drain outflow graph. The variation of drain outflow in the sandy soil is very small, only between 0.2 and 0.6 cm.day^{-1}. This is caused by its high pore volume; at saturation this soil contains 40% moisture and at a moisture tension of 100 cm only 5%, so 35% can be stored in the soil. In the loam soil only 8.5% can be stored. The drain outflow variations are very large here. The 40 cm loam-on-sand soil is intermediate as regards its drain outflow but its behaviour is closer to the loam than to the sandy soil. Apparently it has a moisture storage coefficient which is much lower than that of the sandy soil, although the groundwater table always remained in the sand.

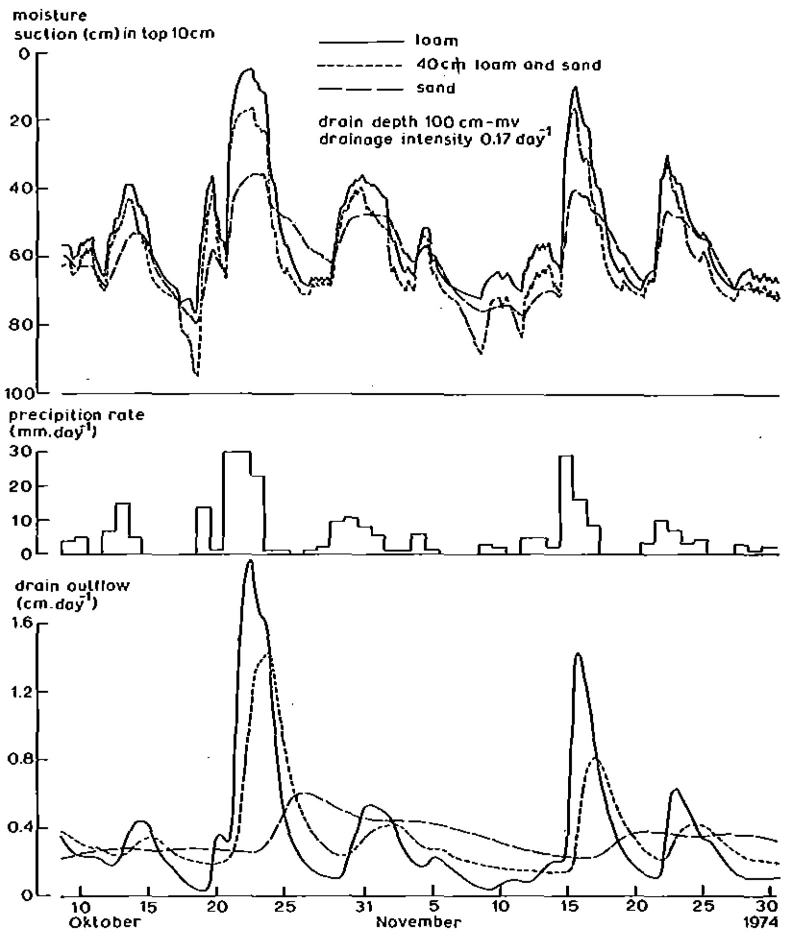

Fig. 10. Differences in behaviour between a loam, a sand and a soil consisting of 40 cm loam on sand

In fig. 11 an example is given of a case with two transition layers. A uniform loam soil (k_0 = 2.8 cm.day^{-1}; α = 0.035 cm^{-1}) is compared with a soil having a compacted layer between 20 en 30 cm depth. This layer was assumed to have the same α and the same $k(\theta)$ relation as the normal soil but a saturated conductivity of one tenth of it, so k_0 = 0.28 cm.day^{-1}. The compacted layer causes the top soil to be wetter in rainy periods because moisture flows very slowly through it. This can be seen in the drain outflow graph where the tops of the soil with a compacted layer are always later and lower than those of the soils without it.

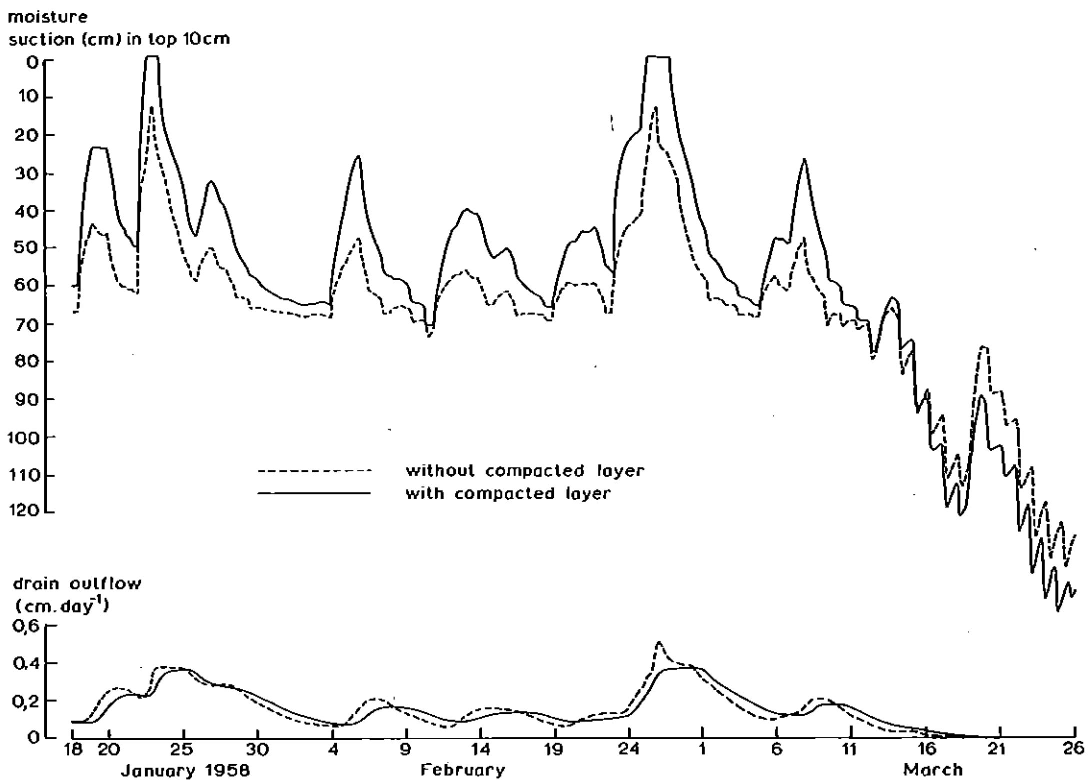

Fig. 11. The effect of a compacted layer on moisture content and drain-outflow

In dry periods with an evaporation as occurs in March, the soil with a compacted layer is drier than the homogeneous soil. The capillary rise is hampered by the low conductivity. So a soil with a compacted layer sometimes can have an earlier workability than without such a layer.

In the examples given only the top layer and the drain outflow were discussed, but of course every layer can be read for its moisture content and unsaturated conductivity.

11. SHORT TECHNICAL DESCRIPTION

Only a short description of the electronic realization of the model, in which more than 200 integrated circuits are used, can be given in this context. More details can be obtained from the second author.

The model is built as a modular cassette system. When inserted in the rack in the proper order the cassettes are interconnected. Every layer has its own cassette. A normal layer has a conductive and a capacitive part (see fig. 2). In the capacitive part a number of functions are combined, two of them are derived from soil properties; two others are caused by the fact that k must be divided by a constant.

An integration of the currents flowing into and out of the layer is needed to calculate the moisture content θ. The voltage representing the moisture content is fed to a function generator simulating the k(θ) relationship as a piece-wise lineair approximation of it. With each set of thumbwheel switches the slope and breakpoint of the line segments can be adjusted (see fig. 5).

In order to be able to present a value $\frac{k}{a}$ to the higher layer and to measure the current through the resistor $\frac{a-1}{a}$, a division by a is necessary. In fact these two values are adjusted by two ganged ten turn potentiometers having a multidial to be seen in fig. 5.

As the summing point of the integrator is at a level of $\frac{k}{a}$ and the current from the lower layer flows to a level k, a special current transformer has been developed. This current transformer is a circuit built with three special op-amps and it has the property that the current I_2 flowing through is measured and comes out of a special output with the same magnitude. This magnitude is independent of the voltage level into which the current flows (I_2^1).

In every cassette, outputs are made to measure the values of k and θ. Normal recording and measuring instruments cannot effect the proper function when using these outputs.

The special layers, e.g. top layer, transition layer and drain layer, are constructed as similar cassettes. As far as possible identical printed circuits are used. In the top layer an integrator is used to simulate the ponding properties and a function generator is modified to represent eq. (9) and the run-off. In a transition layer no accumulation is present, so only a function generator is used. In the drain layer a ln module is used eq. (11). With two sets of thumbwheel switches the function can be adjusted.

In order to have a rough impression of the condition and action of the model several light emitting diodes were added. They light up when specific values are exceeded.

REFERENCES

RIJTEMA, P.E., 1965. An analysis of actual evapotranspiration. Agric. Res. Rep. 159. Pudoc, Wageningen

WIND, G.P., 1972. A hydraulic model for the simulation of non-hysteretic vertical unsaturated flow of moisture in soils. J. Hydrol. 15: 227-246

______ and W. VAN DOORNE, 1975. A numerical model for the simulations of unsaturated flow of moisture in soils. J. Hydrol. 24: 1-20