ANALOG MODELING OF TRANSIENT MOISTURE FLOW IN UNSATURATED SOIL

G.P. WIND

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Analog modeling of transient moisture flow in unsaturated soil

Proefschrift
ter verkrijging van de graad van
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Abstract

Wind, G. P. (1979) Analog modeling of transient moisture flow in unsaturated soil. Agric. Res. Rep. (Versl. landbouwk. Onderz.) 894, ISBN 90 220 0713 8, (viii)+54 p., 26 figs., 9 tables, 30 refs., Eng. and Dutch summaries.
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Hydraulic and electronic analog models are developed for the simulation of moisture flow and accumulation in unsaturated soil. The analog models are compared with numerical models and checked with field observations. Application of soil physical knowledge on a soil technological problem by means of steady state considerations, pseudo-steady calculation, numerical models and analog models are compared.

Some examples of application of analog models on drainage requirements are given. From these it appeared that drain spacing is important to avoid water logging, but that drain depth is more important to obtain workable conditions.

Free descriptors: drainage, workability, simulation, amelioration, precipitation, evaporation, soil moisture.

This thesis without appendices will also be published as Agricultural Research Reports 894

Stellingen

1. De effectieve waarde van bepaalde bodemfysische parameters, die als karakertistiek kunnen worden beschouwd voor een perceel of bodemtype, kunnen eenvoudiger worden bepaald door inductief gebruik van simulatiemodellen dan door de bepaling van deze parameters aan een groot aantal monsters.

Dit proefschrift.
D.R. Nielsen et al. Spatial variability of field measured soil water properties. Hilgardia 42: 215-259 (1973).

2. De nauwkeurigheid, vereist voor de bepaling van het capillair geleidingsvermogen is voor toepassing in dynamische modellen niet groter dan voor gebruik in stationaire formules.

3. Het Nederlandse drainagecriterium bevat ten onrechte geen aanwijzingen betreffende de diepte van de ontwateringsmiddelen.

4. Plantewortels verdichten de bodem.

5. Door woelen van zavelgronden na egalisatiewerken wordt veel meer de gebruiker van de grond psychisch losgemaakt van de gevolgen van bodemverdichting dan de grond zelve.

6. Het gevaar voor winderosie in de Veenkoloniën neemt toe. Het treffen van bodemconserverende maatregelen is daarom noodzakelijk, zelfs als deze in strijd zouden zijn met de streekplannen.

G.P. Wind. Grondverbetering, conservering van veen en winderosie in de Veenkoloniën. Landbouwkundig Tijdschrift / PT 91,3 (1979).

7. Egalisatie van bouwlandpercelen heeft in Nederland slechts zin indien èn de lage delen te nat èn de hoge delen te droog zijn.

8. Voor opvulling van holle wegen in hellende gebieden mag geen specie worden gebruikt uit de flanken van die wegen.

9. Profielverbetering van veenkoloniale grond geeft een groter resultaat dan die van plaatgrond.

10. De bodemdaling die in de westnederlandse veengebieden is opgetreden is veel meer het gevolg van oxydatie van veen dan van dichtheidsverandering.

C.J. Schothorst. Subsidence of low moor peatsoils in the Western Netherlands. Geoderma 17: 265-291 (1977).

11. De verontreiniging van oppervlaktewater door meststoffen is kleiner naarmate de ontwatering dieper is.

12. De betekenis van de oppervlakkige afspoeling na vorstperioden voor de fosfaatverontreiniging van slootwater wordt overschat.

G.J. Kolenbrander. P.A.O. cursus 'Veehouderij en Milieu', 1979.

13. Als naam voor de betreffende sport is 'plankzeilen' te verkiezen boven 'windsurfen'.

Proefschrift G.P. Wind
Analog modeling of transient moisture flow in unsaturated soil
Wageningen, 14 december 1979

Woord vooraf

Dit proefschrift is samengesteld uit twee delen. Het tweede deel bestaat uit overdrukken van vier reeds verschenen artikelen, te weten:

Wind, G. P., 1972. A hydraulic model for the simulation of non-hysteretic vertical unsaturated flow of moisture in soils. J. Hydrol. 15: 227–246

Wind G. P. & W. van Doorne, 1975. A numerical model for the simulation of unsaturated vertical flow of moisture in soils. J. Hydrol. 24: 1–20

Wind, G. P., 1976. Application of analog and numerical models to investigate the influence of drainage on workability in spring. Neth. J. agric. Sci. 24: 155–172

Wind, G. P. & A. N. Mazee, 1979. An electronic analog for unsaturated flow and accumulation of moisture in soils. J. Hydrol. 41: 69–83

Het eerste deel bestaat uit een artikel dat zowel een samenvatting is van de vier artikelen uit het tweede deel, als een beschrijving van de noodzaak en de achtergronden die tot de ontwikkeling van analoge modellen hebben geleid. Bovendien worden de analoge methodieken vergeleken met andere en eerder toegepaste methoden. Ook wordt daarin een nieuwe toepassing beschreven.

De aanleiding tot het onderzoek, dat tot dit proefschrift heeft geleid, was de ontdekking van de mogelijkheid van een hydraulisch analogon. Daarmee werd het mogelijk het tijdsafhankelijke, niet-lineaire complexe systeem van stroming en berging in de onverzadigde zone na te bootsen. Daar deze simulatie kan geschieden in een fractie van de tijd die de processen in werkelijkheid duren, ontstond daarmee de mogelijkheid om cultuurtechnische problemen te onderzoeken in enkele weken of maanden. In veldwerk vitgevoerd, zouden dergelijke onderzoekingen tientallen jaren vergen.

Het onderzoek werd uitgevoerd op het Instituut voor Cultuurtechniek en Waterhuishouding (ICW). De toenmalige directeur, prof. dr. C. van den Berg, onderkende van de aanvang af het belang van deze ontdekking. Hij stelde mij in de gelegenheid de idee nader uit te werken, al lag deze enigszins naast mijn eigenlijke werkterrein, de bodemtechniek. Ik ben u, prof. van den Berg, en ook de tegenwoordige directeur, ir. G. A. Oosterbaan, zeer erkentelijk voor de ruimte die u mij heeft gelaten dit proefschrift te voltooien. U weet dat ik er daarbij naar heb gestreefd mijn hoofdtaken niet te verwaarlozen.

Prof. dr. W. H. van der Molen, u heeft de ontwikkeling van de modellen die in dit proefschrift zijn beschreven meegemaakt; wij waren al in gesprek voordat bij mij de idee ontstond die uiteindelijk tot deze modellen heeft geleid. Onze contacten zijn de stimulans geweest die ik nodig had om dit werk te kunnen

verrichten. Ik ben u daarvoor en voor het feit dat u als promotor wilt optreden bijzonder dankbaar.

Geachte heer W. van Doorne, voor uw ideeën en uw medewerking bij de ontwikkeling van het model FLOW ben ik u zeer erkentelijk. Ir. A. N. Mazee, de snelheid waarmee u zich als elektronicus vertrouwd heeft gemaakt met bodemkundige en cultuurtechnische begrippen heeft mij verrast. U hebt deze begrippen weten te vertalen in elektronische termen; daardoor is het mogelijk geworden het analogon ELAN te ontwikkelen. Die naam is niet alleen een afkorting, ze is ook een typering van onze samenwerking.

Veel medewerkers van het Instituut voor Cultuurtechniek en Waterhuishouding zeg ik dank voor de bijdragen die zij hebben geleverd aan het onderzoek, dat in dit proefschrift wordt beschreven. Speciaal wil ik noemen de heer J. Buitendijk, die mij voortdurend terzijde stond bij de oplossing van veel moeilijke problemen, die de modellen bouwde en bediende, en die samen met ing. J. B. M. M. van Gils de applicatieprogramma's maakte voor de outputverwerking. De heer R. Wiebing maakte het prototype van het hydraulisch analogon; de heer J. Roelofse bracht later zijn technische hulp in. Jacomien Vermeer heeft veel tijd besteed aan de verwerking van modelresultaten. Ook dank ik de heren ir. U. D. Perdok, ing. T. Tanis en ing. M. Telle van het Instituut voor Mechanisatie, Arbeid en Gebouwen voor hun medewerking bij het bewerkbaarheidsonderzoek.

Zeer dankbaar ben ik mijn collega's dr. R. A. Feddes en ir. E. W. Schierbeek, die door hun zorgvuldige en kritische bestudering het oorspronkelijke manuscript zowel inhoudelijk als redactioneel aanmerkelijk hebben verbeterd. Het typewerk werd zeer kundig verricht door Cora van der Wel en Marjo Tichelaar-Wolthuis; ook alle veranderingen zijn door Marjo met veel geduld aangebracht. De heren T. Klaassen en J. van Dijk verzorgden het tekenwerk.

Curriculum vitae

De auteur werd op 12 maart 1924 geboren te Gramsbergen en behaalde in 1942 het diploma HBS-B te Arnhem. Hij studeerde van 1945 tot 1951 aan de Landbouwhogeschool te Wageningen, studierichting Cultuurtechniek.

Van 1951 tot 1956 was hij verbonden aan het Centraal Instituut voor Landbouwkundig Onderzoek (CILO) als onderzoeker op het gebied van de waterhuishouding van grasland. Vanaf 1956 is hij werkzaam op het Instituut voor Cultuurtechniek en Waterhuishouding (ICW) te Wageningen in de eerste jaren als onderzoeker op het gebied van de mechanische grondverbetering, later als hoofd van de hoofdafdeling Bodemtechniek.

Contents

1 Introduction

1.1 Purpose

The moisture content of the soil is important for agricultural as well as other land users. It influences water supply to plant roots, aeration, bearing capacity, workability and many other characteristics.

Ameliorationists try to control moisture contents by means of drainage or soil improvement. The purpose of such works can be to avoid very wet soil conditions during rainfall, to obtain more and earlier falling days in which the soil is fit for seedbed preparation, to lower the number of days that sportsfields cannot be used because of a too low bearing capacity, and so on.

The effects of such ameliorations are investigated at a large number of experimental fields all over the world. Many observations have to be made and recorded in such experiments, which make them fairly expensive. A well known disadvantage of this inductive research is the lack of transferability of such experimental results to other soil and weather conditions. This causes a need for repetition at other sites and in other years. Another and better way to increase transferability is to combine field experiments with a deductive explanation of the results. In the past decades, soil physical knowledge has developed far enough to predict the effect of measures. If such a prediction is checked against field observations it can provide a more general validity than even a large number of field trials can give.

The main problem with these predictions is that the effects of such ameliorations on the moisture content of the soil are indirect. The most important factor, the weather, is not influenced by human interference and rainfall and evaporation are causing alterations in soil moisture content far greater than the effects of drainage or soil improvement will impose.

Prediction methods, therefore, must be able to describe the effects of rain and evaporation on moisture content of the soil, especially the topsoil, and it must be possible to do this quantitatively from day to day.

Such predictions can be obtained from simulation models of the unsaturated zone. Numerical models can simulate what happens with the moisture in the soil. With them the flux at different depths and the accumulation of moisture are calculated just as they are happening in reality, provided that the model and the parameters used are correct. Such models generate the moisture content of each layer at any time, including that of the top layer. The time steps used in such models have to be small in order to avoid instability in the calculations. This implies that much computer time is consumed, which makes the simulation of long time-series too expensive for practical purposes.

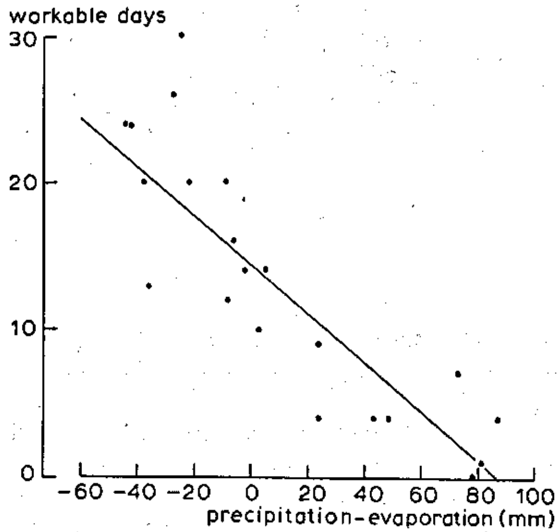

Fig. 1. Relation between precipitation minus evaporation ($0.8E_0$) and number of workable days in March and April in the years between 1951 and 1973 in the Netherlands.

Dealing with long time-series is necessary because of the very irregular distribution of rainfall. Boels & Wind (1975) showed that the unfavourable distribution rather than the high amount of rainfall was responsible for the poor harvesting conditions in the Netherlands in the autumn of 1974. In Fig. 1 the relation is given between precipitation minus evaporation in March and April in the years between 1951 and 1973 and the number of workable days in the same months, as calculated with an analog model. The correlation coefficient is 0.82, which means that the variation in the number of workable days is explained for 67% by the amount of rainfall. The other 33% is to be explained by the rainfall distribution.

Now amounts can be averaged or subjected to a probability analysis, but with distributions this is hardly possible or not at all. In agricultural practice the first date or the first few days with workable conditions in spring is more important than the total number of workable days and the former will be even more dependent on rainfall distribution than the latter. So long time-series must be used to generate data fit for practical application, making numerical models too expensive to use for the prediction of amelioration effects.

This publication deals with the development of simulation models which can be applied to long time-series at very low costs. For this purpose analog models were chosen, because they are continuous in time, having nothing to do with time steps. First a hydraulic analog was developed in which each soil layer is represented by a vessel, its shape being dependent of the moisture characteristic. Unsaturated conductivity is represented by a number of connecting tubes.

Though this model works properly (as will be shown later on), it operates too slow and the adjustment of soil properties is too laborious. In a later stage, therefore, an electronic analog was developed, based upon the analogy between the integrated moisture flux equation and Ohm's law. It operates with great speed and is readily adjustable.

With this instrument most problems about the effect of drainage, soil improvement and other measures on moisture content of soil can be investigated. So, for example, the relation between drain depth and the number of days with insufficient bearing capacity can be studied over a very long period. Drainage requirements of sportsfields in order to avoid days with unplayable conditions of the turf can be determined. The effect of the removal of a compacted layer on water logging and trafficability can be investigated.

Of course the results of the simulations need to be checked against field experiments or field observations, but these can be fairly simple and of short duration because the variability of the results as caused by weather conditions can be calculated with the model.

In this manner long term experiments with many repetitions can be replaced by quick analog simulation in combination with some field checks.

1.2 Basic concepts

For a long time it has been known that moisture conductivity in unsaturated soils depends on moisture content. Before this dependency had been formulated as a mathematical relation, Darcy's law could not be applied to unsaturated soil.

For a basin clay soil Wind (1955a) determined unsaturated conductivity from field observations of moisture content and pressure head. By calculating a regression line between pressure head and unsaturated conductivity the first $k(\psi)$ relation was obtained:

$$k = b(-\psi)^{-n} \tag{1}$$

where k is the conductivity ($\mathrm{cm \cdot day^{-1}}$), ψ the soil moisture pressure head (cm); b and n are constants.

It had some disadvantages (difficult integration and incorrect values of k in the vicinity of $\psi = 0$). Therefore later other expressions were proposed, examples are those of Gardner (1958):

$$k = \frac{d}{(-\psi)^n + c} \tag{2}$$

where c, d and n are constants, and of Rijtema (1967):

$$k = k_0 e^{\alpha\psi} \tag{3}$$

where k_0 is the conductivity at zero pressure head and α is a constant ($\mathrm{cm^{-1}}$).

By using such expressions for the $k(\psi)$ relation, the available knowledge of movement of moisture in unsaturated soils could be applied. For that purpose one of the expressions is substituted in Darcy's law, which reads:

$$v = -k\frac{\delta\phi}{\delta z} \tag{4}$$

where v is the volumetric flux in $\mathrm{cm^3 \cdot cm^{-2} \cdot day^{-1}}$, ϕ is the total moisture potential expressed as energy per unit weight (cm) and z is the vertical coordinate (cm). In this publication ϕ is denoted as the hydraulic head, being the sum of the soil moisture pressure head ψ (negative in unsaturated condition) and the gravitational head z. The coordinate z has its origin at the soil surface and it is taken positive upwards. Then Eq. (4) can be written as:

$$v = -k(\psi)\left(\frac{\delta\psi}{\delta z}+1\right) \tag{5}$$

Substitution of equations like (1), (2) or (3) in Eq. (5) and subsequent integration results in steady state flux equations (see Section 2.1). Although the use of $n = -1.5$ in the power relation (1) causes a fairly unpractical flux equation, it nevertheless provided a quantitative description of the process of capillary rise.

Such steady state solutions made new practical applications possible, of which Wesseling (1961) and Wind (1955b, 1961) gave some examples. This knowledge resulted in a new problem: that of forecasting.

Observations of the gradients $\delta\psi/\delta z$ from the pressure head profiles allowed to calculate unsaturated conductivity. Once the conductivity relation $k(\psi)$ was known, it should—at least in theory—be possible to do the reverse and calculate moisture profiles from weather data and soil properties. This to be done by combination of the continuity equation (6) with the flux equation (5)

$$\frac{\delta\theta}{\delta t} = -\frac{\delta v}{\delta z} \tag{6}$$

where θ is the volumetric moisture content ($cm^3 \cdot cm^{-3}$, mostly indicated in vol. %) and t is time (days). This gives a non-linear partial differential equation, which has two obstacles to obtain a solution, namely the dependence of the two variables θ and ψ and the dependence of k and ψ. Analytical and semi-analytical solutions can only be obtained for specific cases (for examples see Gardner, 1958; Philip, 1969; Braester et al., 1971; Stroosnijder, 1976; Lomen, 1978; Feddes, 1979/80). These solutions have disadvantages with respect to the restrictive assumptions they generally need. Eq. (6) must be supplemented by appropriate initial and boundary conditions. The initial condition (θ or ψ) can be freely chosen. The bottom boundary condition is to be given in a simple form as a pressure head or a flux (e.g. drain outflow rate). The surface boundary condition also has to be simple, e.g. a specified precipitation or evaporation rate.

Numerical solutions at first were not a practical proposition because of the laborious calculations. With the development of computers their importance increased. Numerical solutions require the same first and second boundary conditions, but the surface condition can be freely chosen for every time step; so variable weather conditions can be incorporated. An example of numerical solutions will be given in Section 2.4.

Before dynamic models could be applied, other ways were explored to apply the knowledge of moisture in the unsaturated zone. A makeshift solution was found in the shape of pseudo-steady state sequences. They were used to calculate the amount of moisture which can be extracted by plants below the root zone (Wind & Hidding, 1961). Pseudo-steady state models presume a sequence of steady state situations, with in each situation a new, mostly higher, flux or groundwater depth. They are discussed in Section 2.3.

Dynamic numerical models are now available, but they are only used for short time series. Because of the many calculations needed, they consume so much computer time that they are too expensive to use for long time-series. Nevertheless long time-series are necessary to solve problems like drainage requirements or the effect of groundwater depth on evapotranspiration.

Therefore models other than dynamic numerical ones are used to solve practical problems. The use of the makeshift solution by means of pseudo-steady state models is continued (de Laat, 1976) but also analog dynamic models were developed. The first of these is a hydraulic analog (Wind, 1972) which is discussed in Section 3.1. A recently developed electronic analog by Wind & Mazee (1979) will be treated in Section 3.2.

The greatest advantage of analog models is their negligible operation cost, which makes them appropriate to deal with long time-series. The most striking disadvantage is caused by limitations in the analogy which make them less versatile than numerical models.

A comparison of different calculation methods and models applied to a practical problem is given in Chapter 4. Finally, Chapter 5 deals with the application of the models developed.

1.3 Assumptions

In order to apply soil physical knowledge to practical problems by means of models some schematizations had to be made.

The soils are thought to have physical properties, as $\psi(\theta)$ and $k(\psi)$ relations, which do not change with time. So swelling and shrinking are neglected and compaction is thought not to occur. For soils with cracks the discussed models are not feasible (Bouma, 1977).

The effects of hysteresis are not taken into account. Changes in soil properties as well as hysteresis are of practical importance; neglecting them therefore restricts the applicatility of the models. To include these effects, however, would make the model too complex for practical application at this moment.

Darcy's law is assumed to be valid and only vertical flow is considered. In both the numerical model of Wind & van Doorne (1975) and the electronic analog of Wind & Mazee (1979), the $k(\psi)$ relation is thought to be exponential. According to Rijtema (1965) this confines the applicability of the models to fairly wet conditions. The hydraulic analog of Wind (1972) can be used with $k(\psi)$ relations of any shape.

Flux due to differences in salt concentrations of soil moisture and to temperature gradients is neglected. The flow of moisture to an ice front also is not taken into account. No difference is made between rain and snow in the input of the models. This makes the model output unreliable during frost periods and the differences in the moisture distribution generated in the model and those occurring in reality during frost periods can make the results unreliable for a considerable time after such periods.

In the mentioned numerical and electronic models potential evaporation is used as an input. The effect of a dry top soil, reducing the evaporation in reality, is not taken into account. In the hydraulic model evaporation is made dependent on the moisture suction in the top soil. A device reducing evaporation is being built into the electronic analog. All mentioned models are simulating evaporation at the soil surface; uptake of water by roots is not taken into account. This restricts their validity to bare soil or soils with shallowly rooting crops.

Assumed is a linear relation between drain outflow and hydraulic head, which in reality often is not the case.

1.4 Notation and sign conventions used

- A drainage intensity (day^{-1}), the ratio between drain outflow rate and hydraulic head midway between two drains
- a dimensionless factor $a = e^{\alpha \Delta z}$
- D drain depth in cm below soil surface
- E electric potential (V)
- h height above groundwater (cm)
- i electric current (A)
- k hydraulic conductivity in unsaturated state ($cm \cdot day^{-1}$)
- k_0 conductivity at zero moisture pressure ($cm \cdot day^{-1}$)
- R electric resistance (Ω)
- t time (day)
- v vertical flux ($cm \cdot day^{-1}$) upward is positive
- z vertical distance from soil surface (cm) positive upwards
- z also gravitational head (cm)
- α exponent used in Rijtema's $k(\psi)$ relation Eq. (3) (cm^{-1})
- ψ soil moisture pressure head (cm) in unsaturated zone negative
- ϕ hydraulic head (cm), sum of ψ and z
- θ volumetric moisture content ($cm^3 \cdot cm^{-3}$ or vol. %)

Some other, incidental, symbols are defined in the text only.

2 Some other methods

2.1 General

Before dynamic models of the unsaturated zone were available, calculation of moisture contents in dependence of time was not possible or only possible under simple conditions. Nevertheless investigators did try to predict the effect of some measures on soil moisture content. This was done for example to study the effect of drainage on workability and the effect of soil improvement on available moisture.

Methods to obtain a certain prediction are dealt with in this chapter. They are steady state considerations, pseudo-steady state sequences and a numerical model. Although the latter already is a dynamic model it is discussed here because of its practical unsuitability with regard to long time-series.

Another method, using analytical solutions, is not treated here; reference is made to Stroosnijder (1976). This is done because analytical solutions can only be obtained under rather restrictive assumptions.

2.2 Steady state flux equation

When the $k(\psi)$ relations of Section 1.2 are combined with Darcy's law, flux equations can be developed.

If Rijtema's $k(\psi)$ relation (3) is used Eq. (5) can easily be integrated. This results in:

$$\psi = -h + \frac{1}{\alpha} \ln \left(1 - \frac{v}{k_0} (e^{\alpha h} - 1)\right) \qquad (7)$$

where h is the height above groundwater level.

Integration of Eq. (5) with the use of Wind's (1955) or Gardner's (1958) $k(\psi)$ relation results in less simple equations. Their general shape depends on the value of the exponent n, as Wind (1961) has shown.

As the laboratory or field determinations of k show considerable variability, the choice between adjustions according to Wind, Gardner or Rijtema is arbitrary. Nevertheless the influence of this choice can be important. Fig. 2 gives two $k(\psi)$ relations, a Gardner power curve (8) and a Rijtema exponential curve (9), which cross each other:

$$k = \frac{150}{(-\psi)^{1.5} + 50} \qquad (8)$$

$$k = 3e^{0.03\psi} \qquad (9)$$

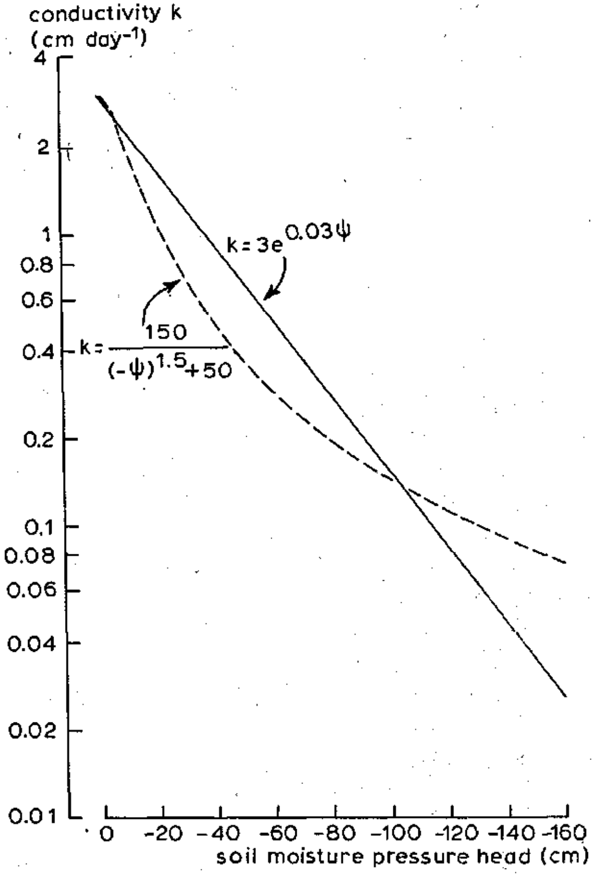

Fig. 2. Exponential and power relation between conductivity and pressure head.

The effect of the difference between those relations on steady state pressure head profiles is shown in Fig. 3. When the flow direction is downward, the power curve, due to its lower conductivity in wet soil, indicates wetter conditions than the exponential curve. For upward flow the reverse holds up to a certain limit, and then the larger conductivity of the power curve plays an important role. Up to a height of 80 cm above groundwater level the differences in ψ between the calculated curves are less than 20 cm. For the wettest curves the difference is 12 cm at a mean value of -40 cm soil moisture pressure head at the surface. So the arbitrary choice can have a major influence in steady state pressure head profiles.

It is to be expected that its influence in the non-steady state is similar but smaller. Then moisture contents are primarily governed by precipitation and evaporation. The secondary influence of soil properties is governed both by the $k(\psi)$ and $\psi(\theta)$ relation. To demonstrate what is remaining of the influence of the choice between a power curve and an exponential curve, a non-steady state calculation has been made. To that end finite difference models were used, based on equations (5) and (8), and (5) and (9) respectively. In both models the slope in the $\psi(\theta)$ relation was constant at 0.1, which means that the moisture loss at

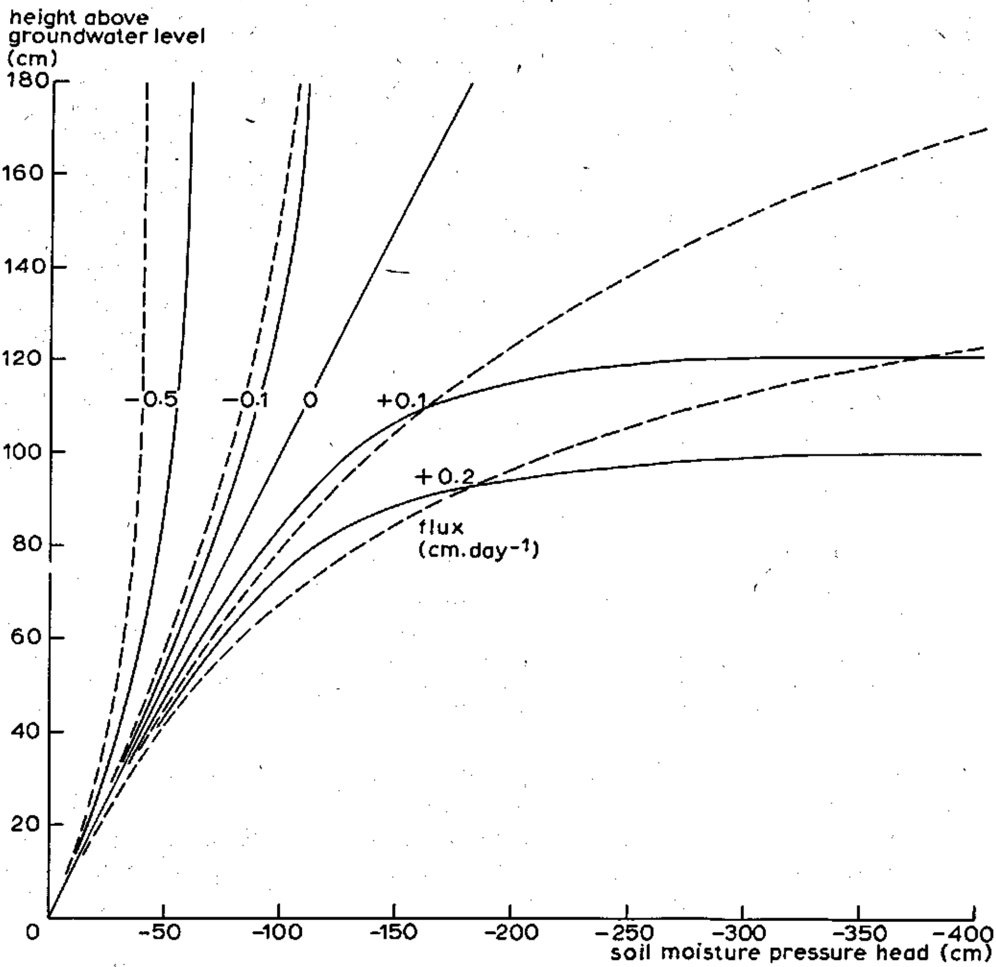

Fig. 3. Steady state soil moisture pressure head profiles calculated with the exponential curve (——) respectively power curve (– – –) of Fig. 2.

$\psi = -100$ cm is 10%. The initial condition taken was a static equilibrium with a groundwater depth of 80 cm; the latter was kept constant at every time. The evaporation-precipitation sequence was: 1 day with 0.5 cm rain followed by 7 days with 0.2 cm $\cdot$ day^{-1} evaporation and then 5 days with an evaporation rate of 0.5 cm $\cdot$ day^{-1}. Fig. 4 gives the result. The differences between the pressure head profiles calculated with the power respectively exponential curve are clearly smaller than in Fig. 3. It can be concluded that steady state considerations require a higher accuracy in the knowledge of soil properties than dynamic models.

With steady state flow equations, the flux between two pressure head values at a certain distance can be calculated. For example between zero pressure head at groundwater level and a soil moisture pressure head of −16,000 cm in the root zone. So if it is known how much moisture per day is required to rise from below the root zone, the appropriate groundwater depth can be calculated.

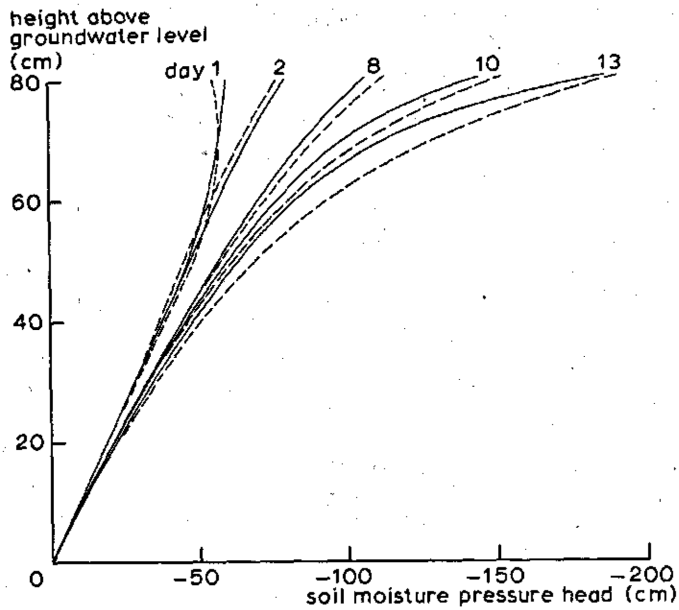

Fig. 4. Non-steady state soil moisture pressure head profiles during a time sequence calculated with finite difference models based on exponential (——) and power (- - -) $k(\psi)$ relations. From static equilibrium at zero time 0.5 cm precipitation until day 1; 0.2 cm · day^{-1} evaporation until day 8 and 0.5 cm · day^{-1} evaporation until day 13.

There are some problems, however. The amount of moisture required changes from year to year and from day to day; the moisture uptake from below the root zone is only partly extracted from below the groundwater table and the latter is seldom at a constant depth. The question which is the best depth of groundwater table is clearly a non-steady state problem and steady state flow equations can only give a rough indication.

So there is need for a method not only dealing with flow of moisture but also with moisture accumulation and depletion, i.e. a non-steady state method.

2.3 Pseudo-steady state sequences

Because steady state considerations are not giving a quantitative solution of soil moisture problems, other methods had to be developed. The need for this was already felt before computers could perform the elaborate calculations required for dynamic simulation models. Wind & Hidding (1961) developed a method to calculate the amount of accessible moisture below the root zone over a certain period. With approximately the same method Wind (1963) calculated the time used to reach a certain moisture content in the top soil at a given evaporation rate.

The principle of this method is the assumption that moisture extraction takes place according to a succession of steady state moisture profiles. Every steady state profile is fully determined by two data; these can be the values of ψ at two depths z, or one such point and the flux. To calculate the amount of accessible moisture below the root zone, the boundary condition $\psi = -16{,}000$ cm at the lower end of the root zone has been chosen. This was combined with a number of decreasing fluxes, each of which yielded a pressure head profile, see Fig. 5. The amount of moisture between two profiles can easily be calculated. If one assumes that the extraction rate is the mean of the fluxes of the two profiles it can be calculated how long the extraction lasts. After calculating this transition time for every pair of fluxes, the total amount of moisture below the root zone accessible within a growing season can be found. Because of the choice of the boundary condition $\psi = -16{,}000$ cm this is the maximum available amount. In reality the amount will be smaller because it takes some time before that boundary condition is reached.

This method was used to calculate either the amount of accessible moisture in a given period or the time required for the drying out of the profile to a certain degree. It can also be used in a simulation model. This has been developed by Rijtema (1970) and de Laat (1976) who called it the pseudo-steady state model.

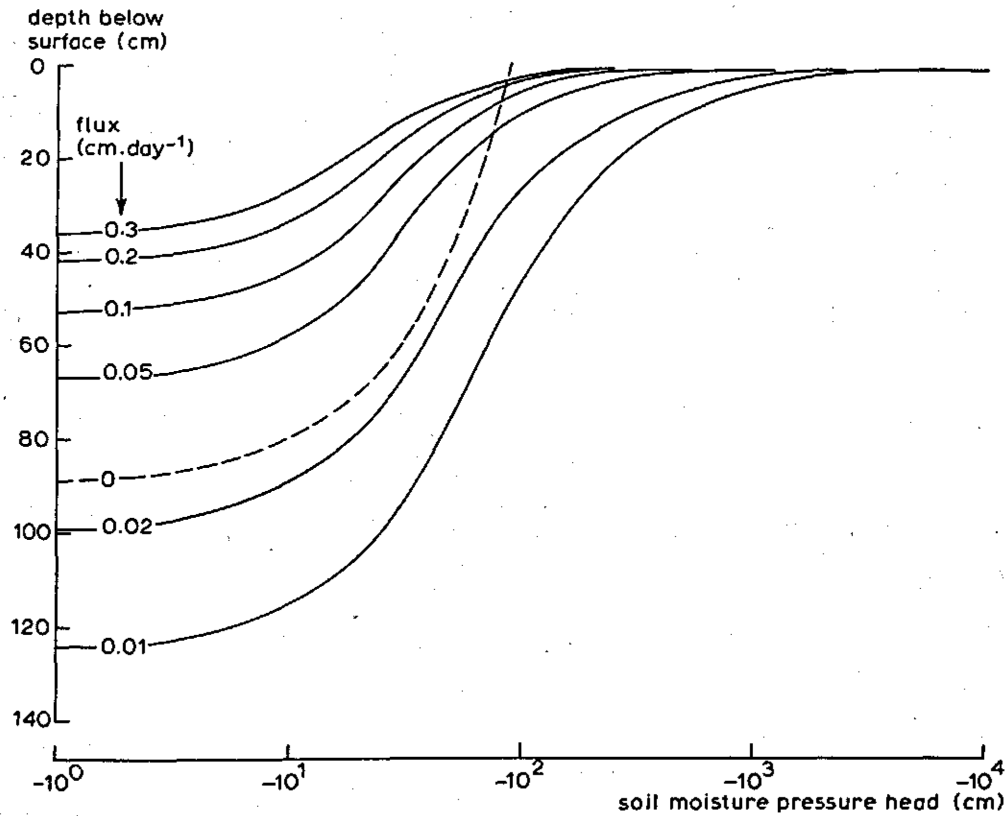

Fig. 5. Soil moisture pressure head profiles in a sandy soil (——) with boundary condition of $\psi = -16{,}000$ cm at zero depth and the corresponding fluxes and one profile (- - -) with boundary condition groundwater table at 90 cm depth and zero flow. After Wind & Hidding (1961).

In steady state flow the flux is constant both in time and depth; in dynamic models flux is varying in time as well as depth. In pseudo-steady state models flux is assumed to be constant in depth but varying in time. Though this assumption is unrealistic, it had to be made to perform quantitative calculations, formerly because dynamic models were not available, now because they are too expensive.

It is not easy to estimate the deviations caused by this erroneous assumption. However, the results of pseudo-steady state calculations can be compared with those of dynamic models; an example of this is the following.

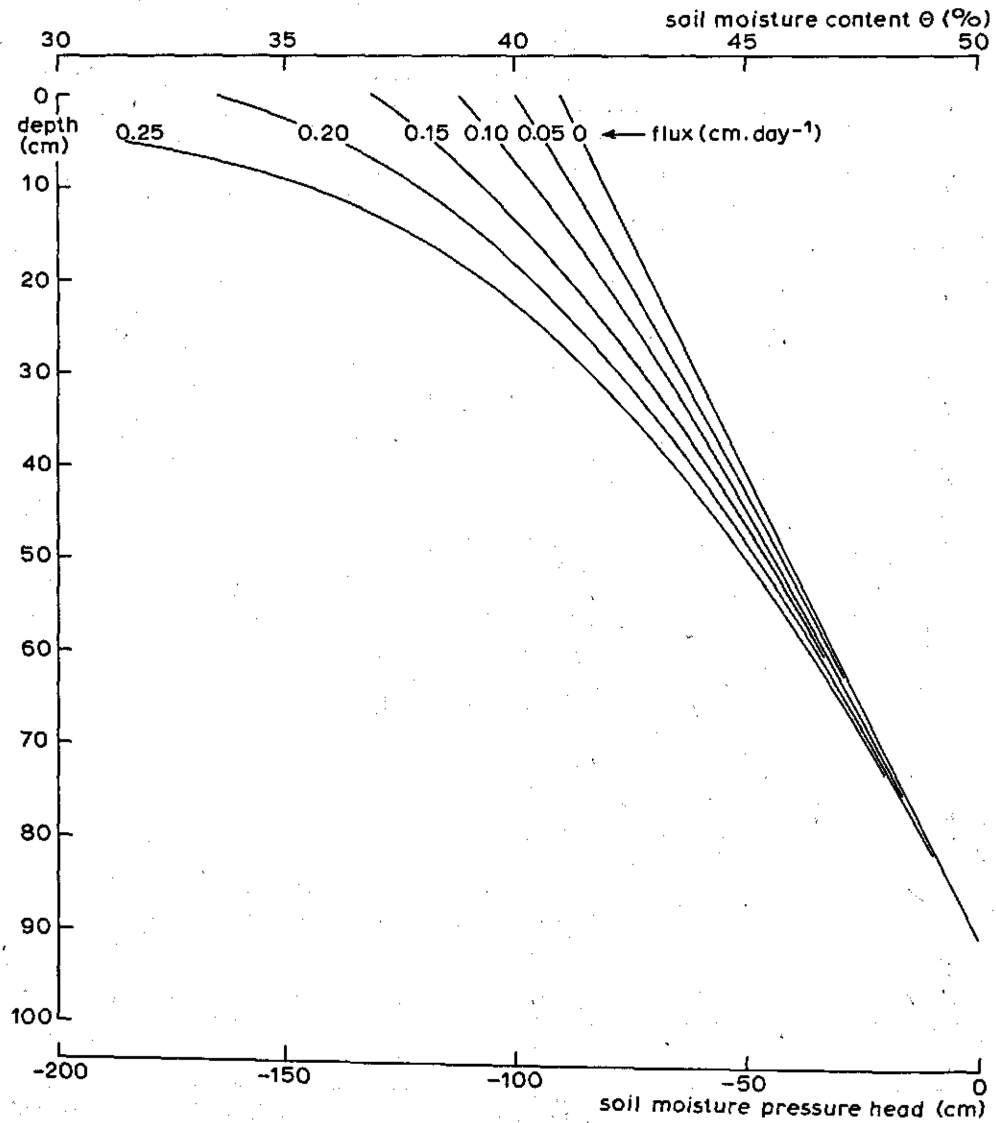

Fig. 6. Steady state moisture profiles for six upward fluxes and a constant groundwater depth at 90 cm.

A soil with a controlled groundwater table at 90 cm depth is evaporating at its surface with a continuous rate of $0.25\ \mathrm{cm \cdot day^{-1}}$. The initial condition at time $t = 0$ is zero flow. The soil physical properties are:

$$\psi = 10\theta - 500 \qquad (\theta \text{ in vol. \%})$$
$$k = 2e^{0.025\psi}$$

The steady state moisture profiles are given in Fig. 6 for $v = 0$; 0.05; 0.1; 0.15; 0.20 and $0.25\ \mathrm{cm \cdot day^{-1}}$. From this figure the total amounts of moisture in the unsaturated zone pertaining to the mentioned fluxes can be read. These are shown in Table 1, column 2. The differences between them are the amounts of moisture which are to be extracted before a new steady state is reached, column 3. The extraction rate, column 4, is the difference between evaporation rate ($0.25\ \mathrm{cm \cdot day^{-1}}$) and mean flux. By dividing column 3 by column 4 the duration of the extraction is found (column 5). The moisture contents at 5 cm depth belonging to the mentioned steady state steps, can be read from Fig. 6 and are given in column 7.

The moisture contents at 5 cm depth also have been calculated with the FLOW-model of Wind & van Doorne (1975) under the same conditions, see column 6. Comparison of columns 5 and 6 shows considerable differences: the pseudo-steady state method overestimates the time to reach a certain moisture content. This is caused by an overestimation of capillary rise which is inherent to this method. In Fig. 7 the vertical fluxes at 90 cm depth are shown as calculated with the FLOW-model and as assumed by the pseudo-steady state method. At first, the pseudo-steady state method overestimated capillairy rise; after 8 days the differences became negligible.

In this example with a constant groundwater table, the drying time is overestimated by the pseudo-steady state method. In another example, see Chapter 4,

Table 1. Moisture extraction by a $0.25\ \mathrm{cm \cdot day^{-1}}$ evaporation rate from the surface of a soil (see text) with a constant groundwater table depth of 90 cm. Data calculated by the pseudo-steady state method and by the dynamic model FLOW.

Flux (cm day^{-1}) 1	Total amount of moisture (cm) 2	Difference (cm) 3	Extraction rate (cm · day^{-1}) 4	Duration extraction: pseudo-steady state method (days) 5	Duration extraction: dynamic model (days) 6	Moisture content at 5 cm depth (%) 7
0	40.950					41.50
		0.265	0.225	1.18	0.50	
0.05	40.685					40.65
		0.306	0.175	2.93	1.66	
0.10	40.379					39.60
		0.368	0.125	5.87	3.80	
0.15	40.011					38.30
		0.451	0.075	11.88	9.23	
0.20	39.560					36.15
		0.840	0.025	45.48	∞	
0.25	38.720					31.30

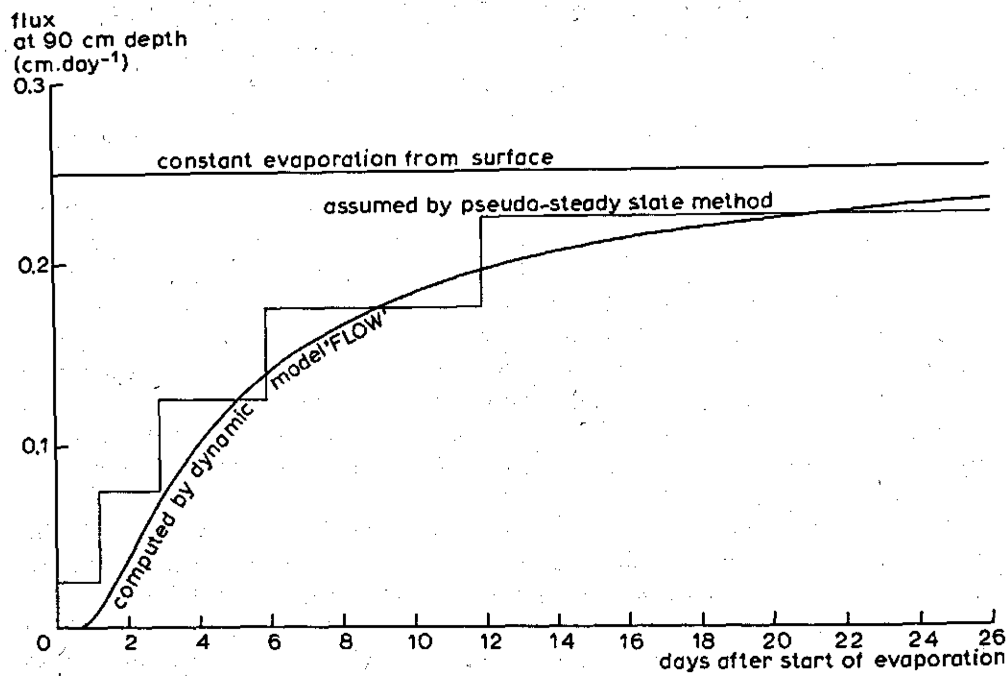

Fig. 7. Capillary rise rates (fluxes at 90 cm depth) as a function of time, computed by dynamic model FLOW and by the pseudo-steady state method.

with a falling water table, drying time is underestimated by this method. So some doubt is indicated about the accuracy of the pseudo-steady state method.

2.4 Numerical model FLOW

For many purposes the effect of actual rainfall on the moisture condition of the topsoil must be calculated. Workability improvement by drainage is such a purpose. It can of course be studied, without calculations, on experimental fields, but because of the tremendous variability in amount and distribution of rainfall such experiments have to last many years. Therefore reliable information from such experiments often comes at too late a time. By calculation one can make use of recorded rainfall data of the past, assuming that the climate is not changing. Thus the results are available in a short time.

To this end Wind & van Doorne (1975) developed a numerical model of the unsaturated zone. The input of the model consists of:
- natural precipitation and evaporation
- maximum value of pool depth (surface storage)
- moisture characteristic in table form
- $k(\psi)$ relation in the form of the expression of Rijtema (1965)
- drain depth
- drain intensity

The output provides information about:
- actual pool depth
- run-off
- depth of groundwater table
- drain discharge
- moisture content and pressure head at every depth

2.4.1 *Averaging conductivity values*

In order to calculate the flux, the discrete shape of flow equation (5) is needed:

$$v = -\bar{k}\left(\frac{\Delta\psi}{\Delta z}+1\right) \tag{5a}$$

A certain depth interval $\Delta z = z_2 - z_1$ is to be selected; the gradient of ψ over this interval is $\psi_2 - \psi_1$. Moreover a certain average value of k has to be chosen somewhere between the k-values pertaining to ψ_2 and ψ_1. The procedure changes the differential equation into a finite difference equation. Van Keulen & van Beek (1971) took for $\bar{k}$ the arithmetic mean, Feddes et al. (1978) are using both the geometric and arithmetic mean.

The model of Wind & van Doorne (1975) makes use of the integrated flux equation:

$$v = \frac{k_2 - ak_1}{a-1} \qquad (a = e^{\alpha\Delta z}) \tag{10}$$

This expression is obtained under the assumptions that ψ is a differentiable function of z, that v is constant over the depth interval during the time interval and that an exponential $k(\psi)$ relation exists. This integrated expression was chosen because of its simplicity, which reduces computer cost.

It appeared that Eq. (10) yields better results than the difference equations with an averaged k. For steady state conditions Eq. (10) is certainly correct; fluxes calculated according Eq. (5a) with a harmonic or geometric mean often differ more than one order of magnitude from the true flux. The arithmetic mean gives the least deviation, but in some cases even the thus calculated velocity is 3 to 7 times higher than it should be.

The errors are counter-balanced by the feed-back system automatically present in such calculations. A flux calculated too high causes gradients to decrease and therefore also decreases flux. However, the errors made are so large that they perceptibly influence the calculated moisture profiles.

The effect of errors caused by averaging conductivity values depends on the depth interval. If this is very small, the effect is negligible. The choice of depth intervals has, however, large consequences with regard to computation costs.

2.4.2 *Choice of time and depth intervals*

Because computing costs decrease with an increase of step size it was tried to maximize the latter. The choice of depth interval depends on the accuracy to be achieved; the time interval can freely be chosen up to a certain limit. Too large time steps cause oscillations in the results with amplitudes increasing at each step.

The assumption that flux is constant during the time interval introduces errors. The cause of oscillation is that these errors are amplified during the next time step. Amplification is proportional to time step size, so the time interval is to be chosen such that the absolute value of the amplifier is smaller than 1. In that case any error made is reduced in the next time steps. The condition for stability is:

$$\Delta t < \Delta z \cdot \frac{e^{\alpha \Delta z} - 1}{e^{\alpha \Delta z} + 1} \cdot \frac{d\theta}{dk} \tag{11}$$

This means that the time interval should be:
- inversely proportional to k_0
- about proportional to α
- about proportional to the square of Δz

2.4.3 Computing costs

The costs of simulation of one day strongly depends on the choice of layer depth and also on the conductivity of the soil. Thin layers and coarse soils give high computing costs. With the discussed numerical model the simulation costs of one day vary between Dfl 1 and 10 (US $ 0.4 to 4.0).

For runs simulating several years computing costs are prohibitive, which means that cheaper ways of calculation must be found.

3 Analog models

3.1 Hydraulic model

The flux in the unsaturated zone in the soil is the product of conductivity and gradient. The main problem in modeling the unsaturated zone is that conductivity does not have a constant value but depends on the moisture content of the soil, the calculation of which is the aim of modeling.

A method to break through this interrelation is to construct a contraption in which conductivity automatically depends on moisture content. Such a solution has been found in a hydraulic model in which the moisture content in the soil is represented by the amount of water in a vessel and in which the conductivity is represented by a number of tubes connecting two vessels at many levels. Nearly full vessels (wet soil) are connected to their neighbour by many tubes (high conductivity). Nearly empty vessels (dry soil) are connected with as many tubes, but only a few of them can conduct water because the other ones are above the water table in the vessels. Fig. 8 gives an outline of the hydraulic model used by Wind (1976) for the simulation of workability.

This is a very simple solution for a problem the author already tried to solve twenty years ago. It is not a question of technical evolution during that time; the analog could have been constructed in 1955 when knowledge about the $k(\psi)$ relation became available, but this particular idea did not come to mind. It later originated when reading a hobby journal containing an article on automation of model railways. An electronic circuit was explained to the readers with the aid of the analogy with a hydraulic example: a vessel that was emptied by openings at three levels. Suddenly the idea came to realize the automatic dependency of moisture content and conductivity with a hydraulic model.

3.1.1 *Development of the model*

Whereas the concept of the model was very simple, some problems had to be solved during the development. The first was that vertical flow deals with two heads; conductivity is controlled by soil moisture pressure head, but flux is controlled by the gradient in hydraulic head. This could be solved by placing the vessels at different heights, so that the height of the water table in a vessel represents the hydraulic head ϕ and the distance between water table and the top of the vessel represents ψ. The possibility to express the two potentials involved in the flow process by one medium is an enormous advantage of the hydraulic analog above an electric one.

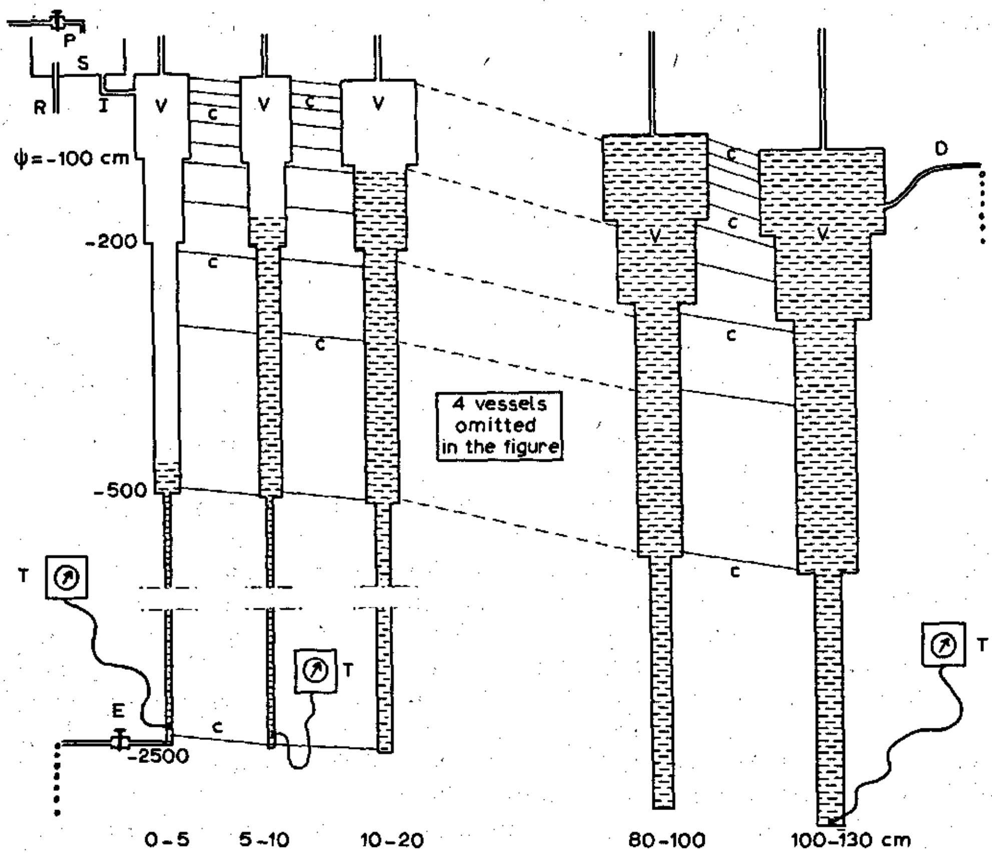

Fig. 8. Outline of a hydraulic analog. V vessels representing soil layers of a certain thickness and moisture characteristic; C connecting tubes, representing hydraulic conductivity; P precipitation valve; E evaporation valve, both valves commanded by a papertape reader; S surface tank; R run-off pipe; I infiltration tube; D drainage tube; T pressure transducer, connected with data logger.

The shape of the vessels had to be chosen such that if the amount of water in them is representing moisture content, the water level represents soil moisture pressure head. A cylindrical vessel represents a straight moisture characteristic, a conical vessel a curved one. Generally the shape of the vessels must be proportional to the first derivative of the moisture characteristic to ψ.

The connecting tubes give a discrete imitation of the continuous relation between conductivity k and soil moisture pressure head ψ. The possibility to choose both length and diameter, as also the number of connecting tubes, allows a good approach. The drainage of the soil can be represented by a tube connected to the vessel at a level representing drain depth. Its length and diameter determine drain intensity. Rainfall is represented by adding water to the first vessel, evapotranspiration by extracting water from it.

3.1.2 Selection scales

Three scales are involved in this model:
S_v, the vertical scale, is the length (m) in the model representing one meter in depth or pressure head in reality;
S_a, the scale for area is the amount of water (cm^3) in the model representing one cm of water in reality. This scale has the dimension l^2, it gives the surface area of a soil column in which the amount of water is the same as in the model;
S_t, the time scale, is the model time representing one real day.

The selection is the result of a number of compromises. A small time scale gives the opportunity to make many observations in short time, but it also causes a turbulent flow in the tubes. A large vertical scale promotes accuracy but it increases model size. The same holds true for the area scale; moreover a large S_a may cause turbulence to appear.

Although there is large degree of freedom in the choice of scales, the time scale cannot be chosen very small because it confines the maximum conductivity which can be represented by the model. Mostly used was a model time of 5 minutes representing one real day. This makes the hydraulic analog a rather slow model. Simulation of 4 months each over 23 years took about 10 days of continuous operation. Once automation of the model was achieved, simulation of long runs was possible with negligible costs.

3.1.3 Two errors in the model

The flow of water through the tubes has the correct value when both ends of the tubes are below the water tables of the vessels they connect. If there is a considerable difference in moisture content between two layers, there is also a large difference in water level in two adjacent vessels. A number of connecting tubes then ends above the water table in the 'driest' vessel. In that case two serious errors are made:
- the gradients in the tubes ending above a water table are lower than they should be;
- the conductivity is determined by the wettest vessel while it should be an average as determined by both vessels.

The effect of each of these errors on the flux can be large but they are working in opposite directions: the gradients are too low, the conductivity is too high. The combined effect of the two can be calculated. The flux in the model is:

$$V_m = \frac{k_2 - k_1}{\alpha \Delta z} - k_1 \tag{12}$$

and it should be:

$$V = \frac{k_2 - k_1}{e^{\alpha \Delta z} - 1} - k_1 \tag{10}$$

With increasing $\alpha\Delta z$ the deviation between the two equations increases at first, but when $\alpha\Delta z$ becomes still larger the influence of the fraction term is becoming less. So there is a maximum deviation of 30% occurring when $\alpha\Delta z = 1.8$ and $k_2 = 0$. The deviation can be confined by the choice of Δz.

The above concerns deviations in steady state situations. In reality they will be much smaller, as they are reduced by a feedback system present both in soil and model. It appeared that a good similarity was found between the moisture distributions calculated by the hydraulic analog and a numerical model in a non steady situation.

In Chapter 5 some applications will be shown in which drain depth and drain intensity are varied, the former seeming more important than the latter. The most striking example, however, is that of a simulation over 6 months of pressure head in the top soil, drain discharge and depth of groundwater table.

There are, however, other problems: the model is fairly slow by simulating one day in 5 minutes and any change of soil properties is laborious. Therefore another model was developed, an electronic analog.

3.2 Electronic analog ELAN

It was tried to find a concept for a model combining the advantages of the hydraulic analog (low costs) as well as of the mathematical model (velocity and easy adjustability). This was realized in an electronic analog, developed in cooperation with the Technical and Physical Engineering Research Service (TFDL, Wageningen), see Wind & Mazee (1979).

At first it was tried to make an electrical model based on the same principles as the hydraulic analog. Moisture pressure head should be represented by electric potential and conductivity by resistors. The connecting resistors could be opened and closed by electronic valves, commanded by the electric potential. It appeared to be very difficult, however, to avoid the occurrence of the same errors as present in the hydraulic analog. There a wrong conductivity and a wrong gradient are compensating each other. In an electrical copy a wrong conductivity would also occur but the gradients will be correct. So a device had to be made to introduce the correct conductivity. This was found, but it made the model too expensive.

Some years later a new idea was born: an electronic analog based on the same integrated flow equation as Wind & van Doorne's (1975) numerical model. In this model the electric potential does not represent pressure head but conductivity. Electric resistors are representing the factor $a = e^{\alpha \Delta z}$. A steady state model of this type could be made in some minutes from a couple of resistors.

The principle of the model is the resemblance between Wind & van Doorne's flow equation:

$$v = \frac{k_2/a - k_1}{\frac{a-1}{a}} \qquad (10)$$

and Ohm's law:

$$i = \frac{E_2 - E_1}{R} \tag{13}$$

If conductivity k_1 is represented by electric potential E_1, flux v by electric current i and the value $(a-1)/a$ by resistance R, the potential E_2 represents the conductivity k_2 divided by a.

3.2.1 *Development and scales of the model*

The division of k by the factor a accumulates with depth, so that in layer number n the electric potential E_n represents k_n divided by a^{n-1}. To avoid this accumulation, in every junction (layer) an amplifier is installed multiplying the potential with the adjustable factor a.

Every layer contains a capacitor, the charge of which represents moisture content θ. As the relation between moisture content and conductivity is not a linear one a function generator is also required. In this procedure the $k(\theta)$ relation is divided into three straight line segments.

In this model the choice of time scale is nearly unconfined; it has been chosen at 2 seconds representing one day to make it possible to use line recorders and tape-writers. A velocity of 1 cm $\cdot$ day^{-1} is represented by 10 μA and a conductivity of 1 cm $\cdot$ day^{-1} by $\frac{1}{3}$ V.

The model contains 10 layers and in each of them the values of conductivity and moisture content can be read without affecting the functioning of the model. Fig. 9 gives a picture of the model.

Rain and evapotranspiration are fed into the model's top layer with a paper tape reader. A device is in preparation which can reduce evaporation in dependency of the moisture condition in the first layer. In off position of the tape reader, an adjustable constant rain rate is applied.

3.2.2 *Special functions*

A device representing the soil surface is called 'top layer'. If the precipitation rate exceeds the infiltration rate, the difference is ponded upon the surface; in the model electricity is stored in a capacitor. A function generator, adjustable for k_0 and α, translates the amount of ponded water into the corresponding value of k.

The lowest soil layer is connected with a device, called 'drain layer'. The drain is thought to be in the middle of the lowest layer. Because the drain discharge rate of a given system in a given soil is governed by the positive pressure head ψ and not by the varying value of k, a logarithmic module is used. The soil values α and k_0 and the drain intensity A can be adjusted. So drain depth and drain spacing can be chosen freely.

The principle on which the model is based, the resemblance between Ohm's law and the integrated flow equation, gives problems with non-homogeneous soils. Between two soil layers of different composition the values of θ and k, both

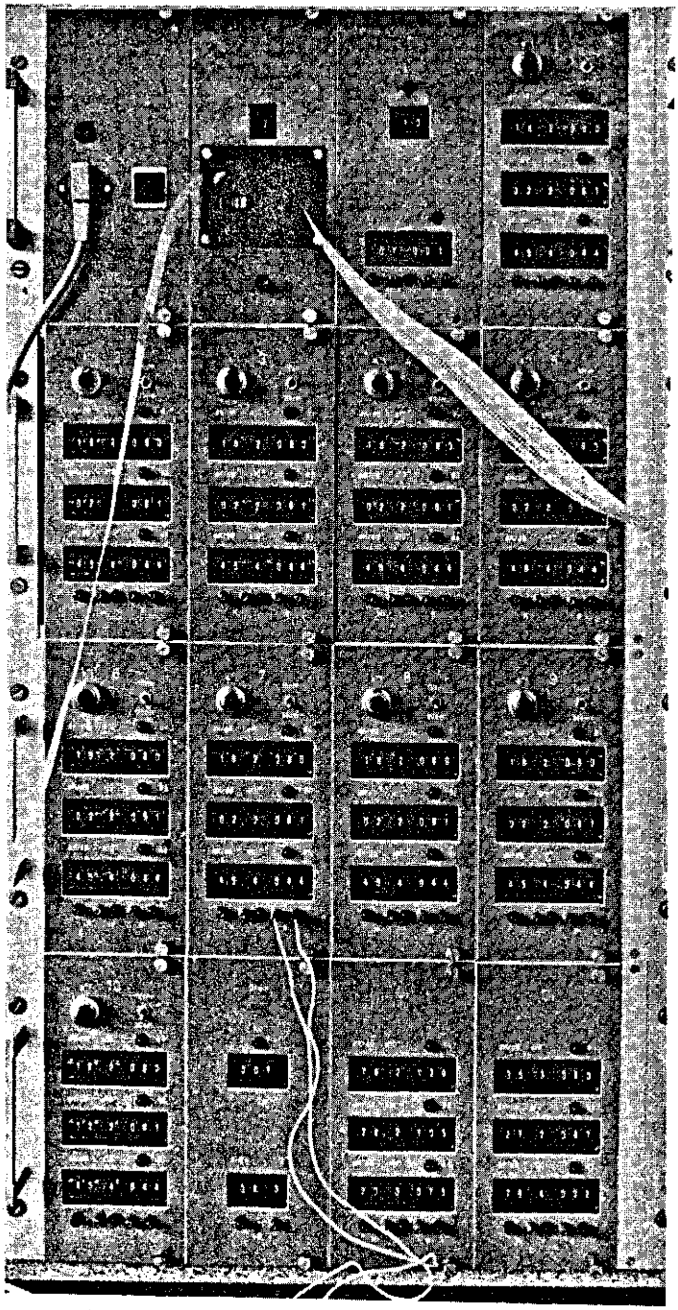

Fig. 9. Electronic analog ELAN. With thumbwheel switches the $k(\theta)$ relation can be adjusted, the potentiometer gives the value of $a = e^{\alpha \Delta z}$.

present in the model, are different at the boundary; the value of ψ is the same, but this factor does not operate in the model.

In the model transition layers are used which translate the value of θ in the upper layer into the value of k that it should have if it had the same composition as the lower layer. The transition layers at the boundary between two differing soil layers consist only of a function generator, whereas resistor and capacitor are absent. The $k(\theta)$ relation in such a transition layer is to be composed of the $\psi(\theta)$ of the upper soil and the $k(\psi)$ of the lower soil.

In principle the electric analog should have an additional circuit for representing saturated flow. Up to now this is not the case. So in saturated conditions the model is operating with the same equations as in unsaturated conditions. The value of θ can not increase above its maximum, i.e. saturation, value. The conductivity, which should be k_0 in this condition, can exceed this value manyfold, however. This results in too deeply calculated groundwater levels. This must be counter-balanced by using a calculated equivalent value of drainage intensity.

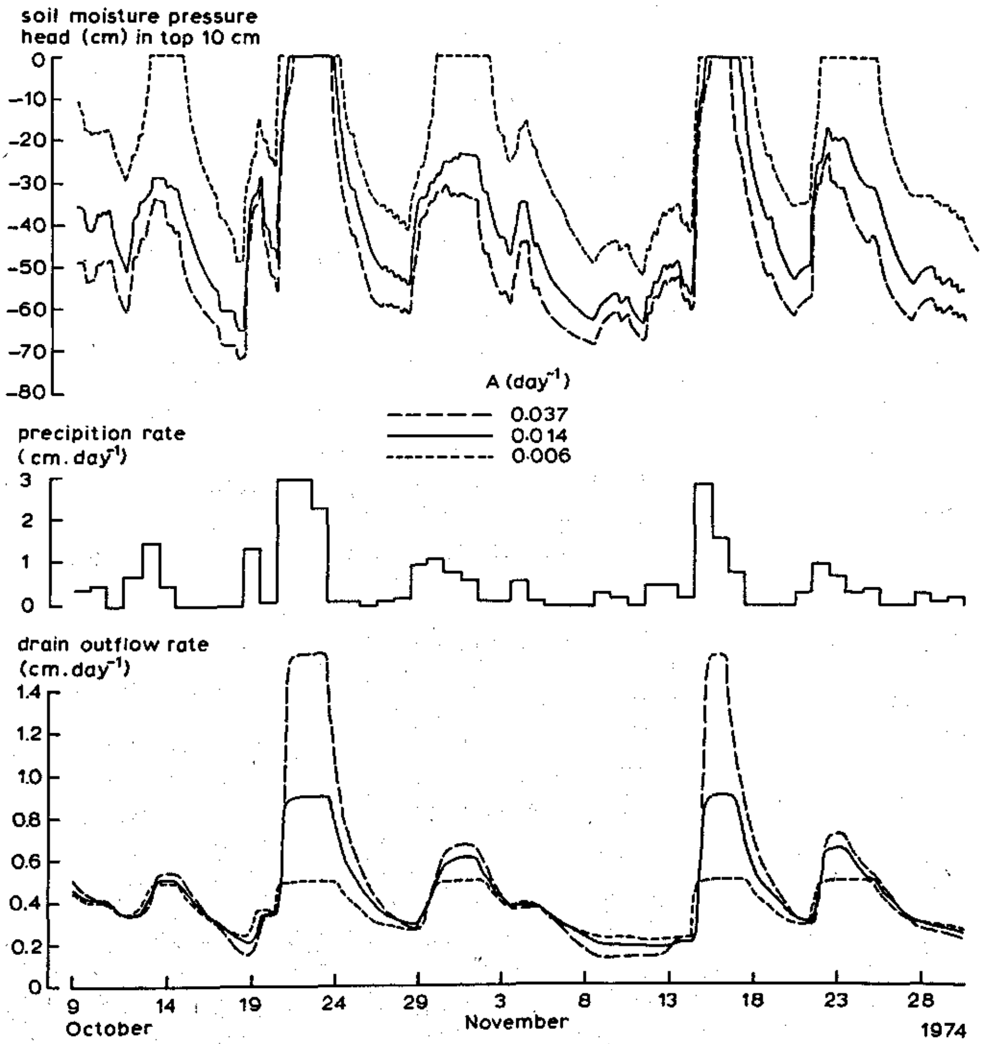

Fig. 10. Effect of drain intensity A on moisture pressure head in the topsoil in the wet autumn of 1974, computed with the electronic analog ELAN.

3.2.3 *Examples of use*

In the paper of Wind & Mazee (1979) examples are given to compare the electronic analog with numerical models; this comparison will be dealt with in Section 5.2.

Three other examples are given here to show some problems to which the model can be applied. The first of these, see Fig. 10, is the simulation of the soil

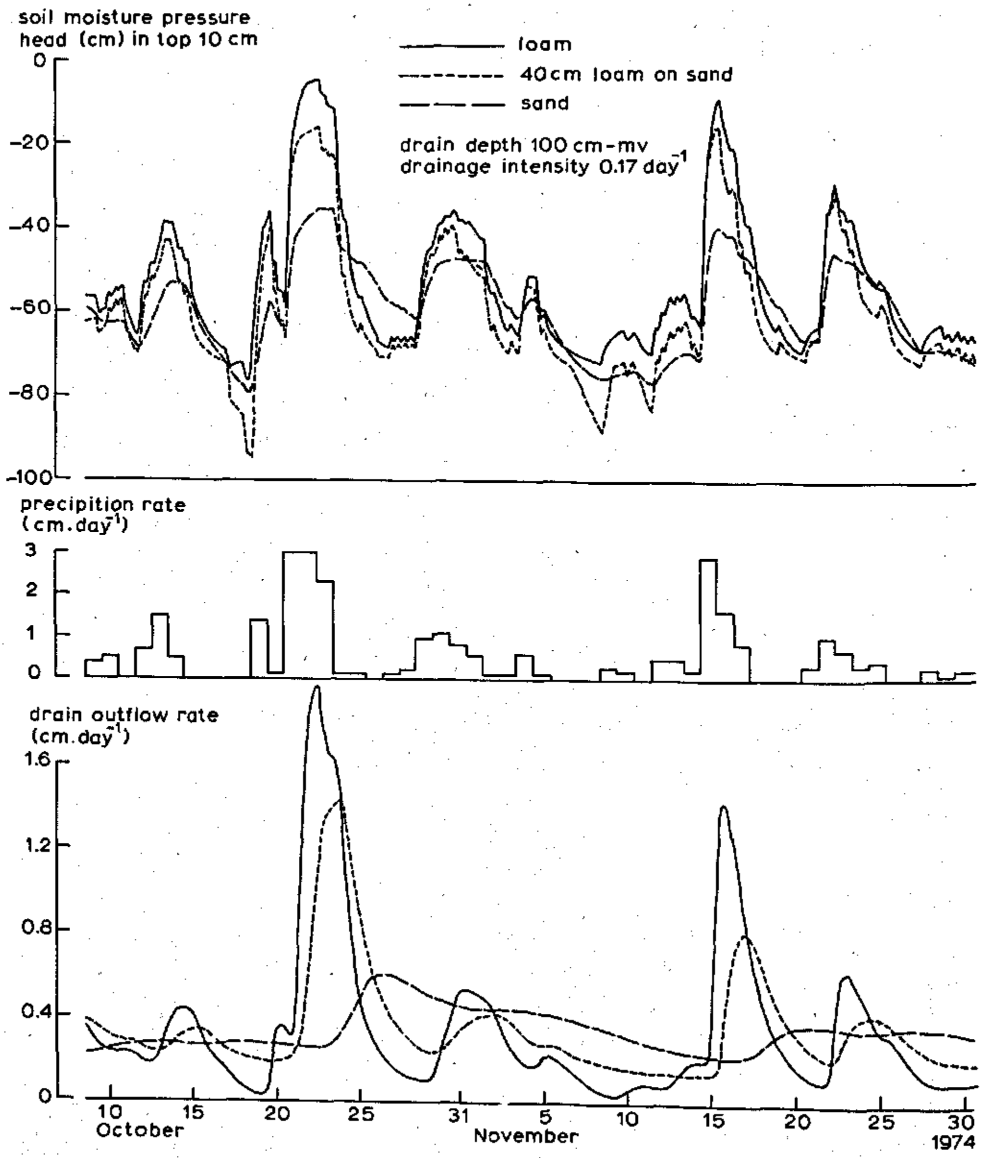

Fig. 11. Precipitation, soil moisture pressure head in the top 10 cm and drain outflow in three soils during the autumn of 1974 as simulated with ELAN.

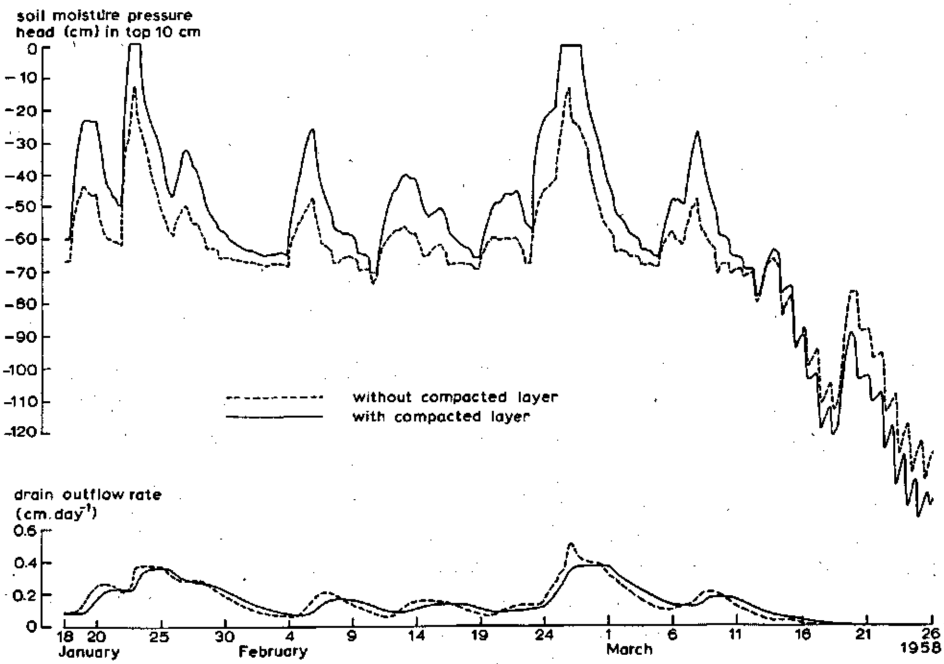

Fig. 12. Effect of a compacted layer between 20 and 30 cm depth on the moisture condition of the topsoil and the drain outflow rate.

moisture pressure head in the top 10 cm of a loam soil during the extremely wet autumn of 1974 for three drain spacings. It shows how the pressure head varies from day to day. Although the soil with the highest drain intensity (0.037 or about 2.5 times normal) was less wet than the others, drier conditions than those corresponding with a pressure head of −80 cm were not observed, while a head of −90 cm is required for the harvesting of potatoes.

Another example, here reproduced in Fig. 11, shows the soil moisture pressure heads as having occurred during the autumn of 1974 in the top layer of three soils, all drained at 100 cm depth with an extremely narrow spacing. The soils used are a loam, a sand and a soil consisting of 40 cm loam on sand (in Dutch: plaatgrond). In wet periods the loam is wettest and the sand the driest soil, the loam on sand has an intermediate moisture content. In dry periods during this wet autumn, the loam on sand is in most cases the driest and the sand often the wettest soil. The differences between the drain outflow graphs of loam and sand are as is to be expected when comparing soils with low and with high moisture storage capacities. Although the groundwater table in the loam on sand never did rise into the loam layer, the drain outflow nevertheless nearly behaved as that of a loam soil.

The last example, Fig. 12, shows the effect of a dense soil layer in winter and spring. During rainy periods the soil with a compacted layer is wetter than the one without it. In dry periods in March the former is drier than the latter because under the given conditions the compacted layer is impeding capillary rise.

4 Comparison of methods

4.1 Example characteristics

With the aid of the methods mentioned in the Chapters 2 and 3, it is possible to predict the effect of changes in soil properties or drainage. A practical example is chosen to illustrate the results obtained with different methods to predict the effects of ameloration.

A soil, LCS, consisting of 40 cm loam on 20 cm heavy clay on sand, drained at 80 cm depth is said to reach the state of workability too late in spring.

Two ameliorations are considered: increasing the drain depth to 110 cm below surface and improvement of the clay layer by subsoiling. It is asked to forecast the effect of each and of a combination of both ameliorations.

The answer will be given:

- with the sole use of steady state considerations (Section 4.2);
- with a pseudo-steady state method (Section 4.3);
- with a numerical dynamic model using standardized weather data (Section 4.4);
- with an analog model using real weather data as input (Section 4.5).

The following assumptions are made. The loam and the clay have an exponential $k(\psi)$ relation, see Eq. (3). For the loam $k_0 = 3\ \mathrm{cm \cdot day^{-1}}$; $\alpha = 0.03\ \mathrm{cm^{-1}}$; for the clay $k_0 = 0.3\ \mathrm{cm \cdot day^{-1}}$ and $\alpha = 0.015\ \mathrm{cm^{-1}}$. The sandy soil is thought to be always in static equilibrium with the groundwater table. So in the sand at any time $\psi(h)$ equals the height h above the groundwater table. The moisture characteristics of loam, clay and sand are as shown in Fig. 13.

With regard to subsoiling, the clay layer then gets the same properties as the loam, its thickness does not change, so the LCS soil changes into a LS soil consisting of 60 cm loam on sand. The drainage is ideal: the groundwater table depth equals drain depth during drain discharge. Except for this, no control of the groundwater table exists, so it may fall below drain depth. The soil is taken to be workable for seedbed preparation when the soil moisture pressure head at surface is -150 cm.

4.2 Steady state considerations

To calculate the effect of drainage and soil improvement on moisture content of a soil in wet periods with steady state considerations, one has to choose a certain constant precipitation rate. In Fig. 14 a downward flux of $0.5\ \mathrm{cm \cdot day^{-1}}$ has been chosen. In the LCS soil this leads to very large gradients in the clay layer, which has a saturated conductivity of only $0.3\ \mathrm{cm \cdot day^{-1}}$.

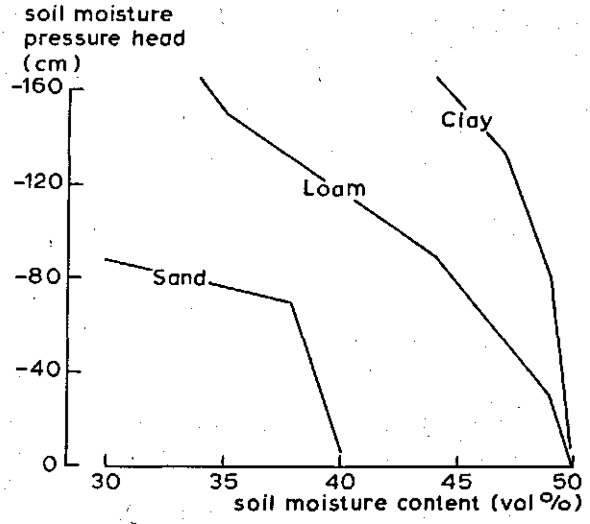

Fig. 13. Moisture characteristics of the example soils L (loam), C (clay) and S (sand).

In Table 2 the calculated pressure heads at the soil surface are given for three fluxes. Both ameliorations, deeper drainage and soil improvement have a favourable effect on the moisture conditions, as with these ameliorations the top soil is less wet than without them. The effect of soil improvement exceeds that of deeper drainage except for the flux of $-0.2\,\mathrm{cm \cdot day^{-1}}$. Especially during very high rainfall rates ($1.0\,\mathrm{cm \cdot day^{-1}}$) the effect of soil improvement is paramount.

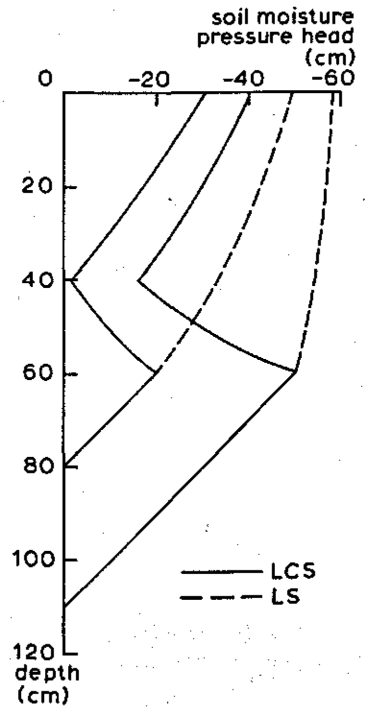

Fig. 14. Soil moisture pressure head profiles in the LCS and the LS soil, drained at 80 and 110 cm below surface, at a steady downward flux of $0.5\,\mathrm{cm \cdot day^{-1}}$.

Table 2. Soil moisture pressure head at the soil surface (ψ_s in cm) for the LCS soil and the LS soil drained at 80 and 110 cm depth respectively for three downward fluxes.

Downward flux ($cm \cdot day^{-1}$)	Soil, respectively drain depth (cm)			
	LCS		LS	
	80	110	80	110
0.2	−53.1	−68.3	−64.1	−79.3
0.5	−30.0	−40.1	−49.0	−57.9
1.0	+3.6	−10.6	−33.2	−38.5

Calculation of the effect of the two ameliorations on the timeliness of field operations is not possible with steady state considerations. Only an approximation can be made. In Fig. 15 steady state pressure head profiles are given as calculated under the assumptions that at the surface $\psi = -150$ cm (workability) and that groundwater depth equals drain depth. The latter assumption is not likely to be true, but every other assumption with regard to groundwater depth is incorrect too.

To reach $\psi = -150$ cm at the surface in the LS soil a higher evaporation rate is required then in the LCS soil. This means that the LS soil will be workable later in spring than the LCS soil, so the effect of soil improvement seems to be negative. On the other hand the LCS soil is likely to be wetter than the LS soil at the beginning of an evaporative period. Which of the two effects is prevailing cannot be concluded from steady state considerations.

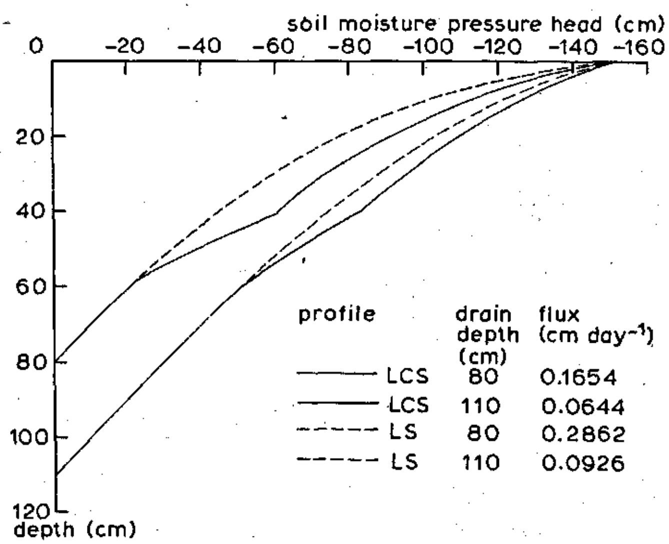

Fig. 15. Steady state pressure head profiles for LCS and LS soil, calculated for a soil moisture pressure head of −150 cm at the surface and groundwater depth equalling drain depth at 80 and 110 cm respectively.

The conclusion is that deeper drainage will work very favourable on the timeliness of field operations. It cannot be said whether soil improvement will cause a negative or positive effect.

4.3 Pseudo-steady state method

To apply the pseudo-steady state method the total amounts of soil moisture have to be calculated for the initial condition, for the end situation and, in this case, for some situations in between. The basis for this calculation are the soil moisture pressure head profiles; for given soil properties these steady profiles are defined by two data. These can be pressure head values at two depths or a pressure head value at one depth and the flux.

The initial condition is fixed by $\psi = 0$ at drain depth and a flux of $-0.5\ \mathrm{cm \cdot day^{-1}}$ (downward) as in Fig. 14. The following sequence of situations has been chosen: -0.5; -0.4; -0.3; -0.2; -0.1 and $0\ \mathrm{cm \cdot day^{-1}}$, all combined with $\psi = 0$ at drain depth. Between these situations moisture is removed both by drainage and evaporation. Between the pressure head profile of $0\ \mathrm{cm \cdot day^{-1}}$ (static equilibrium) and the final situation moisture loss is only caused by evaporation.

The choice of the end situation causes a problem. It makes sense to define the end situation by $\psi_s = -150$ cm at the soil surface (workability requirement) and a flux equalling evaporation rate E_0. However, in some parts of this profile the soil can be wetter than in the initial condition. This is shown in Fig. 16; the curve of $+0.2\ \mathrm{cm \cdot day^{-1}}$ (upward flux) is in equilibrium with a groundwater depth of 73 cm below surface in the LCS soil.

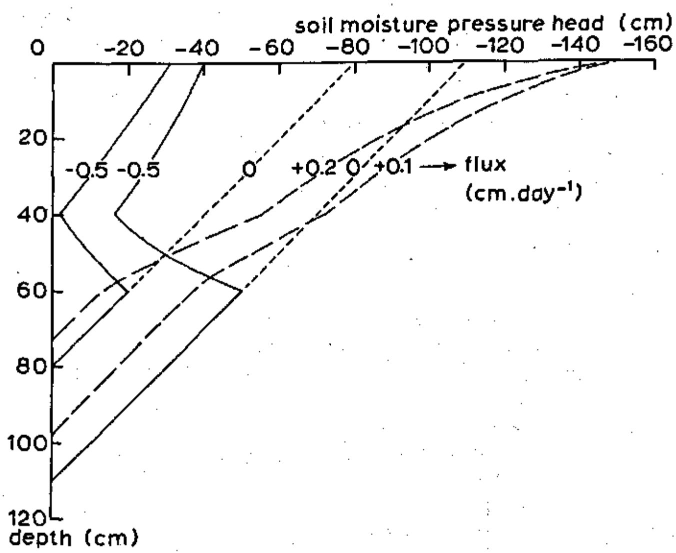

Fig. 16. Pressure head profiles in LCS soil with fluxes of -0.5 and $0\ \mathrm{cm \cdot day^{-1}}$ at two drain depths (80 and 110 cm below surface) and fluxes $+0.1$ and $+0.2\ \mathrm{cm \cdot day^{-1}}$ with a soil moisture pressure head of -150 cm at the soil surface.

Application of this curve assumes an increase in soil moisture in the soil deeper than 50 cm, which certainly is incorrect. To avoid this inconsistency, the [$v = \bar{v}_0$; $\psi_s = -150$ cm]-curve is used to the depth where it crosses the [$v = 0$; $\psi_s = 0$]-curve. For deeper layers the static equilibrium curve is used.

The pressure head profiles are to be translated into moisture content profiles, after which the difference in amount of moisture between these moisture stages is found by determination of the surface area between the curves. Fig. 17 gives an example of these profiles in the LS soil. In Table 3 the differences in amounts of moisture between the seven profiles are given.

These amounts are removed by drainage and evaporation. The duration of extraction is found by dividing the amounts of moisture by the total extraction rate. The total duration is the time required to reach a pressure head of -150 cm at the surface with an evaporation rate of 0.1 $\text{cm} \cdot \text{day}^{-1}$, starting with an initial condition of equilibrium at a precipitation rate of 0.5 $\text{cm} \cdot \text{day}^{-1}$.

The calculations have been carried out for both soils, two drain depths and two evaporation rates. The result is given in Table 4.

From this table conclusions can be drawn about the effects of drain depth and

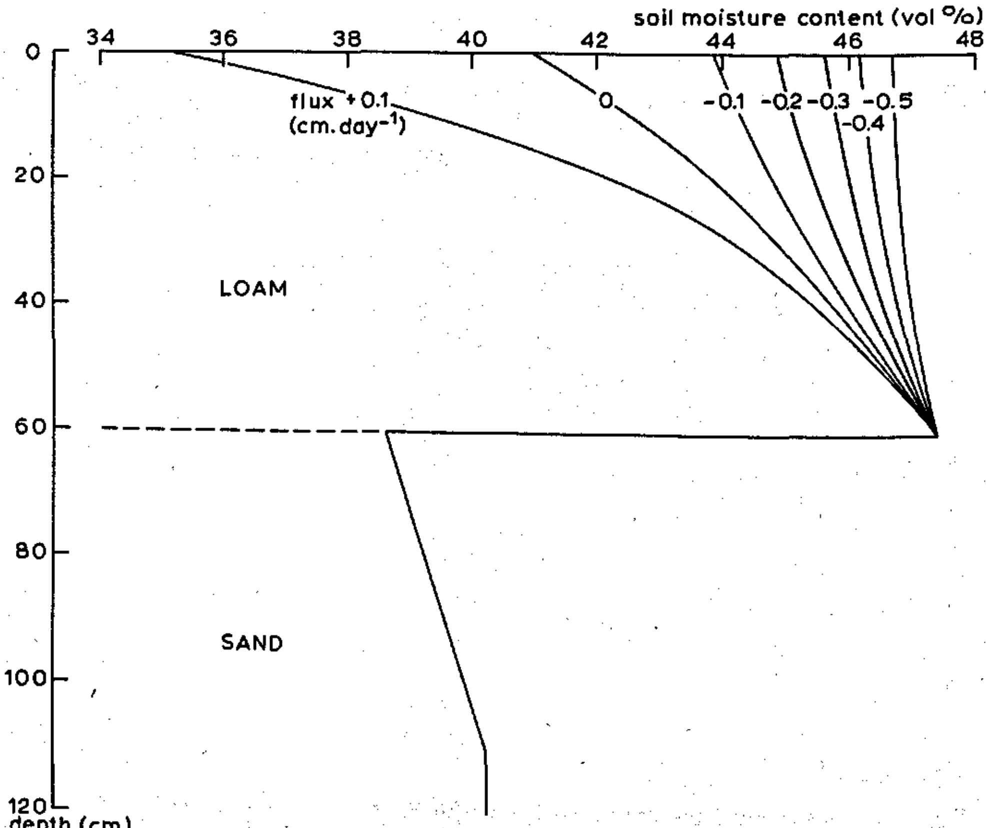

Fig. 17. Steady state moisture profiles in loam on sand soil, LS, drained at 110 cm depth.

Table 3. Differences in amount of moisture between seven steady state soil moisture profiles in the loam on sand (LS) soil drained at 110 cm, as well as mean extraction rates and the duration moisture extraction takes (see Fig. 19).

Steady state flux ($cm \cdot day^{-1}$)	Difference in amount of moisture between this steady state and its forerunner (cm)	Extraction rates by			Duration of extraction (day)
		drainage ($cm \cdot day^{-1}$)	evaporation ($cm \cdot day^{-1}$)	total ($cm \cdot day^{-1}$)	
−0.5 to −0.4	0.170	0.45	0.1	0.55	0.31
−0.4 to −0.3	0.196	0.35	0.1	0.45	0.43
−0.3 to −0.2	0.229	0.25	0.1	0.35	0.66
−0.2 to −0.1	0.283	0.15	0.1	0.25	1.13
−0.1 to 0	0.506	0.05	0.1	0.15	3.37
0 to +0.1	0.781	0	0.1	0.10	7.81
Total	2.165				13.71

soil improvement on the timeliness of field operations. It is clear that deeper drainage has a favourable effect: at a low evaporation rate it about halves the time to reach workability. With a high evaporation rate the effect is less but still positive. Soil improvement has a negative effect at a low evaporation rate, while it is positive at the high evaporation rate; although only small at a drain depth of 80 cm. Combination of deeper drainage and soil improvement is very profitable for both evaporation rates.

The conclusion is that it should be recommended to execute deeper drainage or deeper drainage combined with soil improvement but certainly not soil improvement only.

As regards the earlier reaching of workability conditions, this application of the pseudo-steady state method confirms the conclusion of the steady state method with respect to deeper drainage. As regards soil improvement it shows that this can work out positively or negatively. It gives quantitative information, although it is difficult to select the evaporation rate to be used in the calculations. In March and April, days with 0.1 or 0.2 cm evaporation do not seldom occur, but they are

Table 4. Number of days needed to reach workability in spring ($\psi = -150$ cm), calculated by the pseudo-steady state method. An initial situation of equilibrium with a constant precipitation rate of 0.5 $cm \cdot day^{-1}$ is followed by a dry period with evaporation as indicated.

Drain depth (cm)	Soil, respectively evaporation rate ($cm \cdot day^{-1}$)			
	LCS 0.1	LS 0.1	LCS 0.2	LS 0.2
80	26.5	29.4	10.0	9.4
110	13.1	13.7	7.8	5.2

alternating with rainy days. The effect of such changes is difficult to predict and the pseudo-steady state method gives no opportunity for more refined calculations.

4.4 Dynamic model with standardized weather conditions

A dynamic model has been applied with the same standardized weather conditions as were used in the application of the pseudo-steady method: an initial situation of equilibrium at constant precipitation rate of 0.5 cm · day^{-1}, followed by a period with evaporation rates of 0.1 and 0.2 cm · day^{-1}.

The model used was the numerical model described by Wind & van Doorne (1975). Because the original model can only work with uniform soils it has been modified for a three-layer soil. This modification was necessary in order to calculate the flux v between the upper layer (with conductivity k_1, saturated conductivity k_0 and coefficient α) and the lower layer (with conductivity k_2^*, saturated conductivity k_0^* and coefficient α^*). It was assumed that v and ψ are the same just above and below the boundary between the layers of different composition. When the conductivity at the boundary is called k_b respectively k_b^*, there are 4 equations and 4 unknown values (v, ψ, k_b and k_b^*):

$$\psi = \frac{1}{\alpha} \ln \frac{k_b}{k_0} = \frac{1}{\alpha^*} \ln \frac{k_b^*}{k_0^*} \tag{14}$$

$$v = \frac{k_b - k_1\sqrt{a}}{\sqrt{a}-1} = \frac{k_2^* - k_b^*\sqrt{a^*}}{\sqrt{a^*}-1} \tag{15}$$

where $a = e^{\alpha\Delta z}$ and $a^* = e^{\alpha^*\Delta z}$. From this set of equations the flux v through the boundary can be calculated. For the computations, the same soil properties as in the applications of the pseudo-steady state method were used.

In Fig. 18 the results are given for the loam on sand soil (LS) drained at 110 cm depth at a constant evaporation of 0.1 cm · day^{-1}. The striking differences with Fig. 17 will be discussed later. In Table 5 the time between initial situation and the moment that the pressure head at surface equals −150 cm is given.

The conclusions to be drawn from this table are not entirely identical with those from Table 4. Deeper drainage has a very good effect indeed, but it is smaller than follows from the pseudo-steady calculation. As regards the effect of soil improvement, this only works negatively for the 80 cm drain depth and the low evaporation rate. Although the positive effects of soil improvement are small, Table 5 gives somewhat more doubt with regard to the effect of this amelioration than does the pseudo-steady state calculation.

The largest difference between Tables 4 and 5 is the absolute value of the extraction duration. The dynamic simulation predicts much longer durations than the pseudo-steady state calculations. A priori it was expected that dynamic simulation would give smaller durations, because of a higher extraction rate from the top layers, than is assumed by the pseudo-steady state calculation with its constant fluxes. The reverse turned out to be true. The amount of moisture extracted from deeper layers according to dynamic simulation, is larger than is

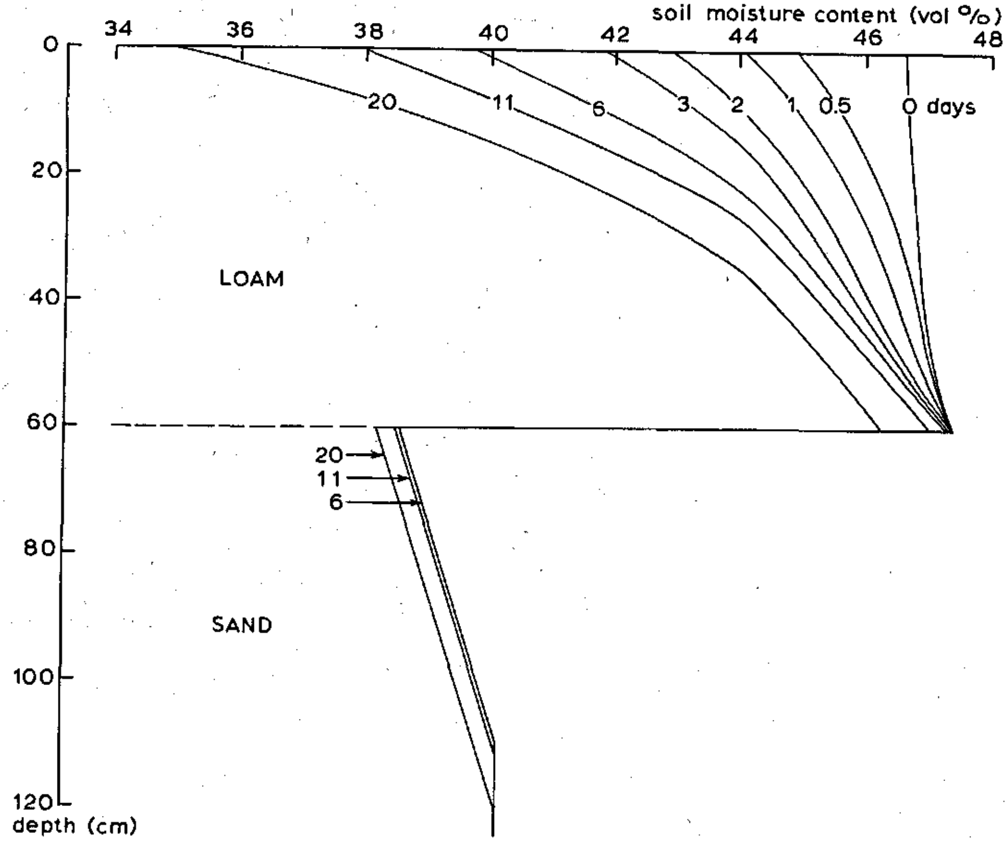

Fig. 18. Moisture profiles in LS soil, drained at 110 cm, as simulated with a dynamic model. The numbers in the figure give the number of days after a constant rain rate of 0.5 cm · day^{-1} did stop. Evaporation rate 0.1 cm · day^{-1}.

calculated with pseudo-steady method. This is shown in Fig. 19 where dynamic and pseudo-steady state moisture profiles are compared. At any time up to 5.9 days the top soil moisture status dynamically simulated is drier than calculated with the pseudo-steady state method. Between 5.9 and 13.7 days this situation changes. On the last mentioned date the pseudo-steady state method calculates a drier top soil than dynamic simulation does. The reason for this behaviour is the assumption in the first

Table 5. Time in days between the initial situation with a constant 0.5 cm · day^{-1} rain rate and the moment that at the surface $\psi = -150$ cm, as calculated with a dynamic model.

Drain depth (cm)	Soil, respectively evaporation rate (cm · day^{-1})			
	LCS 0.1	LS 0.1	LCS 0.2	LS 0.2
80	33.8	37.1	14.0	12.6
110	21.6	19.8	9.4	7.0

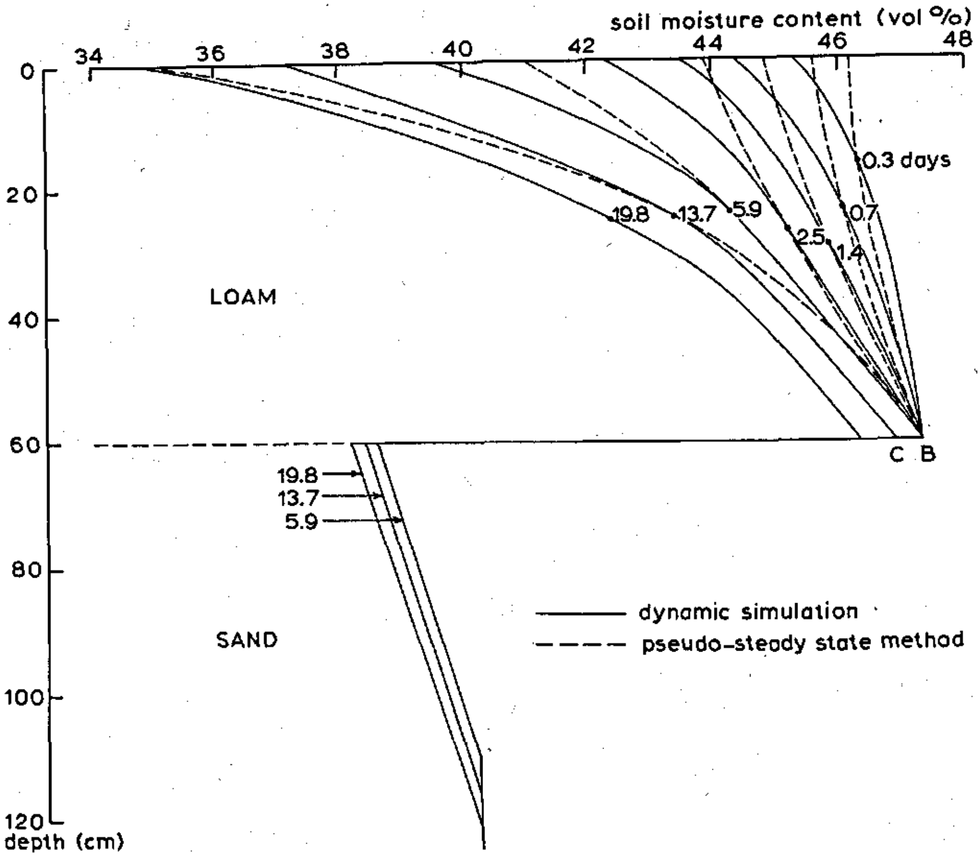

Fig. 19. Comparison of moisture profiles in LS soil, drained at 110 cm, calculated with dynamic simulation and the pseudo-steady state method for the same days after a rain period of 0.5 cm · day^{-1} did stop, given as numbers in the figure. These days correspond with those mentioned in Table 3.

method of a steady flux, in this case 0.1 cm · day^{-1}, which leads to a high moisture content at 60 cm depth when at surface $\psi = -150$ cm is maintained. In reality, and the dynamic simulation pretends to represent reality, the flux decreases from 0.1 cm · day^{-1} at the surface to zero at a certain depth, thus causing a lower moisture content at 60 cm depth, see the points B and C in Fig. 19.

In Table 6 the observed fluxes are given. In the topsoil they are larger than those used in steady state calculation at any time up to day 13.7; then the flux is suddenly smaller than the assumed value of 0.1 cm · day^{-1}. In pseudo-steady state calculations there is no reason to choose another value for the flux than the one equalling precipitation or evaporation rate, but this causes an incorrect gradient in the suction profile. Therefore the pseudo-steady state method is not very fit to calculate absolute values as duration of drying or amounts of available moisture.

As regards the comparison of the two amelioration methods, the results of the pseudo-steady state and the dynamical calculation differ only slightly. The schematic choice of a constant evaporation rate, however, leaves some reason for doubt about the conclusions.

Table 6. Fluxes in cm · day^{-1} as determined by the dynamic simulation of the moisture status of LS soil, drained at 110 cm with an evaporation rate of 0.1 cm · day^{-1} at selected days and depths. The steady state fluxes, as used in the pseudo-steady calculation are also given.

Depth (cm)	Time (days)								
	0	0.3	0.7	1.4	2.5	5.9	13.7	19.8	30.0
10	−0.50	−0.120	−0.048	−0.014	+0.007	+0.054	+0.075	+0.075	+0.074
20	−0.50	−0.268	−0.180	−0.090	−0.040	+0.022	+0.056	+0.057	+0.058
30	−0.50	−0.422	−0.286	−0.166	−0.075	+0.009	+0.041	+0.042	+0.044
40	−0.50	−0.458	−0.366	−0.220	−0.100	+0.001	+0.032	+0.034	+0.031
50	−0.50	−0.483	−0.398	−0.247	−0.115	−0.003	+0.024	+0.026	+0.024
60	−0.50	−0.490	−0.437	−0.269	−0.123	−0.004	+0.018	+0.018	+0.018
Steady	−0.50	−0.400	−0.300	−0.200	−0.100	0	+0.100	—	—

4.5 Analog simulation with real weather data

With the electronic analog model ELAN described by Wind & Mazee (1979) a simulation was made over 6 springs between 1970 and 1975. The two soils, LCS and LS, were taken to have drain depths at 80 respectively 110 cm below surface. The soil properties brought into the model were practically the same as in the earlier calculations. Precipitation and evaporation data were obtained from the Royal Meteorological Institute's station at De Bilt. The evaporation data, available per month, were distributed over the days according to the known radiation data. For every day, lasting 2 seconds in the model, there were 5 readings of the paper tape reader. Rainfall was distributed evenly over these readings; evaporation was situated exclusively in the third reading. This causes the oscillations in the curves of Fig. 20. This figure gives an example of the simulations, which began with an initial condition of a constant downward flux of 0.5 cm · day^{-1} on February 1, 1972.

Initially the LCS soil clearly is wetter than the LS soil with the same drain depth. Near the middle of February this changes and afterwards the LCS soil remains drier than the LS soil for both drain depths. This picture was repeated in all the 6 years simulated. So it is not surprising that, as shown in Table 7, the workability of LCS soil was much better than that of LS soil. This table gives the number of workable days in February, March and April for two criteria: mean soil moisture pressure head in the top 10 cm of respectively −100 and −150 cm.

The conclusion from the former studies that deeper drainage is very profitable for workability, is confirmed. As regards soil improvement the conclusion can be very clear: not advisable.

The simulation with real weather data also allows us to draw quantitative conclusions. For example the mean sowing date can be determined from recordings like Fig. 20. One has to know the soil moisture pressure head at which field operations can be carried out and the number of days required for planting a crop. Let the critical ψ's be −100 cm for sowing spring grains and −150 cm for planting potatoes, while both crops need 5 days for field operations in spring.

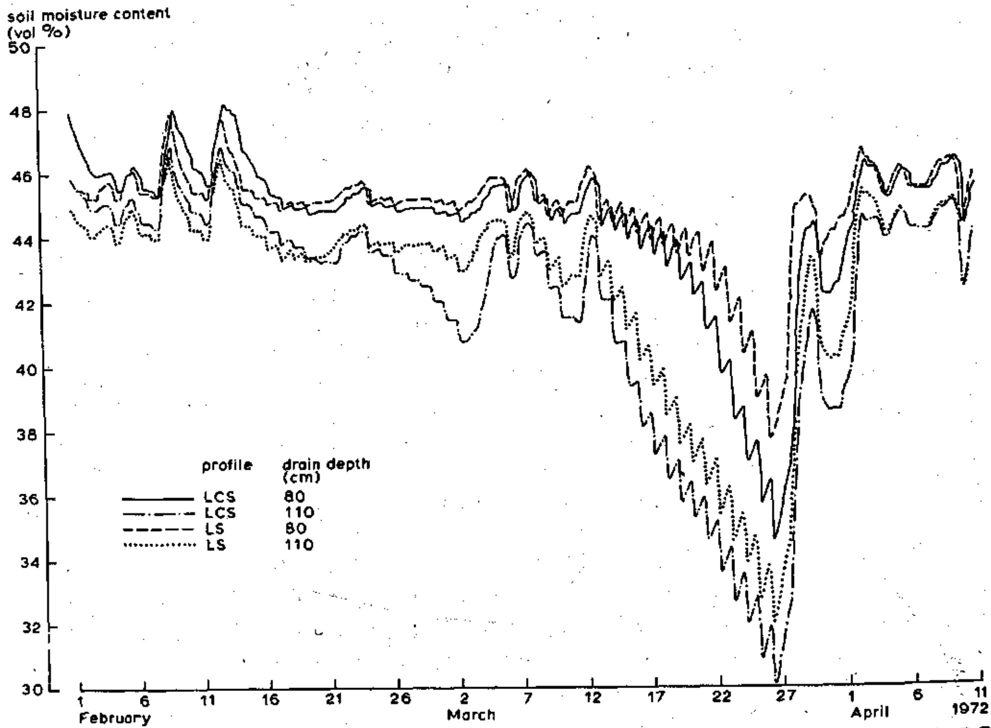

Fig. 20. Moisture content θ (volume %) during the spring of 1972 in the top 10 cm in LS and LCS soil, drained at 80 respectively 110 cm depth, as simulated by the electronic analog ELAN.

Then the mean planting date can be read from the recordings. The data are averaged over 6 years and given in Table 8.

By increasing the drain depth from 80 to 110 cm, planting can be achieved about one month earlier. Soil 'improvement' causes a delay of about half a month.

The question arises whether this soil improvement has any favourable effects. If

Table 7. Mean number of workable days per year for two soils (LCS and LS) and two drain depths in the years 1970 through 1975. Workability defined by soil moisture pressure heads of −100 respectively −150 cm in the top 10 cm in February, March and April.

Month	Drain depth (cm)	Workability criterion ψ, respectively soil			
		−100 cm		−150 cm	
		LCS	LS	LCS	LS
February	80	0.1	0	0	0
	110	6.6	1.4	0	0
March	80	5.8	2.2	0.2	0
	110	23.3	10.2	3.7	1.0
April	80	13.9	10.3	7.3	4.8
	110	20.8	17.8	13.2	8.0

Table 8. Mean planting date of spring grains and potatoes in two soils (LCS and LS) and two drain depths during the years 1970 through 1975. Dates determined under the assumptions given in the text.

Drain depth (cm)	Crop, respectively soil			
	Spring grain		potatoes	
	LCS	LS	LCS	LS
80	March 31	April 15	May 8	May 13
110	February 25	March 14	April 10	April 25

it has, it should be during periods with much rain, considerably more rain than is falling in spring periods. As an example a simulation run was made with weather data from the very wet autumn of 1974. The number of days with a very wet soil is given in Table 9.

During this period with rain rates clearly exceeding the k_0 value of $0.3\,\mathrm{cm} \cdot \mathrm{day}^{-1}$ of the clay layer, removal of this layer has a good effect, even better than deeper drainage has.

4.6 Discussion

During the last decades soil science has rapidly developed. In every stage of development practical investigators have tried to apply the soil physical theory to field problems. They wanted to explain the results of experiments and to predict the results of technical operations. In this chapter it has been shown which possibilities the application of soil science in four stages of development could have given for the solution of such a practical problem: Which is the effect of deeper drainage and the removal of a clay pan on the timelinesss of field operations in spring?

The methods used were (1) steady state considerations, (2) pseudo-steady state calculations, (3) dynamic simulation with standardized weather data and (4) dynamic simulation with real weather data.

With regard to deeper drainage, all methods predict a favourable effect on workability. Steady state considerations are scarcely able to find quantitative

Table 9. Number of days with an air content in the top 10 cm of less then 2 vol. % during October and November 1974 in two soils for two drain depths.

Drain depth (cm)	Soil	
	LCS	LS
80	15.2	7.0
110	9.0	4.6

effects. With the second and third method some indication is found on the magnitude of the effect, but there are fairly large differences between the results of the methods. This stresses the necessity to be careful in applying the pseudo-steady state method. Only simulation with real weather data can give a really quantitative prediction of the effect.

As far as subsoiling is concerned, there is much difference in the conclusions. With the first method hardly any conclusions can be drawn; the second and third method are showing varying effects depending on drainage and weather conditions. Both are leaving some doubt. The fourth method gives a clear conclusion: removal of the clay pan is not beneficial to workability in spring.

The earlier methods can give only poor quantitative information; the conclusions can even be wrong. The simulation with real weather data gives the most information. Its results should be compared with field observations to see whether this information is correct. There certainly will be differences, e.g. the method does not include the effects of hysteresis, nor is allowance made of soil properties varying through the year.

The development of soil science, combined with the possibility of its application by dynamic models, should have influence on the organization of agricultural research. Field experiments can be replaced partly by model investigations. Experimental fields can be set up for a simpler performance or for a shorter duration. The observations to be made will differ from those formerly made because they have to supply data for model application and are to be used to check model results.

5 Applications

5.1 General

The described models simulate moisture conditions in a number of layers, surface run-off and drain outflow. The electronic analog and numerical models can be used in an inductive way to derive soil properties from field observations. They can be applied for studies in which output data are important: effect of drainage, tillage, soil improvement, etc.

A number of limitations was treated in Section 1.3. Another important limitation is that the $\psi(\theta)$ and $k(\psi)$ relationship of the soil must be known. Especially for the $k(\psi)$ relationship few reliable data are available.

5.2 Checking the models

The ultimate check of a model is the comparison between simulated and observed field data over a period not used for calibration. If they do not agree, the model is incorrect. Mostly there is some doubt about the observed moisture contents and about the laboratory determined $k(\psi)$ and $\psi(\theta)$ relationships. By changing these relationships the simulated values often can be brought to approach the observed values. So in most cases it is very difficult to judge whether the model is wrong or the parameters used were incorrect. Another practical problem is precipitation, as there can be a difference between the amount of rainfall at the site of moisture sampling and at the site of the rain gauge. Even the latter can differ from the real rainfall. Moreover, the distribution over the day differs from the distribution which can be applied in the model, and is often unknown.

The most direct check was made on the hydraulic analog where the simulated numbers of workable days were compared with field observations by Hokke & Tanis (1978) over 23 years, see Fig. 21. It shows a good agreement between observed and calculated data. The largest differences, in 1963 and 1964, may have been caused by the fact that the real drain depth in the polder 'Hoeksche Waard' was perhaps less than the 100 cm used in the model. For those two years the effect of drain depth was very large.

A comparison of the results obtained with different models is very well possible. Wind (1972) compared hydraulic model data with those from a simple numerical calculation. Wind (1976) compared hydraulic model data with data computed by van Keulen & van Beek's (1971) numerical model. Fig. 22 shows part of this comparison, including more recent data calculated by the electronic analog. Wind

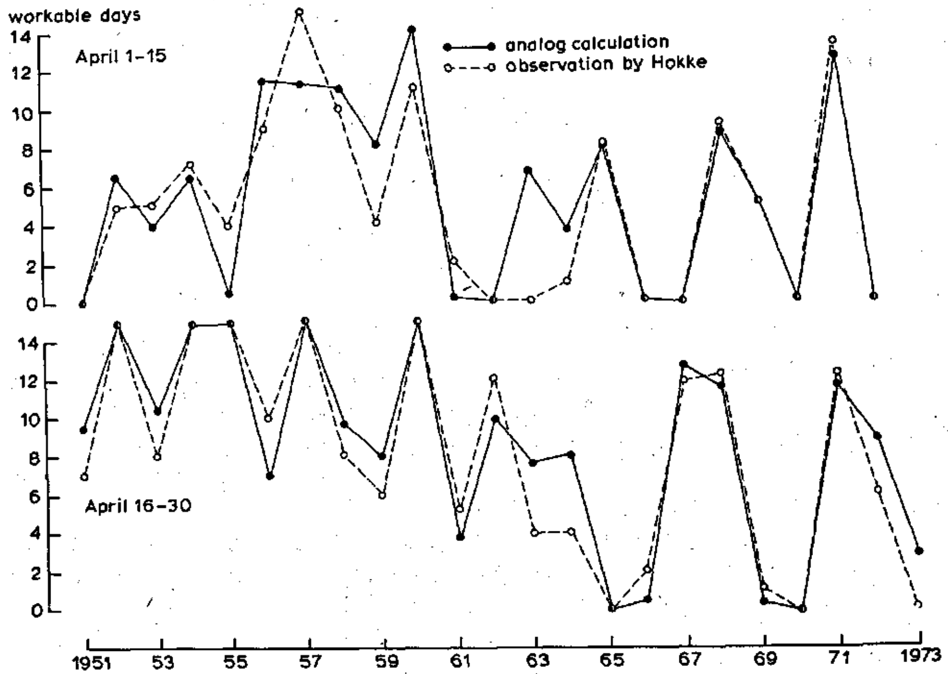

Fig. 21. Simulated and observed number of workable days in the Aprils of 1951 through 1973.

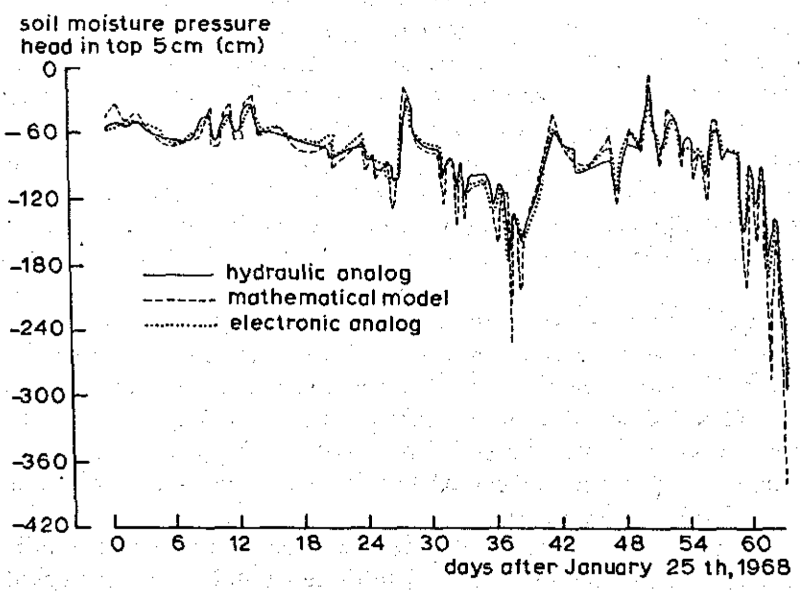

Fig. 22. Soil moisture pressure head in the top 5 cm of a loam soil in spring 1968, as simulated by the numerical model of van Keulen & van Beek (1971), the hydraulic analog (Wind, 1972) and the electronic analog ELAN.

& Mazee (1979) give comparisons of the electronic analog with Wind & van Doorne's (1975) numerical model FLOW.

All these comparisons showed excellent agreement between the results from the models used. This is not surprising because all the models are based on the same principle; they only differ in elaboration. The comparisons therefore did show that only minor mistakes were made in the elaboration, but they did not confirm the validity of the principle. The principle is a combination of Darcy's law and the continuity equation assuming that hysteresis can be neglected. If the models were to be shown incorrect, one or more of these laws or assumptions would also have to be wrong, and this is not to be presumed.

5.3 Inductive use

The purpose of application of these simulation models is to calculate moisture conditions from meteorological data. The soil properties should be known beforehand.

This is called the deductive application of models: field data are deduced from basic knowledge. Since sufficiently accurate data about soil properties often are not available, such a deductive use of simulation models has only restricted value.

If field data about soil moisture content are available, one can try to calibrate the model until its results fit these data. In this process one usually has to calibrate various soil parameters. The more soil parameters are unknown, the more difficult the matching problem will be. The following may serve as an example of such an inductive application. Wind (1976) obtained unsatisfactory results from a simulation run with the hydraulic analog, using for that particular soil values of k_0 and α valid for a comparable standard soil of Rijtema (1969).

According to the simulation, in a certain spring workable days did not occur although in reality that spring had very good workability conditions. Apparently the soil physical data used in the model were incorrect.

Reduction of the used value of k_0 to half its original value had hardly any effect and it was to be expected that this reduction in saturated conductivity had to go to unreasonably low values. Therefore different α-values were tried in the numerical model FLOW. The simulation results were compared with field observations in March 1973. With a certain value of α the model exactly described what had happened in reality.

Because the knowledge of soil physical properties is rather scanty, more attention should be given to this method of inductive use of models in combination with sensitivity analysis to determine the most important factors involved.

5.4 Workability in spring

Reeve & Fausey (1974) state that the most beneficial effect of drainage is early workability of the soil. That gives farmers the opportunity for early planting of crops, thus increasing the length of the growing season. Accurate data on the quantitative effect of drainage on workability are lacking, however.

The availability of a hydraulic analog opened the possibility to investigate this problem (Wind, 1976). The method of inductive use was applied to calibrate the main model specifications so that simulated moisture pressure heads agreed with those observed in 1973 (see Section 5.3).

This study was not only intended to investigate the effect of drainage on workability but also to produce data on the distribution of workable days. These were needed by another institute for labour studies about planting of potatoes. In cooperation with Perdok (1975) the workability limit for potato planting was fixed at a suction of 300 cm in the top 5 cm of the investigated loam soil.

Data of real precipitation and potential evaporation over 22 years between 1951 and 1973, were read from paper tape and used as input for the hydraulic model. Of each year 4 months were taken, beginning at January 1. As the time scale was 5 minutes representing one day, each simulation run lasted about 230 hours. As input 6 drain depths and 4 drain intensities were applied, but not in all combinations and not over all years. Nevertheless, the model had to work continuously for almost a year.

The investigated drain intensities were 0.0011, 0.0033, 0.008 and 0.015 day^{-1}. When drain depth is 100 cm below surface and midway between the drains groundwater depth is 50 cm below surface, these data correspond with drain outflow rates of 0.055, 0.165, 0.40 and 0.75 $cm \cdot day^{-1}$ respectively. The Netherlands drainage criterion of drain outflow under these conditions is 0.7 $cm \cdot day^{-1}$.

The result of the simulation experiment was that drain intensity had hardly any influence on workability in spring. The mentioned lowest intensity was already sufficient. The reason for this lack of influence is that drain outflow rates in spring are fairly small, because rain rates are low and evaporation is already important. So the groundwater table will only slightly exceed drain depth and the differences in groundwater depth for the mentioned drain intensities are small.

The effect of drain depth, on the contrary, was very large. Drain depths of 40, 80, 100, 150 and 200 cm below surface were investigated. The effect of drain depth differed from year to year; in very dry and in very wet springs the effect was small. In 10 of the 22 investigated years drain depth was very important with regard to the number of workable days.

The large effect of drain depth and the small influence of drain intensity lead to the supposition that a deep drainage system with low intensity can be very important for workability. As Ernst (pers. commun.) points out, such a drainage system is often present in practice. Part of excess rainfall is not discharged by the drain tiles but finds its way directly to lower lying ditches or canals.

This effect was investigated for the same soil with the electronic analog ELAN in which a double drainage system was made. A single drainage system at 90 cm depth with an outflow rate of 0.7 $cm \cdot day^{-1}$ at the normative groundwater depth of 50 cm below surface was compared with three double drainage systems. These were composed of a first system at 90 cm depth with an outflow rate of 0.6 $cm \cdot day^{-1}$, and an additional system with an outflow rate of 0.1 $cm \cdot day^{-1}$, both at the normative groundwater depth of 50 cm below surface. The additional drainage systems were situated at depths of 110, 130 and 150 cm below surface.

During February 1972 which had a rainfall surplus of about 24 mm, the results of the four drainage systems seemed to be fully identical. In March, with an

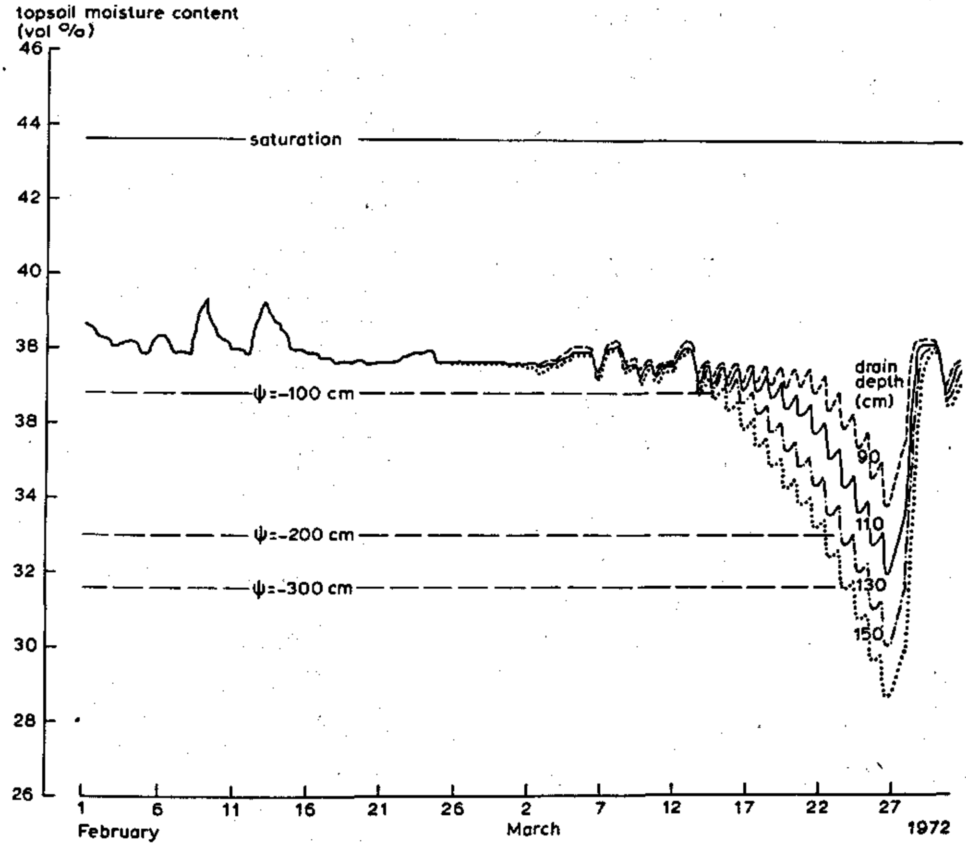

Fig. 23. Effect of double drainage systems on the moisture content in the top 10 cm of a loam soil. Four combinations are shown, each of them with a first system at 90 cm depth with an outflow rate of 0.6 cm · day^{-1} and a second system with an outflow rate of 0.1 cm · day^{-1} at a normative groundwater depth of 50 cm. The depth of the second systems, i.e. 90, 110, 130 and 150 cm, are indicated in the figure.

evaporation surplus, the effect of the additional low-intensity deeper drainage systems clearly appeared to be favourable for the soil moisture contents and hence for the number of workable days, see Fig. 23.

More important than the number of workable days is the first date of soil workability. This determines the date of planting and thus the length of growing season. After a certain date every day of delay causes an increasing depression in crop yield. How the dates of workability are influenced by drain depth is shown in Fig. 24. Deeper drainage results in earlier workability.

Feddes & van Wijk (1977) used the result of this study in combination with a planting date-yield relationship and a pseudo-steady state evapotranspiration model. With this integrated model approach they found that the optimum drain depth for this soil, a sandy clay-loam, ranges between 100 and 150 cm below surface, see Fig. 25.

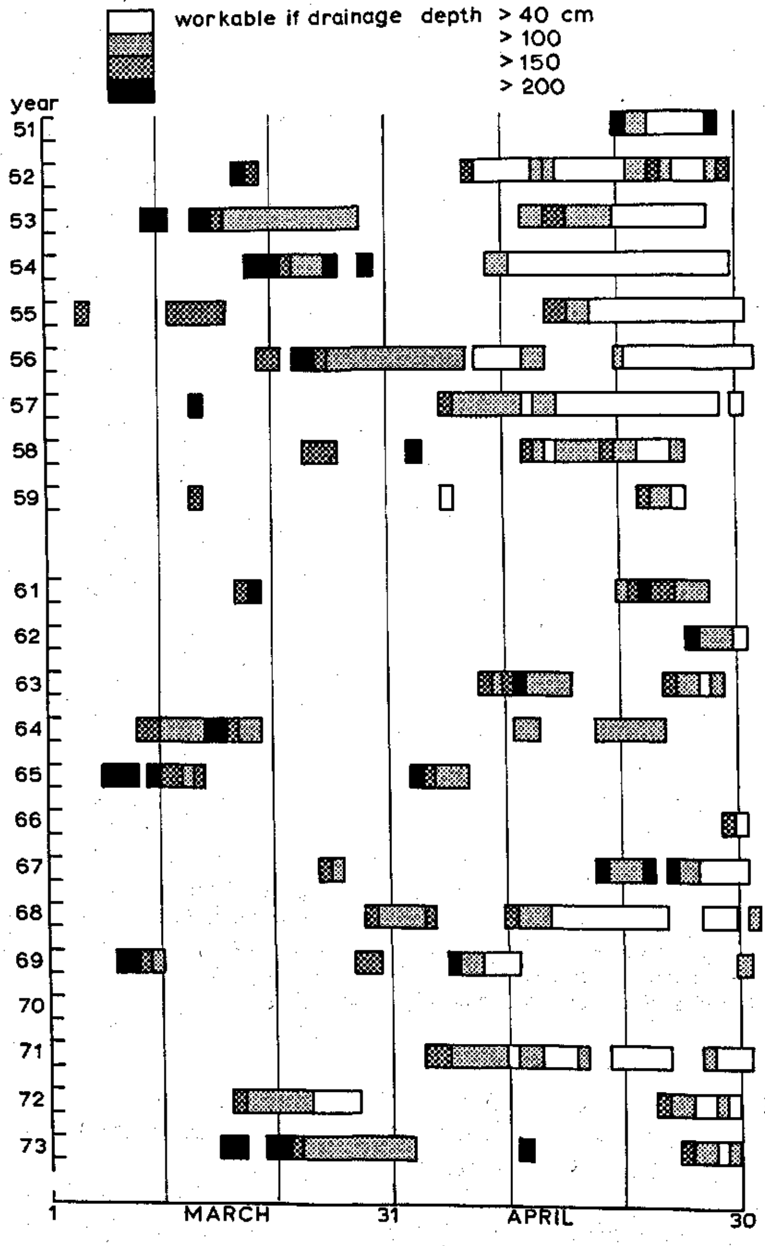

Fig. 24. Periods of workable soil calculated with the hydraulic analog for a loam soil for four drain depths in 22 years. Workability is defined here as a soil moisture pressure head in the top 5 cm being less than −300 cm.

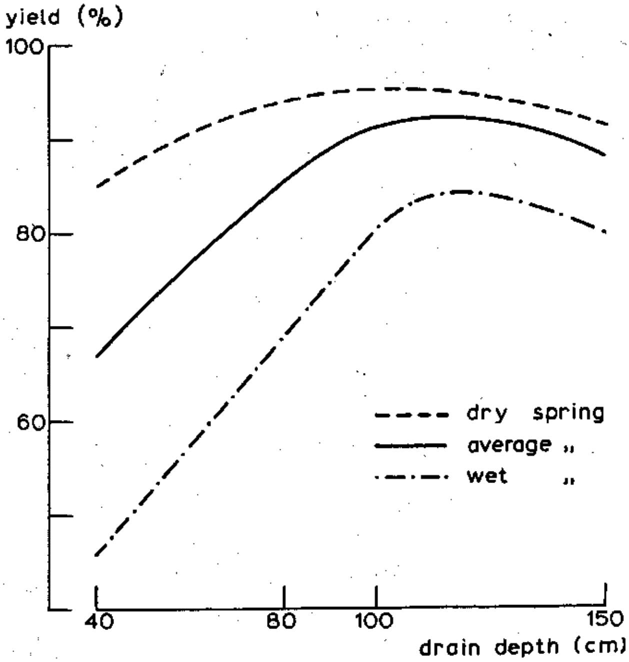

Fig. 25. Influence of drain depth on yield of summer cereals on a sandy clay loam for various types of spring conditions. After Feddes & van Wijk (1977).

5.5 Soil moisture content in dependence of drain depth and drain intensity

5.5.1 General

In Section 5.4 it was found that drain intensity above a certain low value had no influence on workability. Nevertheless drain intensity should be higher than that to serve another purpose of drainage: avoiding too wet conditions at other times. So to obtain low moisture contents in spring drain depth is important, but to avoid water logging in autumn, winter and early spring drain intensity should be high.

In order to see which combination of drain depth and intensity serves both drainage purposes best, an investigation was made with the electronic analog ELAN. Simulations were made with drain depths at 70, 100 and 130 cm below surface. With each depth three intensities were combined; they were chosen in such a way that drain outflow varied between 0.2 and 0.8 $cm \cdot day^{-1}$ when groundwater depth was 50 cm below surface. The simulations were made over 9 months beginning at September 1 in 35 years between 1941 and 1977.

5.5.2 Weather input and soil conditions

Precipitation and evaporation data of the Royal Netherlands Meteorological Institute's station in De Bilt were used. Precipitation data were observed as a daily total; in the model file this amount was distributed equally over the day. When evaporation was larger than rainfall the difference was applied in the middle of the day. Evaporation was calculated according to Penman over periods not much harm. Below that value the number of very wet days increases considerably.

From Fig. 26B it can be seen that the number of wet days with a soil moisture pressure head of more than −50 cm, is about 10 times the number of very wet days with pressure heads of more than −20 cm. To avoid large numbers of wet days both drain depth and intensity are important but both factors are not exchangeable. An insufficient drain depth cannot be compensated by a large intensity.

Figs. 26C and 26D show that to obtain workable conditions drain depth is clearly far more important than drain intensity. Increase of drain depth from 70 to 100 cm causes a considerable increase in dry days (soil moisture pressure head less than −100 cm) in March, the period in which spring grains are mostly sown. For the number of very dry days (pressure head <−200 cm) in April when sugar beets and potatoes are planted, the difference between 70 and 100 cm drain depth is fairly small. This discrepancy between Figs. 26C and 26D is difficult to explain: models give results, not explanations.

The data given in this chapter do not allow to draw a conclusion with regard to optimal drain depth. For this purpose not only the number of workable days is required but also the dates of workability and the effect on the availability of moisture in summer (cf. Section 5.4 and especially Fig. 25).

In this section it has been shown that the electronic analog can handle very long time-series in an efficient way. As regards the effects of drain depth and intensity, it was demonstrated that the drier the reference value of soil moisture pressure head is chosen, the less is the influence of drain intensity.

Summary

The knowledge of the behaviour of water in soils has increased considerably in this century. What happens with rain water falling on the soil surface and which processes then are started is now fairly well known.

The processes involved have thoroughly been studied by many investigators all over the world, with the result that there exists a fairly good knowledge on surface run-off, infiltration, relationship between moisture content and energy status, relationship between moisture content and conductivity, flow processes in both the saturated and unsaturated zone and on the conduction of soil water to drainage systems.

The applicability of all this knowledge formerly was only partial and restricted. The most important item was that it gave soil technologists for their activities a theoretical basis in general terms. When computers became available, this opened the opportunity to actually calculate all these processes in their mutual connection. In this way the behaviour of water in soils now can be simulated. With the aid of numerical simulation models the effect of natural causes or human activities can be forecasted, not only in general terms but also quantitatively and from day to day.

This simulation proved to be so expensive in computer time, however, that it is not well possible to apply it to periods of several decades. The effects of soil technological measures, e.g. drainage, are different from year to year and from day to day, however. Depending on weather conditions a certain activity sometimes can have very large positive effects, in other periods no effect at all and in some years even negative effects. Therefore, in order to investigate the profits of soil technological measures a study over several decades is required. With the aid of analog models such long term investigations can be carried out at relatively low cost.

In this publication the development of some analog models is described, i.e. a hydraulic and an electronic analog. The latter is based upon the analogy between the integrated moisture flux equation and Ohm's law. The models are compared with numerical models and were checked with field observations. It is shown that long time series can easily and correctly be handled. Examples are given about the effects of drain depth and drain spacing on the moisture content of the topsoil. From these it appeared that drain spacing is important to avoid water logging conditions, but that it does not influence workability of the topsoil. To obtain early and good workability conditions for seedbed preparation, drain depth is more important.

It is demonstrated how soil physical knowledge can be applied to reach

conclusions in a soil technological problem. The problem given was to forecast the effects of soil improvement and drainage on workability of a layered soil. Steady state considerations, pseudo-steady calculations and dynamic model computations with standardized weather data, all did leave considerable doubt with regard to the amelioration method to be advised. Only a dynamic simulation with real weather data yielded clear conclusions.

Samenvatting

De kennis van het gedrag van water in grond is in deze eeuw aanzienlijk toegenomen. Tegenwoordig is goed bekend wat er gebeurt met regenwater dat op de grond valt en wat daarvan de invloed is. Een deel van het regenwater infiltreert in de grond, een ander deel verdwijnt door oppervlakte-afvoer. Het infiltrerende water doet het vochtgehalte van de bovengrond stijgen en verandert daarmee de potentiaal van het bodemvocht. Daardoor ontstaat een potentiaalgradiënt, die veroorzaakt dat water naar beneden gaat stromen. De stroomsnelheid wordt niet alleen beheerst door deze gradiënt, maar ook door het geleidingsvermogen van de grond voor water. Bekend is dat dit geleidingsvermogen afhangt van het vochtgehalte, evenals de wijze waarop zij samenhangen.

Bij de neerwaartse beweging bereikt het water de grens tussen onverzadigde en verzadigde zone. In de laatste kan het vochtgehalte niet meer toenemen en het geleidingsvermogen blijft daar constant. Door deze zone vloeit het water deels horizontaal, deels verticaal naar een ontwateringsstelsel. Verdamping doet de bovengrond uitdrogen; daardoor ontstaat een opwaartse waterstroming, vaak capillaire opstijging genoemd.

De processen die de stroming en berging van water in de grond beheersen zijn zorgvuldig bestudeerd door veel onderzoekers over de gehele wereld. Daardoor bestaat een goede kennis van oppervlakte-afvoer, infiltratie, de relatie tussen vochtgehalte en vochtpotentiaal, de relatie tussen vochtgehalte en geleidingsvermogen, stromingsprocessen in zowel de verzadigde als de onverzadigde zone en van de afvoer van water naar ontwateringsstelsels.

De toepasbaarheid van al deze kennis was vroeger slechts partieel en beperkt. Het belangrijkste was dat de cultuurtechnicus door deze kennis een theoretische basis in algemene termen had voor zijn praktische activiteiten. Toen computers beschikbaar kwamen ontstond de mogelijkheid om al deze processen in hun onderlinge samenhang door te rekenen. Zo kan nu het gedrag van water in de grond in modellen worden nagebootst, gesimuleerd. Met behulp van die simulatiemodellen kan het effect van natuurlijke gebeurtenissen of menselijke ingrepen worden voorspeld, niet slechts in algemene termen maar ook kwantitatief en van dag tot dag.

Deze simulatie bleek echter zoveel computertijd te kosten dat ze praktisch niet kan worden toegepast op lange perioden, zoals enige decennia. De effecten van cultuurtechnische maatregelen verschillen echter van jaar tot jaar en van dag tot dag. Afhankelijk van de weersomstandigheden kan een zekere activiteit, bijvoorbeeld drainage, een zeer gunstig effect hebben in bepaalde jaren, in andere perioden totaal geen effect en soms zelfs negatieve effecten. Om nu toch de baten

van cultuurtechnische investeringen te kunnen waarderen is een studie over een lange reeks van jaren nodig. Met behulp van analoge modellen kunnen deze lange termijn onderzoekingen tegen lage kosten worden verricht.

In deze publikatie wordt de ontwikkeling van enige analoge modellen beschreven, namelijk een hydraulisch en een elektronisch analogon. Het laatste is gebaseerd op de overeenkomst tussen de geïntegreerde onverzadigde waterstromingsvergelijking en de wet van Ohm. De modellen zijn vergeleken met numerieke modellen en werden gecontroleerd met veldwaarnemingen. Getoond wordt dat lange tijdseries makkelijk en correct kunnen worden gehanteerd. Voorbeelden worden gegeven over de effecten van draindiepte en drainafstand op het vochtgehalte van de bovengrond. Daaruit blijkt dat de drainafstand zeer belangrijk is om zeer natte omstandigheden te vermijden maar dat deze weinig betekenis heeft voor de werkbaarheid van de grond. Om vroege en goede werkbare omstandigheden te verkrijgen is juist de draindiepte van grote betekenis.

Het effect van de methode waarmee bodemfysische kennis wordt toegepast om conclusies te bereiken met betrekking tot een cultuurtechnisch probleem wordt getoond. Het behandelde probleem was het voorspellen van de betekenis van grondverbetering en drainage voor de bewerkbaarheid van een gelaagde grond. Stationaire beschouwingen, pseudo-stationaire berekening en modelsimulatie met gestandaardiseerde weersgegevens lieten alle aanzienlijke twijfel bestaan over de te adviseren verbeteringsmethode. Alleen dynamische simulatie met werkelijk voorgekomen weersgegevens maakte duidelijke conclusies mogelijk.

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A HYDRAULIC MODEL FOR THE SIMULATION OF NON-HYSTERETIC VERTICAL UNSATURATED FLOW OF MOISTURE IN SOILS

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Abstract: In order to simulate infiltration and drainage processes in soils of low permeability a hydraulic model was built. It consists of a number of vessels, each simulating a layer of soil with its moisture characteristic, connected by a number of tubes, simulating capillary conductivity. It is shown that the model reacts in nearly the same manner as the soil would do, according our knowledge of the flow processes and parameters.

The model can be used to study non-steady state infiltration and drainage processes of which some examples are given.

Notation

A drainage intensity (day^{-1})
a gradient in conductivity with depth (day^{-1})
b constant in (27) (cm. day^{-1})
d depth of the soil represented by one vessel (cm)
d equivalent depth of the aquifer (m) only used in Hooghoudt formula (5)
F ratio between total tube volume and vessel volume (dimensionless)
g acceleration due to gravity (cm. sec^{-2})
h height of the vessels (cm)
i volume of the vessel (ml)
k capillary conductivity (cm. day^{-1})
k permeability of soil for water (m. day^{-1}) only used in Hooghoudt formula (5)
k_0 capillary conductivity at moisture tension 0
L length of tube connecting two vessels (cm)
L drain distance (m) only used in Hooghoudt formula (5)
L_d length of the tube, simulating the drainage in the model (cm)
m difference in soil moisture content simulated by a full and an empty vessel (dimensionless)
m vertical distance between the groundwater in the drains and the groundwater table midway between two drains (m) only used in Hooghoudt formula (5)

n number of tubes connecting two vessels (dimensionless)
p pressure difference in Poiseuille formula (13) (g. cm^{-1}. sec^{-2})
R_1 Reynolds number for gradient 1
r radius of tube connecting two vessels (cm)
s drain discharge velocity (m. day^{-1}) only used in Hooghoudt formula (5)
S_a model scale for area (horizontal scale) (cm^2)
S_t time scale of the model (dimensionless)
S_v vertical scale of the model (dimensionless)
t time in minutes representing one day; model time (min. day^{-1})
V velocity of flow (cm. day^{-1}) positive in upward direction
z vertical distance above a horizontal plane, e.g. soil surface (cm)
α exponent used in Rijtema's relation between k and Ψ (cm^{-1})
ρ density of the fluid used (g. ml^{-1})
Φ total potential expressed as hydraulic head (cm)
Ψ capillary potential expressed as hydraulic head (cm)
Ψ_d capillary potential at drainage depth (cm)
Ψ_e capillary potential simulated by an empty vessel (cm)
Ψ_w soil moisture tension ($\Psi_w = -\Psi$) (cm)
η fluid viscosity (g. cm^{-1}. sec^{-1})

Introduction

Drainage systems serve to improve conditions too wet for the soil as well as the crop. They are mostly designed to regulate groundwater depth, although soil and crop react far more on moisture and air conditions near the surface than on groundwater table depth. This imperfection in drainage design is caused by the lack of a synthesis of saturated and non-saturated flow phenomena.

Despite the existing knowledge on both saturated and unsaturated flow of water in soils and on moisture characteristics it seems hardly possible to calculate moisture contents and groundwater depths from rainfall data, even if all data involved in this process are known.

The reason for this is that the exact solution of non-steady state equations is only possible with distinct initial and boundary conditions. Examples of these are: the soil is homogeneous, the rainfall rate is constant during a certain period, the initial moisture content is the same at every depth, the depth of the groundwater is constant, etc.

Where the real conditions differ from such boundary conditions numerical solutions can be achieved with the aid of computers. It seems, however, that only high capacity computers can solve the problems arising from irregular rainfall rates, such as occur in reality.

This paper deals with a very simple hydraulical analog which can simulate these kinds of problems even when fairly irregular boundary conditions exist. The model simulates the unsaturated vertical flow as influenced by capillary and gravity potential, storage of moisture and saturated horizontal flow to the drainage system.

Unsaturated vertical flow

For both saturated and unsaturated flow it is assumed that Darcy's law is valid:

$$V = -k \frac{d\Phi}{dz}, \tag{1}$$

where V is the velocity of flow in cm. day^{-1}, positive in upward direction; k is the capillary conductivity in cm. day^{-1}; Φ is the total potential expressed as hydraulic head in cm and z the distance in cm above a certain horizontal plane.

The total potential is thought to be composed of the potential due to gravity z and the capillary potential Ψ, both in cm:

$$\Phi = \Psi + z \tag{2}$$

the capillary potential $\Psi = -\Psi_w$ being the negative value of the soil moisture tension, Ψ_w. Below the groundwater Ψ can be positive because there a pressure exists instead of a tension. By combining (1) and (2):

$$V = -k\left(\frac{d\Psi}{dz} + 1\right). \tag{3}$$

For $d\Psi/dz < -1$ the value of V becomes positive, thus indicating upward flow. So downward flow will occur when $d\Psi/dz > -1$. It is known that $V=0$ for

$$\frac{d\Psi}{dz} = -1. \tag{4}$$

The capillary conductivity k is dependent on the soil moisture conditions; it can be expressed as a function of Ψ, e.g.:

$$k = a\Psi_w^{-n} \qquad \text{(Wind, 1955)}$$

$$k = \frac{a}{\Psi_w^n + b} \qquad \text{(Gardner, 1958)}$$

$$k = k_0 e^{-\alpha\Psi_w} \qquad \text{(Rijtema, 1966)}$$

Except k and Ψ_w all symbols used in these expressions are constants. Figure 1 gives some examples of $(k\Psi_w)$ relations for a number of soils.

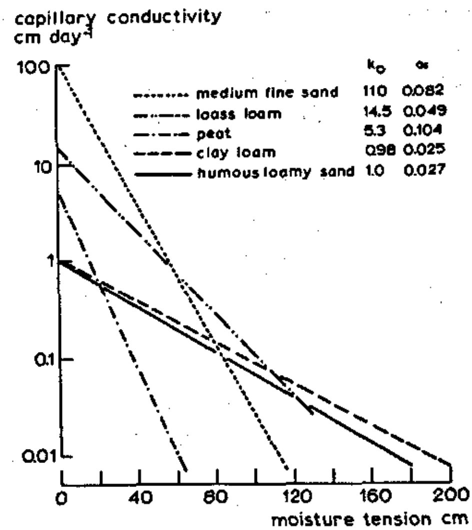

Fig. 1. Examples of $k(\Psi_w)$ relations for five soils (after Rijtema, 1969).

Saturated horizontal flow

Already in 1936 Hooghoudt published the drainage equation:

$$s = \frac{8\,kdm}{L^2} \tag{5}$$

where s is the discharge velocity in m. day^{-1}, k the saturated permeability of the soil for water in m. day^{-1}, d the equivalent depth of the aquifer (in m), m the vertical distance between the groundwater in the drains and the groundwater table midway between two drains, and L the drain distance.*

According to (5) there is a linear proportionality between discharge (s) and hydraulic head (m). Although there are situations in which (5) does not hold because of non-linear radial and entry resistances, it gives in many cases a good approximation.

Equation (5) can be written in the symbols as used in (1) and (3), so V instead of s and Ψ_d instead of m. Then Ψ_d is the capillary potential at drainage depth midway between two drains. Because $\Psi = 0$ at the water level in the drains, Ψ_d must be >0 for a flow from soil to drains. As downward flow is taken negative, flow to the drains will also be negative. So

$$V = -A\Psi_d, \tag{6}$$

* In this sense the symbols s, k, d, m and L are used only in connection with Eq. (5).

where $A = 8kd/L^2$ in day^{-1} indicates the drainage intensity. Many tile drainage systems in The Netherlands have been designed with $A = 0.014$, which means that a hydraulic head of 50 cm will give a drain discharge of 0.7 cm. day^{-1}.

Storage of moisture in the soil

If the vertical flow velocities at two depths in the soil are not equal, moisture will be stored in or removed from the layer between these depths. Every change in moisture content causes a change in moisture tension, so both potential gradient and capillary conductivity will change.

The relation between moisture content and moisture tension is called the soil moisture characteristic. Figure 2 gives examples for a number of soils.

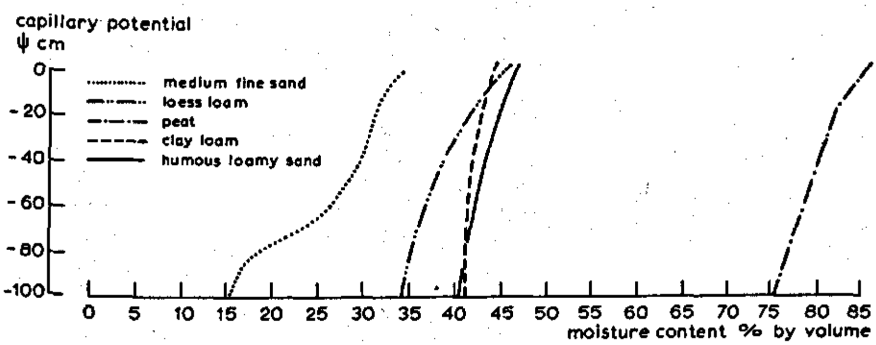

Fig. 2. Moisture characteristics of five soils (after Rijtema, 1969).

There is a wide variation between the curves for different soils. There is also a wide variation between the moisture characteristics of one soil when wetting respectively drying it. This hysteresis phenomenon is no problem if only the proper wetting or drying curve can be used. But in many situations one part of the soil mass is being wetted and another part is drying, making the flow problems under these conditions nearly unsolvable.

As it is not possible to simulate the hysteresis phenomenon in the model to be described, this paper only deals with solutions of problems in which hysteresis can be neglected.

Outline of the model

Figure 3 is a photograph of the model and Fig. 4 gives a schematic outline of it. It consists of a number of vessels representing layers of soil.

The distance of the water table in a certain vessel to the top of the first vessel is a measure for the total soil moisture potential Φ of a particular layer, the amount of water in each vessel for the soil moisture content m of that particular layer and the distance between the water table in a certain vessel and its top the capillary potential Ψ of that layer.

The vessels are connected by a number of tubes thus simulating the possibility of moisture flow from one layer to another. Because the tubes connect

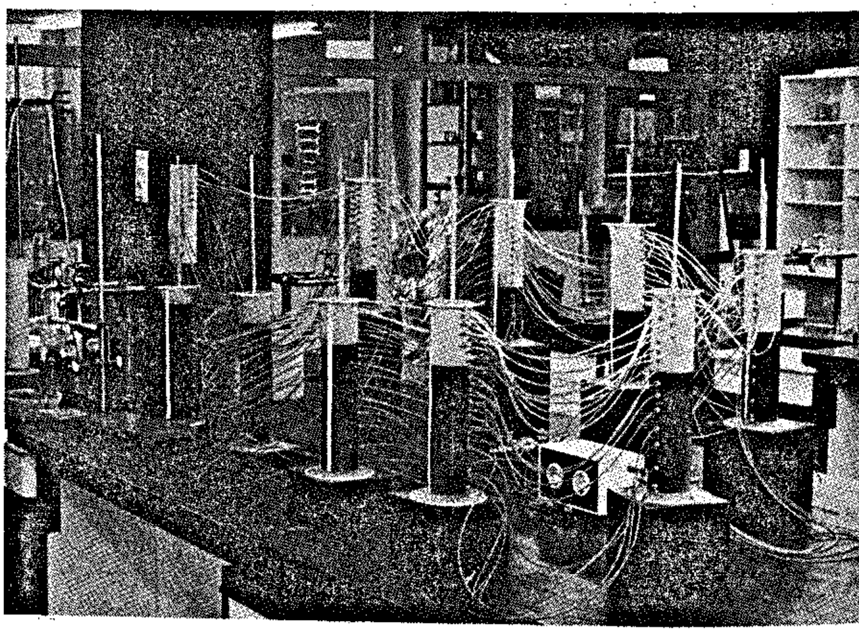

Fig. 3. The model.

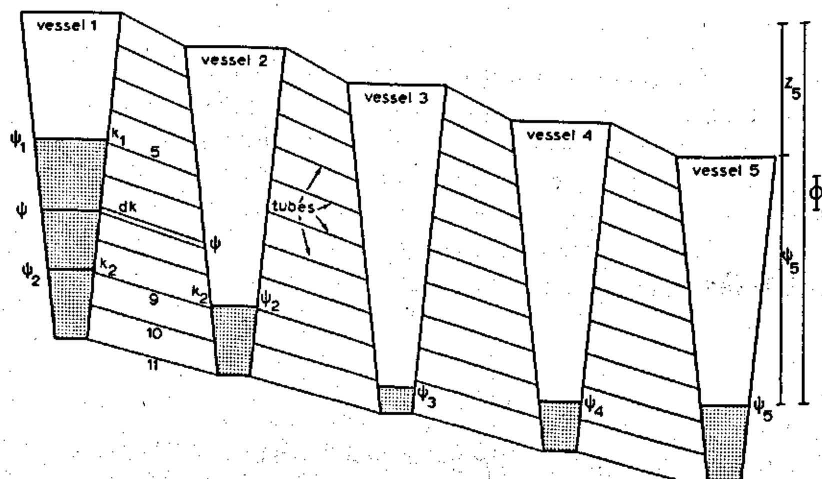

Fig. 4. Scheme of the model. The total potential ϕ is simulated by the distance of the water table in a certain vessel to the top of the first vessel. The distance of the water table in a certain vessel and the top of that vessel is a measure of the capillary potential Ψ. The capillary conductivity k as simulated by the connecting tubes is depending on Ψ.

the vessels at various heights the water transporting capacity depends on the height of the water table in the vessels. By a good choice of height, length and radius of the tubes the $k(\Psi)$ relation of the soil can be built in.

As the moisture content m and the capillary conductivity k are not related to the total potential Φ but to the capillary potential Ψ the vessels must not all be placed at the same height. The difference in height between two connected vessels should be such that the relation

$$\Phi = z + \Psi \tag{2}$$

holds for the model.

To simulate flow and storage processes for moisture tensions to $\Psi = -100$ cm in a soil having an air content of 0.05 at $\Psi = -100$ in layers of 10 cm depth over a surface area of 100 cm^2, the total volume of the vessels should be $100 \times 10 \times 0.05 = 50$ ml. Their height should be 100 cm and the difference in height between two vessels 10 cm. There are two scales involved in this example: the vertical scale being 1:1 and the scale for area 100 cm^2. If it is more convenient to work with less tall vessels the vertical scale can be decreased. If the height of the vessels used is h cm, and the capillary potential simulated by an empty vessel is Ψ_e cm, the vertical scale of the model S_v is:

$$S_v = \frac{h}{\Psi_e}. \tag{7}$$

If the difference in moisture content simulated by a full and an empty vessel is m, the volume of the vessels i (ml) and the depth represented by one vessel d (cm), the area S_a (cm^2) represented by the model is:

$$S_a = \frac{i}{md}\,(\mathrm{cm}^2). \tag{8}$$

To simulate long periods in a short span of time, a time scale is necessary. The time in minutes t representing one day is called the model time, so

$$S_t = \frac{t}{1440}. \tag{9}$$

The field data have to be multiplied by the adequate scales to find the corresponding model data. So a capillary conductivity of k cm. day^{-1}, meaning a flow velocity of k cm. day^{-1} when a gradient 1 exists, is simulated by:

$$k\,\frac{i}{md}\,\frac{1440}{t}\ \mathrm{cm^3/day\ for}\ \frac{hd}{\Psi_e}\ \mathrm{cm}$$

difference in water table between two consecutive vessels. So a tube which gives a flow of

$$\frac{ki\Psi_e}{md^2ht}\ \mathrm{cm}^3/\mathrm{min}$$

when there is a difference of 1 cm in hydraulic head between its ends, represents a capillary conductivity of k cm/day.

An increase in moisture content of the soil Δm is represented in the model by

$$\Delta i = \frac{i}{m}\,\Delta m\,. \tag{10}$$

An increase in capillary potential $\Delta\Psi$ in the soil is represented in the model by

$$\Delta h = \frac{h}{\Psi_e}\,\Delta\Psi\,. \tag{11}$$

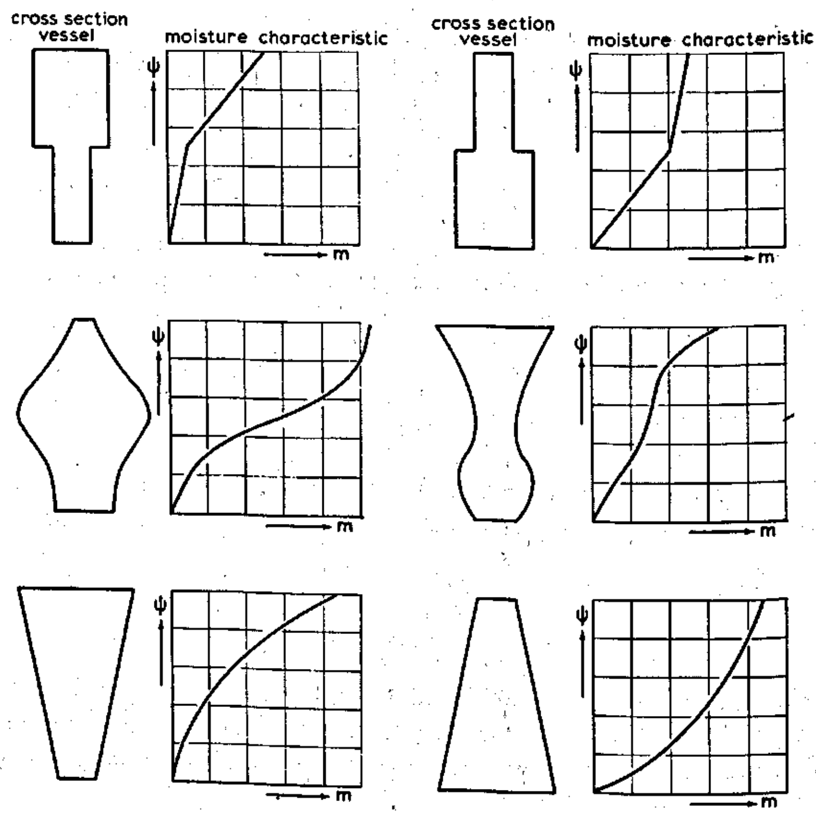

Fig. 5. Examples of moisture characteristics and the cross sections of vessels that represent them in the model.

So the area of the cross-section of a vessel is given by

$$\frac{\Delta i}{\Delta h} = \frac{i\Psi_e}{mh}\frac{\Delta m}{\Delta \Psi}. \tag{12}$$

This means that the shape of the vessels should be proportional to the first derivative of the moisture characteristic to Ψ. Figure 5 gives an example of some vessels with circular cross-sections and the moisture characteristics pertaining to them.

The connecting tubes

Provided the flow is laminar, a tube of L cm length and radius r (cm) gives according to Poiseuille a flow of:

$$q = 60\,\frac{\pi r^4}{8\eta}\frac{p}{L}\,(\text{ml. min}^{-1}), \tag{13}$$

where η is the fluid viscosity (g. cm^{-1}. sec^{-1}) and p is the pressure difference between the two ends of the tube in dyne. cm^{-2}. As the pressure difference is $\rho ghd/\Psi_e$ cm at a gradient 1, the flow is:

$$\frac{60\pi r^4 \rho g}{8\eta}\frac{hd}{\Psi_e L}\,(\text{ml. min}^{-1}), \tag{14}$$

where ρ is the density of the fluid used (g. ml^{-1}) and g the acceleration due to gravity (cm. sec^{-2}). As a flow of 1 ml. min^{-1} represents a real flow of mtd/i cm. day^{-1}, the tube of length L and radius r will represent a capillary conductivity of:

$$k = \frac{60\pi r^4 \rho g}{8\eta}\frac{md^2ht}{\Psi_e iL}\,(\text{cm. day}^{-1}). \tag{15}$$

A certain number (n) of such tubes together represent the total capillary conductivity for zero tension k_0. Placing the model variables at the left and the soil and other constants at the right, one can write:

$$\frac{\rho h}{i}\frac{r^4 n}{\eta L}t = \frac{8k_0\Psi_e}{60\pi gmd^2}. \tag{16}$$

The model variables have to be chosen in such a way that the tubes give a good imitation of the $k(\Psi)$ relation. Figure 6 gives an example of the tube distribution for a clay loam soil. It is clear that the number of tubes (n) should not be too small. For the sake of simplicity it was assumed that all tubes will have the same length (L) and radius (r). When building the model it is convenient, however, to use tubes of larger radius near the top of the vessels, this to avoid a large number of small tubes in that region of low tension where

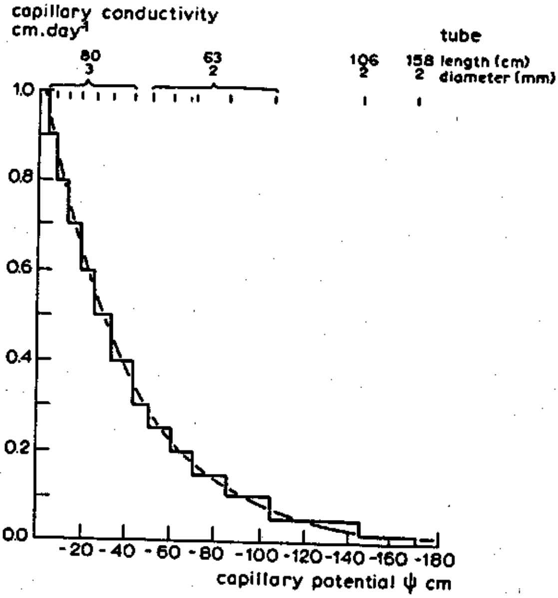

Fig. 6. Simulation of the capillary conductivity of a clay loam soil by means of 14 tubes.

in general the conductivity rapidly increases with increasing Ψ. In the lower parts of the vessel it is advisable to use longer tubes of a smaller radius because the capillary conductivity and its variation with Ψ is generally small.

There are two reasons which limit the freedom in choice of the model factors. The Reynolds number should not be too high, for the linear relationship between mass flow and hydraulic head does not hold for high Reynolds numbers. Further, the fluid content of the tubes should be small in relation to the vessel volume (i). The tubes will not always be full and moreover if they are emptied an amount of fluid which is not always the same will remain in them.

To simulate water removal by means of drains, water has to be removed from the last vessel by a single tube. If the last vessel is representing the soil layer at drainage depth, the outflow opening of the drainage tube must be at the same level as the top of the last vessel. According to Eq. (6) the drainage discharge has to be A cm. day^{-1} if the potential at drainage depth is 1 cm. If this datum multiplied by the three scale factors is brought into Eq. (13) the length L_d of the drainage tube is found:

$$A\frac{S_a}{S_t} = \frac{86400\pi r^4}{8\eta}\,\frac{\rho_g}{L_d}\,S_v$$

or

$$L_d = \frac{60\pi\rho g r^4 hmdt}{8\eta A\Psi_e i}. \tag{17}$$

Choice of the model factors

Aside from the two requirements mentioned above there are some practical arguments which determine the choice of the model factors. For a laboratory scale model the dimensions should not be too large. If the vertical scale is 1, a height of 2 m is necessary for simulation of a soil depth of 1 m and a tension range of 1 m. It is more convenient to confine the total height to 1 m or less to make easy reading possible. On the other hand the vertical scale should not be too small to avoid a serious loss in accuracy. So

$$1 > S_v > 0.2.$$

The choice of S_a mainly depends on the vessel volume i. In order to obtain a high accuracy i should be large, to reduce the laboratory room needed, i should be small. Therefore

$$100 < i < 10000 \text{ ml}.$$

The model time should be as short as possible to make simulation of long periods possible within a short time. Because of the limits of pump capacity and pumping accuracy as well as reading time $t=1$ min is already very short. More convenient is $t=3$ or 5 min.

The tube factors r, n, L and η should fit the requirements of a low Reynolds number and of a low total volume of the tubes. But here also practical reasons influence their value. The length L has to be at least hd/Ψ_e, but in practice the installation of fairly rigid nylon tubes of less than 30 cm is difficult. The number n should be not smaller than about 10, but there is a limit to the number of tubes one can install.

The Reynolds number is the product of density (g. ml^{-1}) of the fluid, velocity (cm. sec^{-1}) and diameter ($2r$ in cm) divided by the viscosity (poise). For gradient 1 the velocity is kS_a/S_t (cm. day^{-1}) divided by $\pi r^2 n$. So the Reynolds number for gradient 1, R_1 is:

$$R_1 = \frac{1440\rho ki}{mdt\pi r^2 n} \frac{2r}{\eta} \frac{1}{1440 \times 60} = \frac{\rho ki}{30\pi\eta mdtrn}. \tag{18}$$

Above a Reynolds number of 1000 the linear relation between hydraulic head and mass flow does no longer hold. As the model is based upon this linear relationship the model factor has to be chosen in such a way that the Reynolds number remains below 1000. In a model with $d=10$ cm and $\Psi_e=100$ the maximum gradient is 11. So the value of R_1 should not exceed 100. But such large gradients occur very seldom and only during a very short time, so the deviations caused by turbulence will not be important if $R_1 < 200$.

For the second requirement the relation F between total tube volume and vessel volume is introduced, so:

$$F = \frac{\pi r^2 nL}{i}. \tag{19}$$

As the tubes are sometimes full, at other times empty and often contain a not constant amount of fluid, F should not exceed 0.1. This value is chosen because the accuracy in determination of the moisture characteristic is about 10%.

It would be possible to construct a model in which $F=1$ or even larger; the ultimate being a model of only tubes at different levels connected by 'vessels' with neglectable volume. In such a model the $m(\Psi)$ relation would have to be simulated by the volume of the tubes. So instead of the tube-volume requirement discussed here there would be another requirement to fulfil.

For a low Reynolds number (Eq. 18) n and r have to be large and they have to be small to obtain a low total tube volume, so:

$$\frac{ki\rho}{30\pi\eta mdtnR_1} < r < \sqrt{\frac{iF}{\pi nL}}. \tag{20}$$

This means that there is a certain maximum k value which can be simulated with this model: it is the value of k for which the left and right hand part of Eq. (20) are the same. So for $R_1=200$ and $F=0.1$ this maximum k is

$$k_{max} = \frac{3350\eta mdt}{\rho}\sqrt{\frac{n}{iL}}\ (\text{cm. day}^{-1}). \tag{21}$$

If the most favourable values to reach a high k_{max} are used ($n=20$, $i=100$, $L=30$) if water is taken as the fluid ($\rho=1$ and $\eta=0.01$) and if further $m=0.1$ and $d=10$ with a model time $t=10$ min, then

$$k_{max} = 27.4\ \text{cm. day}^{-1}.$$

For larger conductivities more viscous fluids or longer model times will have to be used.

Some deviations from the field situation

Aside from the impossibility to simulate hysteresis effects, the model shows some other deviations from the field situation:

a. The capillary conductivity is that of the upper layer (for downward flow). In reality the conductivities of both the upper and lower layer will influence the flow.

b. The potential gradients are not correct if they are > 1. If the fluid has to flow downward through the connecting tubes, the flow can be stopped by air enclosures in the tube which is initially empty in its upper part and filled in its lower. So the tubes have to be as horizontal as possible, which means their beginning and end are at the same distance from the top of the vessels they are connecting. So the potential gradients are varying from the true gradient to gradient 1 (compare tube 5 with tubes 9, 10 and 11 in Fig. 4).

c. The built up of the model profile is in layers of a finite dimension (d), this will cause deviations with a field profile which consists of infinitely thin layers.

A. CAPILLARY CONDUCTIVITY

The solving of the flow processes for which the model is constructed can also be achieved by numerical integration. The flow velocities then are calculated by:

$$V = -k\left(\frac{\Psi_1 - \Psi_2}{d} + 1\right), \tag{22}$$

where $d = z_1 - z_2$, so it is a positive value. The question arises which value of k must be taken. The most convenient solution is to take either k_1 or k_2, but this is incorrect. If it is assumed that over the whole layer between z_1 and z_2 the conductivity is the same, the arithmetic mean can be used:

$$k_a = \frac{k_1 + k_2}{2}.$$

Of course this assumption is not true. A better assumption would be that over the distance from z_1 to $z_1 + d/2$ the conductivity is k_1, and that to depth z_2 it is k_2. To apply this, one has to use the harmonic mean:

$$k_h = \frac{2k_1k_2}{k_1 + k_2}$$

In reality the conductivity will change gradually from k_1 to k_2. According to Rijtema[3]) for not too low values of Ψ

$$k = k_0 e^{\alpha\Psi} \tag{23}$$

holds.

To calculate the flow velocity V we have to integrate

$$V = -k_0 e^{\alpha\Psi}\left(\frac{d\Psi}{dz} + 1\right) \tag{24}$$

between $\Psi=\Psi_1$ and $\Psi=\Psi_2$. This integration results in:

$$z_2 - z_1 = \Psi_1 - \Psi_2 + \frac{1}{\alpha} ln \frac{\dfrac{k_0}{v} + e^{-\alpha\Psi_1}}{\dfrac{k_0}{v} + e^{-\alpha\Psi_2}}.$$

As $z_2 - z_1 = d$, $e^{-\alpha\Psi} = k_0/k$ and $\Psi = 1/\alpha\, ln k/k_0$:

$$e^{-\alpha d} = \frac{k_1 + v}{k_2 + v}$$

or

$$v_d = \frac{k_2 e^{-\alpha d} - k_1}{1 - e^{-\alpha d}} = \frac{k_2 - k_1}{e^{\alpha d} - 1} - k_1. \tag{25}$$

This is the solution for exponential k if a depth interval d is used and which should be compared with the model solution (see Eq. 26).

B. GRADIENT AND CONDUCTIVITY IN THE MODEL

In the model the flow velocity is built up in two parts, for the lower part V_2 where the correct gradient exists because both ends of the tubes are below the water table in each vessel. For the flow in the upper part the beginnings of the tubes are below the water table in the upper vessel; but the ends are above the water table in the lower vessel (see Fig. 4).

So for V_2 the gradient is correct and the conductivity is k_2. But for V_1 the gradient increases from 1 at Ψ_1 to the correct gradient at Ψ_2

$$V_2 = -k_2\left(\frac{\Psi_1 - \Psi_2}{d} + 1\right).$$

For the upper part we assume an idealized infinitely large number of connecting tubes. At a depth Ψ between Ψ_1 and Ψ_2 there is a flow dV over a depth of $d\Psi$:

$$dV = -\left(\frac{\Psi_1 - \Psi}{d} + 1\right) dk = -\left(\frac{\Psi_1 - \Psi}{d} + 1\right) \alpha k_0 e^{\alpha\Psi} d\Psi.$$

By integration from Ψ_1 to Ψ_2 we find:

$$V_1 = k_2\left(\frac{\Psi_1 - \Psi_2}{d} + 1\right) + \frac{k_2 - k_1}{\alpha d} - k_1.$$

By combination of $V_m = V_1 + V_2$:

$$V_m = \frac{k_2 - k_1}{\alpha d} - k_1. \tag{26}$$

It appears that the two deviations of the model conductivity too high and gradient too low, give rise to an expression (26) which, if the values of αd are not too high, hardly differs from the theoretical solution (25).

C. ERRORS DUE TO THE CHOICE OF THE DEPTH INTERVAL

In numerical integration V will also be dependent on the choice of d. There will be a difference in the flow at a depth z between the calculations of:

$$V_d = \frac{k_{z-(d/2)} - k_{z+(d/2)}}{e^{\alpha d} - 1} - k_{z+(d/2)}$$

and

$$V_{d/n} = \frac{k_{z-(d/2n)} - k_{z+(d/2n)}}{e^{\alpha d/n} - 1} - k_{z+(d/2n)}.$$

How big this difference is can only be calculated if something is known about the $k(z)$ relation. If this is a straight line:

$$k = az + b \tag{27}$$

the difference then is:

$$V_d - V_{d/n} = -\frac{ad}{e^{\alpha d} - 1} + \frac{a^{d/n}}{e^{\alpha(d/n)} - 1} - \frac{ad - a^{d/n}}{2}.$$

If n is approaching infinity $V_{d/n}$ approaches the true velocity V_t

$$\lim_{n \to \infty} V_d - V_{d/n} = V_d - V_t = -\frac{ad}{e^{\alpha d} - 1} + \frac{a}{\alpha} - \frac{ad}{2}. \tag{28}$$

As the difference between the model and the theoretical solution is:

$$V_m - V_d = (k_2 - k_1)\left(\frac{1}{\alpha d} - \frac{1}{e^{\alpha d} - 1}\right) = -\frac{a}{\alpha} + \frac{ad}{e^{\alpha d} - 1}. \tag{29}$$

the sum of both differences, that is the total deviation of the model velocity, is:

$$V_m - V_t = V_d - V_t + V_m - V_d = -\frac{ad}{2}. \tag{30}$$

The 'model deviation' is therefore proportional to the depth interval d and the conductivity gradient a. Its relative value depends on the value of the true velocity V_t. As the latter is increasing with an increasing conductivity gradient, a is not the most important factor influencing the relative model deviation. More important are the other factors which decrease the absolute value of V_t such as a low value of k and a high α. Table 1 gives some examples of these relations.

TABLE 1

Values of V_m and V_t in relation to α, k and a with $d=10$ cm

α cm^{-1}	k cm. day^{-1}	a day^{-1}	d cm	V_t cm. day^{-1}	V_m cm. day^{-1}	$\frac{V_m}{V_t}$
0.025	10	1	10	−50	−55	1.10
0.025	1	1	10	−41	−46	1.12
0.025	0.1	1	10	−40.1	−45.1	1.12
0.025	10	0.1	10	−14	−14.5	1.03
0.025	1	0.1	10	− 5	− 5.5	1.10
0.025	0.1	0.1	10	− 4.1	− 4.6	1.12
0.10	10	1	10	−20	−25	1.25
0.10	1	1	10	−11	−16	1.45
0.10	0.1	1	10	−10.1	−15.1	1.49
0.10	10	0.1	10	−11.0	−11.5	1.05
0.10	1	0.1	10	− 2.0	− 2.5	1.25
0.10	0.1	0.1	10	− 1.1	− 1.6	1.45
0.10	1	−0.1	10	0	+ 0.5	∞
0.10	1	0	10	− 1.0	− 1.0	1.00

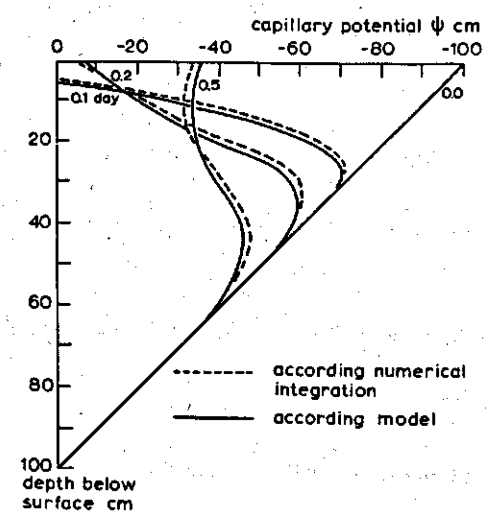

Fig. 7. Simulated moisture tension distribution at three time intervals after the start of a 1 cm rain falling in 0.1 day on a humous loamy sand. $k_0=1$ cm. day^{-1}; $\alpha=0.027$ cm^{-1}; $d=10$ cm.

Fortunately low values of k seldom occur together with high values of α so the relative deviation will mostly be small.

The influence of the model deviation is, that the obtained downward velocity is somewhat too high. This means that within a certain time interval the upper layer becomes too dry and the lower too wet. Consequently in the next time interval the downward velocity in the model will be somewhat less too high, or even too low. The model seems to have a feedback system which reduces the effect of the deviations. Figure 7 gives an example of the small differences between calculations with the model and with numerical integration for layers of $d = 10$ cm.

A special deviation is present when the rainfall intensity is larger than the infiltration capacity. In the field the excess water makes ponds on the surface or gives a surface runoff. In the model it is stored in the upper vessel and causes a moisture content and capillary conductivity which equal the highest values occurring in that soil layer in the field instead of the mean value. It is therefore necessary to simulate the upper layer in more than one vessel, e.g. representing 0–1 cm, 1–4 cm and 4–10 cm depth.

Some applications

A humous loamy sand (for the capillary conductivity and the moisture characteristic see Figs. 1 and 2) is drained at a depth of 100 cm with drainage intensity $A = 0.01$ day^{-1}. At zero time there is equilibrium, so the absolute values of the moisture tension are equal to the height above the groundwater level. Between zero time and 0.1 day there is 1 cm rainfall, after that no more precipitation is received. Figure 8 gives the moisture tension profiles which will occur under these conditions. Every layer is first growing wetter and then is drying out again. Deeper layers start to wet later than higher layers, the wetting velocity is lower and the total increase of Ψ is less. After 2 days the first discharge of drain water begins. After 5 days the moisture tension in the top 10 cm has reached -75 cm.

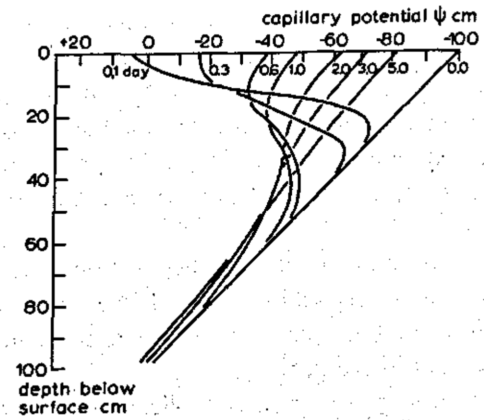

Fig. 8. Simulated infiltration of a 1 cm rain falling in 0.1 day in a humous loamy sand, drained at 100 cm depth with $A = 0.01$ day^{-1}.

Figure 9 shows the same process in a peat soil: the moisture characteristic and capillary conductivity is given in Figs. 1 and 2. Comparison of the peat soil and the humous loamy sand shows that the first wetting does not differ much. The initial velocity of drying out is higher in the peat soil, at time 0.3 the moisture tensions at 5 cm depth are -16 cm in the sand and -37 cm in

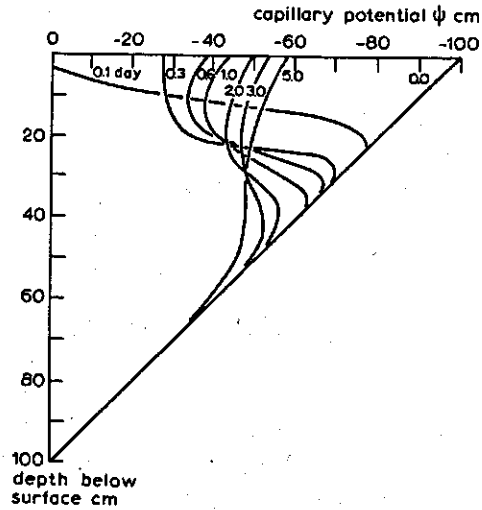

Fig. 9. Simulated infiltration of a 1 cm rain falling in 0.1 day in a peat soil, drained at 100 cm depth with A=0.01 day^{-1}; k_0=5.3 cm. day^{-1}; α=0.1045 cm^{-1}; d=10 cm.

the peat. But later the drying out of the peat soil is going much slower than that of the sand. After 5 days the moisture tension of the peat has reached only -55 cm. The drainage discharge does not begin before the fifth day.

The differences between peat and humous loamy sand are mainly caused by the fact that the capillary conductivity of the peat in fairly dry conditions ($\Psi < -30$ cm) is lower than in the sandy soil. Because of this property the gradients in the peat have to be much larger than in the loamy sand. The high conductivity of the peat under wet conditions did not have much opportunity to show its favourable effects, which will be the case, however, when there is additional rainfall.

In the field the equilibrium condition, which was here assumed to exist at zero time, will not or only very seldom occur. If there is no evapotranspiration it will take more than one month before it is restored in the loamy sand and more than one year in the peat.

Figure 10 gives an example of another application of the model. Humous loamy sand is drained at a depth of 80 cm with a drainage intensity A=0.011 day^{-1}. It is assumed that there is equilibrium at zero time. From then on rain

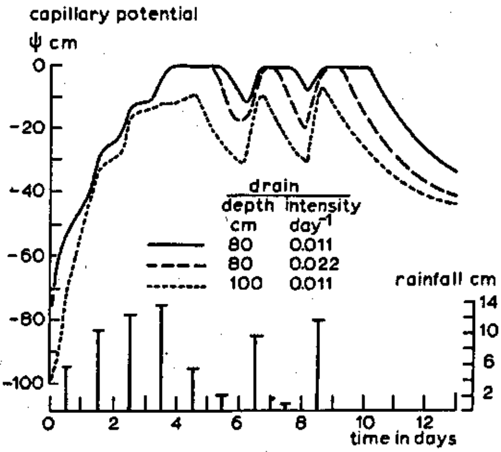

Fig. 10. Simulated course of the capillary potential Ψ under three drainage conditions.

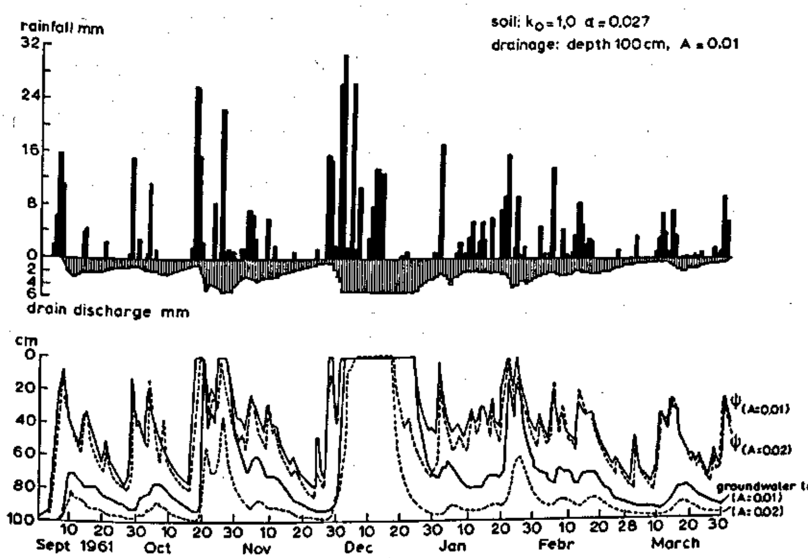

Fig. 11. Simulated course of groundwater table depth and capillary potential Ψ at depth 2.5 cm in a humous loamy sand drained at 100 cm depth with $A = 0.01$ day^{-1} and $A = 0.02$ day^{-1}, as a result of a rainfall pattern as registrated at De Bilt, The Netherlands, during the winter 1961–1962.

is falling as it did in the field in February 1963. The top 10 cm was saturated after 3 rains, falling in 3.5 days. Two ameliorations of the drainage conditions were considered. First to make a new drainage system at the same depth but with an intensity twice as large. From the model observations it appears that the effect is rather small at this rain sequence. The second amelioration plan, keeping the low drainage intensity but at 100 cm depth instead of 80 cm, seems to have considerably more effect. The effect of these two ameliorations will not be the same in every period. Dependent on rainfall intensity and frequency the moisture storage capacity or the drainage intensity will be more important. The model can be used to study this under natural or standardized rainfall conditions. This is shown in Fig. 11 where the rain of September 1961 to March 1962 is applied to the model of the humous loamy sand, drained at 100 cm with $A = 0.01$ and 0.02. An increase in drainage intensity has a considerable effect on groundwater table depth, which seldom comes above 80 cm in the case of $A = 0.02$, but very often in the case of $A = 0.01$. There is hardly any effect on the moisture tension in the top 5 cm, however, except during the 11 days shorter inundation in December with $A = 0.02$.

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A NUMERICAL MODEL FOR THE SIMULATION OF UNSATURATED VERTICAL FLOW OF MOISTURE IN SOILS

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ABSTRACT

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This paper describes a digital model for the simulation of non-steady unsaturated vertical flow of moisture in soils. By using an integrated formula for flow velocity errors due to averaging two conductivity values are avoided. This confines the applicability to situations in which the soil is not too dry.

The model also simulates the drainage process. So it produces the depth of the groundwater table and the discharge data. Other output data are pool depth, runoff, flow velocity at every depth, moisture content and tension for every layer. A discussion is given on the errors caused by averaging conductivity values in models. A chapter is dealing with oscillations caused by too large time steps. The time steps must be chosen in such a way that errors will not be amplified but reduced. The last chapter gives the costs of computation. For soils with low permeability the model seems to be very cheap to run. A digital model as the one described is fit for short runs with many soils. For long runs with one soil an analogous model is more appropriate.

INTRODUCTION

For many purposes a calculation of the effect of actual rainfall on the moisture conditions of the topsoil is needed. Moreover, often an estimate of the effect of ameliorative works on this process is wanted. An example of this is: how will the air content of a topsoil be influenced by rainfall patterns that actually occur and how will these air contents be affected by drainage measures? Another example: how will the bearing capacity of different soils for cattle or traffic, which is a function of their moisture content, be influenced by natural rainfall? A third one: what will be the amount of water stored upon the surface or the amount of runoff under natural rainfall patterns? How will these be affected by soil improvements or drainage?

For these and other purposes field data can be measured and improvements in soil and drainage conditions can be studied on experimental fields. That will cost much labour and give only a small amount of information. Many years are required to find a correct mean. Moreover, the experiments have to be repeated on different soils in order to get a generalization over a certain area.

The analytical knowledge of soil physics and hydrology, however, allows to calculate moisture movements; so the effect of actual rainfall on the moisture conditions of the topsoil can be calculated. This was shown by Van Keulen and Van Beek (1971) with a numerical model and by Wind (1972) with an analog one. Needed is of course knowledge of the moisture characteristic of the soil, the relation between capillary conductivity and moisture tension and the relation between drainage discharge and height of the groundwater table. These relations together with any rainfall pattern can then be put into the numerical model treated in this paper.

The model differs from other models in its very simple, and therefore cheap way of calculation of flow velocities which nevertheless is more accurate than that of other methods.

PRINCIPLE OF THE SIMULATION MODEL

Basic formulas

The basic formulas are:

flow velocity: $$v = -k \left(\frac{\partial \psi}{\partial z}\right) + 1 \quad (1)$$

conductivity: $$k = k_0 \, e^{\alpha\psi} \quad (2)$$

continuity: $$\frac{\partial m}{\partial t} = -\frac{\partial v}{\partial z} \quad (3)$$

drainage: $$v_D = -A \, \psi_D \quad (4)$$

where v = flow velocity (cm day^{-1}), positive in upward direction; k = capillary conductivity (cm day^{-1}); ψ = soil moisture tension (cm), negative in unsaturated soil; z = height above surface (cm), negative for every depth; k_0 = capillary conductivity at $\psi = 0$ (cm day^{-1}); α = soil constant (cm^{-1}); m = moisture content (cm^3 cm^{-3}); t = time (24 h = day); v_D = velocity of drainage discharge (cm day^{-1}), negative when drainage occurs; A = drainage intensity (day^{-1}); ψ_D = moisture tension at drain level (cm), mostly positive; D = drainage depth (cm), negative; d = depth interval ($z_1 - z_2$) (cm), positive.

Formula 1 is the Darcy equation for vertical flow, formula 2 is the Rijtema (1965) expression which is only valid for not too dry soils; 3 is the equation of continuity and 4 is a relation for drainage discharge based on Hooghoudt (1947), only holding midway between two parallel drains if the drainage resistance ($1/A$) is constant.

Vertical flow velocity

The velocity of vertical flow over the distance d between the centres of consecutive layers is supposed to be constant during one time interval. In this model ψ is thought to be a differentiable and thus continuous function of depth. As k varies with depth it is difficult to choose the proper value of k to be used in eq.1. This choice can be avoided by means of the following procedure which is valid during one time step in which the time is treated as a constant.

First eq.2 is differentiated with respect to z, yielding:

$$\frac{dk}{dz} = \alpha k \frac{d\psi}{dz}$$

For given t, substitution in eq.1 leads to the following first-order linear differential equation in which k is a function of z:

$$\frac{dk}{dz} + \alpha k + \alpha v = 0 \qquad (5)$$

This equation holds for a certain depth interval, v being constant. The solution of eq.5 is (see Ayres, 1952):

$$k = Ce^{-\alpha z} - v \qquad (6)$$

Then v is found by substituting the boundary conditions ($z = z_1$, $k = k_1$) and ($z = z_2$, $k = k_2$) and then solving the resulting linear equation for C and v. So

$$v = \frac{k_2 - k_1}{e^{\alpha d} - 1} - k_1 \qquad (7)$$

This expression, obtained under the sole assumptions that ψ is a differentiable function of z, and that v is constant over the depth interval during the time interval, is used in the model. Other methods cause errors which are dealt with later on.

The way in which eq.7 is applied in the computing model will be shown after infiltration and drainage discharge have been discussed.

Infiltration

The same expression 7 is applied for the flow velocity during infiltration at the surface but then d is replaced by $d/2$. This calculation is carried out only during rainfall or irrigation or if there is in any other way water coming into the soil surface. In the model the infiltration velocity is limited by the condition that no more moisture can infiltrate than is equivalent with the depth of the ponded water layer.

Drainage discharge

The velocity of drainage discharge is calculated with eq.4. Therefore ψ_D is to be calculated first. There is an unsaturated flow between the deepest unsaturated layer (depth z_n) and the groundwater table (depth z_g). Its velocity is the same as the discharge velocity. Applying eqs.4 and 7 yields:

$$v_D = -A\,\psi_D = \frac{k_g - k_n}{e^{\alpha(z_n - z_g)} - 1} - k_n \qquad (8)$$

A saturated flow, with the same velocity, exists between the groundwater table ($z = z_g$, $\psi = \psi_g = 0$, $k = k_g = k_0$) and the drainage depth ($z = z_D = D$, $\psi = \psi_D$). Applying eq.1 with constant $d\psi/dz$ gives:

$$v_D = -A\,\psi_D = -k_0\left(1 - \frac{\psi_D}{z_g - z_D}\right) \qquad (9)$$

From eq.9 ψ_D can be solved and substituted in eq.8. So:

$$\frac{k_0 - k_n}{e^{\alpha(z_n - z_g)} - 1} - k_n = \frac{-A\,k_0\,(z_g - z_D)}{k_0 + A\,(z_g - z_D)} \qquad (10)$$

By means of eq.10 the value of z_g has to be determined by iteration. To that purpose it is changed into:

$$z_g - z_D - \frac{k_0}{A\,(k_0 - k_n)}\left[k_n - k_0\,e^{\alpha(z_g - z_n)}\right] = 0$$

From this it is concluded that z_g is the zero z of the function $f(z)$ defined by:

$$f(z) = z - a + b\,e^{\alpha(z - z_n)} \qquad (\alpha,\ b > 0)$$

where a, b and α are constants. The zero of $f(z)$ is found by successive halving of z-intervals choosing the subsequent intervals in such a way that $f(z)$ has different signs at the ends of these intervals (Seidel, 1970).

Changes in the moisture distribution according to the computing model

Starting from a given moisture distribution in soil the value of the capillary conductivity for each layer of soil is calculated by means of $k = k_0\,e^{\alpha\psi}$. Then the vertical flow velocities $v_1, v_2, \ldots, v_{n-1}$ and the velocity of infiltration v_0 during one time interval are calculated according to formula 7. After this the velocity of drainage discharge v_n ($= v_D$) is determined by means of the theory given in the preceding section (eq.9).

From the difference of two velocities the increase of moisture content is calculated according to eq.3. The differences $v_i - v_{i-1}$ have therefore to be divided by the depth interval Δz ($= d$), and multiplied by the time step Δt. We then get:

TABLE I

FORTRAN program for water flow in soils: main program FLOW

```
      PROGRAM FLOW (INPUT,OUTPUT,TAPE2=INPUT,TAPE3=OUTPUT)
C (0)
C *** FOR EACH TIME STEP THE PROGRAM CALCULATES THE VELOCITY OF
C *** MOISTURE FLOW (CM/DAY), MOISTURE CONTENT (CC/CC) AND
C *** MOISTURE TENSION (CM) IN A HOMOGENEOUS SOIL.
C *** THE POOL DEPTH AT THE SURFACE (CM), DRAIN DISCHARGE
C *** VELOCITY (CM/DAY) AND DEPTH OF GROUND WATER (CM) ARE ALSO OBTAINED.
C ***
C *** RAIN FALL/IRRIGATION, SOIL PARAMETERS AND DRAINAGE ARE ACCOUNTED FOR,
C *** BUT HYSTERESIS AND EVAPORATION ARE NOT.
C
C PRINT CONTROL BY HEADER CARD VARIABLES(INDEX1,NA,NB,DPRINT) IS AS FOLLOWS
C     INDEX1=0  FOR PRINTING COMPLETE INFORMATION.
C     INDEX1=1  FOR PRINTING ONLY POOL DEPTH, VELOCITY OF INFILTRATION,
C                             DEPTH OF GROUND WATER AND DRAIN DISCHARGE VELOCITY.
C     NA     =  NUMBER OF TIME STEPS AFTER WHICH OUTPUT HAS TO BE PRINTED
C               IN THE CASE OF POOL FORMATION.
C     NB     =  NUMBER LIKE *NA*, IN THE CASE OF NO POOL FORMATION.
C     DPRINT=   CONSTANT PRINT INTERVAL (DAYS).
C               IF *DPRINT* IS GIVEN, IT DOMINATES *NA* AND *NB*.
C
C READ CONTROL BY HEADER CARD VARIABLE (INDEX2) IS AS FOLLOWS
C     INDEX2=0  FOR STARTING SIMULATION AT AN EQUILIBRIUM SITUATION AND
C               CALCULATING MOISTURE TENSIONS TO DEFINE THE INITIAL PROFILE

C     INDEX2=1  FOR READING MOISTURE CONTENTS TO DEFINE THE INITIAL PROFILE
C     INDEX2=2  FOR READING MOISTURE TENSIONS TO DEFINE THE INITIAL PROFILE
C
      COMMON ALFA,CNDSAT,COND(101),DEPTH,DPRINT,DRAINT,DELTA,FLAY,
     1       FSURF,GRWT,INDEX1,INDEX2,IPER,ISAT,ITE,ITOT,JPT,LUNS,MAXT,
     2       NA,NB,NL,PMAX,POOL,PUNLIM,Q,RAIN(1000),RUNOFF,TENS(101)
     3      ,TENST(501),THLAY,TIME,VELOC(101),WC(101),WCMIN,WCSAT,WCSTEP
     4      ,INSAT(101)
      DIMENSION DT(1000),NT(1000),T(1000)
  101 FORMAT(I3)
  102 FORMAT(2I1,2I3,2F8.3)
  103 FORMAT(2F8.3,F9.4)
  104 FORMAT(15F4.1)
  105 FORMAT(I3/(2F10.4,F12.0))
  106 FORMAT(1H1,/,11H SIMULATION,I3,10H STARTS AT TIME=,F11.3,5H DAYS/)
  107 FORMAT(40H CAPILLARY CONDUCTIVITY AT MOISTURE TENSION 0  =F8.3,7H
     *CM/DAY/40H CONSTANT IN THE EXPONENT OF RIJTEMAS RELATION
     *      /40H BETWEEN CAP. CONDUCTIVITY AND MOISTURE TENSION=F8.3,9H
     *UNITS/CM,/)
  108 FORMAT(40H INITIAL VALUE OF POOL DEPTH                    =F8.3,
     *3H CM,/40H UPPER LIMIT OF POOL DEPTH                      =F8.3,
     *3H CM,/40H INITIAL VALUE OF CUMULATED RUNOFF              =F8.3,
     *3H CM,/)
  109 FORMAT(40H DRAIN DEPTH BELOW SURFACE                      =F8.3,
     *3H CM,/40H DRAINAGE INTENSITY                             =F8.6,
     *10H UNITS/DAY,/)
  110 FORMAT(40H THICKNESS OF EACH LAYER OF SOIL                =F8.3,
     *3H CM,//)
  111 FORMAT(40H LOWER LIMIT OF VOLUMETRIC MOISTURE CONTENT     =F8.3,
     *       /40H UPPER LIMIT OF VOLUMETRIC MOISTURE CONTENT    =F8.3,
     *       /40H STEP SIZE OF MOISTURE CONTENT IN TABLE BELOW  =F11.8//)
  112 FORMAT(40H TABLE OF MOISTURE TENSION VALUES AT EQUIDISTANT
     *        55H MOISTURE CONTENTS (FOR LIMITS AND STEP SIZE SEE ABOVE))
  113 FORMAT(/,(15F8.1))
  114 FORMAT(//38H RAIN FALL OR IRRIGATION IS AS FOLLOWS
     *        ,//40H PERIOD       PERIOD        AMOUNT       TIME STEP
     *        ,/49H NUMBER    LENGTH(DAYS)  OF WATER(CM)  SIZE(DAYS),/)
  115 FORMAT(I5,F16.5,F13.4,F13.6)
  116 FORMAT(F10.7)
  117 FORMAT(1H1,/37H THE INITIAL MOISTURE CONTENT PROFILE,/)
  118 FORMAT(/49H DEPTH OF CENTRE   MOISTURE   MOISTURE   VELOCITY,/,
     *        49H  OF LAYER (CM)     CONTENT    TENSION    OF FLOW,/)
  119 FORMAT(1H1)
  120 FORMAT(/6H TIME=,F8.3,17H DAYS, TIME STEP=,F8.5,19H DAYS, CUM. RUN
     *OFF=,F8.3,16H CM, POOL DEPTH=,F7.3,3H CM/,28H VELOCITY OF INFILTRA
     *TION=,F7.3,28H CM/DAY, GROUND WATER DEPTH=F11.1,18H CM BELOW SURFA
     *CE.17H DRAIN DISCHARGE=,F7.3,8H CM/DAY.)
  121 FORMAT(F8.3,F17.12)
C
C (1) PROGRAM *FLOW* STARTS BY READING THE NUMBER OF SIMULATIONS (*NUMSIM*)
C
      READ(2,101)  NUMSIN
      DO 19 ISIM=1,NUMSIN
C
C (2) READING THE DATA FOR ONE SIMULATION.  *****************************
C     THE INITIAL MOISTURE CONTENT PROFILE HOWEVER, IS DEFINED IN PART(7)*
C                                                                      **
      READ(2,102) INDEX1,INDEX2,NA,NB,DPRINT,TIME                      **
      READ(2,103) CNDSAT,ALFA                                          **
      READ(2,103) POOL,PMAX,RUNOFF                                     **
      READ(2,121) DEPTH,DRAINT                                         **
      READ(2,121) THLAY,DELTA                                          **
      READ(2,103) WCMIN,WCSAT,WCSTEP                                   **

      NUMTEN=(WCSAT-WCMIN)/WCSTEP+1.5                                  **
      READ(2,104)(TENST(I),I=1,NUMTEN)                                 **
C     READ(2,105) NUMPER,(RAIN(I),T(I),DT(I),I=1,NUMPER)               **
C     *****************************************************************
C (3) TIME STEPS ARE SET EQUAL TO CONSTANT *DELTA* IF THE LATTER IS GIVEN.
C     TIME STEPS ARE LIMITED BY THE LENGTH OF THE RAIN/IRRIGATION PERIOD.
C
      DO 1 I=1,NUMPER
      IF(DELTA.NE.0.) DT(I)=DELTA
      IF(T(I).LT.DT(I)) DT(I)=T(I)
    1 CONTINUE
C
C (4) PRINTING DATA.  **************************************************
C                                                                      **
      WRITE(3,106) ISIM,TIME                                           **
      WRITE(3,107) CNDSAT,ALFA                                         **
      WRITE(3,108) POOL,PMAX,RUNOFF                                    **
      WRITE(3,109) DEPTH,DRAINT                                        **
      WRITE(3,110) THLAY                                               **
      WRITE(3,111) WCMIN,WCSAT,WCSTEP                                  **
      WRITE(3,112)                                                     **
      WRITE(3,113)(TENST(I),I=1,NUMTEN)                                **
      WRITE(3,114)                                                     **
      WRITE(3,115)(I,T(I),RAIN(I),DT(I),I=1,NUMPER)                    **
C     *****************************************************************
C
C (5) CALCULATION OF CONSTANTS FOR ONE SIMULATION.
C
      P=ALFA*THLAY
      IF(P.GT.100.) P=100.
      FSURF=1./(EXP(P*0.5)-1.)
      FLAY =1./(EXP(P*1.0)-1.)
      NL    =DEPTH/THLAY+1.5
      TENST(NUMTEN+1)=0.
      Q=0.
      DO 2 I=1,NUMPER
      NT(I)=T(I)/DT(I)+0.5
    2 RAIN(I)=RAIN(I)/NT(I)
C
C (6) DEFINITION OF INITIAL VALUES FOR ONE SIMULATION.
C
      ITE=1
      JPT=NA
      IF(POOL+RAIN(1).LE.0.) JPT=NB
      ITOT=JPT
      LUNS=NL
      DO 3 I=1,NL
    3 VELOC(I)=1.E-6
C
C (7) DEFINITION OF THE INITIAL MOISTURE DISTRIBUTION
C     USING THE HEADER CARD VARIABLE *INDEX2*.
C
      IF(INDEX2.NE.1) GOTO 4
      READ(2,116)(WC(I),I=2,NL)
      GOTO 11
    4 DO 10 IKM=2,NL
      IF(INDEX2.EQ.0) GOTO 5
      READ(2,103) TENS(IKM)
      GOTO 6
    5 TENS(IKM)=DEPTH-(IKM-1.5)*THLAY
    6 DO 7 KIM=1,NUMTEN
      KMI=KIM-1

      IF(TENS(IKM).GE.TENST(KIM)) GOTO 8
    7 CONTINUE
      WC(IKM)=WCSAT
      GOTO 10
    8 IF(KMI.EQ.0) GOTO 9
      VE=(TENST(KMI)-TENS(IKM))/(TENST(KMI)-TENST(KMI+1))
      WC(IKM)=WCMIN+(KMI-1.+VE)*WCSTEP
      GOTO 10
    9 WC(IKM)=WCMIN
   10 CONTINUE
C
C (8) PRINTING THE INITIAL MOISTURE CONTENTS AND MOISTURE TENSIONS.
C
   11 WRITE(3,117)
      WRITE(3,118)
      CALL TABLE
      WRITE(3,119)
C
C (9) CALCULATION OF MOISTURE REDISTRIBUTION BY SUBROUTINE *MOIDIS*,
C     PRINTING OUTPUT BY SUBROUTINE *TABLE*.
C
      DO 18 IPER=1,NUMPER
      DELTA=DT(IPER)
      IF(DPRINT.EQ.0) GOTO 12
      JPT=DPRINT/DELTA+0.5
      IF(IPER.EQ.1) ITOT=JPT
   12 Q=DELTA/THLAY
      N=NT(IPER)
      DO 17 JP=1,N
      TIME=TIME+DELTA
      CALL MOIDIS
      IF(ISAT.EQ.0)                         GOTO 19
      IF(ITE.NE.1 .AND. ITE.NE.ITOT) GOTO 14
      WRITE(3,120) TIME,DELTA,RUNOFF,POOL,VELOC(1),GRWT,VELOC(NL)
      IF(ITE.EQ.1)     GOTO 13
      IF(ITE.NE.ITOT) GOTO 14
      IF(LUNS.EQ.1)    GOTO 14
      IF(INDEX1.NE.0) GOTO 14
   13 WRITE(3,118)
   14 CALL TABLE
      IF(MAXT.NE.0)                         GOTO 19
      IF(ITE.NE.1 .AND. ITE.NE.ITOT) GOTO 16
      IF(ITE.NE.1)                          GOTO 15
      IF(ITOT.GT.ITE)                       GOTO 16
   15 ITOT=ITOT+JPT
   16 ITE=ITE+1
   17 CONTINUE
   18 CONTINUE
   19 CONTINUE
      CALL EXIT
      END
```

TABLE II

Subprograms TABLE and MOIDIS called by the main program FLOW in Table I

```
      SUBROUTINE TABLE
C (0)
C *** THIS SUBPROGRAM CALCULATES MOISTURE CONTENTS AFTER ONE TIME STEP,
C *** USING THE VELOCITIES OF FLOW OBTAINED BY SUBROUTINE 'MOIDIS',
C *** THE CORRESPONDING MOISTURE TENSIONS ARE ALSO CALCULATED.
C *** IF INDEX1=0 (SEE PROGRAM 'FLOW', PART (0)) A TABLE IS PRINTED
C *** WHICH FOR EACH LAYER OF UNSATURATED SOIL CONTAINS
C *** MEAN DEPTH 'DEP', MOISTURE CONTENT 'WC', MOISTURE TENSION 'TENS'
C *** AND VELOCITY OF FLOW FROM THE LAYER CONCERNED ('VELOC').
C
      COMMON ALFA,CNDSAT,COND(101),DEPTH,DPRINT,DRAINT,DELTA,FLAY,
     1       FSURF,GRWT,INDEX1,INDEX2,IPER,ISAT,ITE,ITOT,JPT,LUNS,MAXT,
     2       NA,NB,NL,PMAX,POOL,PUNLIM,Q,RAIN(1000),RUNOFF,TENS(101)
     3      ,TENST(501),THLAY,TIME,VELOC(101),WC(101),WCMIN,WCSAT,WCSTEP
     4      ,INSAT(101)
  101 FORMAT(F11.2,F15.4,2F11.3)
  102 FORMAT(/25H SIMULATION STOPPED AT TIME=F12.6/17H BECAUSE MOISTURE
     *        17H TENSION IN LAYER,I4,20H EXCEEDED THE MAXIMUM VALUE,/
     *        47H ACCORDING TO THE SOIL MOISTURE CHARACTERISTIC./)
C
C (1) MOISTURE CONTENTS AFTER ONE TIME STEP, BOUNDED IF NECESSARY.
C     SIMULATION WILL STOP IF A MOISTURE CONTENT LESS THAN 'WCMIN' IS FOUND.
C
      MAXT=0
      DO 1 I=2,NL
      W=WC(I)+(VELOC(I+1)-VELOC(I))*0*1.E-12
      INSAT(I)=-1
      IF(W.LT.WCSAT) GOTO 4
      INSAT(I)=+1
      W=WCSAT
    4 IF(W.GE.WCMIN) GOTO 1
      MAXT=I-1
      WRITE(3,102) TIME,MAXT
      STOP 1
    1 WC(I)=W
C
C (2) MOISTURE TENSIONS AND PRINTING THE TABLE.
C
      DO 3 I=2,NL
      DEP=(I-1.5)*THLAY
      U=(WC(I)-WCMIN)/WCSTEP+1.
      J=U
      FACT=U-J
      TENS(I)=(1.-FACT)*TENST(J)+FACT*TENST(J+1)
      IF(ITE.EQ.1)      GOTO 2
      IF(ITE.NE.ITOT)   GOTO 3
      IF(LUNS.EQ.1)     GOTO 3
      IF(INDEX1.NE.0)   GOTO 3
    2 IF(I.GT.LUNS)     GOTO 3
      WRITE(3,101) DEP,WC(I),TENS(I),VELOC(I)
    3 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MOIDIS
C (0)
C *** THIS SUBPROGRAM CALCULATES VELOCITIES OF FLOW BETWEEN LAYERS OF SOIL,
C *** POOL DEPTH, DRAIN DISCHARGE VELOCITY AND DEPTH OF GROUND WATER
C *** ARE ALSO OBTAINED.
C
      COMMON ALFA,CNDSAT,COND(101),DEPTH,DPRINT,DRAINT,DELTA,FLAY,
     1       FSURF,GRWT,INDEX1,INDEX2,IPER,ISAT,ITE,ITOT,JPT,LUNS,MAXT,
     2       NA,NB,NL,PMAX,POOL,PUNLIM,Q,RAIN(1000),RUNOFF,TENS(101)
     3      ,TENST(501),THLAY,TIME,VELOC(101),WC(101),WCMIN,WCSAT,WCSTEP

     4      ,INSAT(101)
  101 FORMAT(/25H SIMULATION STOPPED AT TIME=F12.6/14H BECAUSE LAYER
     *      ,I4,44H IS SATURATED WHILE THE DEEPER LAYER IS NOT./)
C
C (1) FINDING THE SEQUENCE NUMBER OF THE DEEPEST UNSATURATED LAYER ('LUNS')
C     EXAMINING WHETHER THE CONDITION THAT SATURATED LAYERS SHOULD LY
C     DEEPER THAN UNSATURATED ONES, IS SATISFIED (ISAT=1) OR NOT (ISAT=0).
C
      ISAT=1
      JSAT=1
      KSAT=1
      LUNS=NL
      DO 2 I=2,NL
      J=NL-I+2
      IF(INSAT(J).LT.0) GOTO 1
      LUNS=J-1
      JSAT=1
      IF(KSAT.NE.0) GOTO 2
      ISAT=0
      JSAT=J-1
      WRITE(3,101) TIME,JSAT
      STOP 2
    1 JSAT=0
    2 KSAT=JSAT
C
C (2) CALCULATION OF  . DRAIN DISCHARGE VELOCITY
C                       IN THE CASE THAT EACH LAYER IS SATURATED ('DISCHA')
C                     . VELOCITIES OF UNSATURATED FLOW             ('VELOC')
C
      IF(LUNS.NE.1) GOTO 3
      U=(DEPTH-0.5*THLAY)*DRAINT
      DISCHA=CNDSAT*U/(CNDSAT+U)
      MAX=2
    3 DO 4 I=2,NL
    4 COND(I)=CNDSAT/EXP(ALFA*TENS(I))
      IF(LUNS.EQ.1) GOTO 9
      MAX=NL+1
      IF(LUNS.LT.NL) MAX=LUNS
      DO 5 I=2,MAX
    5 VELOC(I)=(1.-FLAY)*COND(I)+FLAY*COND(I+1)
C
C (3) CALCULATION OF  . DRAIN DISCHARGE VELOCITY ('DISCHA')
C                     . GROUND WATER DEPTH       ('GRWT')
C
      U=DEPTH-(LUNS-1.5)*THLAY
      H=COND(LUNS)
      X=0.
      X2=U-TENS(LUNS)
      IF(X2**2 .LT. 0.01) GOTO 7
      X1=0.
      F=CNDSAT/(DRAINT*(CNDSAT-H))
      P=ALFA*U
      IF(P.GT.100.) P=100.
      Y1=-F*(H-CNDSAT/EXP(P))
      Y2=X2
      DO 6 NIT=1,15
      X=0.5*(X1+X2)
      P=ALFA*(X-U)
      IF(P.LT. -100.) P=-100.
      Y=X-F*(H-CNDSAT*EXP(P))
      IF(Y**2 .LT. 0.01) GOTO 7
      IF(Y*Y1 .GT. 0.)   GOTO 51
      X2=X

      Y2=Y
      GOTO 6
   51 X1=X
      Y1=Y
    6 CONTINUE
    7 U=X*DRAINT
      DISCHA=CNDSAT*U/(CNDSAT+U)
      GRWT=DEPTH-X
C
C (4) CALCULATION OF  . VELOCITY OF INFILTRATION    ('VELOC(1)')
C                     . VELOCITY OF SATURATED FLOW  ('VELOC(I)')
C                     . POOL DEPTH AFTER ONE TIME STEP ('POOL')
C                     . CUMULATED RUNOFF             ('RUNOFF')
C
      IF(LUNS.GE.NL) GOTO 9
      IF(VELOC(MAX).LT.DISCHA) GOTO 8
      VELOC(MAX)=DISCHA
    8 MAX=MAX+1
    9 VELOC(1)=0.
      POOL=POOL+RAIN(IPER)
      DO 10 I=MAX,NL
   10 VELOC(I)=DISCHA
      IF(POOL.LE.0.) GOTO 11
      VELOC(1)=CNDSAT+FSURF*(CNDSAT-COND(2))
      IF(LUNS.EQ.1 .AND. VELOC(1).GE.DISCHA) VELOC(1)=DISCHA
   11 U=POOL-VELOC(1)*DELTA
      IF(U.GT.0.) GOTO 12
      VELOC(1)=POOL/DELTA
      POOL=0.
      IF(DPRINT.EQ.0.) JPT=NB
      GOTO 13
   12 POOL=U
      IF(DPRINT.EQ.0.) JPT=NA
   13 PUNLIM=POOL
      IF(POOL.GE.PMAX) POOL=PMAX
      IF(LUNS.EQ.1) GRWT=-POOL
      RUNOFF=RUNOFF+PUNLIM-POOL
      RETURN
      END
```

$$\Delta m_i = (v_i - v_{i-1})\frac{\Delta t}{\Delta z} \qquad i = 1, 2, \ldots, n \tag{11}$$

after which the values $m_i + \Delta m_i$ give the moisture distribution at the beginning of the next time interval, which is treated in a similar manner as above.

The choice of Δt is very important: if chosen too small it requires too much computer-time; if chosen too large the calculation sequence looses its stability and the calculation results begin to oscillate. Later in this article the best choice of Δt will be dealt with.

GENERAL REMARKS ON THE SIMULATION PROGRAM

The computer program FLOW (see Table I) written in FORTRAN IV simulates the vertical flow of moisture in soil, with data given on punched cards. Data are described in the paragraph on Input. The soil is thought to be homogeneous and divided into a number of layers of equal thickness.

For the realization of the rainfall pattern the simulation period is divided into sub-periods each with its own length, amount of rainfall and time interval. The initial moisture distribution can be given in three ways: for each layer the moisture content or the moisture tension, or as the general expression zero flow.

The main program FLOW is controlling two subprograms, MOIDIS and TABLE (Table II).

MOIDIS calculates the flow velocities, drainage discharge, depth of the groundwater table and the amount of water upon the surface. TABLE calculates the moisture contents and moisture tensions caused by the flow velocities after one time step. Moreover TABLE prints the tables of moisture contents, tensions and flow velocities. The other part of output is printed by FLOW.

Input

The required number of simulations (NUMSIM) is read first. After that: the beginning time (TIME, days); the initial value of pool depth (POOL, cm); the pool depth at which runoff starts (PMAX, cm); the initial value of accumulated runoff (RUNOFF, cm).

Two soil constants and the drainage data are put in: capillary conductivity at moisture tension zero (CNDSAT, cm day^{-1}); factor α from eq.2 (ALFA, cm^{-1}); drainage depth (DEPTH, cm); drainage intensity (DRAINT, day^{-1}).

On the card which gives the depth step $\Delta z = d$ (THLAY, cm), the factor DELTA can be given. If it is present the whole simulation is done with the fixed time interval Δt = DELTA. Otherwise the time step is read in every sub-period. The moisture characteristic is given in a table of moisture tensions for equidistant values of the moisture content. The latter values are defined by: minimum moisture content (WCMIN, cm^3 cm^{-3}); maximum moisture content

(WCSAT, cm^3 cm^{-3}); step size in moisture characteristic table (WCSTEP, cm^3 cm^{-3})

Also read is the moisture characteristic table (TENST, cm).

The rain and irrigation data are preceded by a punched card of the number of subperiods within the whole simulation (NUMPER).

For each subperiod the following data are given on one card: total amount of rainfall (RAIN, cm); length of the subperiod (T, days); the time step (DT, days) in the case that DELTA was not given previously.

Each set of data for one simulation contains a header card on which control variables are given. INDEX1, NA, NB and DPRINT perform print control. INDEX2 indicates the kind of initial moisture data to be used, see program FLOW, part 0.

To define the moisture situation at zero time (FLOW, part 7) one card per layer is read, containing: moisture content (WC, cm^3 cm^{-3}) if INDEX2 = 1 or moisture tension (TENS, cm) if INDEX2 = 2. If INDEX2 = 0 there is static equilibrium; no data are required.

Output

The output of the program at every printing time gives the information: time (TIME); used time step (DT); accumulated runoff (RUNOFF); pool depth (POOL); infiltration velocity (VELOC(1)); depth of the groundwater table (GRWT); drain discharge velocity (VELOC(NL)).

At times that the soil is saturated completely this is the total output, as it is also if INDEX1 = 1. In other cases the output is completed with a table giving for each unsaturated layer: depth of the middle of the layer (DEP); moisture content (WC); moisture tension (TENS); flow velocity to the next deeper layer (VELOC).

For checking purposes the whole input and the complete output after one time step are printed, independent of the indications on the header card. The choice INDEX1 = 1 decreases the costs and is used if one is merely interested in phenomena concerning the surface, drainage and groundwater table. Computing costs are dealt with in the section on "Estimating computing costs".

SOME EXAMPLES

Fig.1 gives the result of 4 simulations with the same soil and the same rainfall pattern. There were two drainage depths: D = 80 and 100 cm; and two drainage intensities: A = 0.01 and 0.02 day^{-1}. The soil is a humous loamy medium coarse sand for which k_0 = 1 cm day^{-1} and α = 0.027 cm^{-1}. The moisture characteristic is given in (ψ; m): (0; 51.0) (10; 49.5) (20; 47.9) (50; 45.2) (100; 43.0). The rain pattern is shown at the top of Fig.1; the rainfall rate was kept constant at 5 cm day^{-1}. Runoff was supposed not to occur. The initial situation was static equilibrium.

The duration of inundation of the soil drained at 80 cm depth with

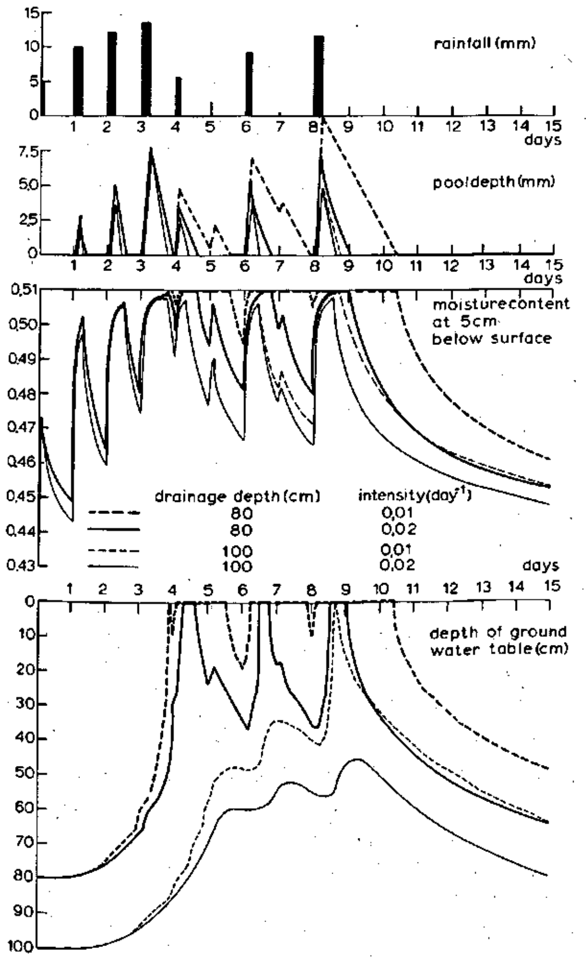

Fig.1. Simulated course of pool depth, moisture content of top layer and depth of groundwater.

intensity 0.01 day^{-1} (80; 0.01) was 7.6 days. A better drainage at the same depth (80; 0.02) gave 3.8 days. But with the deeper drainage at 100 cm depth a still shorter inundation occurred: 1.5 and 0.6 days.

The moisture contents in the top layer are scarcely affected by the drainage depth and intensity during the first 4 days. Only the drainage depth seems to have some influence during the rainless periods. After 4 days the intensity of the shallower drainage is very important. The soil with $A = 0.01$ is fully saturated up to 10.4 days with small exceptions near days 6 and 8. The (80; 0.02) drained soil is less wet. The difference between the shallower and deeper drainage is paramount. In the deeper drained soil there is only small difference in the topsoil moisture contents between the soils with the two drainage intensities.

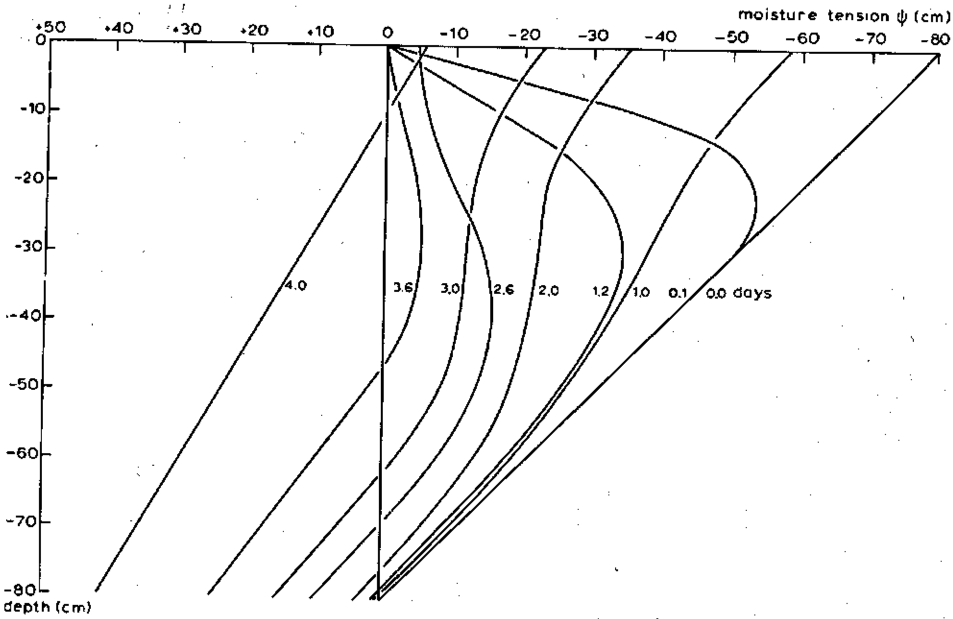

Fig.2. Moisture tension profile at 9 points of time for the first case of Fig.1.

There is a marked influence of drainage depth and intensity on groundwater depth. The influence on groundwater depth is much larger than that on moisture content of the topsoil. The latter has more importance for agriculture and other applications of the soil than the former. Considering the groundwater depth only therefore will cause an overestimation of the effect of drainage improvements and models as the one described can contribute to a better appreciation of the effects of drainage.

One can see a pronounced time lag occurring between the groundwater peak and the rain at day 6. This lag is dependent on the depth of the groundwater; for the highest it is zero, for the lowest, with groundwater at 60 cm, the time lag is about one day.

Slight oscillations can be seen in the course of the groundwater depth between 2 and 5 days. This is a result of the choice of the depth interval of 10 cm. Decrease of this interval wipes out the oscillations but does not affect the calculations significantly.

Fig.2 gives an impression of the distribution of soil moisture tension during the first 4 days.

Errors resulting from averaging conductivity values

According to eq.1, in every model one has to choose an average value of k in the formula:

$$v = -k\left(\frac{\psi_1 - \psi_2}{z_1 - z_2} + 1\right)$$

Mostly the arithmetic mean is chosen. Because the moisture is flowing from z_1 to z_2 over two equal distances ($d/2$) with different conductivities (k_1 and k_2) it is not correct to use the arithmetic mean. If there are really only two conductivities the harmonic mean is to be used.

In electrical models the potentials ψ are readily available, therefore it is then easy to calculate the mean k from the mean ψ. In that case:

$$k = k_0 \mathrm{e}^{\alpha[(\psi_1 + \psi_2)/2]} = k_0^{1/2} \mathrm{e}^{\alpha(\psi_1/2)} k_0^{1/2} \mathrm{e}^{\alpha(\psi_2/2)} = \sqrt{k_1 k_2}$$

This is the geometric mean.

None of the averages mentioned here are correct. The real value of the velocity v is found by integration of formula 1 which results in:

$$v = \frac{k_2 - k_1}{\mathrm{e}^{\alpha d} - 1} - k_1 \qquad (7)$$

if the Rijtema expression is valid. For other cases the correct value of v is to be found by numerical integration between z_2 and z_1.

TABLE III

Flow velocities (in cm day^{-1}) calculated with different averages of k_1 and k_2 for $\psi_1 = 0$, $\psi_2 = -100$ cm, $d = 10$ cm and $k_0 = 1$ cm day^{-1}

Average used	$\alpha = 0.02$	$\alpha = 0.10$
k_1	−11.00	−11.00
k_2	−1.49	−0.00
Arithmetic mean	−6.24	−5.50
Geometric mean	−4.05	−0.074
Harmonic mean	−2.62	−0.001
Hydraulic analog	−5.32	−2.00
True velocity	−4.91	−1.58

In Table III flow velocities are calculated with different averages of k_1 and k_2. The true velocity in this table was calculated with eq.7. It gives the real value of the velocity under these circumstances which is also the value used in the model described in this paper. The hydraulic analog is that of Wind (1972); it determines the velocity according to:

$$v = \frac{k_2 - k_1}{\alpha d} - k_1$$

For low values of α there are small differences between the methods used; for higher values of α, however, the values differ more than one order of magnitude. The harmonic mean is certainly no good proposition; the geometric mean is fairly good for $\alpha = 0.02$ but cannot be used with $\alpha = 0.10$; the arithmetic mean is the best of these three, although it gives 3 times the true velocity for $\alpha = 0.10$.

TABLE IV

Flow velocities calculated with different methods in a sandy clay loam where $\psi_1 = -10{,}000$ and $\psi_2 = -500$ cm at a distance $d = 10$ cm

Calculation method used	Velocity (cm day^{-1})
Upper layer conductivity k_1	0.08
Lower layer conductivity k_2	5.31
Arithmetic mean	2.70
Geometric mean	0.65
Harmonic mean	0.16
Conductivity read at mean potential	0.20
Hydraulic analog	0.51
Numerical integration	0.40

The gradient used in Table III is 11. With lower gradients the differences between the methods are smaller; they are zero for gradient 1. But high gradients do occur; especially in the situations which are subject of study with this and other models. With rain falling on relatively dry soils high gradients initially exist, and the amount of runoff calculated will depend strongly on the accuracy of the velocity calculation.

Large errors are also made in evaporation situations, for which the model described here cannot be applied, because the Rijtema expression is not valid for dry conditions. For a sandy clay loam the value of the upward flow velocity was calculated with $\psi_1 = -10{,}000$ and $\psi_2 = -500$ cm at a distance $d = 10$ cm (see Table IV). For $\psi < -200$ cm the capillary conductivity can be calculated with:

$$k = 33.6\,(-\psi)^{-1.4}$$

From these tables it will be clear that it is rather dangerous to use the arithmetic mean or any other mean of the conductivities. Moreover, there is a formal objection against any averaging procedure as it a priori and tacitly assumes that conductivity is monotonously dependent on depth. The method used in the model, eq.7, merely assumes differentiability from which monotony follows eq.6.

THE CHOICE OF STEP SIZE IN TIME AND DEPTH IN CONNECTION WITH STABILITY

Step sizes may greatly influence computing costs as will be illustrated later on. So it is generally tried to choose the maximum step sizes at which sufficient accuracy is obtained. But in trying to find the proper steps empirically, calculation results appear to oscillate in cases where the time step is chosen too large. The amplitude of the oscillation appears to increase with an increase in the number of steps. Therefore, it is supposed that they are caused by amplification of errors. An error may be introduced by rounding off calculation results or by the finiteness of time and depth steps. Once an error is introduced, it should not be subject to amplification. In order to find a condition for stability we will trace the behaviour of an error one time step after its origination.

Suppose that in the centres of three consecutive layers the moisture tensions according to the calculation model are ψ_1, ψ_2 and ψ_3. Now let any error ϵ occur in ψ_2, so that in subsequent calculations $\psi_2^\star = \psi_2 + \epsilon$ is used instead of the correct value ψ_2. Then the induced conductivity $k_2^\star$ is:

$$k_2^\star = k_0\,e^{\alpha\psi_2^\star} = k_0\,e^{\alpha\psi_2}e^{\alpha\epsilon} = k_2\,e^{\alpha\epsilon} \tag{12}$$

By eq.7 we obtain the velocities of flow between consecutive layers:

$$v_{12} = \frac{k_2 - k_1}{e^{\alpha d} - 1} - k_1 \qquad \text{and: } v_{23} = \frac{k_3 - k_2}{e^{\alpha d} - 1} - k_2$$

From these, applying eq.11, the correct change Δm_2 of the moisture content of the middle layer after one time step appears to be:

$$\Delta m_2 = \left(k_1 - k_2 + \frac{k_1 - 2k_2 + k_3}{e^{\alpha d} - 1}\right)\frac{\Delta t}{d}$$

But in fact $k_2^\star$ is used instead of k_2, so we work with an erroneous $\Delta m_2^\star$. Therefore, in Δm_2 a deviation δm_2 occurs and:

$$\delta m_2 = \Delta m_2^\star - \Delta m_2 = (k_2 - k_2^\star)\,\frac{e^{\alpha d} + 1}{e^{\alpha d} - 1}\cdot\frac{\Delta t}{d} \tag{13}$$

Using eq.12 we get:

$$\delta m_2 = k_2(1 - e^{\alpha\epsilon})\,\frac{e^{\alpha d} + 1}{e^{\alpha d} - 1}\cdot\frac{\Delta t}{d}$$

Now the erroneous and correct values of the moisture content after one time step in the middle layer are equal to $m_2 + \Delta m_2^\star$ and $m_2 + \Delta m_2$, respectively. By subtracting the corresponding moisture-tension values and applying the mean-value theorem of calculus (Ayres, 1950) we obtain after one time step the following expression for the error ϵ' in ψ_2:

$$\epsilon' = \delta m_2\,\frac{d\psi}{dm} \tag{14}$$

where the derivative has to be taken somewhere between m_2 and $m_2^\star$. From the two preceding equations, dropping indices, it follows:

$$\frac{\epsilon'}{\epsilon} = k\frac{d\psi}{dm}\cdot\frac{1 - e^{\alpha\epsilon}}{\epsilon}\cdot\frac{e^{\alpha d} + 1}{e^{\alpha d} - 1}\cdot\frac{\Delta t}{d} \tag{15}$$

This equation can be simplified by expanding the quotient containing ϵ into its MacLaurin series, truncated after two terms (Ayres, 1950). So:

$$\frac{1-e^{\alpha\epsilon}}{\epsilon}=-\alpha-\frac{\alpha^2}{2}\epsilon e^{\theta\alpha\epsilon} \qquad (0<\theta<1)$$

In practice the last term can be omitted because $0<\alpha<0.2$ so its absolute value is smaller than $0.02\,\epsilon e^{0.2\epsilon}$. Therefore even a large error in ψ, for instance $\epsilon=1$, causes a relative deviation of only a few per cents if $-\alpha$ is put into eq.15 instead of the quotient containing ϵ.

Another simplification can be achieved by differentiating eq.2, giving:

$$\frac{dk}{dm}=\alpha k\frac{d\psi}{dm}$$

So eq.15 is equivalent to:

$$\frac{\epsilon'}{\epsilon}=-\frac{dk}{dm}\cdot\frac{e^{\alpha d}+1}{e^{\alpha d}-1}\cdot\frac{\Delta t}{d} \tag{16}$$

From this it may be clear that the initial error ϵ occurring in ψ induces an error ϵ' with an opposite sign. So when $|\epsilon'/\epsilon|<1$ the calculation process starts correcting itself, but when $|\epsilon'/\epsilon|>1$ oscillation will start. This means that the stability condition has the form:

$$\frac{dk}{dm}\cdot\frac{e^{\alpha d}+1}{e^{\alpha d}-1}\cdot\frac{\Delta t}{d}<1 \tag{17}$$

For a given kind of soil α is known as well as the m—k curve. The largest value of dk/dm mostly occurs at saturation. This value should be used in eq.17.

In this chapter we will confine ourselves to the situations in which initially no errors occur in ψ_1 and ψ_3. So theoretically eq.17 merely can give a rough indication of the minimum step size to avoid oscillation. In practice, however, eq.17 appeared to be very useful. Some applications of eqs.16 and 17 are given below.

(a) A humous loamy medium coarse sand ($\alpha=0.0269\ \mathrm{cm}^{-1}$, $dk/dm<18.4\ \mathrm{cm\ day}^{-1}$) is treated in simulation. Taking $d=10$ cm we get according to eq.17:

$$18.4\,\frac{e^{0.269}+1}{e^{0.269}-1}\,\frac{\Delta t}{10}<1 \qquad \text{so} \qquad \Delta t<0.072\ \text{days}$$

(b) A medium fine sand ($\alpha=0.0822\ \mathrm{cm}^{-1}$, $dk/dm<2750\ \mathrm{cm\ day}^{-1}$) gives with $d=10$ cm:

$$2750\,\frac{e^{0.822}+1}{e^{0.822}-1}\,\frac{\Delta t}{10}<1 \qquad \text{so} \qquad \Delta t<0.0014\ \text{days}$$

This means that the time step should be taken about 50 times smaller than that taken for the previous kind of soil.

(c) Suppose a very small error $\epsilon = 10^{-13}$ is present in a moisture tension value in the almost saturated zone of the soil in the preceding example. Let d = 10 cm and Δt = 0.01 days. Now after one time step the error is enlarged according to the ratio

$$\frac{\epsilon'}{\epsilon} = -2750 \frac{e^{0.822}+1}{e^{0.822}-1} \frac{0.01}{10} = -7.06$$

As long as moisture conditions near the saturated zone do not change materially the enlargement factor is about − 7. So after 10 steps (about 2.5 hours) $\epsilon = 3.10^{-5}$, after 15 steps (about 3.5 h) $\epsilon = -0.5$, but after 16 steps $\epsilon = 3.8$.

ESTIMATING COMPUTING COSTS

Formulas which serve to predict the computing costs of one simulation run are given below. They have been developed in a partly empirical way and take into account the quantity of input and output. The formulas give the number of system seconds SS and the number of storage data blocks SDB (units of 1280 characters of information) when using the program FLOW stored in a CDC 6600 computer. Multiplication of SS and SDB by the unit tariffs yields the simulation costs. For each simulation run holds

$$P = \text{NPRINT}\ \{217 + (1 - \text{INDEX1})\ (100 + 52\ \text{NL})\}$$

$$1280\ \text{SDB} = P + 9\ \text{NUMTEN} + 82\ \text{NUMPER} + 100\ \text{NL} + 10\,000 \qquad (18)$$

$$100\,000\ \text{SS} = 2.5\ P + 10\ \text{NL.NCALC} \qquad (19)$$

The variables are defined as follows:

INDEX1 = 0 if complete output is requested, else it is equal to 1 (see section "Output")
NL = number of layers in soil
NUMTEN = number of moisture tension values in the moisture characteristic table
NUMPER = number of periods of rain or irrigation
NPRINT = number of moisture situations to be printed
NCALC = number of moisture situations to be calculated

During simulation, however, some layers of soil, or even all of them, may become saturated by rising of the groundwater. As a considerable part of the calculation work concerns the unsaturated layers and as only unsaturated layers are printed in the tables, a rise of the groundwater will reduce SS and SDB. On the other hand this rise will cause the iterative calculation of groundwater depth to take more computer time. This is one of the reasons why only an estimation of computing costs can be given.

The costs of the first simulation of which the course is given in Fig.1 are estimated as follows. Taken are INDEX1 = 0 (complete output), NL = 8, NUMTEN = 401 (moisture content in moisture characteristic table varies between 0.43 and 0.51 with steps of 0.002). NUMPER = 18 (see Fig.1), NPRINT = 150 (simulation during 15 days, printing after 0.1 days), NCALC = 1500 (time steps of 0.01 days). The formulas 18 and 19 yield:

SDB = 98 (actual value = 61); SS = 4.0 (actual value = 3.4)

using unit prices of HFl. 0.08 per storage data block and HFl. 1.85 per system second, the estimated costs of the simulation are HFl. 14 while the actual costs were HFl. 12. So in this case one day of simulation costs no more than HFl. 1. (HFl. 1 equals about US$0.38 now). From eqs.17, 18 and 19 it will be clear that the chosen thickness of the layers has a very large influence on computing costs.

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Application of analog and numerical models to investigate the influence of drainage on workability in spring

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Key words: analog models, numerical models, drainage, workability, soil moisture, water flow

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Summary

Models of non-steady unsaturated flow of moisture were used to calculate moisture conditions in a top soil from natural rain- and evaporation data over 23 years. The calculations were made with 5 drainage depths and 3 drainage intensities. This was done with an available analog model, mathematical models being anyway too expensive for calculations over such a long period. The most difficult problem in the application of the analog model was to insert the proper soil conditions. This was solved by changing them until the model calculation for one year fitted the workability conditions observed in the field. The results of the then resulting calculations could be verified with field observations available for all 23 years.

The number of workable days and the first date of workability did show large variations over the years. Drainage depth had a pronounced effect on workability, drainage intensity had hardly any effect.

Introduction

Workability in spring for seedbed preparation is a very important topic in arable agriculture. It influences the amount of labour and machinery required and the date of sowing.

Deficient workability in spring causes too late planting dates and thus a decrease in yield. But it also causes problems in the performance of farming operations. There are direct ways to solve these problems (e.g. additional labour, working at night and during the weekend, help by contractors) but production costs increase when the soil shows poor workability and planting cannot be delayed too much. So in a bad spring the farmer has to make his decisions to minimize the total losses resulting from higher costs and lower yields.

Farming practice depends on an equilibrium, found from experience, between number and date of workable days, crop succession, machinery selection and labour force in normal years. In order to find a better equilibrium by optimalization techniques and for example to calculate the benefits of better drainage, more basic knowledge on the occurrence of a good workability is needed.

Workability depends on the amount of moisture in soil and drainage measures will affect this. Workability in spring, however, especially depends on moisture conditions of the top layer which are far more dependent on accidental fluctuations in rainfall and evaporation rate than the amount of moisture in the total soil. The moisture condition of top soil therefore cannot be calculated as simply as the total soil moisture content. With the aid of models of unsaturated flow such calculations have become possible, however.

In order to improve workability in spring by optimum drainage it is necessary to know how drain depth and drain spacing does affect is. This could be studied at an experimental field but this study will take many years as due to the variations in rainfall and evaporation rate the effect of drainage can vary considerably. A good drainage in this respect is to be defined as a drainage which ensures sufficient workable days in nine out of then years on the average. Therefore such an investigation in the field would have to last at least some decades to find statistically reliable information.

With a model the study can be made over a large number of years in a short time.

Models used

With the model the moisture content of the top soil will have to be calculated from input data such as rainfall, evaporation, soil properties and drainage. Therefore it must be a model which simulates non-steady state processes of flow and accumulation of moisture in unsaturated soil; moreover it must incorporate a good function of drainage.

Most models are mathematical, which nowadays means computer-models. For the least expensive computer-model the processing cost is about $ 0.40 per calculated day. Even then calculation of 23 springs of each 120 days for 3 drain depths and 3 spacings would cost $ 10 000. In practice the costs would be still higher.

Another type of model is the analog-model. The real processes which occur in the soil are then represented by analogous processes. Flow of water is for instance replaced by flow of electricity. The condition for a physical analog is that the equations on which the model is based are the same as those describing reality. Such analogues have to be built and automatized, which costs time and money, but once this is done the operation costs are almost negligible. With the hydraulic analog I described (Wind, 1972) investigation could be done cheaply.

The model consists of a number of vessels, each representing a soil layer. The amount of water in a vessel represents the soil moisture content, the height of the water level the soil moisture suction. The vessels are connected by a number of tubes at different elevation. Because the tubes above the water level in the vessels do not transport water, the transport capacity of the tubes depends on the height of the water level. In this way, if they have correct dimensions, the tubes can give a good simulation of capillary conductivity.

The scales of the model are: $S_v = 0.1$ (1 cm of suction is represented by 0.1 cm in the model), $S_a = 900$ cm² (1 cm water layer is represented by 900 cm³), $S_t = 1/288$ (1 day in reality is represented by 5 minutes in the model).

For part of the investigation mathematical models were used. They were described by van Keulen & van Beek (1971) and Wind & van Doorne (1974).

Workability

Workability in spring is the soil condition with which a good seedbed can be made with the usual tools. The soil should fall into small crumbs, not into clods and certainly not be smeared or puddled. According to Perdok (1975) the behavior of soil in this respect is dependent on its mechanical strength. The tillage results obtained and the mechanical strength are highly correlated with the soil moisture tension.

In this paper a mean moisture suction of —300 cm in the top 5 cm, is presumed to be the limit for workability. Wetter conditions impede good tillage. Further it is assumed that a lower limit (for dry conditions), has no practical meaning for professional farmers on the loam soil studied.

The depth of 5 cm was chosen arbitrarily. For the seedbed of grains less depth

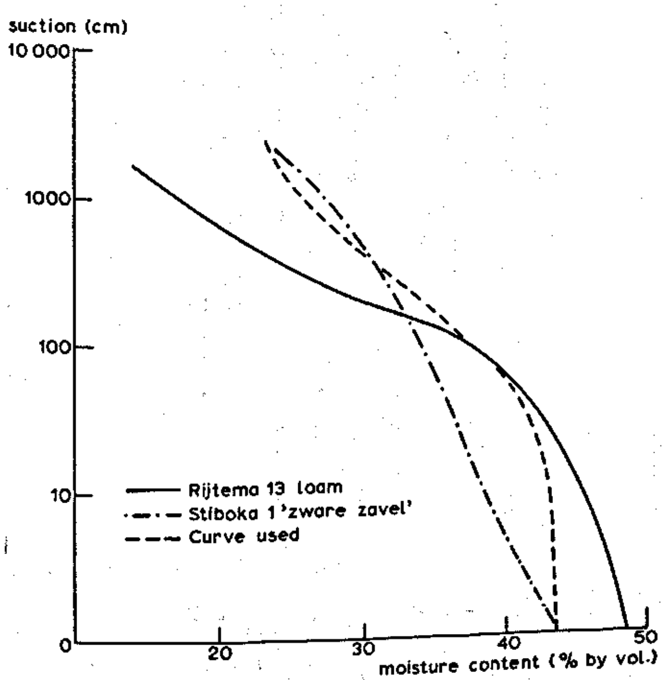

Fig. 1. Soil moisture characteristics of 'Rijtema 13' loam, Stiboka 1 'zware zavel' and the soil used in the model.

is sufficient, for that of potatoes it should be 7 cm (Perdok, 1975). For sugarbeet about 5 cm must be tilled.

Soil properties

The study was made for loam soil. The 'model'-soil was meant to represent the properties of the arable soil called 'zware zavel' in Dutch, which occurs in the Hoekse Waard. Some observations on moisture content and workability in this polder were made in 1973 by Perdok (1974).

As no data on capillary conductivity, the k (ψ) relation, were known, the standard soil no. 13, loam of Rijtema (1969) was used for this purpose. A hydraulic model was built based on the moisture characteristic shown in Fig. 1 and on the k (ψ) relation where $k_0 = 5$ cm · day^{-1} and $\alpha = 0.023$ cm^{-1}. The first test-run made

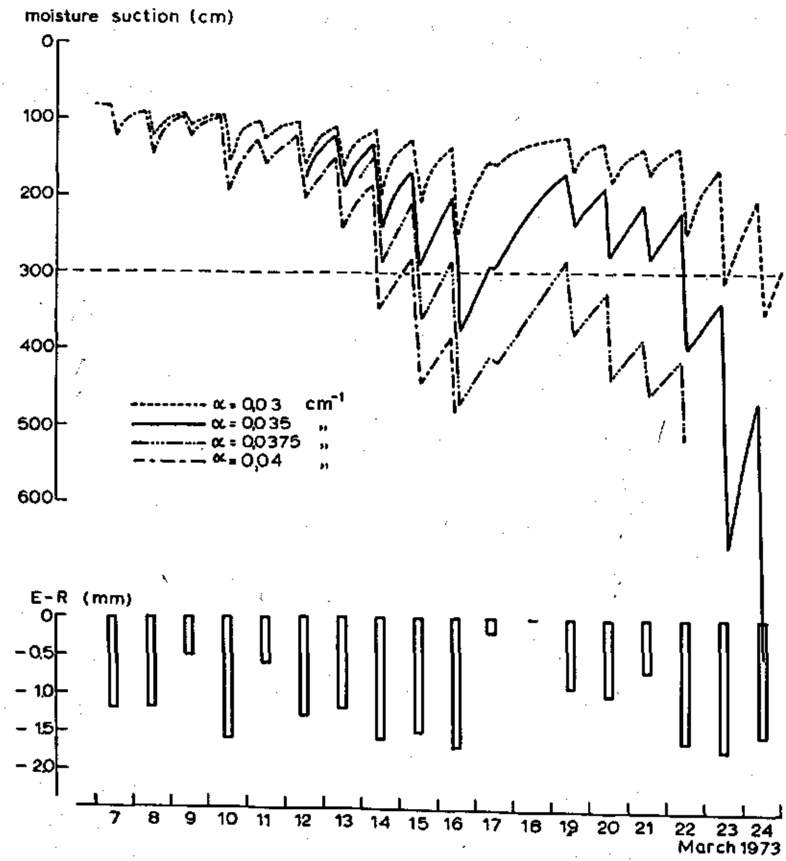

Fig. 2. The value of α in the relation $k = k_0 e^{\alpha\psi}$ was found by trying what value fitted best the field observations which indicate that the topsoil was drier than 300 cm during a short time on 16 March and during a longer period beginning 22 March.

with the meteorological data of 1960 was a disappointment. This year was known for its favourable tillage conditions in spring, but the model showed non-workable soil conditions up to 30 April.

After checking the model specifications the cause seemed to be the use of wrong soil properties. These could simply be changed by increasing the scale for area (S_a) of the model, as both k_0 and the moisture contents are inversely proportional to S_a.

A small change of S_a from the original value of 500 to 600 cm² was not sufficient. Nor was an increase to 1000 cm² even when combined with a very deep drainage. In none of the test-runs for 1960 was workability reached. Apparently the discrepancy with the actual soil properties was large and it was thought it concerned the exponent α of the k (ψ) relation.

It was then decided to relate the soil properties, especially α, to actual workability data in the Hoeksche Waard as known for 1973. First the moisture characteristic of the model, based on the 'Rijtema 13' soil, was reduced to the curve given by

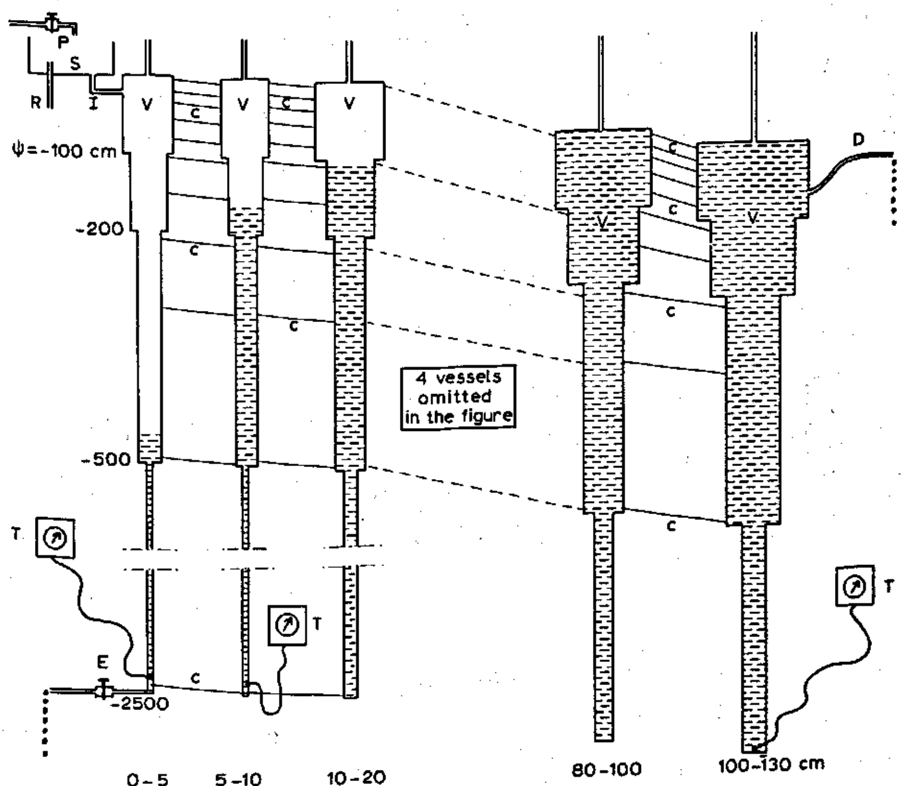

Fig. 3. Outline of the hydraulic analog used. V: vessels representing a soil layer of a certain thickness and a certain moisture characteristic; C: connecting tubes, representing capillary conductivity; P: precipitation valve; S: surface tank with R: run-off pipe, I: infiltration tube; E: evaporation valve; T: pressure transducer, connected with recorder and data logger; D: drainage tube.

Anon. (1971). This was done by increasing the scale of S_a from 500 to 900 cm^2. In this way adjusting the moisture contents in the model fitted closer to the Stiboka moisture characteristic. This is shown in Fig. 1.

Determination of the coefficient α from field observations by inductive use of a mathematical model

The rainfall data of 1973 from the experimental farm 'Westmaas' on the island Hoeksche Waard showed much precipitation up to 7 March, with a mean of 0.2 $cm \cdot day^{-1}$ in the last 10 days. On 7 March a clear period began with evaporation rates between 0.05 and 0.17 $cm \cdot day^{-1}$, which lasted until 17 March. Then light rainfall occurred so that the difference between evaporation and rainfall was nearly zero on 17 and 18 March. After that a new evaporation period started (Fig. 2).

On 17 March the soil was nearly workable, but workability ended next day and it returned on 22 March. A soil parameter α had to be found with a value such that

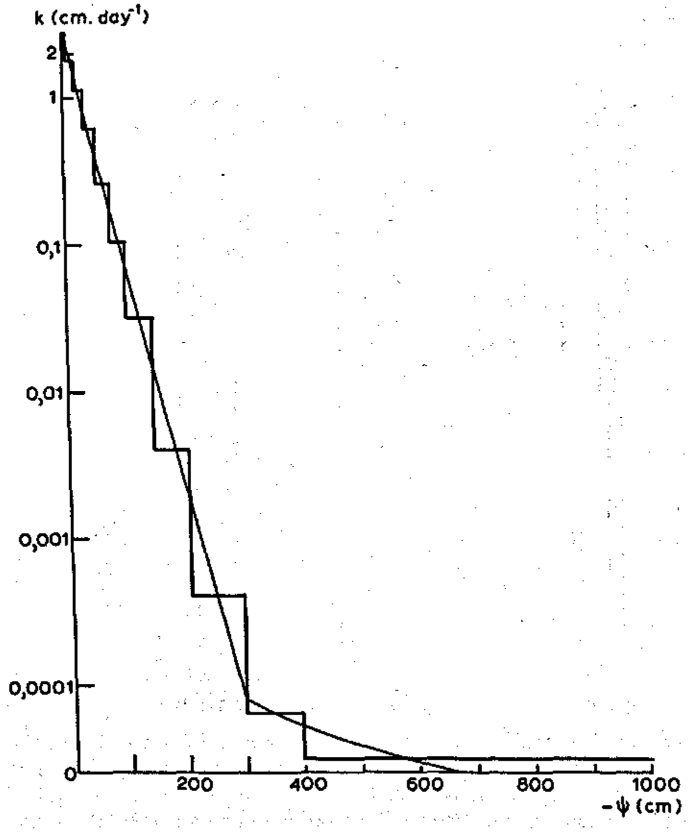

Fig. 4. The k (ψ) relation is simulated in the model by a step-curve.

the given weather input would give an output equal to the workability situation observed in the field.

The mathematical model 'Flow' of Wind & Van Doorne (1975) was used to calculate the moisture suctions in the top 5 cm. The initial condition on 7 March was a dynamic equilibrium at 0.2 $cm \cdot day^{-1}$ downward flow. This means a depth of the groundwater table 80 cm below surface, with a drainage depth of 100 cm and intensity of 0.01 day^{-1}. The conductivity at zero suction (k_0) was 2.8 $cm \cdot day^{-1}$, the moisture characteristic as given in Fig. 1 (curve used). Evaporation rates were used as shown in Fig. 2. It was assumed that the evaporation had a constant rate for 0.2 days in the middle of each day and that during the rest of the day no evaporation occurred.

The first run was made with $\alpha = 0.03$ cm^{-1}. Workability was reached on 24 March, two days too late. The second run with $\alpha = 0.04$ cm^{-1} gave workability on 14 March, two days too early. The third run with $\alpha = 0.035$ cm^{-1} described exactly what did happen in the field: a nearly workable soil on 17 March and definitely so on 22 March. An additional run with $\alpha = 0.0375$ gave conditions that were too dry. So the value of α was set at 0.035 cm^{-1}.

Apparently, such models can be used to obtain soil property data from simple field data.

Specifications of the model

After determination of α, the model (Fig. 3) was rebuilt to a capillary conductivity described by

$$k = 2.78\ e^{0.035\,\psi}\ (\psi > -300\ \text{cm})\ \text{and}\ k = 0.225\ \psi^{-1.4}\ (\psi < -300\ \text{cm})$$

This was realized in a step-curve by tubes at suction values of 5, 20, 35, 55, 80, 105, 210, 300, 500, 1000, 1500, 2000 and 2500 cm. See also Figs 3 and 4.

The scale for area was made 900 cm^2, so a water layer of 1 mm was represented by 90 ml water. The vertical scale was made 0.1, so a difference of 10 cm in moisture suction was represented by 1 cm in the model. As the maximum height of the room was 2.50 m, the driest condition was $\psi = -2500$ cm.

In order to follow the moisture characteristic (curve used) of Fig. 1 the vessels were divided in 4 sections according to Table 1.

For vessels representing layers of different thickness the volume was changed in

Table 1. Volume, length and diameter of the sections of a vessel representing a layer of 10 cm.

Suction range (cm)	Moisture content (% by volume)	Volume (ml)	Length (cm)	Diameter (cm)
0 - 100	43.6 - 36.8 = 6.8	612	10	8.8
100 - 200	36.8 - 33.0 = 3.8	342	10	6.6
200 - 500	33.0 - 28.8 = 4.2	378	30	4.0
500 - 2500	28.8 - 23.2 = 5.6	504	200	1.8

proportion. In total there were 11 vessels representing the layers 0-5, 5-10, 10-20, 20-30,30-40, 40-60, 60-80, 80-100, 100-130, 130-160 and 160-190 cm depth. The time scale was 0.00347, which means that one actual day was represented by 5 minutes in the model.

Rainfall

The input on paper tape was read in every minute in 5 periods per day. As nothing was known about the distribution of rain over the day, the rain was distributed equally over the periods in the day. The tape-reader opened and closed a valve by which water entered the top of the model at a rate of 2700 ml per 5 min (30 mm rain per day). If less rain was read, the valve opening time was reduced proportionally to multiples of 2 seconds.

Rainfall and evaporation data were used which had been observed in the meteorological station of De Bilt in the centre of the Netherlands.

Evaporation

As no daily evaporation data were available, data of 10 or 30 day totals had to be used. These data could not be distributed evenly over the days. On sunny days there should be more evaporation than on cloudy days. As radiation is the most important factor governing evaporation, daily radiation data, observed in De Bilt were, by linear interpolation, used for the distribution. The evaporation was presumed only to occur during the third of the 5 periods in the day. The tape-reader opened and closed a valve near the bottom of the model by which water left the upper vessel (0-5 cm deep soil layer) of the model at a rate of 270 ml in one minute, equivalent to 15 $mm \cdot day^{-1}$. For smaller evaporation rates the opening time was reduced. Evaporations of more than 3 mm were distributed over the third and fourth period of the day.

By placing the evaporation valve at the lowest place of the model, i.e. at suction value of 2500 cm, the real evaporation was lower than the potential, according to:

$$E_r = E_p \frac{2500 + \psi}{2500} \; [\psi > -2500]$$

This function was chosen because it can easily be realized in a hydraulic model. Hoogmoed (1974) tried another function in a mathematical model, based on the CSMP-model of van Keulen & van Beek (1971) and found only small differences (Fig. 7). This function reduced potential evaporation in proportion to the ratio of moisture content and maximum moisture content of the top 5 cm. It gives larger reductions in wet conditions and smaller ones in dry conditions than the function used in this article.

The function and mechanism used here reduce high evaporation rates more than low ones.

Run-off

In rainy periods the infiltration rate of the model may be lower than the precipitation rate. In order to simulate ponding, a special tank was made on top of the model.

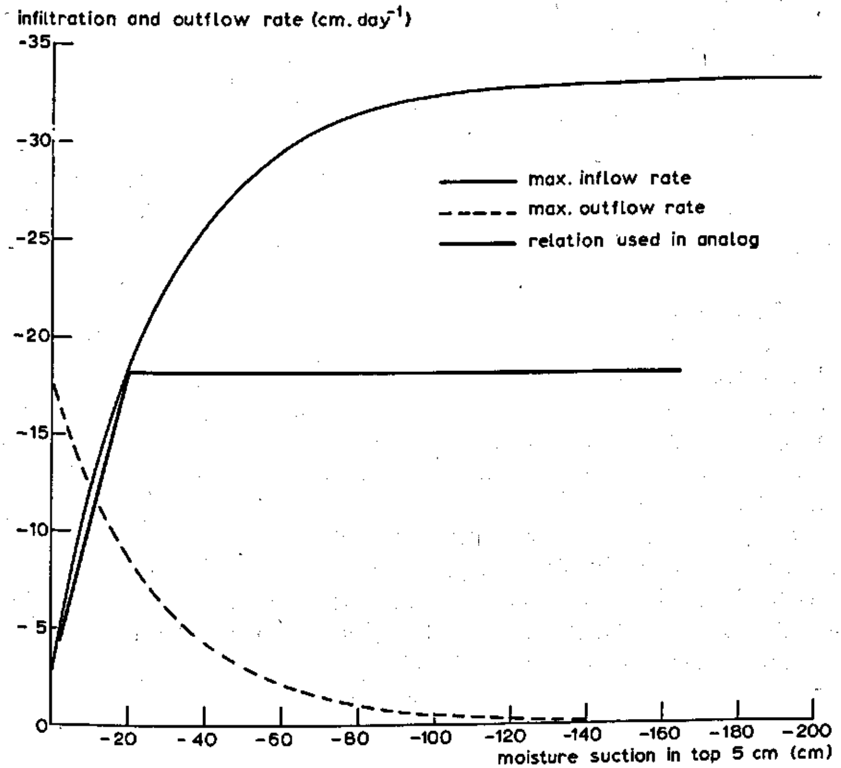

Fig. 5. Maximum inflow rate and maximum outflow rate as related to suction in the top 5 cm, and the relation used in the model.

Its content was 270 ml (0.3 cm water layer). An excess could flow out of that tank and thus simulate run-off. This occurred only with poor drainage.

Infiltration
Infiltration depends on the suction in the first layer, as shown in Fig. 5. It was simulated in the model, in a way given in the same figure, by a tube connecting the surface tank with the first vessel at a point corresponding with $\psi = -20$ cm. In drier conditions the infiltration rate is very high compared with the maximum flow velocity between the two first vessels. Then the moisture content of the top vessel increases so fast that even a large mistake in the infiltration rate is unimportant for the problem studied. The maximum flow rate at 5 cm depth was calculated under the assumption that the layer 5 to 10 cm has a moisture suction of -2500 cm.

Drainage
It was supposed that the flow of water from surface to drain was vertical to drain depth and horizontal from that depth to the drain. Therefore the drainage tube was

always connected to the vessel representing the layer in which the drain was thought to be situated. The model was supposed to represent a place midway between two drains.

Registration
The height of the watertable was read in the two top vessels and in the drainage vessel with pressure transducers. Their readings were recorded by line-recorders and by a paper tape writer. The tape could be fed into a computer.

Check on the calculations

An unexpected opportunity to check the results derived with the model occurred when it became known that at the Agricultural Extension Service in the polder 'Hoeksche Waard', Mr Hokke had made daily notes on the soil condition for more than 25 years. Every day he had made a note whether the top soil was very wet, wet, moist, dry or very dry.

In spring the notations 'dry' and 'very dry' meant that the soil was fit for tillage. In Fig. 6 the number of workable days are given according to Hokke's observations and according to the analog model. For that purpose the model was fed with the relevant rainfall data of the Hoeksche Waard, taking a drainage depth of 100 cm below surface and a drainage intensity A of $0.012 \cdot day^{-1}$.

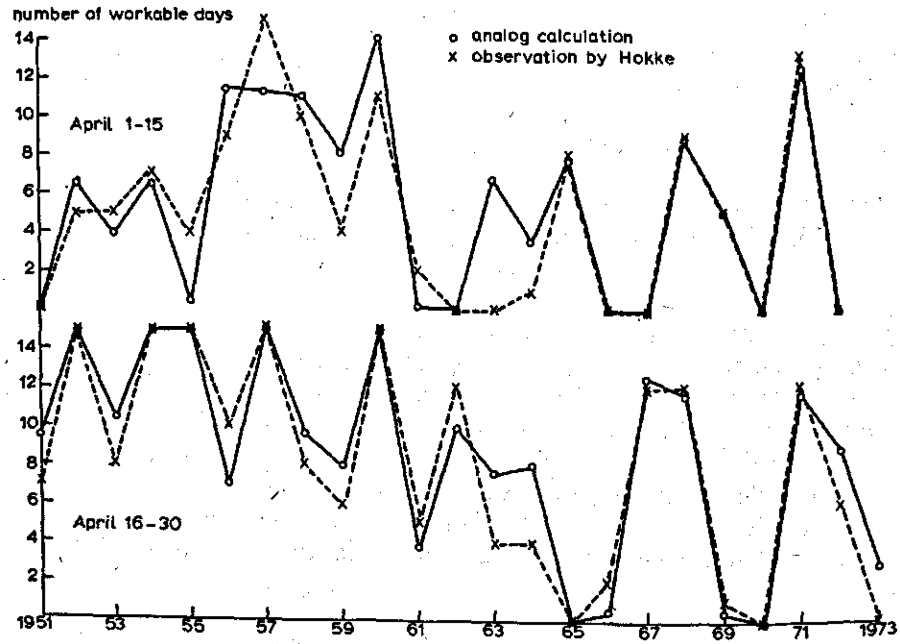

Fig. 6. Comparison of the calculated and the by Mr Hokke observed number of workable days in the Aprils of 1951 through 1973.

Table 2. Drain depths and intensities used in the model observations.
A: over all years between 1951 and 1973 except 1960; S: over some years only.

Depth (cm below surface)	Intensities (day^{-1})			
	0.0011	0.0033	0.008	0.015
0				S
40				A
80		S	S	S
100				A
150	S	S	S	A
200				A

Observed and calculated data agree quite well. The deviations can be caused by a number of factors, e.g. the subjective interpretation of soil condition by Hokke, and incorrect drainage input, an unequal distribution of rain over the polder or differences in the soil physical properties. The differences in workable days between

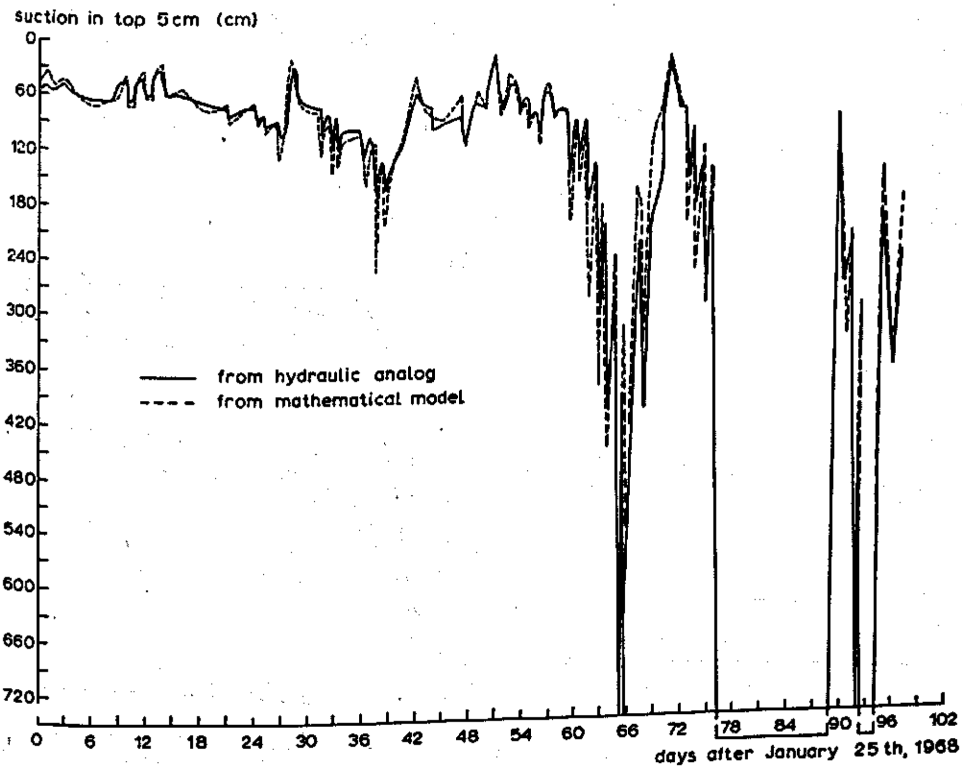

Fig. 7. Moisture suction in the top 5 cm in the spring of 1968 as calculated by hydraulic analog and by mathematical model of van Keulen & van Beek (1971) and Hoogmoed (1974).

the model – and Hokke's observations were always positive when small amounts of rainfall occurred during a workable period. Hokke could have seen the surface as moist, but the mean amount of moisture in the top 5 cm measured by the model would be less. The differences are fairly large in 1963 and 1964. It later appeared that both years drainage depth had pronounced effect on workability (Fig. 11).

Observations

The number of workable days was determined with the aid of the analog during the months March and April of the years between 1951 and 1973. Evaporation and rainfall data were used from the meteorological station De Bilt in the centre of the Netherlands. Each simulation started on 1 January. The initial condition was derived from a rough calculation based on precipitation and evaporation in the preceding summer and autumn. In most years the effect of the initial condition on the moisture condition of 1 March appeared to be negligible, except for 1960 when a very dry summer was followed by a dry autumn and winter. The results of that year were therefore omitted.

The data on workability were determined for a number of drainage depths and intensities as shown in Table 2.

Results

For each year and each combination of drainage depth and intensity a graph was

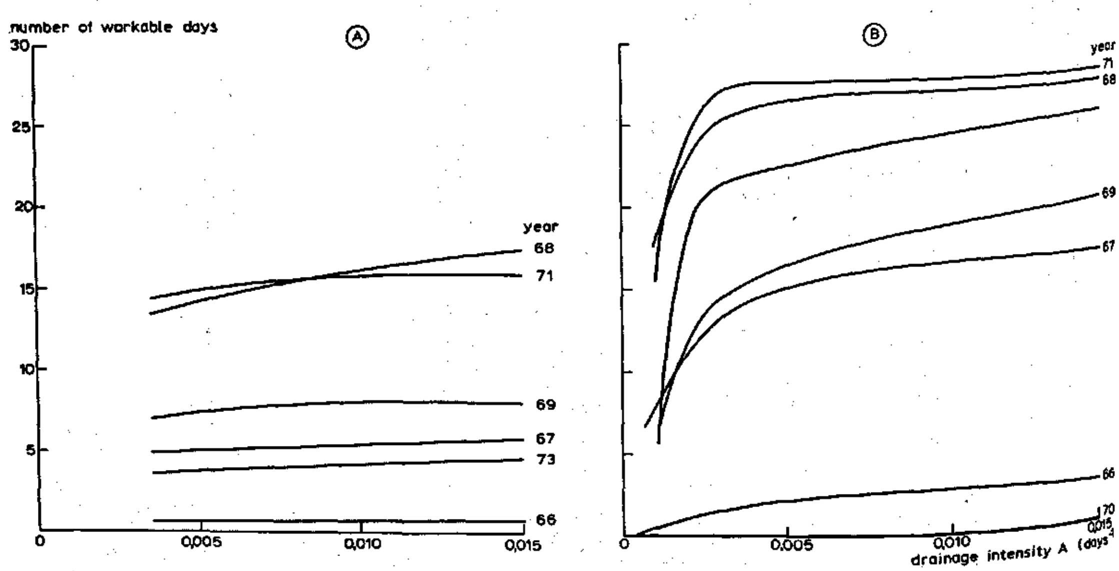

Fig. 8. A: relation of workable days before 1 May and the drainage intensity for a drain depth of 80 cm; B: the same for a drain depth of 150 cm.

produced as Fig. 7, giving the moisture tension in the top 5 cm. The number of workable days could be read from these graphs or from the punched tape of the recorder.

In Fig. 8A for the six mentioned years the relation is given for the number of workable days and drainage intensity for a drainage depth of 80 cm. The influence of the drainage intensity seems to be small. Only for very low intensities is the influence important. But such intensities are not important because there are other reasons than workability which require an intensity larger than 0.0033. With that intensity a soil drained at 80 cm depth has a watertable situated at less than 20 cm depth even during the mean winter rainfall of 0.2 cm·day^{-1}.

For a 150 cm drain depth, as shown in Fig. 8B, nearly the same conclusions hold: the influence of the drainage intensity is large only for intensities lower than 0.0033. This intensity gives a discharge rate of 0.33 cm·day^{-1} if the watertable is 50 cm below surface (neglecting the vertical resistance), so it is about half the intensity required by the Netherlands' drainage criterion. The latter requires for arable soils a discharge rate of 0.7 cm·day^{-1} if the watertable is 50 cm below surface.

Wesseling (1969) calculated that this criterion causes a groundwater depth of 25 cm with a probability of once a year. Apparently the requirement for water

Table 3. Total number of workable days in March and April in relation to drainage depth.

	Drainage depth (cm)			
Year	40	100	150	200
1951	5	7	7	9
2	14	20	24	25
3	8	26	29	33
4	19	24	25	30
5	13	13	23	23
6	15	30	33	35
7	16	24	25	26
8	4	12	17	18
9	2	4	6	6
1961	0	4	8	10
2	1	4	4	5
3	1	9	12	13
4	0	14	17	19
5	0	4	8	13
6	1	1	2	2
7	4	10	11	14
8	13	20	23	23
9	3	7	10	13
1970	0	0	0	0
1	11	20	22	22
2	7	16	18	18
1973	1	14	16	21
Average	6,2	12,5	15,4	17,1

removal in winter is more severe than that for workability in spring. The conclusion therefore is that the requirement of good workability in spring hardly influences the choice of the drainage criterion.

The influence of drainage depth on workability (Table 3) is very important, as is shown in Fig. 9. There seem to be three types of spring:

1. In years such as '66 and '70 there is a poor workability regardless of drainage depth. This is caused by a very small number of rainless days ('70) or by only short periods without rain ('66).
2. In years like '68 and '71 even at shallow drainage depths a high workability exists. Although there is a marked influence of drainage depth on workability, even shallow depths give enough workable days.
3. In many years drainage depth is very important for the spring cultivations. To this type belong 10 of the 22 investigated years.

The number and distribution of workable days in spring is important for the choice of type and size of farm machines and for the organization of farm work. Mostly it only slightly affects labour costs because the labour peak is not in spring but in autumn. More workable days primarily do not mean an improvement of the

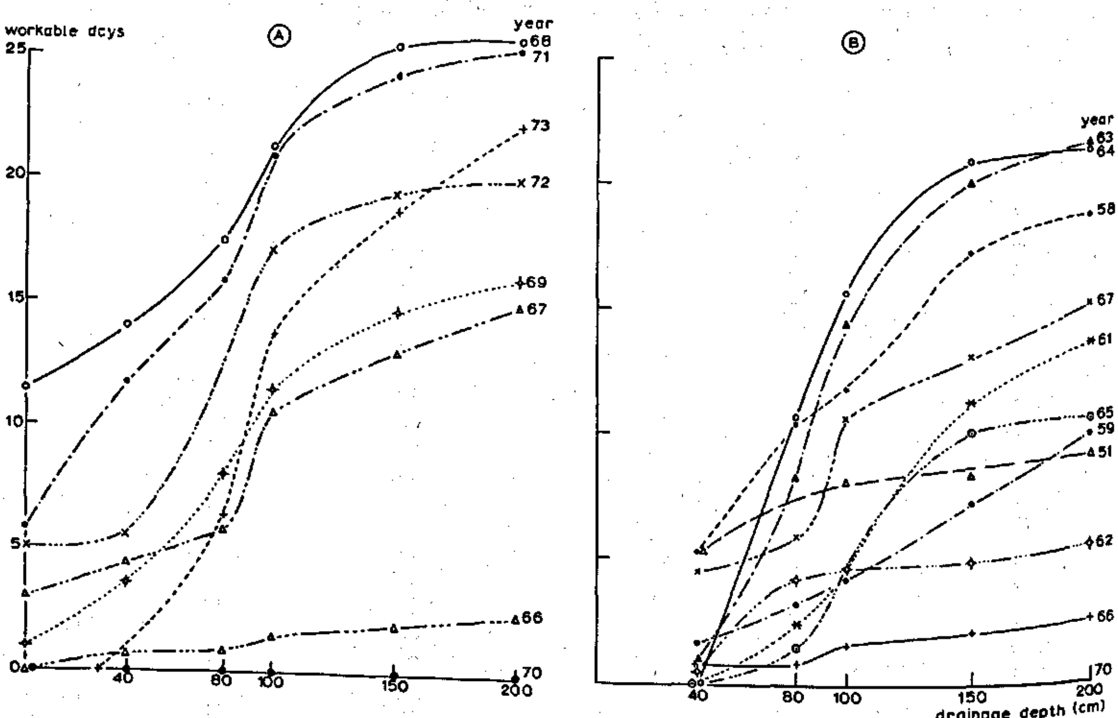

Fig. 9. A: number of workable days before 1 May in the last 8 years for different drainage depths and a constant drainage intensity of 0.015 day^{-1}; B: the same for some years with a low workability at shallow drainage depths.

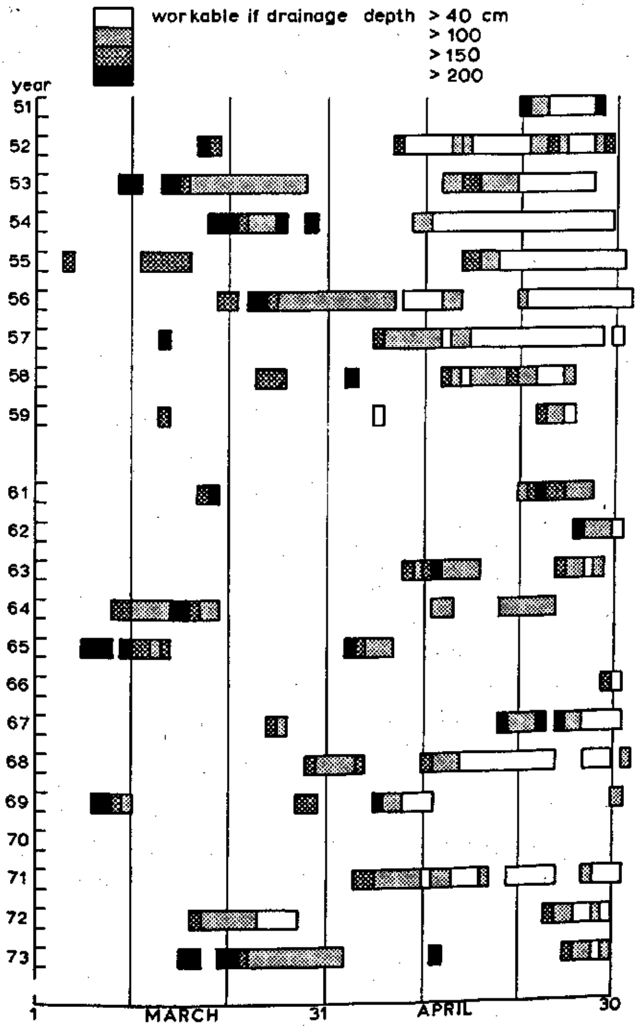

Fig. 10. Periods of workability for 4 drain depths in 22 years. Workability defined as: suction in the top 5 cm of the profile $\leqslant -300$ cm.

time scheme, but earlier workability and thus earlier planting in spring. As Reve & Fausey (1974) stated, this is perhaps the greatest benefit of drainage.

Fig. 10 gives the periods of workability in 22 years for 4 drainage depths. For all depths the same drainage intensity, 0.015 day^{-1}, was used. The year 1960 was omitted because of its unreliable initial condition. The year 1970 gave no workable days in March and April; 1959, 1962 and 1966 had only a few workable days. The experiment did not include the month of May, which afterwards proved to be a mistake.

In a few years (see for example 1951) there is only a small difference in the date

Table 4. Yield reduction on a sandy loam soil in the Netherlands in % of maximum yield, as caused by too late planting (mean over the years 1951 through 1973) and drought damage in a model spring of these 22 years, both as dependent on drainage depth (van Wijk & Feddes, 1975).

Drainage depth (cm)	Planting date		Drought damage		Total reduction	
	summer grains	potato and sugarbeet	summer grains	potato and sugarbeet	summer grains	potato and sugarbeet
40	36.3	11.7	0.5	2.1	36.7	13.5
80	17.8	6.4	1.1	3.8	18.7	9.0
100	11.0	5.4	1.8	3.6	12.6	8.8
150	8.0	3.8	7.5	7.2	14.9	10.8

of first workability between the 4 drainage depths. In principal the differences are due to differences in drainage depth, but in most years they are amplified by rain storms. Therefore planting dates on shallowly drained soils are usually much later than on deeply drained soils. The effect of planting date on yield has often been studied by agronomists. Wind (1960) calculated from literature data the losses caused by later planting. On basis of that paper and the results of the experiment described in the present paper, van Wijk & Feddes (1975) calculated the significance of the benefit to be obtained by drainage. They combined the effect of planting date and that of drought damage. Their results are given in Table 4. This table shows that a drainage depth in the investigated sandy loam soil should preferably be between 100 and 150 cm, and it confirms that the common practice of a drainage depth of about 100 cm is correct. Shallower drainage is certainly not advisable for this soil.

Discussion

The most remarkable result of the investigation presented here is the conclusion that drainage depth has a considerable and drainage intensity only a small effect on workability in spring. The effect of drainage depth can easily be explained. Deep drainage causes a drier top soil at the end of a wet period than shallow drainage, so less moisture has to evaporate. Moreover the conductivity in dry soil is low, so that the rate of capillary rise in deeply drained soils is lower than in shallowly drained soils. The time required to obtain workability is therefore evidently dependent on drain depth. Why drainage intensity has a lesser effect on workability than drainage depth is less evident. The main explanation is that after a few dry days the effect of drain intensity on depth of watertable is slight compared with the effect of drainage depth. A workable period in spring occurs mostly in a dry period following a rainy one. The depth of watertable at the beginning of the dry period is important for the time required to obtain workability.

Table 5. Depth of the groundwater table (cm below surface) after a 6-day period of 0.6 cm rain per day for three drainage depths (D) and three intensities (A).

D (cm)	A (day^{-1})		
	0.005	0.010	0.015
80	25	38	46
100	45	58	66
150	95	108	116

Groundwater depth can be calculated with the Hellinga-de Zeeuw formula (1958) using a constant moisture storage coefficient:

$$h_t = \frac{i}{A}(1 - e^{-\frac{A}{\mu}t}) + h_0 e^{-\frac{A}{\mu}t}$$

where h is the height of the watertable above drainage depth (cm), i is the rainfall rate ($cm \cdot day^{-1}$), A the drainage intensity (day^{-1}), μ the moisture storage coefficient. Calculating the depth of the groundwater after a 6-day period with 0.6 cm rain per day, following an initial steady state vertical flow of 0.1 $cm \cdot day^{-1}$ at three drain depths and three drain intensities, both ranging from poor to very good, and a storage coefficient of 0.05, will give the results in Table 5.

The differences in groundwater depths thus calculated at the same drainage intensity equal, of course, the differences in drainage depth. The difference in groundwater depth for intensities of 0.005 and 0.015 day^{-1} was 21 cm, which is small when compared with the 70 cm groundwater depth *difference* between the drainage depths 80 and 150 cm.

In most cases the difference in depth of the watertable due to different drainage intensities will be smaller than in the example chosen, since a rainfall of 3.6 cm in 6 days, as taken in the example, has a probability of about 0.2 in March and April. For a 0.5 probability the rate or the duration is smaller and so the magnitude of the differences also will be smaller.

There are other aspects which favour the effect of drainage depth more than of drainage intensity, e.g. the moisture storage coefficient and the drain discharge during a dry period. All these causes taken together can explain why drainage depth has more influence on workability than drain intensity. To calculate those effects quantitatively, however, a model is required.

An important consequence of the results of the research described is that a low intensity but deep drainage will have a favourable effect on workability. This phenomenon can be observed on many sites where low lying fields, although with an appropriate tile drainage, are later in a workable stage than higher lying fields.

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[4]

AN ELECTRONIC ANALOG FOR UNSATURATED FLOW AND ACCUMULATION OF MOISTURE IN SOILS

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ABSTRACT

Wind, G.P. and Mazee, A.N., 1979. An electronic analog for unsaturated flow and accumulation of moisture in soils. J. Hydrol., 41: 69—83.

An electronic analog model of the unsaturated zone has been developed, based on the similarity between the integrated flow equation (5) and Ohm's law. The main difference between the two equations is compensated for by amplifiers. The model simulates one day in 2 s. There are ten normal layers, each with adjustable magnitude. Moreover, there is a top layer in which infiltration, ponding and runoff are simulated, and a drain layer with adjustable drain intensity. The normal layers contain an adjustable resistor for the connection with another layer and a function generator for the $k(\theta)$ relation. There are two transition layers which have to be placed at the boundary between two layers of different soil properties.

Under saturated conditions the model is acting incorrectly, so that a too small thickness of the saturated layers is calculated. This can be compensated for by using an equivalent drain intensity which is lower than the real one.

The model can be used to simulate the effect of natural rainfall and evaporation on moisture content at every depth. Soil physical properties and drainage conditions can be adjusted. Homogeneous as well as layered soil can be represented by the model. In particular, it can be applied to investigate the drainage requirements of soils.

Some examples of the applicability of the model and short technical description are presented.

INTRODUCTION

Non-steady unsaturated flow and accumulation of moisture in soils can be described by models. This has to be done if one intends to forecast effects of drainage, tillage or soil improvement on the behaviour of soil moisture under natural quickly changing weather conditions.

Numerical simulation models are the most versatile. They have few restrictions, except computer costs; these are prohibitive for long time series. Application of Wind and Van Doorne's (1975) numerical model costed about 1 US

$ per calculated day. Wind's (1972) hydraulic analog method has also very low operation costs. However, this is a slow model with a small flexibility, because the variation of soil properties is laborious.

This paper describes a model which combines the advantages of both models mentioned but lacks the disadvantages of both. It is quick, flexible and has very low operation costs. This electronic model simulates the processes of infiltration, runoff, unsaturated flow and accumulation of moisture and drainage outflow for the case of an exponential relation between unsaturated conductivity and pressure head. The relation between conductivity and moisture content can have any shape; it is approached by three line segments. In a number of soil layers (usually ten, but more are possible) the soil properties $k(\psi)$ and $\psi(\theta)$ of each layer can be adjusted. Drainage can be simulated in the model by one or more drain layers in which the drainage intensity can be chosen. Rainfall and evaporation input are given on paper tape. The time scale is one day per 2 s, so θ and k output can be read on line recorders and magnetic tape.

NOTATION AND SIGN-CONVENTIONS USED

A	= drain intensity	(day^{-1})
a	= dimensionless factor $a = \exp(\alpha \Delta z)$	
E	= electric potential	(V)
I	= electric current	(A)
k	= unsaturated conductivity	(cm day^{-1})
k_0	= conductivity at zero moisture pressure	
k_d	= conductivity at drain depth	
k_s	= conductivity at soil surface	
R	= electric resistance	(Ω)
V	= vertical flux, upward is positive	(cm day^{-1})
V_d	= drain outflow, negative	(cm day^{-1})
V_{ro}	= runoff, negative if present	(cm day^{-1})
z	= height above soil surface, negative below surface	(cm)
z_d	= drain level	(cm)
z_g	= level of groundwater	(cm)
Δz	= distance between the centers of two layers	(cm)
α	= exponent used in Rijtema's (1965) relationship between ψ and k	(cm^{-1})
ϕ	= total potential expressed as hydraulic head	(cm)
ψ	= capillary potential, pressure head in unsaturated zone negative	(cm)
ψ_d	= pressure head at drain level	
ψ_s	= pressure head at soil surface	
θ	= moisture content (volume fraction)	
θ_{ro}	= maximum depth of water ponded on soil surface	(cm)
θ_s	= depth of water ponded on soil surface	(cm)

GENERAL PRINCIPLE OF THE MODEL

In unsaturated soil the conductivity k in Darcy's law:

$$V = -k\ (d\phi/dz) \tag{1}$$

is a function of soil pressure head ψ. In the wetter part of the moisture range Rijtema's (1965) expression can be used for this function:

$$k = k_0 \exp(\alpha\psi) \tag{2}$$

The total head ϕ is composed of the soil pressure head ψ and the head due to gravity z:

$$\phi = \psi + z \tag{3}$$

Combination of eqs. 1, 2 and 3 gives:

$$V \exp(\alpha z)\ dz = -\ k_0 \exp\ (\alpha\phi)\ d\phi \tag{4}$$

Integration of eq. 4 from z_1 to z_2 and from ϕ_1 to ϕ_2 under the assumption that V is constant over $\Delta z = z_1 - z_2$ gives a simple formula:

$$V = \frac{k_2/a - k_1}{(a-1)/a} \tag{5}$$

In this equation the factor a represents:

$$a = \exp(\alpha\Delta z) \tag{6}$$

The denominator of eq. 5 is therefore independent of k. There is some resemblance between eq. 5 and Ohm's law:

$$I = (E_2 - E_1)/R \tag{7}$$

If one represents the flow of moisture V by the flow of electricity I, the numerator $(a-1)/a$ by an electrical resistance R and the conductivity k_1 by an electric potential E_1 then the electric potential E_2 represents the conductivity k_2/a.

In this way an electric analog of unsaturated flow can easily be built (Fig.1). In this model the electric potential does not represent pressure head, or total head, but unsaturated conductivity.

To make this model fit for non-steady processes, a capacitor has to be installed in each junction. Its charge represents the moisture content θ. With a function-generator and an amplifier the corresponding value of k/a is presented at the upper-side of the layer and the value of k at the lower side. This ampli-

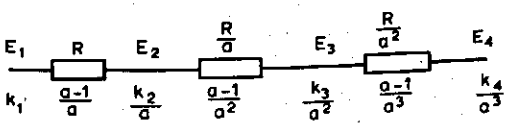

Fig.1. Outline of an electric analog for unsaturated vertical flow; steady-state model.

fication in each junction has the advantage that all resistors can have the same value, in contrast with Fig.1. Moreover, also the relation between E and k is now the same for all depths. Fig.2 gives the outline of one layer of the model. A more precise description is given in a next section. In every layer the values of k and θ can be read without affecting the working of the model.

The soil properties are given in each layer by the value of a and the $k(\theta)$ relation. This curve is represented by three straight lines, the $dk/d\theta$ and the maximum value of θ of which are given. Fig.3 gives an example of such a curve, Fig. 4 shows the whole model and Fig.5 is a photograph of one layer.

SCALES

In order to enable the use of line recorders and tape-writers for the model's output the time scale has been chosen so that 2 s represent one day. A velocity of 1 cm day^{-1} is represented by 10 μA and a conductivity of 1 cm day^{-1} by 0.333 V. From these three scales it can be calculated that 1 cm moisture is represented by 20 μC and that the connecting adjustable resistors have a resistance of about 10^5 Ω.

The model-output, i.e. the moisture content and the conductivity of each layer, the runoff and the drain outflow, are given in volts. For flow velocities and conductivities, 1 V represents 3 cm day^{-1}; for moisture contents 1 V stands for 10%.

RAIN AND EVAPORATION

Rain and evaporation are fed into the model's top layer with a paper-tape reader. The tape is read five times per day (= 2 s); the ASCII-code is used. If the tape-reader is in the off-position, a constant rainfall rate variable between

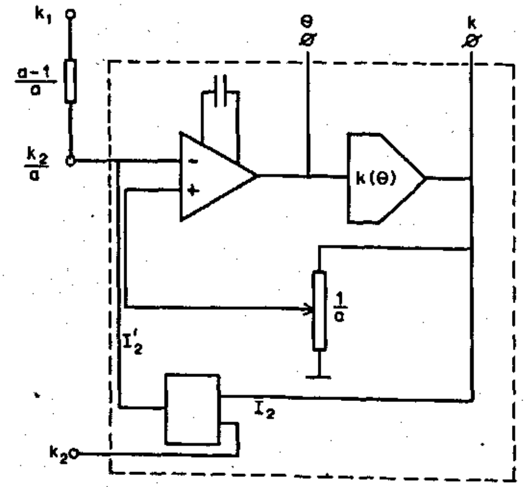

Fig.2. Outline of the wiring diagram of one layer of the model.

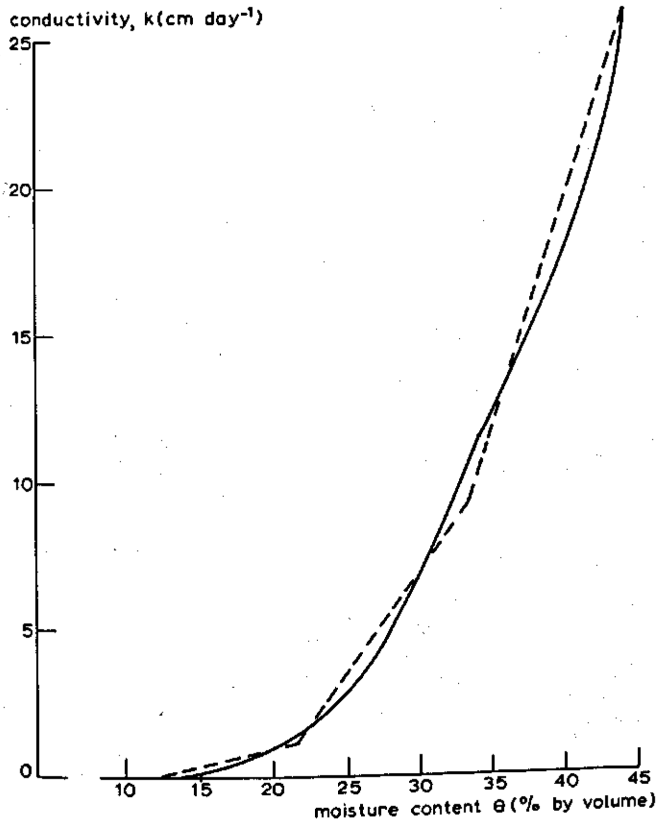

Fig. 3. Example of a $k(\theta)$ relationship and its realization in the model as three straight lines.

0 and 9 mm day^{-1} is applied. It is possible to change some scales so that more or fewer than five readings per day can be realised. For the normal scales the rain can be chosen in steps of 0.2 mm up to a maximum of 30 mm. Evaporation is mostly applied in the middle of the day, in steps of 0.1 mm up to 3.0 mm. If more readings are used, higher evaporations are possible.

TOP LAYER

The top layer represents the soil surface. It receives electrical currents from the tape-reader representing rain and evaporation. The top layer is connected to the first layer with a resistor as is also the case for normal layers. In the top layer a value of $a^{0.5}$ instead of a has to be adjusted because the distance Δz is half the layer's magnitude.

If rain rates are exceeding infiltration rates, water is ponded upon the surface. In the model electricity has to be stored in the top layer's capacitor. The value of ψ_s, indicating the moisture tension at soil surface, equals the amount of moisture θ_s. So the conductivity is:

$$k_s = k_O \exp(\alpha\theta_s) \tag{8}$$

It should be noted that k_s is exceeding k_0 now, which obviously is not correct. The same occurs in saturated conditions in the subsoil, which will be discussed later. Because the values of $\alpha\theta_s$ are small, eq. 8 can be transformed into:

$$k_s = k_0 + \alpha k_0 \theta_s \tag{9}$$

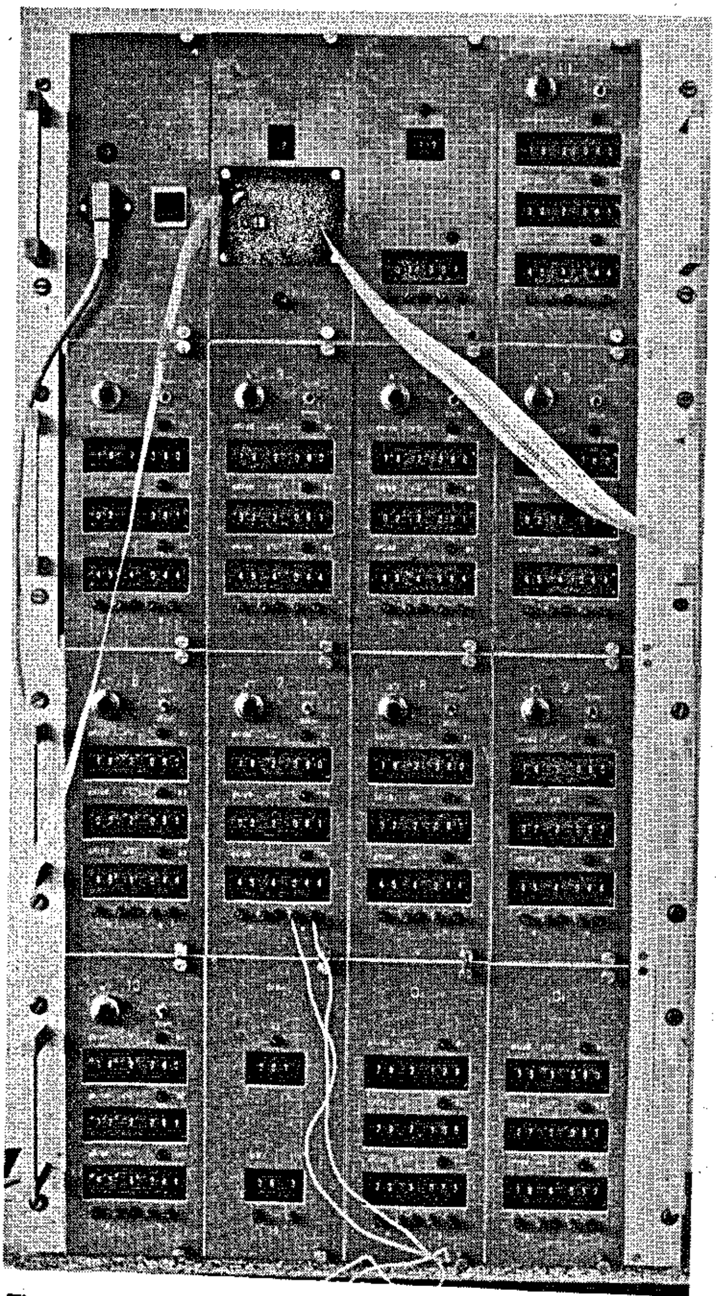

Fig.4. Front of the electronic analog.

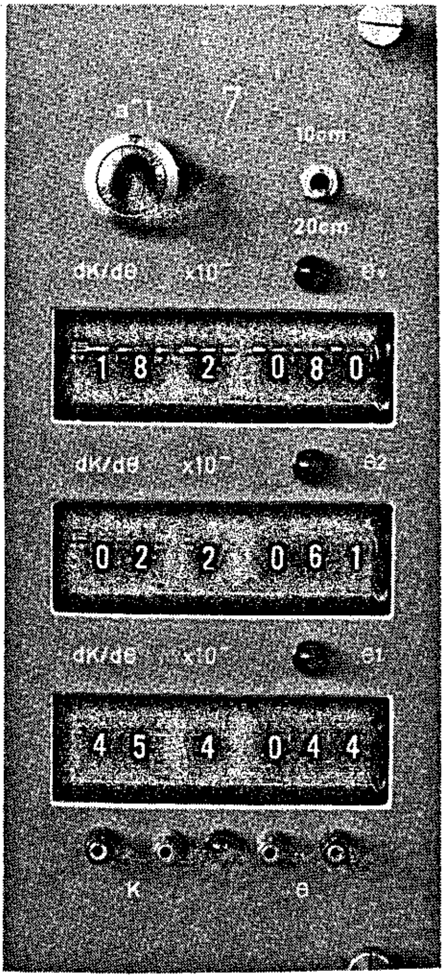

Fig. 5. Example of one normal layer.

This equation is brought into the top layer by a very low fixed capacitor working between zero and k_0 and a variable capacitor working from k_0 upward. The values of α and k_0 have to be adjusted in the top layer. A third value to be adjusted in this layer is that of the maximum ponding depth, θ_{ro}, above which runoff occurs. The value of θ_{ro} can be adjusted in steps of 0.1 cm to a maximum of 2.9 cm. The values of k_s and runoff rate V_{ro} can be read.

Not yet present is a device which reduces evaporation rates in dependence of the first layer's moisture condition.

DRAIN LAYER

The drain outflow V_d (taken negative) is assumed to be dependent on the (positive) moisture potential at drain depth ψ_d as:

$$V_d = -A\psi_d \tag{10}$$

where A is the drainage intensity (day^{-1}) and ψ_d the potential at drain depth, expressed as hydraulic head. As a value of ψ is not present in the model, but only that of k_d, the following equation derived from eq. 10 is applied in the drain layer of the model:

$$V_d = -(A/\alpha)\ln(k_d/k_0) \tag{11}$$

The values A/α and k_0 have to be adjusted. The drain outflow V_d can be read.

TRANSITION LAYER

At the boundary between two layers with different soil properties the values k and θ of the upper and lower layer are different. The value of ψ, which is continuous at the boundary, is not present in the model. So the transition to a layer with different soil properties is a problem which should be solved. The solution that has been chosen is to make transition layers without capacity and resistance but with a same function generator as normal layers.

These have to be placed at the depth where the soil properties, $k(\psi)$ and $\psi(\theta)$ are changing. In a transition layer such a k value is generated as if the upper layer had the $k(\psi)$ relation of the underlying soil. Because no value of ψ is present in the model, this k value has to be generated by a $k(\theta)$ relation in the transition layer, using the θ value of the upper layer. This $k(\theta)$ relation is composed of the $\psi(\theta)$ relation of the upper layer and the $k(\psi)$ relation of the underlying soil. This is illustrated in Fig.6 for the case of medium fine sand on silty clay loam.

Transition layers have no capacity, so their value of $\Delta z = 0$. Therefore they do not replace normal layers but are placed between them. They have also no resistance, so their value of θ equals that of the overlying normal layer.

SATURATED CONDITIONS

The main drawback of the model is that it is based on an unsaturated flow equation. In many studies of the unsaturated zone, drainage plays a role and saturated conditions prevail normally near drain depth. Under saturated conditions ψ is positive; the model continues to act in the unsaturated mode and calculates k-values exceeding k_0. Instead of eq. 12, which should be used for saturated flow:

$$-V = k_0\,[(\psi_2 - \psi_1)/(z_2 - z_1) + 1] \tag{12}$$

eq. 5 is used. Flow velocities calculated with eqs. 5 and 12 can differ considerably. But the model continues application of eq. 5 whether the soil is saturated or not. For example, if $\psi_1 = \psi_2 = +\,20$ cm, $\Delta z = 10$ cm and $\alpha = 0.1$

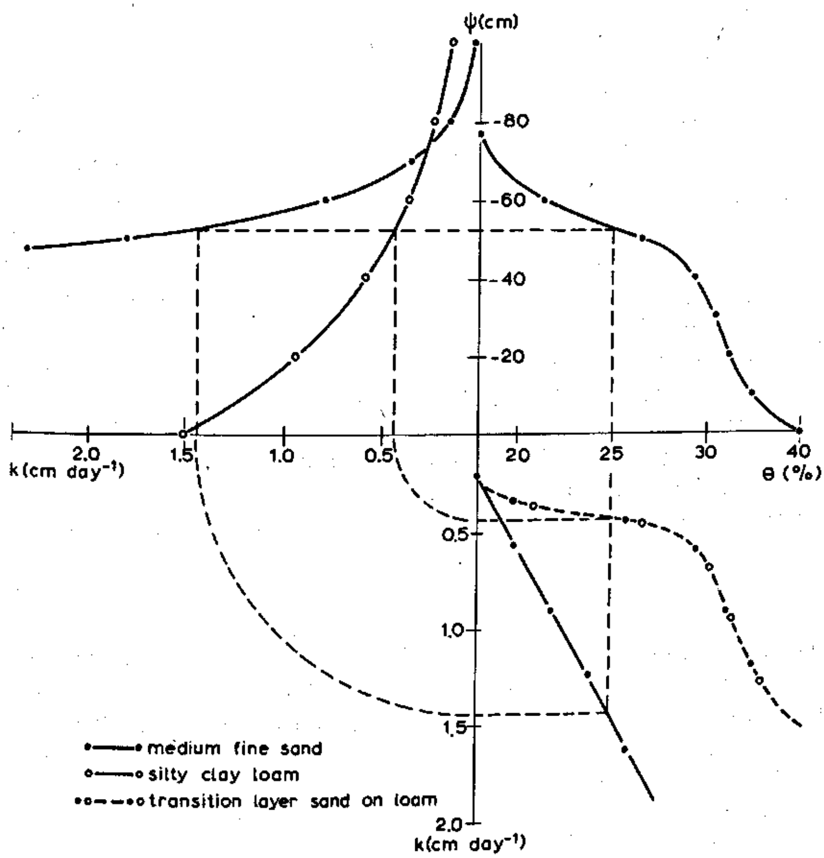

Fig. 6. The $k(\theta)$ relation of a transition layer has to be composed of $\psi(\theta)$ of the upper layer and the $k(\psi)$ of the lower one.

cm^{-1}, the flow velocities calculated with saturated eq. 12 and unsaturated eq. 5 are $-k_0$ and $-7.4\ k_0$, respectively.

This is a large difference but calculations and comparison of flow velocities do not have much sense. Although Darcy's law is suggesting that the gradient and the conductivity are the cause and the velocity their effect, the reverse is usually true. Flow velocities are controlled by rain and evaporation, the causes, and the moisture conditions are their effect.

If the thickness of the saturated layer above the drain level is calculated correctly, the model functions well. By the use of the unsaturated equation (5), however, its thickness is estimated too small.

According to the saturated equation (12) and drainage function (10) the correct magnitude of the saturated layer is:

$$(z_g - z_d)_{correct} = -k_0\ V/[A\ (k_0 + V)] \tag{13}$$

By using the unsaturated equation (5) combined with the drainage function

(10) the model finds:

$$(z_g - z_d)_{\text{model}} = \frac{1}{\alpha} \ln \left(\frac{\exp(-\alpha V/A) + V/k_0}{1 + V/k_0} \right) \tag{14}$$

The differences between the values of $z_g - z_d$ calculated with these equations are dependent of $\alpha V/A$ and V/k_0. The higher α, A and V, and the lower k_0, the larger is the difference. The most important factor is V/k_0.

The error can be counterbalanced by adjusting a value $A^x < A$ in the model. It can be calculated from eqs. 13 and 14 by equaling these two. Therefore a certain value of V should be chosen, normally near the highest value to be expected. Then for lower flow velocities the thickness of the saturated layer will come out somewhat too large, but the error cannot be large. Moreover, its effect is counterbalanced by a feedback in the system, for a too high water table requires a certain amount of water. If this is not available, the water table drops automatically. Fig.7 shows that, provided that a good A^x is chosen, moisture content and drain outflow can be calculated well with the model.

SOME EXAMPLES

In Fig.7 the moisture content of the top 10 cm and the drain outflow are shown, calculated with the numerical model FLOW of Wind and Van Doorne (1975) and with the electronic analog. The drain intensity used here was A = 0.014 day^{-1}; the adapted value calculated with eqs. 13 and 14 for a saturated

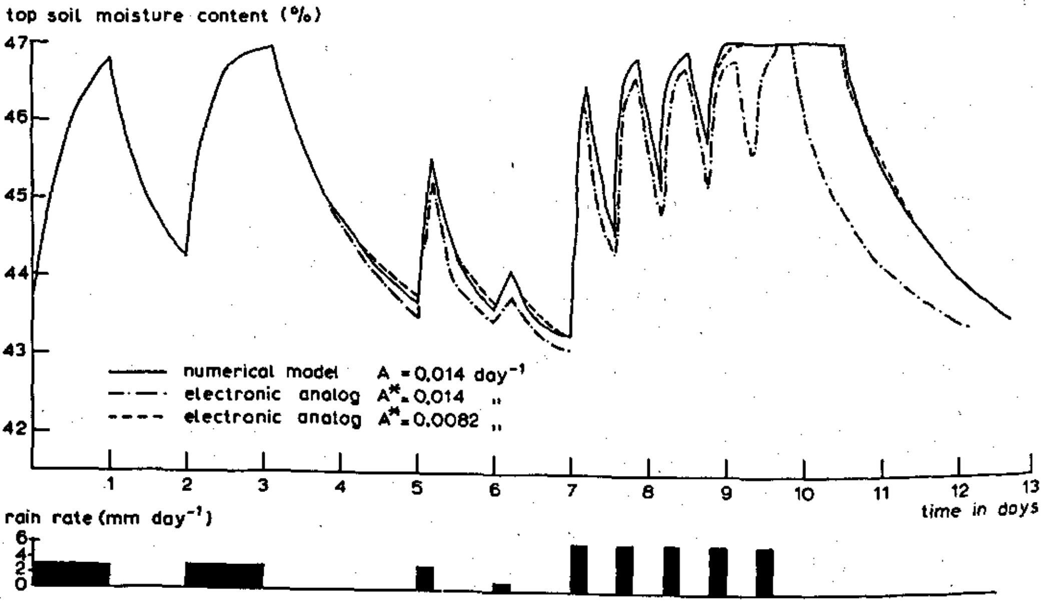

Fig.7. Moisture content of top soil and drain outflow calculated with a numerical model and with the electronic analog.

zone of 100 cm was $A^x = 0.0082$ day^{-1} for the electric model. With this value, the differences between the two calculations were insignificant.

In the example of Fig.7 the three $k(\theta)$ line segments (e.g., the example in Fig.3) used in the model were a close approximation of the $k(\theta)$ function used in the computer. Usually the deviations are larger and this causes differences between the results of the two calculations. In Fig.8 this is shown for a homogeneous soil with $k_0 = 2$ cm day^{-1} and $\alpha = 0.03$ cm^{-1}. The $k(\theta)$ relationship and its approximation by three straight-line segments are shown in the inset. The drain depth is 100 cm, the intensity $A = 1$ in the electric model and infinitely large in the numerical calculation. Some differences in the moisture content at 5 and 35 cm depth can be seen, but they are smaller than 0.5% moisture.

In the very wet autumn of 1974 farmers in The Netherlands had a severe problem in harvesting potatoes, sugar beets and onions. The moisture suction in the top soil must be about 90 cm or drier to enable harvest on loam soils. Fig.9 shows how the moisture suction of the top soil varied during a part of this autumn in dependence of weather and drain-spacing. The soil did not become dry enough for harvesting operations, regardless the drain intensity. The lines of Fig.9 were produced by the electric analog in about 2 min for a loam soil with $k_0 = 2.8$ cm day^{-1} and $\alpha = 0.035$ cm^{-1} and a drain depth of 100 cm. In the drain outflow graph one sees that the best drainage has a larger outflow than a poor drainage in wet periods. During dry periods the reverse can be

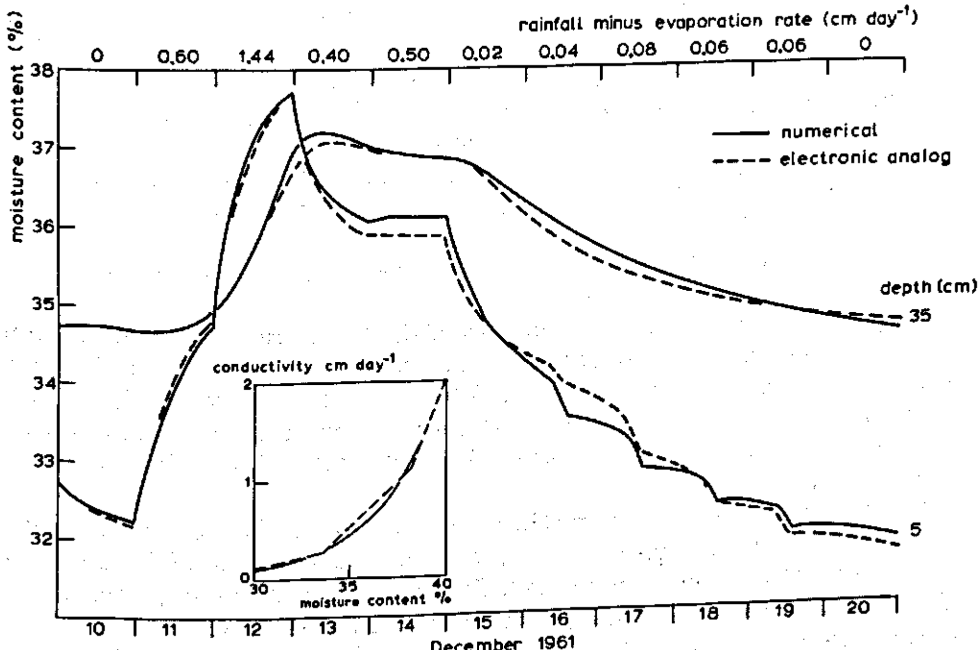

Fig.8. Comparison of numerical and analog calculations of moisture content. Inset: $k(\theta)$ relationship and line segments used.

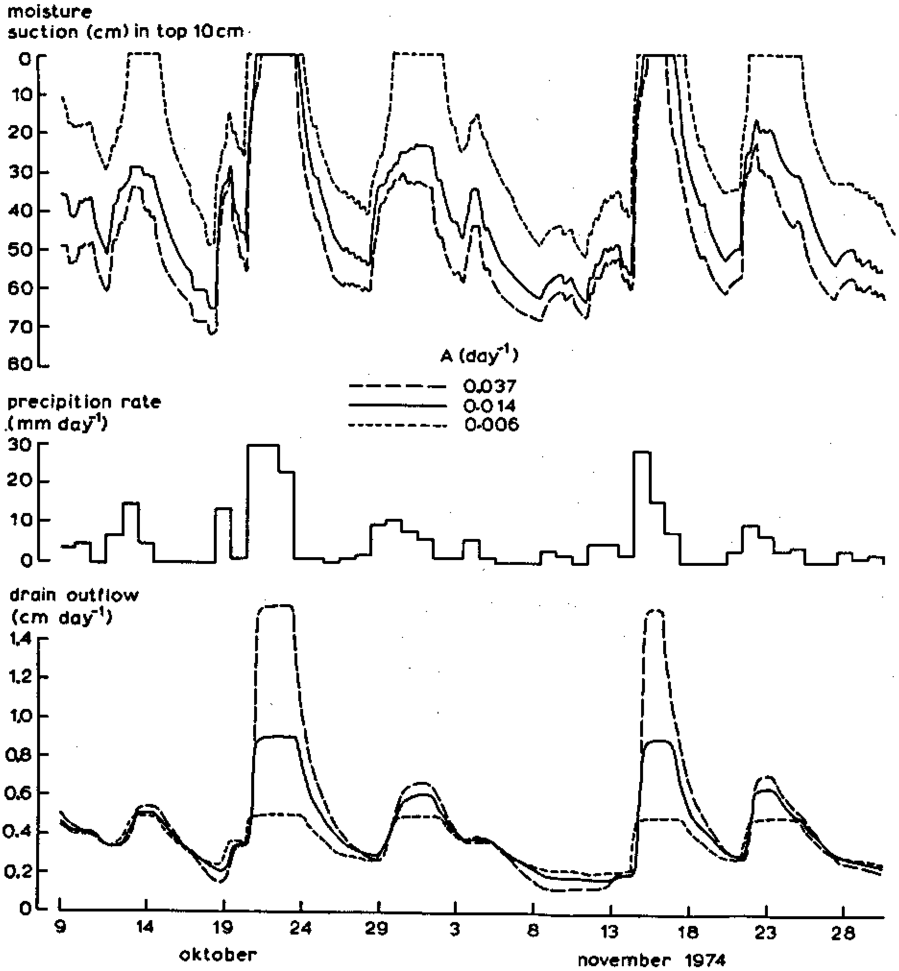

Fig.9. Effect of drain-spacing on moisture suction of the top soil in the wet autumn of 1974.

true, e.g. on October 19. Total discharges of the three drain intensities are not equal because considerable surface runoff occurred. Runoff in the analog was set to occur when there was more than 0.3 cm water on the soil surface. If a larger amount had been chosen, the differences between the three drain-spacings would have been larger, although the soil for all spacings should have been wetter at any time.

An example of the use of a transition layer is given in Fig.10. For the wet autumn of 1974 we compared the behaviour of a loam soil, a sandy soil and a soil consisting of 40 cm loam-on-sand. For the sandy soil the properties k_0 = 10 cm day^{-1} and α = 0.05 cm^{-1} were used; the loam is the same as in Fig.9. The soils were taken to be drained nearly infinitely well (A = 0.17 day^{-1}, at a depth of 100 cm; about 12 times drainage design criterion of The Netherlands).

In wet periods the loam soil is the wettest and the sandy soil the driest; the loam-on-sand soil then takes an intermediate position. In relatively dry periods the latter soil is usually the driest and the sandy soil the wettest. The drain outflow graph is striking. The variation of drain outflow in the sandy soil is very small, only between 0.2 and 0.6 cm day^{-1}. This is caused by its high pore volume; at saturation this soil contains 40% moisture and at a moisture tension of 100 cm only 5%, so 35% can be stored in the soil. In the loam soil only 8.5% can be stored. The drain outflow variations are very large here. The 40 cm loam-on-sand soil is intermediate with regards to its drain outflow but its behaviour is closer to that of the loam than to that of the sandy soil. Apparently it has a

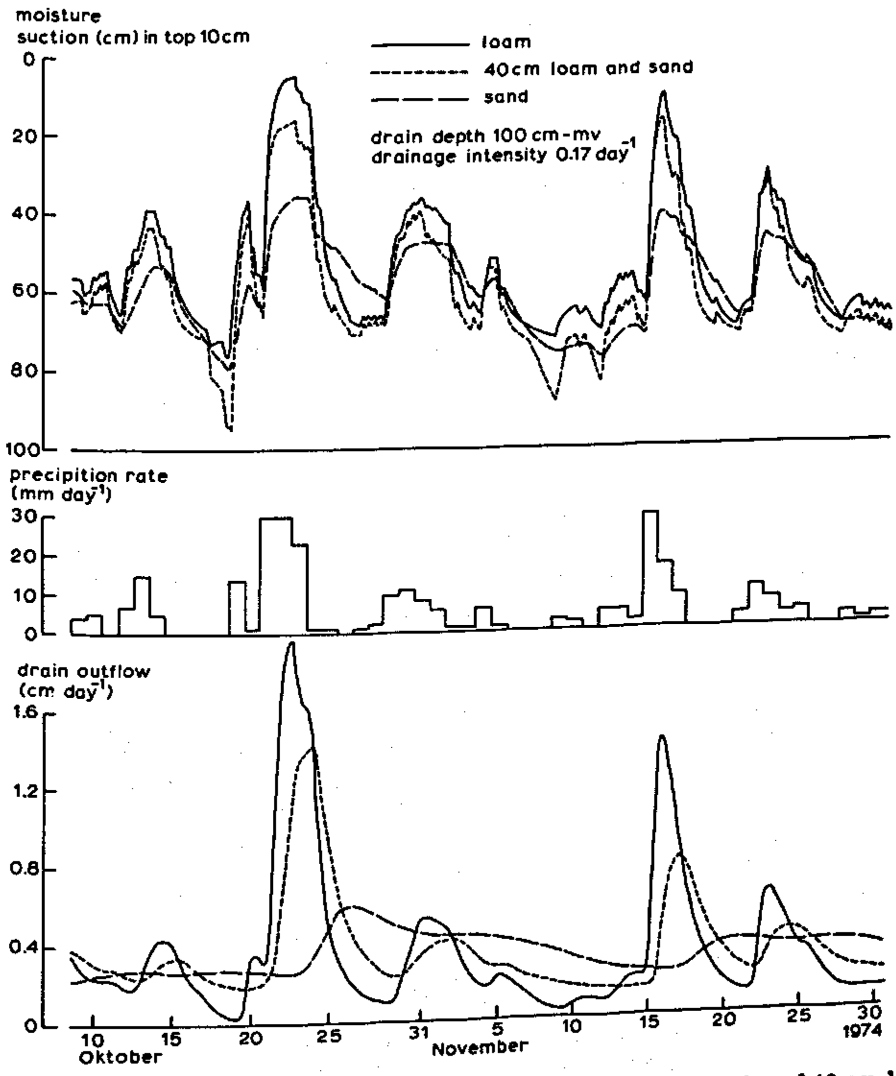

Fig.10. Differences in behaviour between loam, sand and soil consisting of 40 cm loam-on-sand.

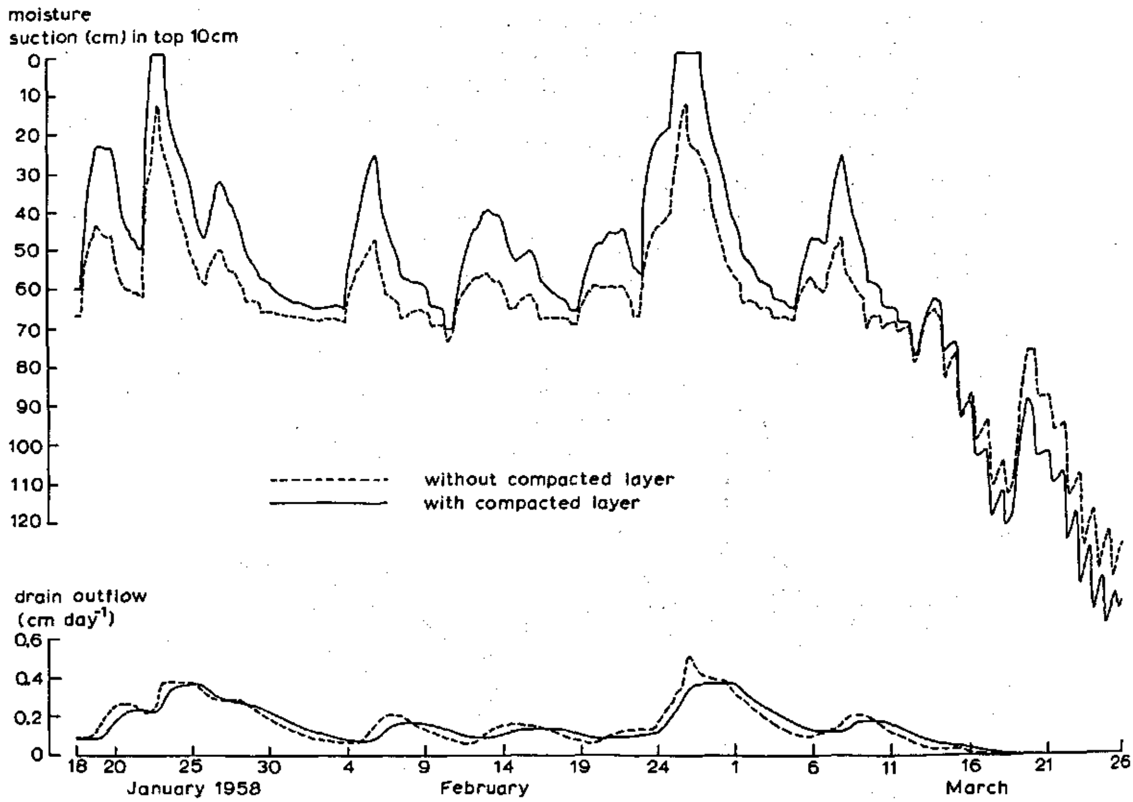

Fig.11. The effect of a compacted layer on moisture content and drain outflow.

moisture storage coefficient which is much lower than that of the sandy soil, although the groundwater table always remained in the sand.

In Fig.11 an example is given of the case with two transition layers. A uniform loam soil ($k_0 = 2.8$ cm day^{-1}; $\alpha = 0.035$ cm^{-1}) is compared with a soil having a compacted layer of between 20 and 30 cm depth. This layer is assumed to have the same α and the same $\psi(\theta)$ relation as the normal soil but a saturated conductivity of one tenth of it, so $k_0 = 0.28$ cm day^{-1}. The compacted layer causes the top soil to be wetter in rainy periods because moisture flows very slowly through it. This can be seen in the drain outflow graph where the maxima for the soil with a compacted layer are always later and lower than those of the soils without it.

In dry periods with an evaporation such as that in March, the soil with a compacted layer is drier than the homogeneous soil. The capillary rise is hampered by the low conductivity. So a soil with a compacted layer sometimes can have an earlier workability than without such a layer.

In the examples given only the top layer and the drain outflow were discussed, but of course every layer can be read for its moisture content and unsaturated conductivity.

SHORT TECHNICAL DESCRIPTION

Only a short description of the electronic realization of the model, in which more than 200 integrated circuits are used, can be given in this context. More details can be obtained from the second author.

The model is built as a modular cassette system. When inserted in the rack in the proper order the cassettes are interconnected. Every layer has its own cassette. A normal layer has a conductive and a capacitive part (see Fig.2). In the capacitive part a number of functions are combined, two of them are derived from soil properties; two others are caused by the fact that k must be divided by a constant.

An integration of the currents flowing into and out of the layer is needed to calculate the moisture content θ. The voltage representing the moisture content is fed to a function generator simulating the $k(\theta)$ relationship as a piecewise linear approximation of it. With each set of thumbwheel switches the slope and breakpoint of the line segments can be adjusted (see Fig.5).

In order to be able to present a value k/a to the higher layer and to measure the current through the resistor $(a-1)/a$, a division by a is necessary. In fact these two values are adjusted by two-ganged ten-turn potentiometers having a multidial to be seen in Fig.5.

As the summing point of the integrator is at a level of k/a and the current from the lower layer flows to a level k, a special current transformer has been developed. This current transformer is a circuit built with three special op-amps and it has the property that the current I_2, flowing through it, is measured and is connected with a special output with the same magnitude. This magnitude is independent of the voltage level into which the current flows (I_2').

In every cassette, outputs are made to measure the values of k and θ. Normal recording and measuring instruments cannot affect the proper function when using these outputs.

The special layers, e.g. top layer, transition layer and drain layer, are constructed as similar cassettes. As far as possible identical printed circuits are used. In the top layer an integrator is used to simulate the ponding properties and a function generator is modified to represent eq. 9 and the runoff. In a transition layer no accumulation is present, so only a function generator is used. In the drain layer a ln module is used, eq. 11. With two sets of thumbwheel switches the function can be adjusted.

In order to have a rough impression of the condition and action of the model several light emitting diodes were added. They light up when specific values are exceeded.

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