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ABSTRACT

A new, analytical, physically based, model of the vertical flow velocity profile and the hydraulic roughness of submerged vegetation has been developed. For the vegetation layer and the surface layer, different turbulence models have been applied. Model simulations correspond well with results from flume experiments reported in literature. Because the analytical model includes only one empirical relation, it potentially has a wide range of applicability. Due to lack of data, the model is not yet validated for field conditions.

INTRODUCTION

As a consequence of nature rehabilitation, which has become an important aspect of river management world wide, vegetation characteristics of flood plains are expected to change in the future. In order to assess the effects on river functions such as the safe conveyance of flood waves, it is important to know how the hydraulic roughness of the flood plains will be affected. In this paper, results from studies on hydraulic roughness of vegetation reported in literature are used for development and verification of a physically based model of the vertical flow velocity profile and hydraulic roughness of submerged tall vegetation such as reeds. The emphasis is on an analytical expression, which can be easily incorporated in numerical hydraulic software packages.

VELOCITY PROFILE OF SUBMERGED VEGETATION

The velocity profile of submerged vegetation (illustrated in figure 1) is treated separately for the vegetation layer and the surface layer. The two profiles will be smoothly matched through boundary conditions at the interface.

VEGETATION LAYER

The momentum equation, assuming uniform and steady flow, reads:

$$\frac{\partial \tau(z)}{\partial z} = F_D(z) - \rho \cdot g \cdot i \qquad (1)$$

with: τ = shear stress (kg/ms^2), ρ = density of water (kg/m^3), z = vertical co-ordinate (m), g = acceleration due to gravity (m/s^2), i = energy gradient (-) and in which the drag-force $F_D(z)$ on the vegetation is defined by:

$$F_D(z) = m \cdot D \cdot C_D \cdot \frac{1}{2} \cdot \rho \cdot u(z)^2 \qquad (2)$$

with: m = vegetation elements per m^2 (m^{-2}), D = diameter stem vegetation element (m), C_D = drag coefficient (-), $u(z)$ = flow velocity at level z (m/s).

The turbulent shear stress can be described by the concept of Boussinesq:

$$\tau(z) = \varepsilon \cdot \frac{\partial u(z)}{\partial z} \qquad (3)$$

with: ε = turbulent viscosity (kg/ms) = $\rho \cdot \nu_t$ and ν_t = eddy viscosity (m^2/s).

In conformity with the turbulence models described in e.g. Rodi (1980), ν_t is assumed to be characterised by the product of a velocity scale and a length scale of large scale turbulence, which is responsible for the vertical transport of momentum. In conformity with Tsujimoto and Kitamura (1990), the characteristic velocity scale is assumed to be represented by the flow velocity $u(z)$. The characteristic length scale α is assumed to be independent of z. The turbulent shear stress (3) then reads:

$$\tau(z) = \rho \cdot \alpha \cdot u(z) \cdot \frac{\partial u(z)}{\partial z} \tag{4}$$

The momentum Equation (1) now transforms to:

$$u(z) \cdot \frac{\partial^2 u(z)}{\partial z^2} + \left(\frac{\partial u(z)}{\partial z}\right)^2 = \frac{m \cdot D \cdot C_D \cdot u(z)^2}{2\alpha} - \frac{g \cdot i}{\alpha} \tag{5}$$

which has the following analytical solution:

$$u(z) = \sqrt{C_1 \cdot e^{-\sqrt{2 \cdot A} \cdot z} + C_2 \cdot e^{\sqrt{2 \cdot A} \cdot z} + u_{s0}{}^2} \qquad (0 < z < k) \tag{6}$$

with: k = vegetation height (m) and

$$A = \frac{m \cdot D \cdot C_D}{2\alpha} \tag{7}$$

$$u_{s0} = \sqrt{\frac{2 \cdot g \cdot i}{C_D \cdot m \cdot D}} \tag{8}$$

u_{s0} is the characteristic constant flow velocity in non-submerged vegetation, which also follows directly from (5) with all velocity gradients set on zero. Constants C_1 and C_2 in (6) follow from boundary conditions. At the bed (z=0) the bottom shear stress is neglected and the flow velocity is assumed to be equal to u_{s0}. At the top of the vegetation layer the boundary condition is determined by the shear stress:

$$\tau(k) = \rho \cdot g \cdot (h - k) \cdot i \tag{9}$$

with: h = water depth (m)

With these boundary conditions, the following values for C_1 and C_2 are derived:

$$C_1 = \frac{-2 \cdot g \cdot i \cdot (h - k)}{\alpha \cdot \sqrt{2 \cdot A} \cdot (e^{k \cdot \sqrt{2 \cdot A}} + e^{-k \cdot \sqrt{2 \cdot A}})} \tag{10}$$

$$C_2 = -C_1 \tag{11}$$

The velocity profile for the vegetation layer is now established. The only unknown parameter is the characteristic length scale α.

SURFACE LAYER

For the surface layer, Prandtl's mixing length concept is adopted resulting in the well known logarithmic flow velocity profile. The virtual bed of such a profile does not coincide with the top of the vegetation but appears to lie at a distance h_s under that level. The flow velocity profile thus can be written as:

$$u(z) = \frac{1}{\kappa} \cdot u_* \cdot \ln\left(\frac{z - (k - h_s)}{z_0}\right) \qquad (k < z < h) \tag{12}$$

with: κ = Von Kármán's constant (-)

h_s = distance between top of vegetation and virtual bed of surface layer (m),

z_0 = length scale for bed roughness of the surface layer (m)
u_* = virtual bed shear stress for the surface layer: $u_* = \sqrt{g \cdot (h-(k-h_s)) \cdot i}$

h_s and z_0 follow from the continuity condition that both actual value and gradient of the flow velocity of the vegetation and the surface layer should be equal at the interface (z=k). These conditions result in the following values for h_s and z_0:

$$h_s = g \cdot \frac{1+\sqrt{1+\dfrac{4 \cdot E^2 \cdot k^2 \cdot (h-k)}{g}}}{2 \cdot E^2 \cdot k^2} \tag{13}$$

$$z_0 = h_s \cdot e^{-F} \tag{14}$$

with:

$$E = \frac{\sqrt{2 \cdot A} \cdot C_3 \cdot e^{k \cdot \sqrt{2 \cdot A}}}{2 \cdot \sqrt{C_3 \cdot e^{k \cdot \sqrt{2 \cdot A}} + u_{v0}^{\ 2}}} \tag{15}$$

$$F = \frac{k \cdot \sqrt{C_3 \cdot e^{k \cdot \sqrt{2 \cdot A}} + u_{v0}^{\ 2}}}{\sqrt{g \cdot (h-(k-h_s))}} \tag{16}$$

$$C_3 = C_2 / i \tag{17}$$

$$u_{v0} = u_{s0} / \sqrt{i} \tag{18}$$

HYDRAULIC ROUGHNESS OF SUBMERGED VEGETATION

From the average flow velocity in the vertical U, which follows from the integrals of (6) and (12), the hydraulic roughness expressed as the value of Chézy ($m^{1/2}$/s) can be obtained via $C=U/\sqrt{(h.i)}$. To integrate (6) analytically, the first term under the square root sign is neglected in comparison to the second term. The impact of this simplification is only noticeable at extremely low vegetation densities for which the model is not developed. The following value of Chézy is then obtained:

$$C = \frac{1}{h^{3/2}} \cdot \left\{ \begin{aligned} & \frac{2}{\sqrt{2 \cdot A}} \cdot \left(\sqrt{C_3 \cdot e^{k \cdot \sqrt{2 \cdot A}} + u_{v0}^{\ 2}} - \sqrt{C_3 + u_{v0}^{\ 2}}\right) + \\ & \frac{u_{v0}}{\sqrt{2 \cdot A}} \cdot \ln\left(\frac{\left(\sqrt{C_3 \cdot e^{k \cdot \sqrt{2 \cdot A}} + u_{v0}^{\ 2}} - u_{v0}\right) \cdot \left(\sqrt{C_3 + u_{v0}^{\ 2}} + u_{v0}\right)}{\left(\sqrt{C_3 \cdot e^{k \cdot \sqrt{2 \cdot A}} + u_{v0}^{\ 2}} + u_{v0}\right) \cdot \left(\sqrt{C_3 + u_{v0}^{\ 2}} - u_{v0}\right)} \right) + \\ & \frac{\sqrt{g \cdot (h-(k-h_s))}}{k} \cdot \left((h-(k-h_s)) \cdot \ln\left(\frac{h-(k-h_s)}{z_0}\right) - h_s \cdot \ln\left(\frac{h_s}{z_0}\right) - (h-k) \right) \end{aligned} \right\} \tag{19}$$

The hydraulic roughness thus can be calculated analytically when vegetation characteristics (m, D, C_D, k), water depth h and characteristic length scale of large scale turbulence α are known. The only unknown parameter α will be analysed next.

MODEL VERIFICATION

The performance of the analytical model is assessed in two successive steps:

1. Comparison with measured flow velocity profiles from flume experiments by varying the characteristic length scale α in such a way that the shape of the measured velocity profile is represented;
2. Comparison with measured hydraulic roughness values from flume experiments.

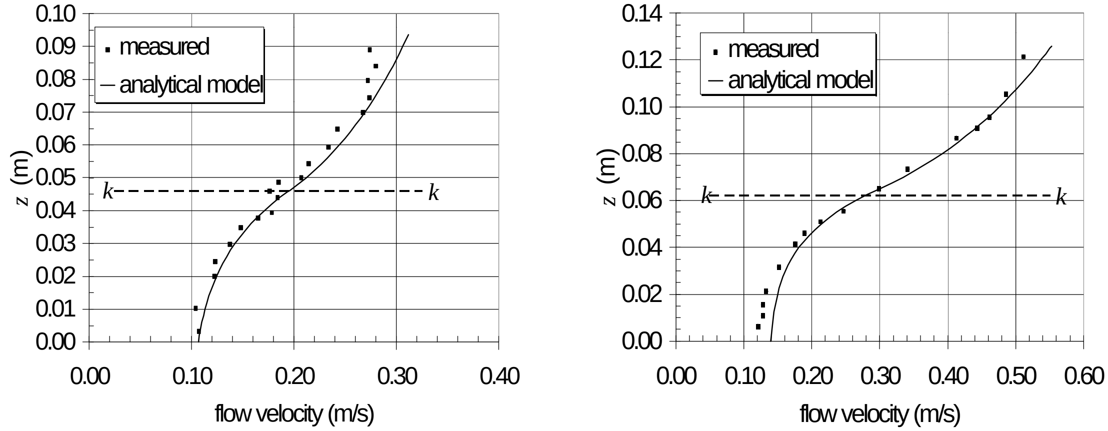
Experimental results used for this are summarised in Table 1. Equal drag coefficients have been applied for all verification tests: C_D=1.4 for cylinders/reed and C_D=2.0 for strips. These are average values from the papers of Table 1. Figure 1 shows two typical results of the first verification step, for which in total 23 measured flow velocity profiles have been used. This verification step shows that values of α can be selected in such a way that calculated and measured flow velocity profiles are in good agreement. This means that the assumption that α is independent of z does not have to be withdrawn. To make the analytical model generally applicable, α has been correlated to hydraulic and vegetation characteristics, with the following best-fit result (Figure 2):

$$a = 0.0793 \cdot k \cdot \ln \frac{h}{k} - 0.00090 \quad \text{and } \alpha \geq 0.001 \qquad (20)$$

With this relation for α, the performance of the analytical model is tested against hydraulic roughness values which follow from the flume experiments of Table 1. The results of this second verification step are shown in Figure 3. Taking into consideration that constant drag coefficients are applied, the performance of the analytical model is good.

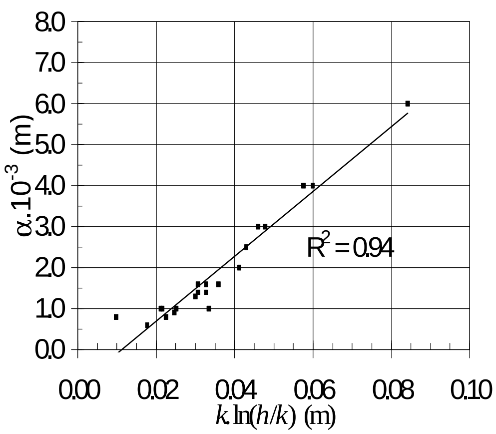
Paper	vegetation characteristics			
	shape	m (m^{-2})	D (m)	k (m)
Tsujimoto and Kitamura (1990)*	cylinders	2,500	0.0015	0.0459
Shimizu and Tsujimoto (1994)*	cylinders R	10,000	0.0010	0.041
	A	2,500	0.0015	0.046
Starosolsky (1983)	reed	220	0.0046	0.15|0.25
Tsujimoto, Okada and Kontani (1993)*	cylinders sphere on top	10,000	0.00062 0.003	0.065
Nalluri and Judy (1989)	cylinders 1C	400	0.006	0.15
	2C	200	0.006	0.15
	strips 5B	833	0.005	0.16
	6B	833	0.005	0.16
Kouwen et al (1969)	strips	1,000	0.005	0.10
* including flow velocity profiles				

Table 1: Flume experiments used for model verification.



a. Run A31 (Shimizu & Tsujimoto, 1994) b. Run BZ11 (Tsujimoto et al., 1993)

Figure 1 Measured and calculated flow velocity profiles



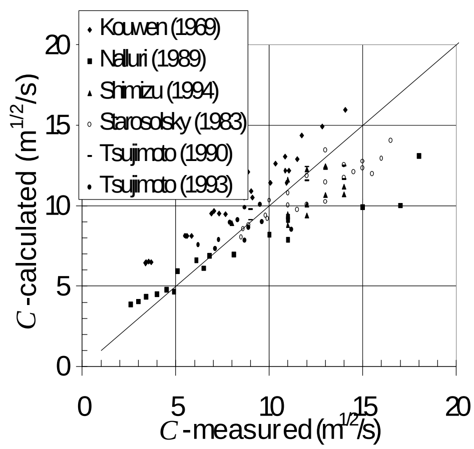


Figure 2 Relation for α

Figure 3 Measured and calculated C- values

DISCUSSION

The new analytical model for the vertical velocity profile and the hydraulic roughness of submerged vegetation appears to be an important step towards a generally applicable analytical model (HKV CONSULTANTS, 1996). Hydraulic roughness values calculated with the model correspond well with results from flume experiments. However, the model can not yet be validated for field situations due to lack of data. Model results for field situations (see Table 2) show that under certain conditions (i) α exceeds the values for which the relation for α was fitted, (ii) the calculated virtual bed level for the surface layer is below the actual bed level (i.e. $h_s > k$) and (iii) the length scale for the bed roughness z_0 is of the order of magnitude of h_s. This, in combination with the resulting low Chézy-values, illustrates the need for additional research on the validity of the modelling concepts, as well as the relation for α. This should be combined with a profound field measurement program (or large scale flume experiments) so as to validate the study results.

	Input parameters				Calculated parameters			
Test	h (m)	k (m)	D (m)	m (m^{-2})	C (m$^{1/2}$/s)	h_s (m)	z_0 (m)	α (m)
1	5.0	0.5	0.005	100	17.5	0.74	0.26	0.09
2	5.0	2.0	0.005	100	8.7	1.14	0.46	0.14
3	5.0	0.5	0.005	500	16.9	0.46	0.22	0.09
4	5.0	2.0	0.005	500	7.4	0.69	0.37	0.14

Table 2 Results analytical model for field situations with reed.

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