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SURFACE HYDROLOGY*

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ABSTRACT

Latteral flow components can be caused by a vertical force in an anisotropic sloping soil. Such a latteral flow can cause water accumulation in concave parts of the landscape and outcrop of water by seepage at some roadcuts. This, in turn, if proved correct could explain experimental observations of moisture accumulation, runoff at low intensity rains, various erosion phenomena by seepage forces and a number of other hydrological phenomena. This article is an introduction to a series that will prove in details the existence of such latteral flows.

1. HISTORICAL NOTE

The series of articles to be published in the following summs up work that has started around 1964. The senior author has been asked to review the regulations given in literature and used in practice to prevent erosion in water channels dug in soil. The study of many field observations has indicated that something is wrong with the present notions about erosion. The importance of seepage forces and piping has been realised. It led to the measurement of piping in cohesive soils (ZASLAVSKY and KASSIFF, 1965 and KASSIFF et al., 1965). Observations in later years indicated that the mechanism of piping by seepage forces may be quite important in field erosion as well. However, it was impossible to explain how it can occur in semi-arid zones without the presence of groundwater. Seepage out of the soil was beyond our understanding in such places where seemingly the only flow could be into the soil because it is unsaturated.

In 1968 the senior author had the opportunity of serving as a guest scientist at the Hydrograph Laboratory of S.W.C. in the A.R.S.

of the U.S. Department of Agriculture. One of the main tasks was to evaluate the approach to surface hydrology and mainly rain - runoff relations from the premises. Serious doubts has been raised as to the present approaches to the problem and mainly as to the soundness of its elementary physics and mathematics. First hints towards a new approach to surface hydrology have been published by the U.S. Department of Agriculture in a report (ZASLAVSKY, 1970). It indicates the existence of horizontal flow component as a result of rain in the soil rather than above it. Among other very interesting results it explains also how seepage forces that cause field erosion can be formed. The rain enters in the soil first and then accumulates in some points and seeps out to form both runoff and erosion.

Back in Israel a series of grants by the U.S. Department of Agriculture through P.L. 480 and the Israel - U.S.Binational Science Foundation coupled with aid by the Israely Soil Conservations Service made it possible to make some more detailed studies. Several graduate students participated in this work. G. Shacham and E. Sabach did some sork on erosion and splashing raindrops. Dr. Gideon Sinai was most instrumental in several parts of this work, but his main contribution is in the numeral solution of the transient flow near the soil surface.

Finally the senior author has been invited to the State Agricultural University in Wageningen and the Institute for Land and Water Management Research (ICW) there, where he had the opportunity to present the whole work in a series of lectures and summarize them up. In view of many reactions of scientists and engineers having a large volume of field experience it seems more and more that the theory offered in the following is of a universal interest. It suggests a straight forward explanation for an increasing number of situations. It offers a rational approach for many engineering solutions.

It is especially difficult to bring the many field observations that have convinced us in the soundness of our approach. For example Dr. J. Morin of the Soil Conservation Service has made numerous and systematic studies throughout Israel on infiltration runoff and erosion. His enthusiastic support after a long experience with field observations has a special weight in letting us to believe that the material is ripe for publication.

2. RAIN AND RUNOFF

The classical model that serves the hydrologists universally to date, is that runoff is formed in one of two ways:

a. the rate of rain exceeds the infiltration capacity of the soil at a point, implying that at a lower rate of rain there will be no runoff;
b. there is a buildup of groundwater table or of perched water table that eventually flows out of the soil. This outflow of groundwater is considered strictly for streamflows or baseflow of large delay, certainly not during one rainstorm.

These concepts may be represented by MEINZER (1923, 1942), LOWDERMILK (1926), SHERMAN (1932), HORTON (1935), BARNES (1939), ROUSE (1950) and many that followed. A statistical organization of these models such as by SCHREIBER and KINCAID (1967), DISKIN (1970), CLARKE (1973) do not really get away from the basic notions that runoff is constituted of rain minus infiltration. One can cite stochastic models such as by CHOW and RASASESHAN (1965), GRACE and EAGELSON (1967), MATALS (1967), BURAS (1972), VEN TE CHOW (1964) and VISSER (1967) or deterministic models by KISIEL (1969), VEN TE CHOW (1964), CRAWFORD and LINSLEY (1962, 1966), JAMES (1970). The two basic notions will prevail.

FREEZE (1969, 1971, 1972a, 1972b, 1973, 1974, 1976), AMERMAN (1973) introduced a more rigorous treatment of saturated and unsaturated flow in the soil. Still they remained within the same two notions that the water will run off either by not being able to penetrate the soil or by accumulating in the groundwater. Where the groundwater seeps out of the soil the runoff may be formed. However, it is delayed long after the rain.

Actual observation of runoff that occur within the rainstorm with a delay as short as few minutes or a portion of a minute indicate that the above notions draw at best a very partial picture. Certainly, rain that exceeds the infiltration capacity runs off. However, why sometimes quick runoffs are formed by rains which are much lower than the infiltration capacity. All kinds of excuses have been invented to explain this fact. They are often short even of admitting the phenomenon.

The infiltration capacity is presumably a unique figure. This primitive concept which prevails so many years does not allow for dependence of runoff on antecedent moisture. In more sophisticated treaties, a more realistic picture of unsaturated flow in the soil is admitted. We shall not refer here to a number of articles that relate the rate of infiltration to the water storage in the soil. The real phenomenon can best be understood by the work of BRAESTER (1973). According to this work, the surface moisture gradually increases during the rain. The infiltration capacity in a uniform soil is simply its hydraulic conductivity. The soil approaches saturation after a long time if the rate of rain equals, or surpasses the hydraulic conductivity (in a non-uniform soil a different definition is necessary). Even if the rain exceeds the infiltration capacity (in a uniform soil the hydraulic conductivity), there is a need for certain time to reach surface saturation and flooding. This time will depend on the rain intensity, on soil properties (not only the saturated hydraulic conductivity) and on antecedent moisture.

A high antecedent moisture alone cannot account for runoff phenomena. In view of unsaturated vertical flow, a very short time after the end of a rain, the soil everywhere reaches a more or less fixed moisture known as the field capacity. A few days between rainstorms are sufficient to evaporate only few millimeters of water from the soil (and often not even that). Neither the previous wetting, nor the drying that can be refilled in few minutes of rain, can possibly explain the cumulative effect of rainstorms in gradually increasing the runoff during the rain season. It is quite common experience that in many regions little or no runoff occurs before a few hundred millimeters of rain have occurred. The time distribution hardly affects this phenomenon. The intensity of a given storm affects the runoff only in addition to the total cumulative rain and antecedent moisture. These well known experiences encourage investigators only to invent statistical tricks and fudging factors. The worst part is that afterwards they give names that intend to insinuate true physical entities. The basic dilemma remains: How does runoff form when the rain does not exceed the infiltration capacity over the whole field.

At least two more ideas should be mentioned that attempt to explain runoff. One is the formation of a surface crust (SEGINES and MORIN, 1970), which has been shown to develop in direct correlation with the accumulative rain (MORIN, 1976, personal communication). For example some wind blown loess soils of Israel can start at infiltration capacity of 30 to 40 millimeters per hour and end at 3 to 5 millimeters per hour after a cumulative rain of some 200 millimeters. However, after each drying period there will be some recovery of surface permeability. In any case the initial rate of infiltration for any new rain will be at least 10 to 20 mm per hour. Only the final rate of infiltration which is obtained after a portion of an hour will be very low. However the runoff starts much earlier. The curst formation can thus explain only part of the problem.

The other concept which should be mentioned here is that of a partial contribution or partial area which states that small parts of the soil surface have a very low infiltration capacity and thus contribute considerably to runoff while the other do not at all. One cannot prove or disprove this concept. It is only another way of saying that there must be some reason for runoff despite the fact that the rain seemingly does not exceed the infiltration capacity. There can certainly be parts of the area where the rain exceeds the infiltration capacity.

In the following we shall show how some parts of the landscape contribute runoff. However, they are related mainly to the topografic configuration and not to parts which are less permeable.

3. MORE RESERVATIONS ABOUT THE COMMON CONCEPTS OF RUNOFF (following ZASLAVSKY, 1970)

Scalars and vectors

Traditionally the infiltration into the soil has been almost synonymous with vertical flow. In reality it is only one out of three (or at least two) flow components. The horizontal flow component cannot be added to the infiltration as if both were scalars. The commonly used equation

$$R = P - I \quad (1)$$

where R = runoff
P = precipitation
I = infiltration

can be used at most as an overall scalar balance over an area where it is not really measured at a point but is the difference between the measured precipitation and the outflow through a well defined and measurable river or channel (assuming the boundaries of the drainage basin to be determinable by the topography alone). Thus eq. (1) cannot be considered an equation at a point and is not one to predict runoff but to calculate net recharge over a field.

Errors in P and I

Precipitation can be measured with a limited accuracy (e.g. ±20%). Infiltration capacity can be measured or estimated in a very rough manner. It can change within a storm (SEGINER and MORIN, 1970). It is not a constant in time or space. A change by a half order of magnitude is not uncommon.

It is therefore unrealistic to expect any reasonable accuracy in predicting the runoff R which is most commonly 5 to 10% of the P (precipitation). Eq. (1) or any similar equation of differences, sophisticated as it may look, cannot be seriously considered as a tool for prediction based on actual measurements.

Is it possible to measure runoff?

The question of measuring runoff R depends on its definition. If it is the outflow through a well defined channel, then it is reasonably meaningful and measurable. However, at a point in the field or as overland sheet flow or as sometimes more carefully called runoff supply it defies unique measurement as well as definition. The difficulty of defining the surface runoff is as it is difficult to define the soil surface itself. This problem of definition will be treated later in this report. It is reasonably clear that at the soil surface there is a transition between the soil bulk and the air. The porosity as well as the hydraulic conductivity gradually increase in a direction from the soil bulk outward. Some may find it hard to accept this very fundamental argument about the transitive nature of the soil surface. They may then appreciate the practical problem of intercepting the runoff for measurement. The result will depend strongly on the depth at which such interception will be performed. A common falacy is to produce a 'deep enough' cutoff and let 'every drop of water' climb above it. This type of measurement definitely affects the entity which is to be measured and undoubtably tends to increase the apparent runoff. The alternative is a very thin horizontal threshold that supposedly divides between the runoff and the flow within the soil. The question is how thin is the threshold and at what elevation. As the soil surface is irregular, to which size irregularities should the divider between the soil bulk and the air conform.

It is much more sensible, and in fact feasible, to measure the horizontal flux component or, easier still, the horizontal discharge (by vertical integration of the horizontal fluxes). In fact, that is what one measures near a vertical cutoff. Stagnation near such a wall causes sometimes part of the water to overflow and part of it to underflow the cutoff. The horizontal flow can be within the soil or outside the soil. There is no way to tell. The problem is not that of a technical limitation but a fundamental one.

In summary, the notion of runoff as a point value over the soil area is fundamentally wrong and practically impossible.

As popular as it is (from kindergarden and up), the model of eq. (1) still lacks a real demonstration of relevancy to either the physical understanding, the consistent mathematical formulation, or to practical measurement.

4. CONCENTRATION OF WATER IN CONCAVE AREAS

There is a phenomenon of moisture concentration in concave areas. By concave we mean, not only the slope bottom or valleys but any transition from a steep to a moderate slope. This is a phenomenon that cannot be explained by any existing hydrologic models. Such a concentration has been observed in an area of sand dunes with 70 mm rain per year, and infiltration capacity of some 500 mm per hour. It occurred both on steep slopes and moderate ones. In the southern part of Israel one has to travel following the rain and see the green of the seasonal grass and shrubs painting concave parts of the landscape. Beduins have been used to plant their barley only in concave parts of the landscape. The accumulation has been observed in areas where no surface runoff could possibly be observed, where no water table was present and where no highly impermeable layer and perched water were obvious.

In the flowerbulb sand area near Lisse (The Netherlands) it has been observed in a soil cross-section that under concave surface the sand was wetted to a considerably greater depth (personal communication van der Valk and Knottnerus). This in turn had its effect on moisture availability to plants and on wind erosion patterns.

Looking at fields under rain or after a rain, water often appear in some very shallow concave parts either in the form of small puddles or just as shining soil surfaces.

Concave parts in fields often suffer from wetness, traficability problems and even aeration problems. Shallow water ways accumulate moisture and stay wet for a long time even where there is no water table.

Any model attempting to explain surface hydrology should be compatible with this phenomenon.

5. MAIN OBSERVATIONS IN THE BEER SHEVA EXPERIMENT

Curvature of the soil surface has been measured geodetically through the elevation z at different points according to the formula

fig. 1

$$\nabla^2 z = (z_{i+1,j} + z_{i-1,j} + z_{i,j+1} + z_{i,j-1} - 4z_{ij})/h^2 \qquad (2)$$

Moisture contents have been measured at 20 and 40 cm depth over 70 x 70 meters area. The field has been planted more or less parallel to contours. The main slope was 12%. Yields have also been measured. In brief, the results are shown in fig. 2 where the concavity is estimated by $\nabla^2 z$. The correlation between the moisture contents 2 weeks after the rain and the curvature has been found at r = 90% to be

$$C = 8.67 + 50.4\nabla^2 z \qquad (3)$$

fig. 2

We shall not cite here the details of measured yields (that were exactly correlated with the moisture). It reached more than 2 tons per ha in the concave part and as little as 0.2 - 0.3 tons per ha in some convex parts.

No runoff was observed in the usual sense. The soil was a loess-loam which was plowed and disced in a regular commercial manner. No water table, perched water or impermeable layer was found anywhere near. The rain totaled some 250 mm. Diagnostic tests other than the moisture content have been run (salinity, fertility, clay content, etc.) without any visible trends.

6. SOIL FORMING PROCESSES (following ZASLAVSKY, ROGOWSKY, 1969)

The concentration of water in concave parts of the landscape can explain some soil forming processes. The pedologic 'genetic' formation of the 'B' horizon is pronounced on a flat land but thickest on concave parts (excluding hydromorphic alluvial bottom land). Upper on the slope at convex parts the development of B horizon is smallest.

Many theories related this differentiation to overland flow and erosion. They cannot explain how concave slopes (nevertheless relatively steep slopes) 'catch' more water or clay to form a thick B horizon. The existing theories cannot explain how erosion, that carry away any A and B horizon leave a distinct B horizon, though faintly developed. Is it that there are some tens or hundreds of years of B horizon developments and then some of erosion?

The development of loamy B horizon of dune sand parent material serve as a perfect model to shake existing theories and offer some new insight. It is perfectly clear that the clay is imported into the sand by rainwater and settling dust. Water is certainly the vehicle by which the clay accumulates in the B horizon. The dune sand has a hydraulic conductivity of some 10^3 cm/day. Only rains of few minutes may have an intensity that will exceed this hydraulic conductivity. However, the actual infiltration capacity of such short term rain spurts is certainly several times larger than this hydraulic conductivity (BRAESTER et all, 1971; BRAESTER, 1973). In short, it is very unlikely to get any runoff and surface flow on sand dunes. Concentration of water in concave parts of the landscape can serve an explanation.

7. A SUGGESTED EXPLANATION

ZASLAVSKY (1970) introduced the concept of lateral flow in the unsaturated soil which is not caused by boundary conditions but by the soil anisotropy. The anisotropy is caused by soil layering. When the layers are horizontal the main driving force (gravity) is orthogonal to the layers and so is the flow which is straight down. When the layers are at an angle to the horizon the gravity force points downstream from the orthogonol. It will therefore cause a horizontal flux component.

It was therefore reasonable to assume (and in fact later to prove) that at least under steady state the average horizontal flux $\overline{q_h}$ is proportional to the vertical average flux $\overline{q_v}$ to the slope tan α and to a coefficient of anisotropy U.

$$\overline{q_h} = \overline{q_v}\, U \tan\alpha \qquad (4)$$

This simple notion leads to many possible explanations. In a concave landscape the incoming slope is larger than the outcoming one. Therefore the horizontal incoming flux is higher than the outcoming one. In other words, there will be moisture accumulation in concave parts. In mathematical terms in two dimensional problems (z vertical, x horizontal)

$$\tan\alpha = -\frac{\partial z}{\partial x} \qquad (5)$$

$$-\frac{\partial q_h}{\partial x} = \frac{\partial c}{\partial t} + q_v{}^* = \overline{q_v}\, U - \frac{\partial^2 z}{\partial x^2} \qquad (6)$$

assuming $\overline{q_v}$ and U not to be functions of x as a first approximation and c the moisture content and $q_v{}^*$ outcoming water by more vertical flux. In short, any concave part of the landscape $(\partial^2 z/\partial x^2) > 0$ leads to higher moisture contents and a higher share of vertical infiltration. With more water there is more development of B horizon. Furthermore, on convex parts of the landscape there is a lack of moisture and smaller vertical flow $(\partial^2 z/\partial x^2 < 0)$. In fact, as the B horizon develops the anisotropy U develops and $q_v{}^*$ is negative more and more. The development of the B horizon in convex parts of the landscape stops of its own accord. This new explanation is interesting as it also interprets the fainter B horizon on straight and convex slopes in genetically mature soil catenas. This is without the questionable crutch of erosion and runoff theories.

The latteral flow component and moisture accumulation in concave parts of the slope could explain the Beer Sheva experiment. Furthermore if proven correct and of proper magnitude it could explain saturation in some parts of the field, seepage out of the soil and the formation of overland flow. The partial area contribution would get a new meaning. Any rain, falling on areas with surface seepage will not infiltrate into the ground. This would not be because of the limited infiltration capacity, the value of which is totally irrelevant in this case.

If the rain can get into the ground (at least to a shallow depth) before turning into overland flow then it has a different effect on leaching. The longer term accumulation of rain water may have now an effect on runoff.

If the rain gets first into the ground and then seeps out in concave parts then it can explain field erosion by seepage forces. Furthermore various depressions in the soil surface are often a starting point of erosion. This is due to the high local accumulation of moisture and concentration of streamlines that produce high seepage forces.

Road cuts truncate the soil layering. The latteral flow reaches the soil surface but cannot seep out as the soil is unsaturated. The streamlines bend down and accumulate until seepage is formed (usually followed by erosion).

Concentration of rain water in concave points can explain net recharge in some areas of limited rain. This is a natural form of 'water harvesting' where certain parts of the landscape obtain several times more rain water than the average. This is totally contrary to the partial area contribution theories that stipulate that these parts are excluding most of the rain water to form runoff. Evidently the latteral flow concept is more likely to be of physical significance.

It is suggested that every soil, without any exception, has a more permeable layer at its surface. This by itself will produce a latteral flow component. It is also suggested that splashing rain-drops will produce a real latteral flow component, very much like in eq. (4). However, it will not produce seepage of water coming out of the soil.

If eq. (4) is proven to be physically sound then it has another fringe benefit in bookkeeping. Certain errors in measuring the rain $\overline{q_v}$ will produce only the same relative errors in the horizontal flow component. There is no amplification of the relative error because of the smaller value of the runoff relative to the rain and the infiltration as in eq. (1). In fact eq. (4) assumes no such things as runoff and infiltration. Every drop of rain may be supposed to be at the soil surface and not above it or below it.

8. FORMATION OF GULLIES AND RILLS BY WATER EROSION (ZASLAVSKY, 1970)

The formation of gullies and rills is evident in areas of little runoff. In fact, the evolvement is mostly at their upper tip where the quantity of the overland flow is the smallest. The most baffling observation is that backward advance of erosion channels is by undermining that seems to be due to water that comes out of the soil. Such undermining is followed by caving in and then by a gradual transportation by overland flow.

The explanation of this and other erosion phenomena depends on the introduction of two processes:

a) a mechanism by which outcoming water erode the soil
b) a mechanism by which water comes out of the soil

The first mechanism is undoubtably that of seepage forces. At sharp concave points very high hydraulic gradients can be formed due to convergence of streamlines. The drag forces enacted by the outflowing water can then detach soil particles overcoming even high cohesion.

The first mechanism of seepage forces is not possible without water coming out of the soil. When water comes out of the soil it must be at positive pressures (at least somewhat higher than atmospheric). At positive pressures, the soil must be saturated or nearly saturated.

In people's mind saturated soil is related to one of the two cases:

a) high water table or perched water table above an impermeable layer
b) overland flow that forms when the rain exceeds the soil infiltration capacity

Our enigma was how can water outflow be formed where there is no water table or perched water table and where the rain does not exceed the infiltration capacity.

A suggestion has been made that there is a latteral flow component that can occur at any rain and in unsaturated soil. This horizontal flow is within the soil and adjacent to the soil surface. It causes moisture accumulation at concave parts of the landscape. It is possible that such a moisture accumulation can reach even saturation. Saturation can be followed by outflow from the soil, by erosion and runoff.

A badly gullied valley around Nahal Bohu in the Israeli Negev was made a subject to a soil conservation and reforestation. In the preparation, two air pictures taken 20 years apart have been compared. The tips of some of the gullies were advancing at an average rate of some 1 meter per year, invariably by undermining of a tunnel followed by a caving in. The advancing gullies were almost invariably at concave parts of the landscape where the topography is amfitheaterlike. The measures suggested against further erosion were underground drainage flow barriers and filters. Although there was no obvious water table or another zone of saturated soil, such drains would let out water.

9. MORE OBSERVATIONS AND THE STRAW ROOF

After presentation of some of these ideas in an experiment station in Ohio (USA) (1968) the senior author has been shown an amfitheater-like drainage basin with a spring at its mouth. There was no obvious impermeable layer. Measurements did not indicate saturated flow around or below the small area that was seeping out. The seepage continued long after the rain. Since then many such places have been observed with evidences on erosion and seepage in agricultural field and in roadcuts.

The straw roof story is probably best to shake up some of the older concepts and look for a better one. An 'expert' would have measured its infiltration capacity, and found it too high to serve as a roof. Despite the expert's opinion no rain gets through the roof within the building's area. Every drop of rain comes off but not a single drop runs above the roof as an 'overland flow'. This case, though extreme, indicates some of the limitations of present day surface hydrology. No builder in his right mind would make a straw roof flat, the effect is related to the slope and probably to the anisotropic nature of the medium.

10. CONCLUSION

In the future chapters the detailed unsaturated flow regime near the soil surface will be studied. First we shall study splashing raindrops, then the transition layer of the soil surface and finally the layered soil. The theory of erosion and its application will also be elaborated. The report will include theoretical as well as some preliminary experimental evaluations.

The report is based on the two notions:

a) that latteral flow component is formed by rain near the soil surface and it accumulates in concave parts of the landscape;
b) seepage forces are a major cause of field erosion. For seepage forces rainwater must come out of the soil.

11. REFERENCES

ABBOTT, M.B., A.F. ASHAMALLA and G.S. RODENHUIS. 1972. On the numerical computation of stratified groundwater flow. Bulletin of the Intern. Assoc. of Hydrol. Sci., Vol. 17, no. 17: 177-182.

AMERMAN, C.R. 1973. Hydrology and Soil Science in field soil water regime. SSSA special publ. no. 5: 167-180.

AMERMAN, C.R., A. KLUTE, R.W. SKAGGS and R.E. SMITH. 1975. Soil water. Reviews of Geophysics and Space physics. Vol. 13, no. 5: 451-454.

BARNES, B.S. 1939. Structure of discharge recession curves. Trans. Amer. Geophys. Union 20: 721-725.

BRAESTER, C., D. ZASLAVSKY, S.P. NEUMAN and G. DAGAN. 1971. A survey of the equations and solutions of unsaturated flow in porous media. First annual report, part 1, project no. A10-SWC-77. Hydraulic Eng. Lab., Technion, Haifa.

BRAESTER, U. 1973. Moisture variation of the soil surface and the advance of the wetting front during infiltration of constant flux. Water Resour. Res., vol. 9, no. 3: 687-694.

BURAS, N. 1972. Syntific allocation of water resources. American Elsevier, New York.

CHOW, VEN TE. 1964. Handbook of applied hydrology. Chow Ven Te, edt. McGraw Hill Book Co., New York.

CHOW, VEN TE and S. RASASESHAN. 1965. Sequential generation of rainfall and runoff data. Jour. Hydraul. Div. ASCE, HY4: 205-223.

CLARKE, R.T. 1973. Mathematical models in hydrology. Irrigation and drainage paper, no. 19. FAO, Rome.

CRAWFORD, N.H. and R.K. LINSLEY. 1962. The synthesis of continuous streamflow hydrographs on a digital computer. Stanford Univ., Dept. Civil Eng., Techn. Rep. 12.

CRAWFORD, N.H. and R.K. LINSLEY. 1966. Digital simulation in hydrology. Stanford watershed model IV. Stanford Univ., Dept. Civil Eng., Techn. Rep. 39.

DISKIN, M.H. 1970. Definition and uses of the linear regression model. Water Resour. Res., vol. 6: 1668-1673.

FREEZE, R.A. 1967. The continuity between groundwater flow systems and flow in the unsaturated zone. In: Proc. of Hydrol. Symp. no. 6, held at Univ. of Saskatchewan on November 15 and 16, 1967: 205-240.

FREEZE, R.A. 1969. The mechanism of natural groundwater recharge and discharge. 1. One-dimensional, vertical unsteady, unsaturated flow above a recharging or discharging groundwater flow system. Water Resour. Res., vol. 5, no. 1: 153-171.

FREEZE, R.A. 1971. Three-dimensional, transient, saturated-unsaturated flow in a groundwater basin. Water Resour. Res., vol. 7, no. 2: 347-366.

FREEZE, R.A. 1972. Role of subsurface flow in generating surface runoff. 1. Upstream source areas. Water Resour. Res., vol. 8, no. 5: 1272-1283.

FREEZE, R.A. 1972. Role of subsruface flow in generating surface runoff. 1. Base flow contributions to channel flow. Water Resour. Res., vol. 8, no. 3: 609-623.

FREEZE, R.A. 1974. Streamflow generation; reviews of geophysics and space physics. Vol. 12, no. 4: 627-647.

FREEZE, R.A. Simulation of subsurface flow in watershed models. IBM seminar on regional groundwater hydrology and modelling, Venice, Italy.

GRACE, R.A. and P.S. EAGLESON. 1967. A model for generating synthetic sequences of short-time interval rainfall depths. Proc. Intern. Hydrol. Symp. (Fort Collins, Color.) 1: 268-276.

HORTON, R.E. 1935. Surface runoff phenomena, analysis of the hydrograph. Horton Hydrol. Lab. Publ. 101. 73 p.

HORTON, R.E. 1940. An approach toward a physical interpretation of infiltration capacity. Proc. Soil Sci. Soc. Amer., vol. 5: 399-417.

IZZARD, C.S. 1946. Hydraulics of runoff from developed surfaces. Highway Res. Board Proc., vol. 26.

JAMES, L.D. 1970. An evaluation of relationships between streamflow patterns and watershed characteristics through use of OPSET. A self calibrating version of the Stanford waters shed model. Univ. Kentucky Water Resour. Inst. Res. Rep., no. 36.

KASSIFF, G., D. ZASLAVSKY and J.G. ZEITLEN. 1965. Analysis of filter requirements for compacted clays. Proceedings of the 6th ICSMFE Division, 6, Montreal: 495-499.

KISIEL, C.C. 1969. Time series analysis of hydrologic data. In: Advances in hydroscience. Academic Press.

LINSLEY, R.K. JR., M.A. KOHLER and J.L.H. PAULHUS. 1958. Hydrology for Engineers. MacGraw Hill, New York.

LOWDERMILK, W.D. 1926. Forest destruction and slope denudation in the Province of Shansi. China J., vol. 4: 127-135.

MATALAS, N.C. 1967. Mathematical assessment of synthetic hydrology. Water Resour. Res., vol. 3: 937-945.

MEINZER, O.E. 1923. Outline of groundwater hydrology, with definitions. U.S. Geol. Survey (Water supply paper 494). 71 p.

MEINZER, O.E. 1942. Hydrology. Dover Publ., New York. 712 p.

ROUSE, H. 1950. Engineering hydraulics. Wiley, New York. 1039 p.

SCHREIBER, H.A. and D.R. KINCAID. 1967. Regression models for predicting on-site runoff from short duration convective storms. Water Resour. Res., vol. 3: 389-395.

SEGINER, I. and J. MORIN. 1970. A model of surface crusting and infiltration of bare soils. Water Resour. Res., vol. 6, no. 2: 629-633.

SHERMAN, L.K. 1932. Streamflow from rainfall by unit-graph method. Eng. News. Rec., no. 01: 501-505.

SINAI, G. and D. ZASLAVSKY. 1974a. The effect of lateral flows on the yields. Results of an experiment at Be'er Sheva. Paper presented at the ISAE meeting.

SINAI, G., P. GOLANY and D. ZASLAVSKY. 1974b. Influence of anisotropy in soil permeability on surface runoff. Publ. 232 Faculty of Agric. Eng., Technion, Haifa.

VISSER, W.C. 1974. The aim of modern hydrology. Techn. Bull. 90, Institute for Land and Water Management Research, Wageningen.

ZASLAVSKY, D. and G. KASSIFF. 1965. Theoretical formulation of piping mechanism in cohesive soils. Geotechnique, vol. XV.3: 305-316.

ZASLAVSKY, D. and A.S. ROGOWSKI. 19 . Hydrologic and morphologic implications of anisotropy and infiltration in soil profile development. Soil Sci. Soc. Amer. Proc., vol. 33, no. 4: 594-599.

ZASLAVSKY, D. 1970. Some aspects of watershed hydrology. Special report to the US Dept. of Agriculture, Agric. Res. Serv., ARS.

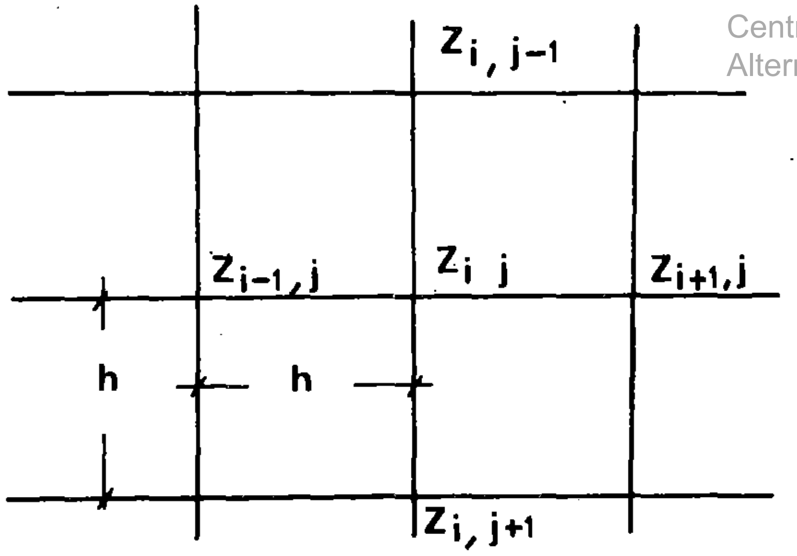

Fig. I.1. A two-dimensional grid for geodetically measured soil surface elevation Z_{ij}

SOIL MOISTUR CONTENT AGAINST SOIL SURFACE CONCAVITY

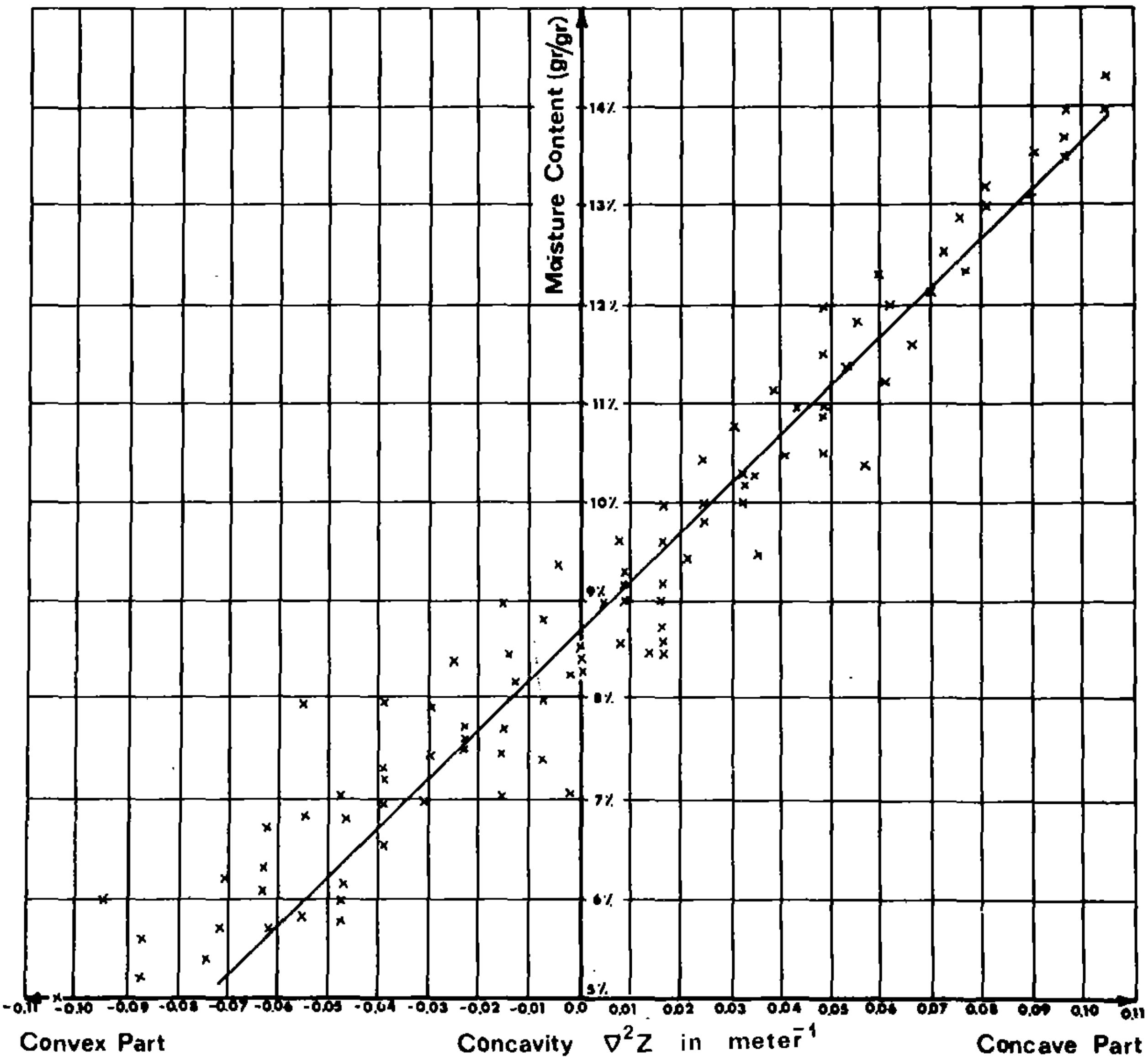

Fig. I.2. Correlation of moisture content at 20 and 40 cm depth 10 days after rain with soil surface curvature

II. LATERAL FLOW DUE TO RAIN DROP SPLASHES

ABSTRACT

Several phenomena of surface hydrology could be explained by stipulating lateral horizontal flow proportioned to the rain itself and to the soil surface slope. Here this mechanism is shown to exist as a result of raindrop splashing. Both theory and measurements prove it. As a result it is anticipated to have excess rain in parts of the landscape, proportioned to their concavity. The effective rain in concave parts of the landscape can reach several times the average rain up in the air. This concavity can be measured geodetically. It is roughly the local slope divided by the surface drainage density. Erosion by splashing of soil material is also dependent on the same mechanism and could be calculated there from.

1. WHY DO RAINDROP SPLASHES PRODUCE REAL LATERAL FLOW

If the soil is sloping the splashes downhill will travel further away than uphill. The center of gravity of the original raindrop will be found downhill of the first hitting point. This means that given a certain distribution of rain intensity at some higher horizontal surface the eventual 'effective' rain distribution on the soil surface will be a result of a downhill translation. The horizontal discharge amounts to the rate of rain times this horizontal change in the center of gravity of the raindrops. An observer watching the splashes passing will count more passing downhill than uphill. The net difference is a very real net lateral flow. At least for moderate slopes we may stipulate that the bias downhill increases with slope. Over a long and uniform slope the result of the horizontal flow will not be recognised. The final rain distribution will be unchanged. At the top, at the bottom and at any point of change in the slope the effects of the horizontal flow will be felt. In the following we shall present three more or less independent parts. A. An experimental evaluation of the net downhill flow as a function of slope. B. Demonstration of moisture concentration at the bottom of slopes. C. A theory that attempts a prediction of lateral flow due to raindrop splashes.

2. A MODEL FOR RAINDROP SPLASHES

A raindrop when hitting a water surface produces a crown of splashes (fig. 1). This phenomenon was investigated by many but especially by MUTCHLER (1967, 1971). They come out at a fixed angle β with the horizon. The size and distance of splash flight has been found to be symmetrical to the initial flight path. However the experiments were of vertical flight only. Splashes occur from a non saturated soil as well.

fig. 1

We adopted first the convenient assumption that the exit angle is uniform around the drop and that the exit velocity V_o is uniform on the average. Notably both the final solution and conclusions are not sensitive to some deviation from these assumptions.

A single splinter of initial velocity V_o of angle β will be assumed to describe a parabolic path (with no air resistance and over a flat gravity field). The components of velocity are then

$$V_x = V_o \cos \beta \tag{1}$$

$$V_z = V_o \sin \beta \tag{2}$$

The equation of flight path is

$$z = x \tan \beta - \frac{g x^2}{2V_o^2 \operatorname{Cos}^2 \beta} \tag{3}$$

where:

z - the vertical coordinate positive upward

$z = 0$ at the intial hitting points

x - horizontal coordinate in the plane of splashflight

$x = 0$ at the initial hitting point of the raindrop

g - gravity acceleration

β, V_o - angle with the horizon and value of the exist velocity

x_d and x_u will be the hitting points of splinters downstream and upstream respectively (fig. 2). If the slope angle is α

$$x_d = \frac{2}{g} V_o^2 \cos^2\beta \, (\tan\beta + \tan\alpha) \tag{4}$$

$$X_u = \frac{2}{g} V_o^2 \cos^2\beta \, (\tan\beta - \tan\alpha) \tag{5}$$

fig. 2 Consider now a three dimensional picture: (x, y horizontal coordinates and z upward vertical coordinates with the origin at the hitting point). A mass m of a raindrop becomes a mass of splashes $m' = \Sigma m_i$. A single splinter forms an angle Θ_i with the vector of slope $\tan\alpha$ (fig. 3). The range of splashing r_i of a mass m_i is simply obtained from (4) and (5) by adjusting the slope ($\tan\alpha$) to ($\tan\alpha \cos\Theta_i$)

$$r_i = \frac{2}{g} V_o^2 \cos^2\beta \, (\tan\beta + \tan\alpha \cos\Theta_i) \tag{6}$$

where:

Θ_i - the angle between the horizontal projections of the slope and the splinter flight

V_o, β - as before the speed and the angle of the exit velocity vector

$\tan\alpha$ - slope of the soil

Each splinter at an angle Θ_i then has on the average conjugate at $\Theta_i + \pi$. The difference in range is:

$$\Delta_i = r_{(\Theta_i)} - r_{(\Theta_i + \pi)} \tag{7}$$

or explicitly

$$\Delta r_i = \frac{4}{g} V_o^2 \cos^2 \beta \tan \alpha \cos \Theta_i \qquad (8)$$

The component in the direction of slope (x axis) is:

$$\Delta x_i = \Delta r_i \cos \Theta_i = \frac{4}{g} V_o^2 \cos^2\beta \ \cos^2 \Theta_i \tan \alpha \qquad (9)$$

The average translation of the center of gravity is defined by

$$\overline{\Delta x_i} = \frac{\Sigma m_i' \ x_i/2}{m'} \qquad (10)$$

fig. 3

remembering that x_i has been calculated for a pair of masses that m_i represents the mass of a splinter while m is the original and m' the total mass of splinters of the raindrop. The explicit translation according to the above assumption is obtained by substituting (9) into 10.

$$\Delta\bar{x}' = \frac{2}{m'g} V_o^2 \cos^2\beta \tan \alpha \sum_{i=1}^{n} m_i' \cos^2\Theta_i \qquad (11)$$

For many drops (in time and space) we pass in the limes to the integral assuming equal probability for all angles Θ_i

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \cos^2\Theta_i = \frac{1}{\pi} \int_{\Theta=0}^{\pi} \cos^2\Theta\alpha\Theta \qquad (12)$$

$$\Delta\bar{x}' = \frac{2}{g} V_o^2 \cos^2\beta \tan \alpha \frac{1}{\pi} \int \cos^2\Theta \frac{m'(\Theta)}{m} d\Theta \qquad (13)$$

Note that the center of gravity change has been calculated for the mass of splinters m' and not for the mass of the rain.
The solution of (13) is

$$\Delta\bar{x}' = \frac{1}{g} V_o^2 \cos^2\beta \tan \alpha \frac{\Sigma m_i'}{m'} \qquad (14)$$

In words, the translation of the center of gravity of the raindrop due to splashing is proportional to the specific kinetic energy of the splashes $V_o^2/2g$ or to the maximum possible hight to which these splashes can jump. This kinetic energy is probably related in some manner to that of the original downdrop velocity less some losses due to friction due to pick up of soil particles and the production of new water surfaces. Consideration of the momentum conservation requires the following two equations to be fulfilled (For a raindrop which falls vertically)

$$\Sigma m_i' V_{xi} \sin \Theta_i = \Sigma m_i' V_{oi} \cos \beta_i \sin \Theta_i = 0 \qquad (15)$$

$$\Sigma m_i' V_{xi} \cos \Theta_i = \Sigma m_i' V_{oi} \cos \beta_i \cos \Theta_i = 0$$

where V_{xi} is the horizontal velocity component in direction Θ of splinter i, with a mass m_i'. In passing to the limes of many drops there may be maintained a symmetry of the horizontal momentum with respect to two orthogonal lines. Continuity in the function of Θ and the requirement that any distribution would have the slope and a direction normal to it as principal axes leave a very small number of possibilities with respect to velocity and mass distribution of the splinters (or splashes) around the first hit of the raindrop. The derivation above certainly fulfills the eq. (15). A more rigorous derivation of (14) will assume in eq. (9) a V_o and β varying from one splinter to another just as Θ_i and m_i'. Furthermore on averaging for many drops one can include also tan α_i. The averaging or summation should read then

$$\Delta \bar{x}_i' = \frac{2}{m'g} \Sigma m_i' V_{oi}^2 \cos^2\beta_i \cos^2\Theta_i \tan \alpha_i \qquad (16)$$

This will only amount to the assignment of average values to all terms in eq. (14).

3. THE LATERAL FLOW DUE TO SPLASHING

A single drop provokes on the average a mass m' with a translation $\overline{\Delta x}'$. The lateral flow can be found by counting the number of drops passing through a vertical control surface. Clearly this is the amount of splinters per unit time, times the distance $(\overline{\Delta x})$. This is the distance upstream over which drops fall and can still pass through the control surface.

The horizontal flow Q_x is then simply

$$Q_x = \overline{\Delta x}\, P = \overline{\Delta x}' P \frac{\Sigma m_i'}{\Sigma m_i} \qquad (17)$$

$$\overline{\Delta x} = \overline{\Delta x}' \frac{\Sigma m_i'}{\Sigma m_i} = \varepsilon\, \overline{\Delta x}'$$

where $\overline{\Delta x}'$ is the time and area average translation, P the rate of rain and $\Sigma m_i'/\Sigma m_i = \varepsilon$ is the ratio between the splashing mass and the original mass of rain and $\overline{\Delta x}$ is the weighted equivalent translation of rain drops.

Clearly $\overline{\Delta x}$ depends on the slope (eq. 14) on the rain energy, on the type of soil, but also on the total rain and the intensity itself as they determine the wetness conditions at the soil surface. The ratio between splinters mass and rain mass may be changed from zero to more than a unity. In horizontal soil $\overline{\Delta x}$ will vanish. It is probably monotonic with the slope, at least on small slopes.

In the expression for Q_x in (17) let us introduce the explicit values of $\overline{\Delta x}'$ as in (10)

$$Q_x = P\varepsilon \frac{\Sigma m_i' \, \Delta x_{i/2}}{\Sigma m_i'} \qquad (18)$$

and writing the values for Δx_i from (9)

$$Q_x = P \tan\alpha \, \frac{2}{g}\, \varepsilon \left[\frac{\Sigma \frac{1}{2}\, m_i' \, V_{oi}^2 \, \cos^2\beta_i}{\Sigma m_i'}\right] \qquad (19)$$

The expression in brackets is proportional to the kinetic energy due to the horizontal component of the exit velocity per unit width of the slope

$$\tfrac{1}{2} V_o^2 \cos^2\beta = \frac{\Sigma \tfrac{1}{2} m_i' V_o^2 \cos^2\beta_i}{\Sigma m_i'} = \frac{\Sigma \tfrac{1}{2} m_i' V_o^2 \sin^2\beta / \tan^2\beta_i}{\Sigma m_i'} \qquad (20)$$

We may introduce a new parameter expressing the maximum hight of the flight trajectory of a single splinter

$$\delta_i = \frac{1}{2g} V_{oi}^2 \sin^2\beta_i \qquad (21)$$

where V_{oi} and β_i are the speed and angle of the exit velocity vector. Eq. 19 now reads

$$Q_x = P \tan\alpha \; 2\varepsilon \; \bar{\delta}/\tan^2\beta \qquad (22)$$

$$\frac{\bar{\delta}}{\tan^2\beta} = \frac{\Sigma m_i' \delta_i / \tan^2\beta_i}{\Sigma m_i}$$

The parameter $\bar{\delta}$ may be estimated from measurements. Looking at the muddy staining of walls we can observe qualitatively the hight. It is typical for the soil and rain and gives us an order of magnitude of the jumps. A more accurate estimate may be found by the measurement of drop density in the air or by sponge paper stained with methyl blue. From the area of the stained blue one can estimate the mass of the drops at every hight.

Introducing the jump hight into the above formulas one finds:

$$Q_x = P\overline{\Delta x} = P \tan U\bar{\delta} \qquad (23)$$

$$U = \frac{2\varepsilon}{\tan^2\beta}$$

$$\overline{\Delta x} = \tan\alpha \, . \, U\bar{\delta}$$

Equation 23 is similar to the fundamental stipulation in the previous part of this report (Zaslavsky Sinai 1978) eq. (4). The rain is the rate of vertical flow. Due to the extremely short delay and small storage of the splashing phenomenon eq. (23) may be considered a quasi steady state. The parameters $\bar{\delta}$ and U are due to the rain energy and soil condition. They remind in their form a layer thickness and coefficient of anisotropy.

Preliminary Measurements of lateral flow

The first quantitative experiment was made with impermeable and relatively smooth surfaces at various slopes. Rain was provided in a raintower of 17 meter hight where the drop flight is on the average vertical. The rain is reasonably uniformly distributed in time and space and the flight velocity is very near the end velocities in air. Measurements have been made of the actual splash distribution over a distance upstream and downstream of an edge of a wide slope. Still a considerable amount of splashes fell off the sides of the slope. Thus the absolute values of lateral flow quoted here are on the low side and can be in reality at least 20 - 30% higher.
From the measurements actual mass moments could be deduced and change in center of gravity could be calculated. However, a simpler check could be made by comparing the net downward splash discharge from the edge of the slope with the net upstream splash discharge across the upper edge of the slope. The difference is simply the lateral discharge Q_x. The results are given in fig. 4 as a function of the slope. They are given in terms of eqs. (17) or (23) where $\Delta\bar{x}$ can be calculated from the measured discharge Q_x and the rate of rain P.

fig. 4

The conclusions from the measurements are:

a. There is a considerable lateral flow due to raindrop splashing
b. Within the range of our experiment the lateral flow due to raindrop splashing increases monotonically with the slope. Furthermore up to some 20% slope it increases linearly with the slope.

One can convince himself about the order of magnitude of $\overline{\Delta x}$ by observing the hight of raindrop splashes on vertical walls. Typically it reaches 30 cm hight. In equation (23) this can be taken for $\overline{\delta}$. A typical value for β is $45^o - 50^o$ so that $\tan^2\beta$ = 1 - 1.4. Let us assume also $2\varepsilon \sim 2$ we have then $U \simeq 1.4 - 2$.

$$\text{so that} \quad \overline{\Delta x} \simeq 40 - 60 \tan\alpha \quad \text{in cm} \tag{24}$$

From the experiment reported above one gets (up to a slope of 20%)

$$\overline{\Delta x} \simeq 66 \tan\alpha \quad \text{in cm} \tag{25}$$

so that either the existing angle of the splashes β is somewhat smaller or the associated mass thrown up by the raindrops splashing is larger so that $\varepsilon > 1$. or both. This figure will probably vary with the roughness, aggregate strength of the soil and the soil moisture content. Similarly it may change with the rain intensity not only as a result of changing the moisture regim at the soil surface but also through the increase in raindrop specific kinetic energy which is associated with increased rain intensity (an actual increase in the final raindrop velocity).

A possible ratio between lateral flow and rain

On a uniform long slope the contribution of a lateral flow may be negligible. It is a constant that does not depend on the slopes's length. Consider a rain discharge Q_p over a slope of unit width and length L:

$$Q_p = P.L \tag{26}$$

The horizontal splash discharge is Q_x from eq. (17) or (23).

The ratio of lateral discharge to the rain discharge is then

$$\frac{Q_x}{Q_p} = \frac{\Delta\bar{x}}{L} = \frac{U\bar{\delta}\tan\alpha}{L} \qquad (27)$$

$\Delta\bar{x}$ in the above experiment was found to be approximately (66 tan α) in centimetres. The ratio then depends on the slope steepness and length. Consider rills gullies and depressions every 2 metres so that a typical value of the slope length is L ≃ 1 meter and a slope not exceeding 10%. $\bar{x}/L$ can reach a value of 7% i.e. 7% of the rain flows towards the depression due to splashes only. This is a considerable amount of lateral flow.

The term (tan α)/L has a significant physical meaning. Clearly the concentration of rain in concave places will be proportional to this ratio. It is the slope times the drainage density of the landscape. Geometrically it is the curvature of the landscape. It will be simpler to understand it by considering a simple model of soil surface as a sinusoidal shape where the elevation z is

$$z = \bar{Z} + \frac{A}{2}\sin\left(\pi\frac{x}{L}\right) \qquad (28)$$

A/2 is the amplitude of the sine wave and L is half cycle length. The first derivative of z is the slope. It's maximum value is $\frac{\pi^2}{2}\frac{A}{L}$ and its average is $\bar{s} = A/L$. The curvature is estimated by the second derivative. It's maximum value is $\frac{\pi^2}{2} \cdot \frac{A}{L}{}_2$. In terms of the average slope $\bar{s}$ it is $\frac{\pi^2}{2}(\bar{s}/L)$.

E x c e s s R a i n i n C o n c a v e P l a c e s

In the above it has been proved that the lateral flow due to raindrop splashing is quite significant may be responsible for accumulation of rain in concave points. The expression for excess rain may be obtained more rigorously from eq. 23. Consider tan α the slope to be a vector of two components in the x and y directions z being the elevation.

$$-\tan\alpha = 1_x\frac{\partial z}{\partial x} + 1_y\frac{\partial z}{\partial y} \qquad (29)$$

$\underline{Q}_h$ is then also a vector parallel to tan α (if the soil surface behave isotropically and the rain is vertical).

$$- \underline{Q}_h = (P.U\overline{\delta})(1_x \frac{\partial z}{\partial x} + 1_y \frac{\partial z}{\partial y}) \tag{30}$$

The excess rainwater in a given point can be simply obtained by conservation equation

$$P^* = - \text{div } \underline{Q}_h = (PU\overline{\delta})(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}) \tag{31}$$

Assuming of course that the rain P and the coefficients $U.\overline{\delta}$ (eq. 23) are independent on the coordinates x and y. If not, one should add a term to (31) grad (z).grad $(U.\overline{\delta})$. The chances are that the two vectors are parallel anyhow so that (31) is exact. The total amount of rainwater landing on a soil would then be

$$P_t = P + P^* = P[1 + U\overline{\delta}\ \nabla^2 z] \tag{32}$$

where $\nabla^2 z$ is the curvature that can be measured geodetically (see first article of the series Zaslavsky and Sinai 1978).

It is interesting to note the order of magnitudes of the term in the brackets of (32). It has been shown that the term $U.\overline{\delta}$ can reach a value of 66 cm (at least in the experiment reported above). For slopes of 1 meter length and elevation differences of 0.1 m only, the curvature is of the order of 5×10^{-3} cm^{-1}. Thus the rain excess at such concave points can be 33/100. The effective total can then be nearly 1.5 times the original rain.

To have in some spots a precipitation higher than the average rain is an extremely significant topographic effect from an hydrological and agrotechnical point of view.

The curvature at some points in the field can be very high. Theoretically at a meeting of two plane slopes the curvature (second derivative of elevation) tends to infinity. Does this mean that one should expect there an infinitely higher effective rain water. This problem has not been studied in details. There is certainly a scale effect which depends on typical distances of splash flight and the minimum wavelength to be considerd in the shape of soil. This is in assuming that the soil surface can be described as a Fourier series. Infinit excess rain at a spot of an infinit curvature does not contradict in any way the physical reality as its spreading over an infinitesimal area. We ran experiments with a V shaped 10% symmetrical slope and runoff and erosion appeared almost immediately at the bottom edge. However this phenomenon may be explained by secondary splashing and by the flow in the surface transition layer.

A note about erosion by raindrop splashes

The complete problem of raindrop action on soil structure will not be treated here. It is sufficient for the present discussion to know that the raindrop splashes carry with them soil material which is measurable. Typical figures that have been measured by us had 1 - 10% of splashed soil in the rainwater by weight in a Loess soil. Typically a treatment of the soil surface by a soil conditioner reduced this figure to 0.25 - 0.5% or at least reduced the flight distance. Each mm of rain gives in the example calculated above, of 1 m slope with 10 cm elevation differences, 70 grams of raindrops flowing to the depression. 1 - 10% splashed soil gives 0.7 - 7 grams of soil per square meter splashed towards the depression. This is a considerable amount. It can explain the accumulation of splashed material in soil depressions that one can see almost in every soil after any rain. It can explain considerable erosion if there is an actual runoff coming out of a depression that is capable of carying away the splashed soil.

A typical annual rain of 500 mm is capable of carrying away in the above example 0.3 - 3.5 millimeters of soil and rework more than 10 times this amount.

4. CONCLUSIONS

There is a horizontal flow due to splashing of raindrops on a sloping soil. This splashing has been shown in theory and in some preliminary experiments to be proportioned to the slope (at least for moderate slopes up to 20%). The accumulation of rain in concave parts of the landscape increases with the curvature of the soil surface. The local effective rain intensity can be much higher than the original average rain. This phenomenon involves possible runoff, and increased groundwater recharge that will be discussed in the next part of this report. It can explain and help calculate a range of erosion phenomena by splashing of soil material.

REFERENCES

MUTCHLER, C.K., 1967. Parameters for describing raindrop splash. Soil and water conservation. Vol 22 no. 3.

_____ 1970. Splash of a waterdrop at terminal velocity. Soil Science Vol. 169: 1311-1312

_____ 1971. Splash amounts from waterdrop impact on a smooth surface. Water resources research. Vol. 7, no. 1.

ZASLAVSKY, D. and SINAI, G., 1978. Surface Hydrology I. Field observations that require explanation.

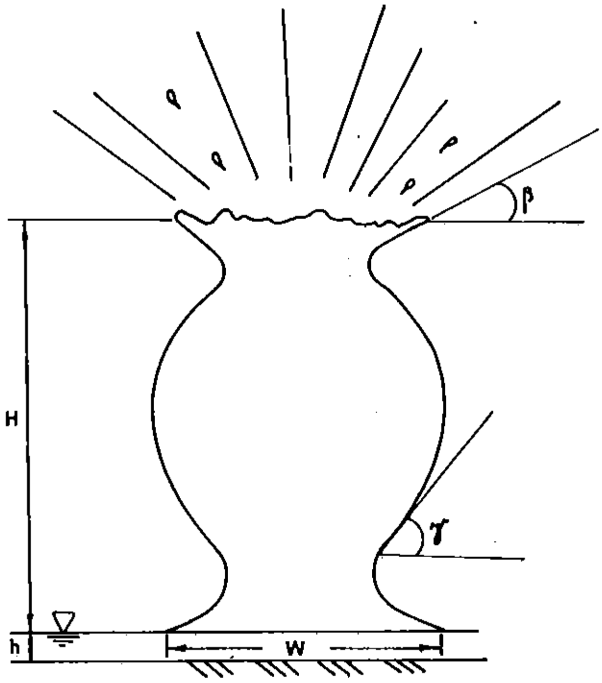

Fig. II.1. Typical splash shape caused by a raindrop hitting a soil covered with a water layer of depth h (after CALVIN and MUTCHLER, 1967)

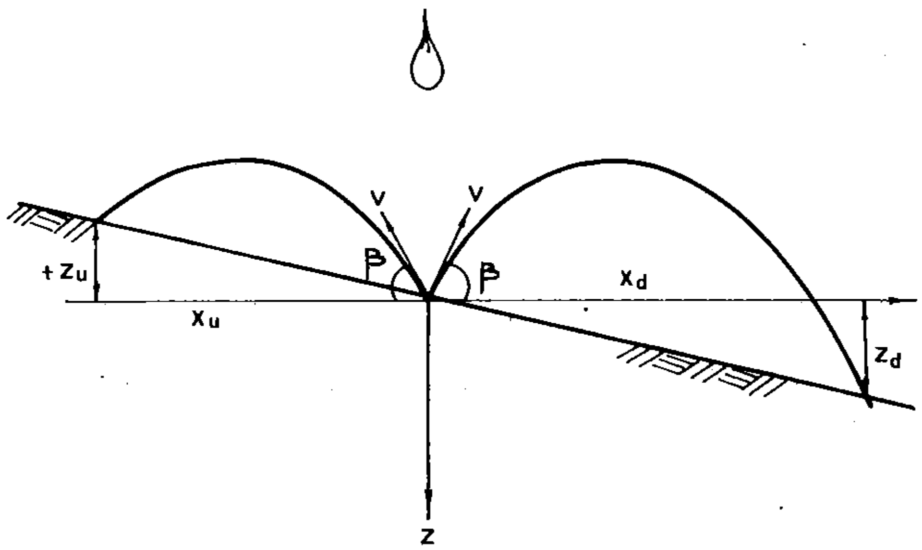

Fig. II.2. Splashes trajectories downhill and uphill

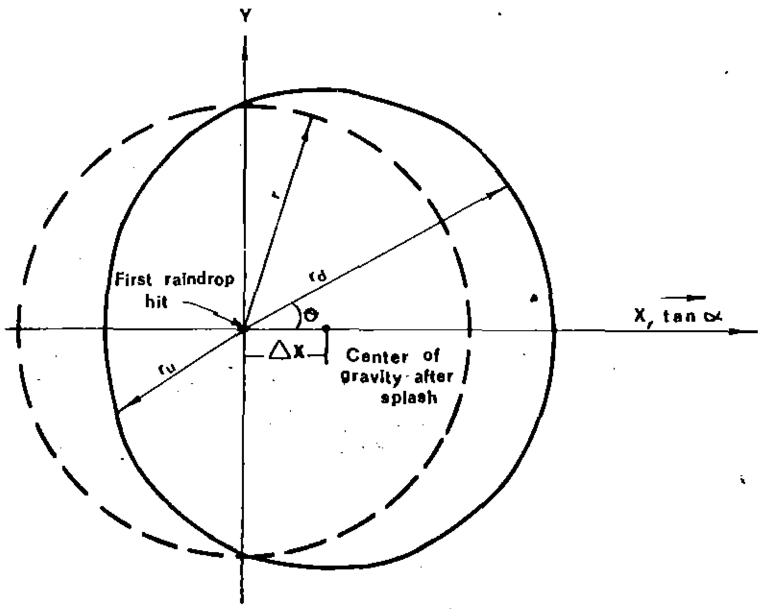

Fig. II.3. Change of Splash Range (r_i) on a slope (plane view). Drawing is for slope α = 20^o exit angle β 50^o

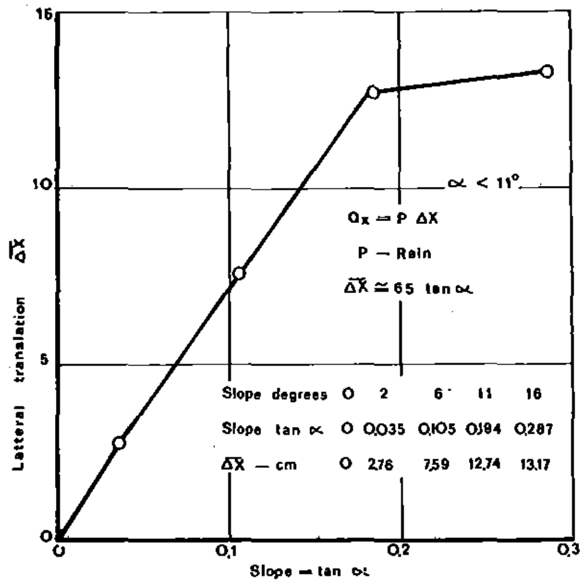

Fig. II.4. Latteral discharge Q_x due to splashing rain of intensity P over an impermeable smooth surface of slope α

III. RAINFALL EFFECTIVE DISTRIBUTION

ABSTRACT

The rain passing through a high horizontal plane is not uniformly distributed in time and space. Slanting of the rain flight causes a further variance in the distribution of the precipitation on the land. This is due to slopes of different aspects relative to the rain flight aspect. Latteral flow due to raindrop splashing cause concentration of rain in concave parts of the landscape. All three sources of fluctuations when averaged produce runoff or net water recharge at very low average rains and in general, non linear relations between them. The soil surface roughness is defined as the mean squar of the local curvature and or of the local slope. They are very important hydrological parameters determining the extent of precipitation distribution variance. The boundaries of a surface drainage basin or watershed has a clear mathematical definition in terms of latteral flow due to raindrop splashing.

1. INTRODUCTION

In the previous part of this report (ZASLAVSKY and SINAI 1978 II) it has been shown that there is a horizontal flow vector $\underline{Qh}$ due to raindrop splashing (in volume per unit time and unit width)

$$\underline{Q_h} = P.U.\overline{\delta}\,\underline{\tan\,\alpha} \tag{1}$$

where P is the rain intensity in length per unit time the slope is a two dimensional vector

$$-\,\underline{\tan\,\alpha} = 1_x \frac{\partial z}{\partial x} + 1_y \frac{\partial z}{\partial y} \tag{2}$$

and $U.\overline{\delta}$ is a quantity that could be formulated theoretically and measured experimentally. In some experiments it has been found

$U\,\overline{\delta} \simeq 70$ cm. There is an excess moisture accumulated in concave places p^* calculated by the divergence of Q_h in (1). It has the same units as the rain itself. The total effective rain in a point is $P_t = P + P^*$ is given by

$$P_t \simeq P(1 + U.\overline{\delta}\,\nabla^2 z) \qquad (3)$$

In the following the phenomenon of rain excess accumulation a. its consequences will be studies.

2. SHARP V SHAPED SLOPES

It has been mentioned that at lower edge of a V shaped slope the curvature tends to infinity. The physical consistency of eq (3) is not disturbed as the area of high curvature must diminish as the degree of curvature increases. An extension of the mechanical theory brought in the previous part of the report gets quite tedious algebraically. However it can show that the rain concentration over a V shaped slope will be continous and will have no tendency for infinit rain excess at infinit curvatures.

However, the actual physics may be different in view of some secondary splashing. The secondary splashing may be of much smaller distances however with large masses of rain. Such repeated splashes should increase the concentration of rain in sharp V shaped slopes.

An experiment was run in such a slope with a rain simulator. Runoff and erosion appeared almost immediately at the sharp edge. However this observation may also be related to other phenomena. One is that of a lateral flow in the surface transition zone that will be discussed in future parts of the report. The other would be a simple surface runoff due to the already increased rain concentration. This subject should be studied further. In the soil surface roughness there are probably wavelengths that are too small to be relevant for raindrop splashing. The distance of splashing can reach about 1 meter and this is probably the order of minimum wavelength which is still relevant to this process.

3. SUMMATION OVER THE FIELD

Eq. (3) may be integrated over the whole field. For maintaining of conservation of rainwater there must be

$$\iint P_t \, dxdy = \iint P \, dxdy + V_o \qquad (4)$$

From equation 3 this means

$$PU.\overline{\delta} \iint \nabla^2 z \, dxdy = V_o \qquad (5)$$

This is easily proven true if V_o is some finite contribution or losses of rainsplashes over the boundaries that become relatively negligible for large enough areas. Equation 5 reads after first integration (fig. 1)

fig. 1

$$P.U.\overline{\delta} \int \left[\left(\frac{\partial z}{\partial x}\right)_2 - \left(\frac{\partial z}{\partial x}\right)_1 \right] dy + \int \left[\left(\frac{\partial z}{\partial y}\right)_4 - \left(\frac{\partial z}{\partial y}\right)_3 \right] dx = V_o \qquad (6)$$

the term

$$P.U.\overline{\delta} \left[\left(\frac{\partial z}{\partial x}\right)_2 - \left(\frac{\partial z}{\partial x}\right)_1 \right] \qquad (7)$$

is the net rain addition by splashing in the x direction. The other defines the net gain in the y direction. The definition of drainage basins is by boundaries along which the slope normal to the boundary is zero. Therefore over surface drainage basin $V_o = 0$. There must be another way to express the curved nature of the field and its impact on hydrological phenomena.

The type of averaging depends on the function involved and the form of its dependence on the rain concentration.

4. OTHER FORMS OF RAIN NON UNIFORMITY

The accumulation of rain in concave places is an important form to cause non uniformity in the final distribution of the precipitation water. There is another form which has been published by ZASLAVSKY (1970). Consider a slanting rain (fig. 2) with an angle β with the vertical over a slope α of the soil and with an azimuth of the slope γ and the azimuth of the rain ω. The effective rain is then (from simple geometrical considerations)

$$P \text{ effective} = P\left[1 + \eta \tan \alpha\right] \tag{8}$$

$$\eta = \tan \beta(\cos \gamma \cos \omega + \sin \gamma \sin \omega) \tag{9}$$

It can be demonstrated that $\eta \tan \alpha$ can easily be ± 1, thus doubling the effective rain on one slope and diminishing it to zero on the other. The extreme of eq. 8 is easily checked when γ and ω have the same value i.e. a two dimensional case where the slope and the rainflight have the same aspect then eq. 8 reads

$$P_{eff} = P\left[1 \pm \tan \alpha \tan \beta\right] \tag{9}$$

When $\alpha = \beta = 45^{o}$ $P_{eff} = 2P$ or zero. For a vertical wall a finite rate of rain accumulates on an infinitesimal point ($\tan \alpha \rightarrow \infty$). It is interesting to combine the two mechanisms of concentration in concave areas (eq. 3) and slanting rain (eq. 9).

A further complication of the behaviour is anticipated due to variations in the rain intensity P itself at a higher level in the air.

5. ANTICIPATED CHANGES IN $\bar{\delta}$ AND U WITH THE RAIN

The formulation of the latteral flow due to splashing remains simple enough if the anisotropy U and the jump hight $\bar{\delta}$ are independent on the rain. However, it is probable that the ratio of splashed mass to the rain, as well as the recoverable kinetic energy will change with the rate of rain. Investigations (VEN TE CHOW 1969, MUTCHLER, 1967) indicate that larger rain drops occur with increased storm intensity. They certainly reach higher final velocities. In a given soil it has been observed that the specific kinetic energy of the splashes is proportional to the original kinetic energy of the rain drops. The product $U.\overline{\delta}$ is expected to be proportional to the recoverable kinetic energy of the splashes. Therefore it is anticipated that the change in $U.\overline{\delta}$ (P) is somewhat like in fig. 3. Therefore the horizontal discharge $\underline{Qh}$ will be related to the rain intensity P by a positive power k:

$$\begin{aligned} &\underline{Qh} \sim (P^{1+K}) \qquad 0 < K < 1 \\ &U\delta(P) \sim P^{K} \end{aligned} \tag{10}$$

fig. 3 This is a correction over eq. (1) where $U.\overline{\delta}$ being considered a constant. No experiment has been run to prove eq. (10).

However it is quite reasonable to stipulate it.

This in turn indicates that on averaging latteral flow due to fluctuations in rain intensity we shall find a net contribution of the intensity variance as well as that of the average rain (over time and space). It means that short and strong and even local bursts of rain can produce strong latteral flows more than proportional to the intensity. Higher intensities will be associated with more extreme concentration in concave places.

6. RAIN CONCENTRATION RUNOFF AND GROUND WATER RECHARGE

The main conclusion of the discussion above is that even under uniform rain in time and space the effective rain in some points on the soil surface can be higher than the average. This is due to slanting rain and due to splashing that produce downhill flow of raindrops.

Furthermore, a non uniform rain in time and space can increase the amplitudes of the fluctuations of the precipitation over the field. Let us assume that at least under some cases the eventual local precipitation can exceed, the infiltration capacity and that the excess will turn into runoff. The same model may apply to ground water recharge. A net recharge may occur only after a certain rain quantity has reached the ground. The simplest mathematical form of the model will be that of a difference equation.

$$R = P - I \tag{11}$$

where R is some net effect (may it be runoff or recharge or crop yield). A common error is to write the average $\overline{R}$ as the difference between the averages of $\overline{P}$ and $\overline{I}$

$$\overline{R} = \overline{P} - \overline{I} \tag{12}$$

The averages $\overline{P}$ and $\overline{I}$ are measured and then $\overline{R}$ is calculated.
In a corrected form we should calculate

$$\overline{R}_{eff} = \overline{P - I} \quad \text{for} \quad (P-I) > 0 \tag{13}$$

In the above it has been demonstrated that P can have large fluctuations between zero and several times P. If P has an expected value P and a variance σ_P^2 and so is I and σ_I^2 then as a first approximation (neglecting higher statistical moments)

$$\overline{R}_{eff} = \overline{R} + f(\overline{R}\ \sigma_P^2,\ \sigma_I^2,\ \sigma_{PI}) \tag{14}$$

where σ_{PI} is the correlation between fluctuations in P and I. Most often such a correlation will exist. For example if I is surface retention and R is either runoff or water recharge then there is a positive correlation. Surface crust forms as a result of rain accumulation and especially in concave places. Thus if I is infiltration there is a negative correlation with P as far as runoff is concerned.

A case is possible where $\overline{R} = 0$ i.e. the average rain is equal the average infiltration (plus retention etc.). Runoff may nevertheless occur due to local concentration of rain as expressed by the statistical terms of 14. Similarly very slight rains may produce net water recharge in some concave places of the landscape due to latteral flow. ZASLAVSKY (1970) calculates the eq. (14) assuming a normal distribution for both P and I. It is possible to check that the result is

$$\overline{R}_{eff} = \overline{R} + \frac{\sigma\overline{R}}{\sqrt{2\pi}} \exp(- \overline{R}^2/2\sigma_R^2) - \frac{\overline{R}}{2} \operatorname{erfc} \left(\frac{\overline{R}}{\sigma_R\sqrt{2}}\right) \qquad (15)$$

$$\sigma_R^2 = \sigma_P^2 + \sigma_I^2 - 2\, \sigma_{PI}\, \sigma_P\, \sigma_{\underline{T}} \qquad (16)$$

$$\overline{R} = \overline{P} - \overline{I}$$

It is not necessarily true that P and I have a normal distribution. It is even clear that P and I cannot have a perfectly normal distribution at least for negative values of P and I. However, other distributions while being mathematically more complicated, will produce qualitatively similar conclusions. It is interesting to learn about the shape of eq. 15 with different values of $\overline{P}$ and σ_R^2. For $\overline{R} = 0$.

$$\overline{R}_{eff} = \frac{\sigma_R}{\sqrt{2\pi}} \qquad (17)$$

As P fluctuations can be of the same order of magnitude as the average value of P and larger then σ_R can be about P/2 and even more and the runoff or net recharge can still be a significant part of the rain (¼ and more). This is totally due to local concentration of rain water by latteral flow.

At very high values of P (and $\overline{R}$) the second term in (15) vanishes and $\overline{R}_{eff}$ increases proportionally to R. This is true only if the fluctuations remain unchanged. However, it has been found that the fluctuations in effective P increase with its average (eqs. 3,8). Therefore the terms $\overline{R}^2/2\sigma_R^2$ and $\overline{R}K_R$ in eq. 15 do not increase only so much as I remains a constant. At higher values of R the ratio will tend to become a constant. Thus in eq. (15) probably all three terms increase in a similar fashion. Contrary to what ZASLAVSKY (1970) suggests the term of local concentration of precipitation does not become negiligible with increased rain intensity. It is even possible that the effect of the fluctuation increases (e.g. in view of eq. 10).

A most interesting conclusion may be drawn here. The simplest lineair equation like (13) has produced on averaging a non linear relation like (15). This is due to a combination of fluctuations and a treshold I. Equation (15) reminds more closely experimental relations between rain and runoff that are far from linear with $\overline{R}$ (ZASLAVSKY, 1970).

7. THE FIELD SLOPE AND ROUGHNESS

In most hydrologic models the field slope is considered to be an important feature. It has been proved so far, at least as far as the raindrop splashing is concerned, that the local curvature is just as an important entity.

The specific form of averaging depends on the explicit formulation of the end function. As an example the oldest and most common hydrological model is examplified in eq. (12) and somewhat corrected in eq. (13). A specific statistical distribution is assumed in eq. (15). The variance of P has been related to variations in slope (8) or variations in curvature (3).

Let us calculate the variance of P, σ_P^2 from equation (3) assuming constant U and $\overline{\delta}$ (only concentration in concave spots)

$$\sigma_{P_1}^2 = P^2 U^2 \overline{\delta}^2 \frac{\iint (\nabla^2 z)^2 \, dxdy}{\iint dxdy} \tag{18}$$

Clearly the roughness in this case is the average of the squared curvature. Another variance is due to slanting rain (eq. 8).

$$P_2^2 = P^2 \frac{\iint \eta^2 \tan^2 \alpha \, dxdy}{\iint dxdy} \tag{19}$$

Note that η changes from point to point as the aspect of the slope changes. This means that the roughness of the field may be different for slanting rains of different aspects. Still more complicated cases can be obtained by a combination of the two mechanisms (of slanting rain and concentration in concave spots). In the above only integration over the space has been registered. Fluctuations of the rain intensity P over space and time and dependence of $U.\bar{\delta}$ on P would require the calculation of the variance as follows.

$$\sigma_{P_3}^{2} = \iiint \left[(P-P) + PU\bar{\delta}\nabla^2 Z\right]^2 dxdydt/t\iint dxdy \qquad (20)$$

This is for the splashing effect. Luckily as the curvature is time independent and possibly of no correlation with the rain fluctuations, the actual calculation may be somewhat simpler.

It is important to point now the need for measuring different topographic parameters and rain parameters that have not been considered in the past. Unquestionably they have a decisive effect on local and temporal concentration of rain that can lead to ground water recharge, runoff or at least parts of the soil that are wetter than others.

As an illustration let us produce a two-dimensional sinussoidal landscape. With the elevation Z fluctuating around the average $\bar{Z}$ with an amplitude $A_i/2$ and half cycle $x = L_i$.

$$Z = \bar{Z} + \frac{A_i}{2} \sin \left(\pi \frac{x}{L_i}\right) \qquad (21)$$

then the slope is

$$-\tan \alpha = \frac{\partial Z}{\partial X} = \frac{A_i}{2}\frac{\pi}{L_i} \cos \left(\pi \frac{x}{L_i}\right) \qquad (22)$$

and the curvature is (droping the index i for one cycle only)

$$\frac{\partial^2 Z}{\partial_x^{\ 2}} = -\frac{A}{2}\left(\frac{\pi}{L}\right)^2 \sin \left(\pi \frac{x}{L}\right) \qquad (23)$$

The roughness or variance according to eq. 18 is found by

$$\sigma_P^2 = \frac{1}{L} P^2 U^2 \bar{\delta}^2 \left(\frac{A}{2}\right)^2 \left(\frac{\pi}{L}\right)^4 \int_o^L \sin^2\left(\pi \frac{x}{L}\right) dx =$$

$$= \frac{\pi}{2} \left(\frac{\pi}{L}\right)^4 \left(\frac{A}{2}\right)^2 P^2 U^2 \bar{\delta}^2 \tag{24}$$

The standard deviation of the precipitation at the soil surface is

$$\sigma_{P1} = PU\bar{\delta} \frac{A}{L^2} \left(\frac{\pi}{2}\right)^{5/2} \tag{25}$$

As mentioned earlier the term $\frac{A}{L^2} \frac{\pi^2}{2}$ is the maximum curvature. (A/L) is the average local slope while 1/L is the drainage density. For L $\sim$ 1 and A only 0.1 m $U\bar{\delta}$ can be 0,50 m (as measured in earlier part of this report ZASLAVSKY, SINAI, 1978). The standard-deviation can be around 0.2 P.

A second example can be of a slanting rain (eqs. 19 and 22). Assuming $\eta = 1$ at a 45° slanting rain

$$\sigma_{P2} = P \frac{A}{L} \left(\frac{\pi}{2}\right)^{3/2} \tag{26}$$

and for (A/L) 0.1 the value is about 0.18 P. Much higher values may be obtained for a steeper relief of the soil and for shallower slanting rain.

A rough estimate of the final variance of the precipitation on the soil surface is probably an addition of the separate variances. This means that the total standard deviation can easily be 50% of the average rain intensity.

8. CONCLUSIONS

The study of the average hydrological behaviour over the field requires the measurement of the rain statistical terms, the local and themperal fluctuations of its intensity and the aspect and slope of the rain flight. In addition one has to express the soil surface roughness in terms of the mean square of the local curvature or the local slope magnitude, depending on the phenomenon in question.

The mean squar local curvature expresses the degree of precipitation concentration in concave places by raindrop splashes. The soil surface is to be expressed as a Fourier series (or double series). Each wavelength contributes linearly to the variance of the precipitation. If the amplitude is $\frac{A_i}{2}$ and half wavelength is L_i then the contribution is proportional to A/L_i^2. However there is a physical limit on the wavelength which is contributing to this process. Very sharp changes in soil slope have Fourier harmonics of short wavelength. If L_i is smaller than a typical splash distance of the raindrop it may not contribute to the precipitation accumulation. This is a subject that should be further studied.

The order of magnitude of the effective precipitation in some spots can be much larger than the average rain. Thus runoff and water recharge can become a non linear function of the average rain. Runoff can be formed even when the average rain is lower than the infiltration capacity. Ground water recharge can be formed even when the average rain is lower than the potential evaporation and runoff. Ground water recharge can be formed even when the average precipitation cannot but wet the top soil.

REFERENCES

CHOW, VEN TE, 1964. Handbook of applied Hydrology; CHOW, VEN TE, edt. McGraw Hill Book Co., New York

MUTCHLER, C.K., 1967. Parameters for describing raindrop splash. Soil and Water Conservation. Vol. 22, no. 3

——— 1970. Splash of water drop at terminal velocity. Soil Sci 169: 1311-1312

——— 1971. Splash amounts from water drop impact on a smooth surface. Water Resources Res. Vol. V, no. 1

ZASLAVSKY, D., 1970. Some aspects of watershed hydrology. Special report U.S.D.A. ARS 41157

——— and G. SINAI, 1978a. Surface Hydrology I. Field observations that require explanation

——— 1978b. Lateral flew due to raindrop splashes

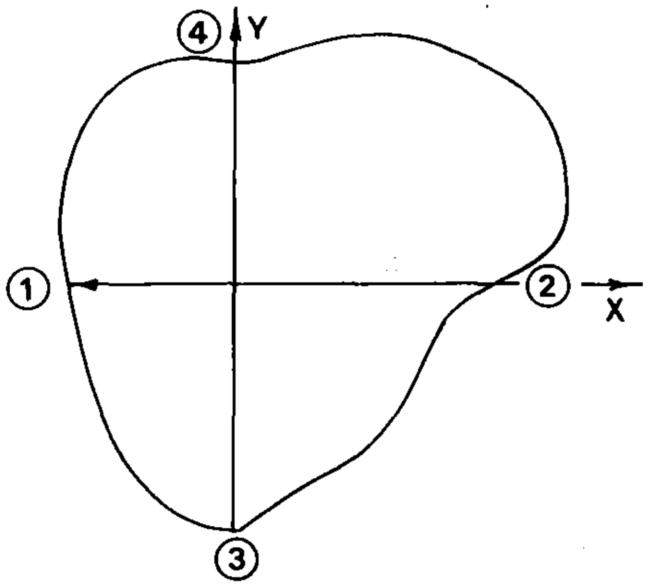

Fig. III.1. Boundaries of a surface drainage basin

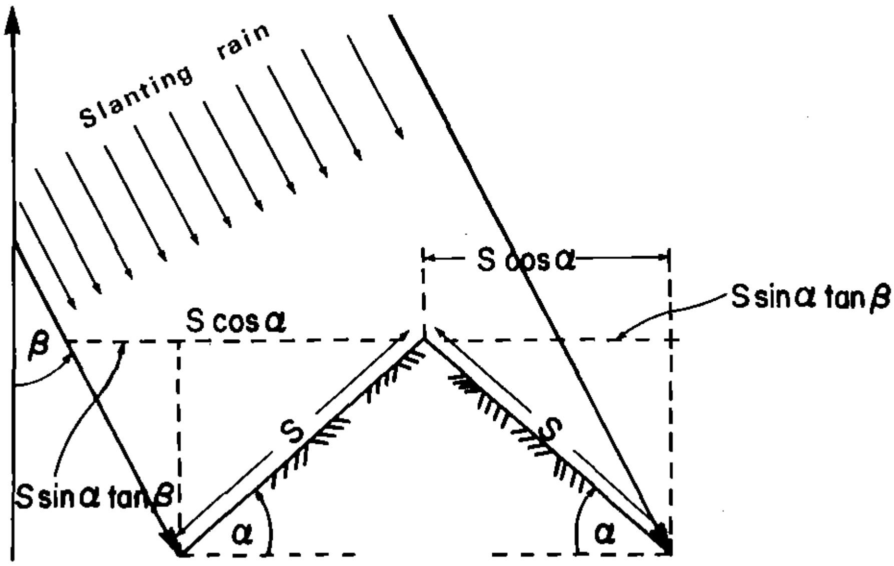

Fig. III.2. Slanting rainfall into microrelief (after ZASLAVSKY, 1970). Rain angle β, soil slope α

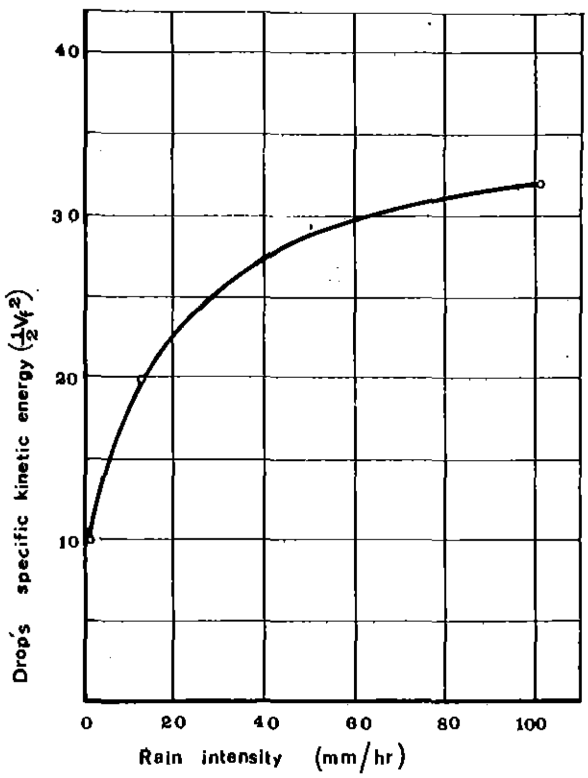

Fig. III.3. Median specific kinetic energy as a function of rain intensity. Calculated from hydrological data (after VEN TE CHOW, 1964)

IV. LATERAL FLOW IN THE SOIL SURFACE - QUALITATIVE CONSIDERATIONS

ABSTRACT

In this part of the report a second mechanism is explained by which latteral flow is formed during rain, followed possibly by the concentration of moisture in concave places and possibly leading to runoff, erosion and other physiographic phenomena.

Streamlines that enter the soil vertically tend to curve downstream on a transition to a more permeable layer. In unsaturated flow a more permeable layer can be produced by a local water accumulation. Such an increase in moisture, pressure and conductivity can occur within a layer which has a higher saturated hydraulic conductivity overlaying a layer which has a slightly smaller hydraulic conductivity. There can be a slight moisture accumulation or the formation of perched water table. In either case latteral flow component is associated with the vertical infiltration. The latteral horizontal flow is proportional to the slope. The soil surface is defined as a transition from the soil to the air with an extremely permeable layer at the top. Thus latteral flow occurs in every sloping soil and with any rain even a very small one. Every rain, even a very high one penetrates completely into the ground. Concentration of rainwater in concave parts of the landscape can now be explained by two consequent mechanisms, the splashing of rain drops and latteral flow in the soil surface transition layer. A plow layer is a special case of a thick transition layer.

While the process of moisture accumulation due to raindrop splashing increases with concavity only up to a certain value, the concentration due to flow in the transition zone can tend to very high local values at high concavities. On the other extreme the flow in the surface transition layer remains important at very moderate slopes and curvatures. The process is significant for uniformity of irrigation and for errosive processes. Concentration of moisture in concave places continues during drainage and evaporation. It may explain the long

term cumulative effects of the precipitation on the eventual formation of runoff in a given storm.

1. INTRODUCTION

In the first part of this report (ZASLAVSKY and SINAI, 1978I) it has been suggested that there exists a horizontal flow component as a result of precipitation over the soil surface. This horizontal flow component is to substitute the concept of surface runoff flow and similar terms used to describe a situation where the rain exceeds the infiltration capacity.

It has already been shown (ZASLAVSKY and SINAI, 1978II and III) in theory and experiments that raindrop splashes actually provide a considerable horizontal flow component proportional to the rain itself, to the first power or higher, and to the slope of soil surface. The result is that the rain accumulates in concave parts of the landscape. Local and temporal fluctuations in the precipitation can produce moisture excess even at low rates of rain or low total rain depths.

It is the intention of the present part of the report to demonstrate a similar phenomenon at the soil surface after the raindrops rested and entered the ground. Under a uniform rain there will be a horizontal flow component downstream which is similarly increasing with the rain intensity and is proportional to the soil slope. Here the analysis will be limited to relatively simple deductions intended more towards qualitative conclusions and an insight into the process. A demonstration of the existence of such a phenomenon is very simple. Many, after reading the present discussion, will identify observations they saw in nature that cannot be explained otherwise. The Beer Sheva experiment reported in the first part of this report (ZASLAVSKY and SINAI, 1978I) is such an observation. Some of our deductions here will be based on a steady state analysis. The results of the Beer Sheva experiment show a non steady state regime which is very much like the steady state. Its analysis has been done by numerical methods and is postponed to some following parts of the report.

2. THE CURVING OF STREAMLINES

When a streamline moves from one medium to another with the respective conductivities K_1, K_2 it will form different angles γ_1, γ_2 with the orthogonal to the interface so that (fig. 1) -(BEAR et all., 1969)

fig. 1

$$\frac{\tan\gamma_1}{\tan\gamma_2} = \frac{K_1}{K_2} \tag{1}$$

Consider fig. 1 as an example with two soil layers and an angle α with the horizon. On passing from a less permeable to a more permeable layer the streamline will turn from a vertical direction to a diagonal direction having a horizontal component. If as in our case initially $\gamma_1 = \alpha$ the ratio between horizontal and vertical fluxes will change to

$$\frac{q_x}{q_z} = \frac{\tan\alpha\,(\frac{K_2}{K_1} - 1)}{1 + \frac{K_2}{K_1}\tan^2\alpha} = \frac{\frac{1}{2}\sin 2\alpha U'}{1 + \sin^2\alpha U'} \;;\quad U' = (\frac{K_2}{K_1} - 1) \tag{2}$$

as can be shown by simple trigonometric considerations.

At small slopes (angles α), $\frac{K_2}{K_1}\tan^2\alpha \ll 1$ so that

$$\frac{q_x}{q_z} \sim \tan\alpha U' \tag{3}$$

or $\frac{1}{2}\sin 2\alpha \sim \sin\alpha$ and $\sin^2\alpha \ll 1$, so that

$$\frac{q_x}{q_z} \sim \sin(\alpha) U' \tag{4}$$

We shall see later that in more complex cases of unsaturated flow one obtaines the same result except that the coefficient of anisotropy U' takes a somewhat different form (e.g. K_2 and K_1, being the weighed averages of horizontal and vertical conductivities, respectively).

If q_z happens to be the rate of infiltration I which is equal to some steady rain then $K_1 = I$. Substituting it in eq. (2) or (3) one gets the horizontal flow component as a function of the rain or the steady rate of infiltration and the slope.

In a medium of a gradually varying conductivity the change in flow direction follows exactly the same rules. The angles of the streamline are then related to the direction of the vector (grad K) and a change in tan γ is expressed by its scalar product with an elementary path length ds.

$$\frac{d(\tan\gamma)}{\tan\gamma_1} = \frac{\text{grad } K . ds}{q_1} \qquad (5)$$

By simple observation of fig. 1 and eq. (5) the following conclusion can be drawn with respect to a uniform slope with a more or less vertical entry of the water:

a) if the hydraulic conductivity increases with depth the streamlines will curve downstream to form a horizontal flow component;
b) if the conductivity decreases with depth the streamline may turn upstream but not beyond a direction normal to the soil surface. Thus one can conclude that there will be always a flow component parallel to the soil surface. This component can diminish to $q_1 \sin\alpha$ in uniform soil and to zero in a highly impermeable part of the soil. The parallel component is always downstream;
c) as the horizontal flow component is proportional to the slope it can explain concentration of moisture in concave places.

In using components q_s and q_n in fig. 1 parallel and normal to the interfaces one gets the simple formula for any K value

$$\frac{q_s}{q_n} = \tan\gamma = \frac{K}{K_1} \tan\gamma_1 \qquad (6)$$

and in the case that q_1 is vertical and $\gamma_1 = \alpha$ the soil surface slope, $q_{s_1} = q_1 \sin\alpha$; $q_{n_1} = q_1 \cos\alpha$ (assuming q_n and q_z positive pointing down) then

$$\frac{q_s}{q_n} = \frac{K}{q_1} \tan \alpha \tag{7}$$

From eqs. (6) and (7) one can further validate the above conclusions.

3. CURVING OF STREAMLINES ABOVE THE WATER TABLE

The simplest case is that of a steady accretion to a phreatic surface by rain (or negative accretion by evaporation). If the soil is thick enough above the phreatic surface and it is uniform then the streamlines will enter the soil vertically. Also $K_1 = q_1$ (fig. 1) (see for proof eq. 14 with $z \to \infty$). The saturated hydraulic conductivity K_s is near the phreatic surface. The slope is that of the water table $\alpha \equiv \gamma_1$. The streamlines will enter the water table at an angle. According to eq. (7)

$$\frac{q_s}{q_n} = \frac{K_s}{q_1} \tan \alpha \tag{8}$$

Or in the xz coordinates by eq. (3)

$$\frac{q_x}{q_z} = \left(\frac{K_s}{q_1} - 1\right) \tan \alpha \tag{9}$$

remembering that by conservations $q_z = q_1$ at every depth

$$q_x = (K_s - q_1) \tan \alpha \tag{10}$$

Interestingly at smaller vertical fluxes the horizontal component will be relatively larger. One can actually calculate the horizontal span of a streamline while coming down from the soil surface. Assume a saturation surface at elevation zero. The equation for the pressure head of ψ is found from Darcy's equation

$$dz = - \frac{d\psi}{1 - \frac{q_1}{K}} \tag{11}$$

remembering that q_1 is in a negative z direction.

Assuming an experimental relation between the conductivity K and the pressure head ψ one can integrate (11). For example

$$K = K_s \exp\left[a\ (\psi + \psi_o)\right]; \quad \psi + \psi_o \leqslant 0 \tag{12}$$

a and ψ_o being experimental coefficients and K_s the saturated hydraulic conductivity.

Introducing into eq. (11)

$$z - z_o = -\frac{1}{a} \ln \frac{\frac{K}{K_s} - \frac{q_1}{K_s}}{1 - \frac{q_1}{K_s}} \tag{13}$$

solving for K

$$K = q_1 + (K_s - q_1) \exp\left[-a(z - z_o)\right] \tag{14}$$

z_o is the elevation of saturated soil ($\psi = -\psi_o$). With the help of eq. (9) where K_s is substituted by any K from eq. (14) one can calculate the horizontal translation of a streamline (which is identical with a path line under steady state).

$$dx = (\frac{K}{q_1} - 1) \tan\alpha\, dz = (\frac{K_s}{q_1} - 1) \exp\left[-a(z-z_o)\right] \tan\alpha\, dz \tag{15}$$

on integration

$$x_H = (\frac{K_s}{q_1} - 1) \tan\alpha\ \frac{1}{a} (1 - e^{-a(H-z_o)}) \tag{16}$$

where $H-z_o$ is the height between the surface of saturation in the soil and the soil surface over which the total horizontal movement is x_H. In the case of a large term $a(H-z_o)$, eq. (16) is approximated by

$$x_H = \frac{1}{a} (\frac{K_s}{q_1} - 1) \tan\alpha \tag{17}$$

In some drainage problems the slope is 0.01 to 0.1. The hydraulic conductivity can be 10 to 100 mm per day while the rate of drainage may be 2 to 3 mm per day. A typical value of the coefficient (a) can be 0.01 cm^{-1}. Clearly the horizontal movement x_H can be from few centimetres to few metres within the unsaturated zone. This is a striking result especially for some shallow water problems of waterways and drains. Here, even in a flat land, the groundwater slope increases and the latteral flow within the unsaturated soil will become significant.

This special case of non uniformity in conductivity and of lateral flow is due to the boundary conditions of the problem and has already been recognised by FREEZE (1967-1976). Instead of the numerical technique used by him it has been preferred here to have a simple analytic derivation that demonstrates better the principal nature of the phenomenon and its order of magnitudes. The interesting point is that the unsaturated flow regime induces variations in the hydraulic conductivity that in turn cause a significant horizontal downstream flow component above the water table. This simple case is introduced as an intermediate step towards the more general and more significant case where the latteral flow is induced by a layering of the soil. It has been known that a layered soil behaves anisotropically on the average under saturated flow (BEAR et al., 1969). In the following unsaturated flow will be considered.

4. A TWO LAYER PROBLEM (FOLLOWING ZASLAVSKY, 1970)

Consider a permeable layer of saturated conductivity K_{s1} and
fig. 2 thickness D_1 overlying a less permeable layer K_{s2},D_2 (Fig. 2). The rain is of intensity q_1, which is smaller than the hydraulic conductivity of the top layer K_1. Thus the flow at this layer will be unsaturated, under negative pressure. If D_1 is long enough the flow regime at the top will approach asymptotically $K_1 = q_1$ and the hydraulic gradient will approach a unity (see eq. 14 with $z \to \infty$). Approaching the interface between layers the pressure will increase gradually. The hydraulic conductivity within the top layer will gradually increase towards saturation $K_1 \to K_{s1}$. In fig. 2 two adjacent vertical sections are observed. Compare the pressure head curve on the upper one with that on the lower one. At two points along a horizontal line C-D the

elevation is the same. The pressure head ψ is higher at the upper section (curve 1) than in the lower section (curve 2). Therefore there must be a flow component in the horizontal direction, i.e. downstream.

Let us study this problem in view of the streamline equations (3) and (4). At the top of layer 1 (fig. 2) $K_1 \rightarrow q_1 < K_{1s}$. At the bottom of layer 1 $K_1 = K_1''$, it increases and may approach K_{1s}. As a result of the less permeable layer and the unsaturated flow there is a build up of moisture, pressure and conductivity above the interface. The flow direction in the top layer will then change from vertical ($q_x = 0$) to

$$q_x = (\frac{K_1''}{q_1} - 1) \tan (\alpha)\ q_1 \tag{18}$$

(from eq. 3 identifying $K_1'' = K_2$, $q_1 = K_1$). It becomes clear that if there exists a more permeable layer at the soil top and if the flow is unsaturated streamlines will bend downstream. It is stipulated that every soil without an exception has a more permeable layer at its surface. Therefore in every sloping soil under prolonged rain the streamlines will bend downstream. In other words, at the surface of every sloping soil there will be a horizontal flow component downstream.

This has been shown to be under non-saturated conditions as well as with the presence of a water table. Furthermore one may conclude:

a) under steady state flow the vertical flow component is the same at every depth. The horizontal flow component is therefore explicitely proportional to the vertical flux at every depth. So is the total horizontal discharge. Implicitely the horizontal flux increases also due to the coefficient of an isotropy $(\frac{K_1''}{q_1} - 1)$ which in turn increases also with the rate of vertical flow. Thus the horizontal flow depends on the steady rain to a power higher than a unity;
b) if there is a change in the slope so that the landscape is concave there is also a concentration of moisture. This is because the incoming horizontal flux is higher than the outcoming one. A very high hydraulic conductivity at saturation (K_s) usually means a fast reduction of the conductivity due to suction e.g. having a higher value of the coefficient (a) in eq. (12). This means that relatively

small depth of soil ($z-z_o$ in eq. 14) is sufficient to make the conductivity equals the rate of infiltration and the flow vertically down. Thus the very high conductivity at every soil surface validates our assumption of initial rain penetration to be vertical.

5. THE TRANSITION AT THE SOIL SURFACE

In the first part of this report (ZASLAVSKY and SINAI, 1978I) a criticism has been passed as to the possibility of measuring surface flow because there is no unique definition where the soil surface really is. ZASLAVSKY (1968) argues more generally that in soil physics meaningful entities are averages over time and space that produce a continuum. The same should be applied to the soil surface. For example, the porosity can be measured over a finite sampling volume or over an area. High in the air it will be 100%. Somewhere in the soil it may be 50%. Anywhere inbetween it changes gradually. The hydraulic conductivity will change in a similar way from some finite value well in the soil bulk to a very high value at the air (e.g. change in K_s of eq. 12). In a similar way the air entry value ψ_o will be reduced to zero passing from the soil bulk to the air ($\psi_o \rightarrow 0$ in eq. 12). Finally the rate of K reduction under suction will increase (coefficient a in eq. 12). (see dictionary of soils by MUALEM and DAGAN, 1976). The air can be considered as some limiting form of the porous medium itself (very similar to a very coarse gravel). The pressure of the raindrops is always atmospheric. The rate of flow is the rate of rain and the unsaturated conductivity is conveniently equal to the rate of rain. The surface transition is far from being just a mathematical artifact. It may be very thin in some uniform and smooth sand but can be several decimetres thick in most cultivated soils.

The soil surface is defined by a transition of the properties. Its direction is defined by a surface normal to the property gradient. Out of many such surfaces one can be chosen to represent the surface through some convenient conservation demand (e.g. that the total porosity will be unchanged). Then the property may change abruptly at this representative surface. We may conclude that a unique soil surface is more of a mathematical artifact.

An important consequence of the above is that every rain, intense as it may be, penetrates completely into the soil (at least into its surface transition zone). There is no such a thing like a surface run-off because the rain can never exceed the hydraulic conductivity of the uppermost part of the soil surface (i.e. the air itself). Saturation due to high intensity of rain when it occurs will always appear first within the surface transition zone.

The splashing of raindrops proved to cause a horizontal flow component proportional to the slope explicitely proportional to the rate of rain and implicitely to some fractional power of the rain. Evidently a steady flow through the surface transition layer is related exactly in the same way to the rain. The response time of the splashing raindrops is measured in fractions of seconds. It can therefore be considered quasi steady (following exactly the rain itself). The flow in the transition layer can be delayed, depending on its thickness. It is expected that thin transitions will react faster.

At the wetting front the flow motivating force is mainly the pressure gradient which is anticipated to be normal to the surface. Therefore the flow will tend to be normal to the soil surface ($q_s \rightarrow 0$, fig. 1) and even have a slight upstream flow component ($q_x < 0$, fig. 1). Well behind the wetting front the main force will be gravity and downstream horizontal flow ($q_x > 0$, fig. 1) will be formed.

The flow in the surface transition layer can sometimes be observed as tiny trickles of water or shiny soil surfaces. Concave parts of a very small dimension where water concentrates can be considered in details as such or be averaged out as part of a thicker transition layer. The exact limit depends arbitrarely on the chosen scale of observation.

6. ORDER OF MAGNITUDES OF FLOWS IN THE SURFACE TRANSITION

Consider first a well cultivated heavy soil with a good and stable structure. The hydraulic conductivity at the top (even not near the air) can reach 10^{-1} cm/sec and even 1 cm/sec. These have been actually measured in a drainage research field in Hazorea, Israel. In the subsoil the hydraulic conductivity may reduce to 10^{-6} cm/sec and less. This means that the anisotropy coefficnet can change all the way from nearly zero at an extremely low rate of rain and up to 10^{6}. One can actually observe lateral flows of few metres or few tens over a vertical infiltration of few decimetres. Water puddles form after rain or irrigation at slightly concave spots. This example is of course extreme and almost trivial. The observations reported earlier (ZASLAVSKY and SINAI, 1978I) north of Beer Sheva is much less trivial and fits the above analysis. Even a change of 2 orders of magnitudes in the hydraulic conductivity and a slope of 1% can produce a horizontal flux equal to the vertical one. Higher slopes were in Beer Sheva (more nearly 10%).

We have no directly measured data of the field anisotropy. This and some other entities should be the subject of future research efforts.

The two processes of raindrop splashing and flow in the surface transition layer join to produce latteral flow and moisture concentration. Under laboratory conditions one may try the second one only separately by applying the moisture without the high kinetic energy of the raindrops. In nature it will be difficult to distinguish between the two. In the previous part of this report (ZASLAVSKY and SINAI, 1978III) an experiment has been mentioned with a V shaped soil slope. An almost immediate runoff started at the sharp bottom edge. Raindrop splashing alone cannot explain it. As there was no impermeable layer or thorough saturation of the soil directly by the average rain then the phenomenon may be related to flow in the top transition layer. This deduction is supported also by the fact that the runoff at the 'V' sharp edge involved also liquification of the soil and erosive flow. Such a flow can occur only if water is coming out of the soil. Raindrop splashes remain outside the soil. However, the rain enteres the transition zone and can then seep out.

It seems that the water flow in the surface transition zone can be quite significant where the raindrop splashing is less. It can respond to more extreme curvatures M in the soil surface $M > 1\ m^{-1}$ where it can produce runoff and erosion with very small amounts of rain and in relatively short times. On the other extreme it can accumulate moisture at relatively moderate curvatures $M < 0.1\ m^{-1}$ (in the Beer

Sheva experiment). There, the effect of raindrop splashing becomes negligible.

7. SOME NOTES ON THE OCCURRENCE OF SURFACE TRANSITION AND ITS SIGNIFICANCE

By definition any particulated material has a transition surface layer which is at least several times the dimension of a particle. The thickness of the transition will depend not only on geometry but also on effects like the transfer of momentum in liquid flow (SUFFMAN, 1971).

However most soils will have a more loose structure at the surface with some aggregates root holes and other disturbances. Newly exposed soil cuts will develop such thicker transitions over some time. Development of surface latteral flow and erosive mechanisms will develop accordingly.

Some surface layers may be very similar to a straw roof. This may be the case in litter covered forest soil and possibly even in some grass covered area where the old growth may have a marked orientation parallel to the soil surface.

The concept of surface transition may apply in an interesting way to some other water flow problems such as outcrop of water on a seepage face. Accordingly the flow medium will be described by highly permeable layers at the surface. The concept of a seepage face becomes redundant. The streamlines simply bend downstream in the transition layers. In what has been called seepage face the flow is more or less parallel to the surface but within the soil. It is significant not only in contributing to the physical consistency of our flow analysis. It can explain how some small surface geometrical irregularities can cause local outflows and erosion due to seepage forces.

It is significant that downstream horizontal flow component occurs whenever the pressure head reduces with elevation. This is certainly the case during drainage and drying of the soil surface by evaporation. A surface transition makes the effect more significant. Thus the latteral flow and moisture concentration in concave parts of the landscape will continue long after the rain has stopped. This process will keep the concave parts wetter over long periods and shorten the delay to runoff on the next rain. It seems that the importance of the latteral flow becomes quite universal in saturated and non saturated flow, during prolonged rain, drainage and drying of the surface. It occurs in seepage faces. It occurs in the natural, exposed soil and possibly with litter covered soil.

Irrigation in cultivated soils must be affected by latteral flow of water. Farmers have been emphasising the importance of leveling the fields. It seems that the term leveling is a semantic error accompanied with a misinterpretation of the mechanism. Leveling of fields is in practice the provision of plane surfaces (though with smaller slopes too). It may be that leveling is not as important as 'planing'. Local, very small scale high curvatures such as furrows cause non uniformities in the moisture distribution that are averaged out by the soil itself and the plants. Still sharp edged furrows always involve erosion and fast development of runoff. Some thought may be given to the shape of the furrow in view of the above analysis. More moderate curvatures still cause non uniformities in the moisture distribution. However they can be of a scale that cannot be evened out by the size or the root volume of a single plant or by latteral redistribution of water in deeper soil. Thus the plane shape of the field becomes essential for an even distribution of rain or irrigation water.

REFERENCES

BEAR, J., D. ZASLAVSKY and S. IRMAY (ed.). 1968. Physical principles of water percolation and seepage. UNESCO, Paris.

FREEZE, R.A. 1967. The continuity between groundwater flow systems and flow in the unsaturated zone. In: Proc. of Hydr. Symp., no. 6, held at Univ. of Saskatchewan on 15 and 16 Nov.: 205-240.

FREEZE, R.A. 1969. The mechanism of natural groundwater recharge and discharge. 1. One-dimensional, vertical unsteady, unsaturated flow above a recharging or discharging groundwater flow system. Water Resour. Res. 5.1: 153-171.

FREEZE, R.A. 1971. Three-dimensional, transient, saturated-unsaturated flow in a groundwater basin. Water Resour. Res. 7.2: 347-366.

FREEZE, R.A. 1972. Role of subsurface flow in generating surface runoff. 1. Upstream source areas. Water Resour. Res. 8.5: 1272-1283.

FREEZE, R.A. 1972. Role of subsurface flow in generating surface runoff. 2. Upstream source areas. Water Resour. Res. 8.5: 1272-1283.

FREEZE, R.A. 1972. Role of subsurface flow in generating surface runoff. 1. Base flow contributions to channel flow. Water Resour. Res. 8.3: 609-623.

FREEZE, R.A. 1974. Streamflow generation. Reviews of geophysics and space physics 12.4: 627-647.

FREEZE, R.A. 1976. Simulation of subsurface flow in watershed models. IBM Seminar on regional groundwater hydrology and modeling, Venice, Italy.

MUALEM, Y. and G. DAGAN. 1976. Methods of predicting the hydraulic conductivity of unsaturated soils. USA-Israel (BSF) Res. Project no. 442, Hydraulic Lab., Technion, Israel Inst. of Techn., Haifa, Israel.

MUALEM, Y. 1974. Hydraulic properties of unsaturated porous media. P.M. 38/74 Res. no. 012-295. Technion, Israel Inst. of Techn., Haifa, Israel.

SUFFMAN, P.G. 1971. On the boundary condition at the surface of the porous medium. Studies in applied mathematics. Vol. L, no. 2: 93-101.

ZASLAVSKY, D. and G. SINAI. 1978. Surface Hydrology. I. Field observations that require explanation. II. Lateral flow due to raindrop splashes. III. Rainfall effective distribution.

ZASLAVSKY, D. 1968. The average entities in kinematics and thermodynamics of porous materials. Soil Sci. 106: 358-362.

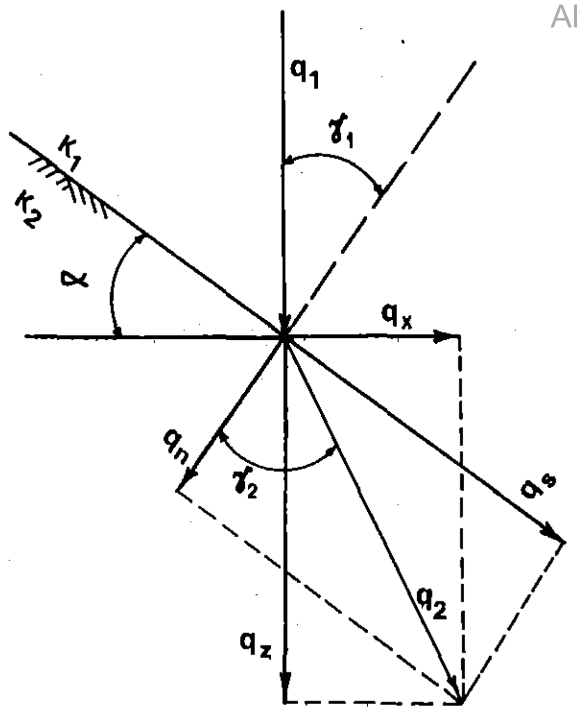

Fig. IV.1. Turning of streamlines across interlayer surface. The hydraulic conductivities $K_2 > K_1$

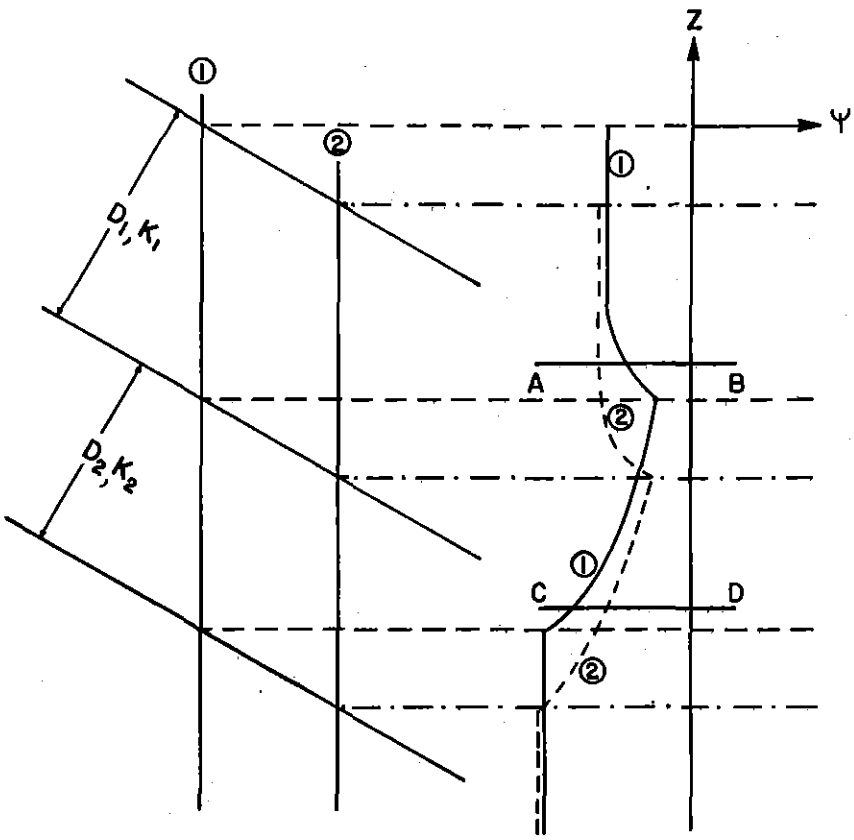

Fig. IV.2. Pressure head ψ distribution for a vertical flow through a sloping two-layered soil system. Comparison of two neighbouring cross-sections 1 and 2 indicate downstream horizontal flow component when ψ increases with depth and upstream horizontal flow when ψ decreases with depth

V. STEADY LATTERAL FLOW IN A LAYERED SOIL

ABSTRACT

Considering a layering in the soil inherited properties or in the hydraulic regime, it is proved generally that on the whole the soil behaves as an anisotropic medium. When the layers are sloping the gravity force produces latteral flow components downstream. The flow is proportional to the slope, to the vertical or normal flux and to a coefficient of anisotropy that can be calculated. In a steady state flow the problem becomes simpler and the latteral flow can be related to the rate of rain or the net groundwater recharge. On the whole the horizontal flow component increases with the rain more than to the second power. In a cyclic layered soil it is relatively simple to calculate the coefficient of anisotropy and its change with the rate of rain. A surface transition of the soil hydraulic properties could also be described as a sequence of layers.

The latteral flow can produce water concentration in concave parts of the layers relief or in places where the layers are truncated. The latteral flow should be taken into consideration in studies of pollution. A mound or convex layers could be used to prevent water flow into structures or through sources of leachable pollutants. The latteral flow due to soil layering add up to those due to raindrop splashing and surface transition that have been studied previously.

1. INTRODUCTION

In previous parts of the report it has been demonstrated that latteral flow (parallel to the soil surface or in horizontal direction) will occur above the soil due to raindrop splashing and within the soil as a result of a layer at the soil surface of a higher saturated conductivity. Every soil has such a layer at its surface.

In the present part more specific calculations will be made for a layered soil. First it will be proved in general that non-uniform soil will behave on the average as non-isohopic. Then a cyclic layered soil will be calculated in details and expressed as a non-isotropic medium with a conductivity higher parallel to the layers.

There is some repetition in the presentation. However, it has been found easier to follow this way. In addition somewhat different routes of derivation have been found more or less plausible to different readers.

2. BASIC ASSUMPTIONS

Consider a non-uniform soil with the soil properties changing

Fig. 1 in a direction n. The parallel direction is s (Fig. 1). n and s make an angle α with z and x coordinates respectively. Consider now the flux q_o to be known somewhere in the medium especially vertical in the positive z direction as evaporation or in the negative direction as infiltration.

To solve an actual problem one needs the hydraulic properties of the medium and some boundary conditions. The general case is of very little interest. We are interested here mainly in two cases:

a) with uniformity along s, e.g. $\partial\Psi/\partial S = 0$, $\partial K/\partial S = 0$ etc.
b) with a uniform slope as above and a steady state flow

While these are not the most general cases they are sufficient to provide some of the more important conclusions or at least as good starting points lending us an insight to hydrological processes.

3. THE BASIC EQUATIONS OF FLOW AND THE GENERAL PROOF OF ANISOTROPY

The fluxes in the n, s directions are by Darcy's law (Fig. 1):

$$q_s = K \sin \alpha \qquad (1)$$

$$q_n = -K(\frac{\partial\Psi}{\partial n} + \cos\alpha) \tag{2}$$

where K (n,Ψ) is the hydraulic conductivity that can vary with the pressure head Ψ and the location along n, explicitely.

The fluxes expressed in the x,z system are found by simple geometrical transformation from eqs. (1) and (2):

$$q_x = q_s \cos\alpha + q_n \sin\alpha = -K \frac{\partial\Psi}{\partial n} \sin\alpha \tag{3}$$

$$q_z = -q_s \sin\alpha + q_n \cos\alpha = -K(\frac{\partial\Psi}{\partial n} \cos\alpha + 1) \tag{4}$$

At a point we assume the medium to be isotropic. Therefore q_x and q_z can also be directly written from the Darcy equation (the n and s directions were the principal axes and did not raise any problem of isotropy)

$$q_x = -K \frac{\partial\Psi}{\partial x} \tag{5}$$

$$q_z = -K(\frac{\partial\Psi}{\partial z} + 1) \tag{6}$$

Eqs. 5 and 6 check well with (3) and (4) if we transform the gradient vectors in the uniform slope condition from the n,s to the x,z systems.

At every point it is assumed that the force and flux are parallel so that the ratio of flux components is exactly equal to the ratio of the force components (simply divide 1 by 2 with the same K at both or 3 by 4 or 5 by 6). This is the essence of assuming an isotropy at a point.

The first question is whether the medium as a whole behaves isotropically or not. To arrive at a general conclusion consider path lines (that are identical with the streamlines only under steady state). The ratios of path components in the s and n directions are found by dividing (1) by (2) and integrate over n and divide for averaging by $\int dn$.

$$\frac{\int \frac{q_s}{q_n}\, dn}{\int dn} = \frac{\int \frac{\sin\alpha\, dn}{\frac{\partial\Psi}{\partial n} + \cos\alpha}}{\int dn} \tag{7}$$

The ratio q_s/q_n is the angle of the flux vector and the l.h.s. integral is the overall parallel component in the s direction. The r.h.s. is found by expressing q_s, q_n from (1, 2). Clearly on the r.h.s. if $\partial\Psi/\partial n$ is variable with n then the overall slope of water path vary with n. The overall ratio of parallel force $\bar{F}_s$ to the normal force $\bar{F}_n$ is found by integrating the gradient components separately over n and dividing by the integrated normal component over n

$$\frac{\bar{F}_s}{\bar{F}_n} = \frac{\sin\alpha \int dn}{\int(\frac{\partial\Psi}{\partial n} + \cos\alpha)dn} \tag{8}$$

Clearly the ratios in (7) and (8) become identical only if $\partial\Psi/\partial n = 0$. In every other case they differ. Another case where the ratios become identical is trivially for $\sin\alpha = 0$, i.e. horizontal soil layering. Clearly if the ratios of the overall flux component s is different then the ratio of the overall force component s then the soil behaves on the whole as non-isotropic.

The ratio between (7) and (8) is

$$(\frac{\bar{q}_s}{\bar{q}_n})/\frac{\bar{F}_s}{\bar{F}_n} = \frac{\int\left[(\frac{\partial\Psi}{\partial n} + \cos\alpha)dn\right]\left[dn/(\frac{\partial\Psi}{\partial n} + \cos\alpha\right]}{(\int dn)^2} \geqslant 1 \tag{9}$$

The proof of the inequality is that the harmonic average is smaller or equal to the arithmetic one. In effect it means that on the average the conductivity in the direction (s) is higher than that in the direction (n). The equality to unity in (9) is obtained only when the numerator equals $(\int dn)^2$. This can be so only if $\partial\Psi/\partial n = 0$ i.e. in a uniform soil and flow regime. It is interesting that this anisotropy on the average is induced not only by soil layering but also by changes in Ψ due to boundary conditions.

In a cyclic medium Ψ and $\partial\Psi/\partial n$ fluctuate around some value. Thus the first integral on (9) reduces over a complete cycle to $\cos\alpha\, dn$ and (9) reduces to

$$(\frac{\bar{q}_s}{\bar{q}_n} / \frac{\bar{F}_s}{\bar{F}_n})_{cyclic} = \frac{\int \frac{dn}{\frac{1}{\cos\alpha}(\frac{\partial\Psi}{\partial n}) + 1}}{\int dn} \geqslant 1 \tag{10}$$

Care must be taken in the case of horizontal soil as (9) and (10) have been obtained by dividing (7) and (8) both of which vanish in this case.

In summary where $\partial\Psi/\partial n \neq 0$ and $\alpha \neq 0$ there is always a flux downstream to the force. If the force is vertical there is a net horizontal component downstream.

4. PRESENTATION OF THE STEADY STATE, TWO LAYERS PROBLEM (FIG. 2)

The case presented throughout is that of a uniform slope. This means that on two parallel planes (whether vertical or diagonal) the fluxes are identical at the same levels along the n-axis. Adding the condition of a steady state the same fluxes cross the soil surface and any other surface parallel to it. On a control surface of Fig. 2

$$-p \cos \alpha \, dA = q_n \, dA \tag{11}$$

P being the rain intensity over a horizontal surface above the soil.

Facoring out dA one gets:

$$q_n = -p \cos \alpha \tag{12}$$

But in eqs. (1) and (2) q_n and q_s has been found by Darcy law. They may be rewritten here for convenience with (12)

$$q_n = -p \cos \alpha = -K(\frac{\partial \Psi}{\partial n} + \cos \alpha) \tag{13}$$

$$q_s = K \sin \alpha \tag{14}$$

These three equations (two in 13 and one in 14) provide the whole basis for our further calculation. First more specific expressions may be found for (7), (8), (9), (10). (7) reads

$$\frac{1}{dn}\int \frac{q_s}{q_n} \, dn = -\tan \alpha \frac{\int K \, dn}{p \int dn} \; ; \tag{15}$$

(8) reads

$$\frac{\bar{F}_s}{\bar{F}_n} = \tan\alpha \frac{-\int dn/p}{\int \frac{dn}{K}} \tag{16}$$

The ratio between the overall flow direction and the overall force direction is

$$\frac{\int K\, dn}{(\int dn)^2} \int \frac{dn}{K} \geq 1 \tag{17}$$

which is in effect the ratio between the parallel average conductivity $\bar{K}$ and the serial average $\bar{K}_s$. Clearly this is the proof that on the average the medium on the whole is nonisotropic in the unsaturated state as well as in the saturated state and if the force deviates from the normal to the soil layers the flux will deviate even more.

The flux components in the x, z directions are by 13, 14 and 3, 4 for the steady state case

$$q_x = -K \frac{\partial\Psi}{\partial x} = \tfrac{1}{2}(K-P) \sin 2\alpha \tag{18}$$

$$q_z = -K(\frac{\partial\Psi}{\partial z} + 1) = -(P \cos^2\alpha + K \sin^2\alpha) \tag{19}$$

By simple geometrical relations remembering $(\frac{\partial\Psi}{\partial s} = 0)$

$$\frac{\partial\Psi}{\partial x} = \frac{\partial\Psi}{\partial n} \sin\alpha \tag{20}$$

$$\frac{\partial\Psi}{\partial z} = \frac{\partial\Psi}{\partial n} \cos\alpha \tag{21}$$

As a result of 20 and 21

$$\frac{\partial\Psi}{\partial x} = \frac{\partial\Psi}{\partial z} \tan\alpha = -(\frac{\partial\Psi}{\partial z}) \frac{\partial z_o}{\partial x} \tag{22}$$

if z_o is the elevation of some surface n = constant.
By substituting 22 into 18 we get still for q_x:

$$q_x = -K \frac{\partial\Psi}{\partial z} \tan\alpha \tag{23}$$

Now $\frac{\partial\Psi}{\partial z}$ can be substituted from 19 in one of two forms

$$\frac{\partial \Psi}{\partial z} = -(\frac{q_z}{K} + 1) = -(1 - \frac{P}{K}\cos^2\alpha - \sin^2\alpha) \tag{24}$$

Thus 23 becomes for the steady state case

$$q_x = (q_z + K)\tan\alpha = (K-P)\cos^2\alpha\tan\alpha \tag{25}$$

Note that during rain q_z has a negative value pointing down.

Integration of q_x over z gives the total horizontal discharge

$$Q_x = \int q_x\, dz = \tan\alpha\left[\int K\, dz + \int q_z\, dz\right] \tag{26}$$

or

$$Q_x = \int q_x\, dz = \tan\alpha\cos^2\alpha\left[\int K\, dz - p\int dz\right] \tag{27}$$

For convenience we define the averages

$$\bar{K}_x = \frac{1}{D}\int K(\Psi)dz \qquad -\bar{q}_z = \frac{1}{D}\int q_z\, dz \tag{28}$$

$$\bar{q}_x = \frac{1}{D}\int q_x\, dz$$

Then from 26:

$$\bar{q}_x D = D\,\bar{q}_z(\frac{\bar{K}_x}{\bar{q}_z} - 1)\tan\alpha = \bar{q}_z\, U\tan\alpha\, D; \tag{29}$$

$$U = (\frac{\bar{K}_x}{\bar{q}_z} - 1)$$

where $\bar{q}_z$ is the averaged downward vertical flow. From 27 one gets

$$\bar{q}_x\, D = P\, U^*\tan\alpha; \qquad U^* = (\frac{\bar{K}_x}{P} - 1)\cos^2\alpha \tag{30}$$

Eq. (29) gives the basic ratio between the horizontal and vertical components of flow, the basic subject of our discussion. Eq. (30) gives a similar ratio between the rain and the horizontal average flux which applies only on steady flow.

The horizontal flow component is really the needed figure. It is related to a unique coordinate system and can be integrated over a map. We shall therefore have to continue using it although the use of n, s coordinates is far more elegant.

In eqs. 29 and 30 there appeared again the basic relations that have been stipulated in the first part of the report (ZASLAVSKY and SINAI, 1978I) and later proved for splashing raindrops and surface tranzition zone (ZASLAVSKY and SINAI, 1978 parts II - IV).

Strictly speaking eq. 29 is proper also for non-steady state. Eq. 30 that relates the horizontal flux to the rain is valid only under steady state conditions. So far the uniformity of the slope is limited to the demand that $\partial\Psi/\partial s = 0$ or that this term is neglibible in the calculation of the first approximation of the vertical flow regime. Eqs. 29 and 30 are not limited to two-layered cyclic soil but to any soil with varying conductivity along the n-axis (whether because of soil properties or boundary conditions).

5. EXAMPLES AND ORDERS OF MAGNITUDE

5.1. Net recharge into deep groundwater

The net recharge is introduced into the soil in a form of seasonal pulses. However, below several wavelengths the pulses damp and the flow becomes practically steady. In the various equations P must be considered not as a rain but as the net groundwater recharge. If the net recharge is P mm per year and the moisture content is C then the wavelength is about P/C e.g. if the moisture content is 30% and the net recharge is about 300 mm the wavelength is 1 meter. A net recharge of 300 mm/year is about 10^{-6} cm/sec.

Consider now a series of soil layers changing from clay or rock with a conductivity around the rate of net recharge. In eq. 15 the ratio between the parallel and normal flux component is determined by $\int K\,dn$. It can be at least tan α when K = P in a perfectly uniform flow and up to several orders of magnitude if K changes from highly impermeable to very permeable layers as it is the case in some alluvial deposits.

5.2. Flow behind the wetting front

This flow can be nearly steady in relatively short times. Then as a first approximation the formulas using the rain P can be used.

5.3. The effect of a cumulative rain

The cumulative horizontal flux over a time will depend mainly on the cumulative rain. One can convince himself by expressing K in eq. 1 by q_n and $\partial\Psi/\partial n$ in eq. 2. Similarly in the first part of eq. 25 remembering that K increases roughly with $(-q_z)$ or with P. The cumulative vertical flux q_z in a point is related to the cumulative rain and thus also the cumulative horizontal flux.

It is thus expected as a rough approximation that regardless of the precipitation regime the horizontal flow will depend first and formost on the total precipitation. If this will be found true then there will be an explanation to the fact that in many areas runoff starts after a certain amount of rain has precipitated. This is to a great extent irrespective of the rain distribution and intensity.

To actually calculate the flow regime and the coefficient of anisotropy we shall have to introduce boundary conditions.

6. THE TWO LAYERS PROBLEM

From eq. 13 we get by solving for dn and integrating (steady state)

$$\cos\alpha\int dn = \int\frac{K(\Psi)}{P-K(\Psi)} \tag{31}$$

From eq. 19 one gets similarly along the z coordinate

$$\cos^2\alpha\int dz = \int\frac{K(\Psi)}{P-K(\Psi)} \tag{32}$$

figs. 3-5 The boundaries of integration are determined in figs. 3 to 5 that describe two layers at four flow stages.

Stage	First layer		Second layer	
	n-boundary	Ψ-value	n-boundary	Ψ-value
A. top and bottom layers	0	Ψ_1	$-D_1$	Ψ_2
completely unsaturated	$-D_1$	Ψ_2	$-(D_1+D_2)$	Ψ_1
B. saturation appears in	0	0	$-D_1$	Ψ_2
the top layer so that	$-D_1$	Ψ_2	$-(D_1+D_2)$	0
$\Psi_1 = 0$				
C. partial saturation so	0	Ψ_1	$-D_1$	Ψ_2
that $\Psi_2 < 0$; $\Psi_1 > 0$;	$-n_1$	0	$-(D_1+D_2)$	0
$c-n_1$ is saturated	$-D_1$	Ψ_2	$-(D_1+D_2)$	Ψ_1
D. both layers are comple-	0	Ψ_1	$-D_1$	0
tely saturated	$-D_1$	0	$-(D_1+D_2)$	Ψ_1
$K_{1s} < P < K_{2s}$				

The actual values of Ψ_1 and Ψ_2 can be found from an implicit equation as a function of the rain P.

The solution can be easily found by introducing a relation $K(\Psi)$ for the two layers then $\Psi(n)$ or $\Psi(z)$ is found as well as $K(n)$ or $K(z)$. From these, in turn, one can calculate $\bar{Q}_x$ from (27) or $\bar{Q}_s$ by integrating (14).

As an example consider the experimental relation for the i^{th} layer:

$$K_i = K_{si} \exp\left[a_i(\Psi+\Psi_o)\right] \qquad \text{for } \Psi+\Psi_o < 0$$

$$K = K_{si} \qquad \text{for } \Psi+\Psi_o \geqslant 0 \tag{33}$$

Introducing (33) into (31) one gets:

$$\cos\alpha \int dn = \frac{1}{a} \ln\left[P - K_o\ a(\Psi-\Psi_o)\right]_q^r \tag{34}$$

where q and r are two consequetive points.

The unsaturated conductivity at any point R is expressed as a function of the elevation difference from a known point q and as a function of the conductivity at that point.

$$K = K_q + (P-K_q)\left\{1 - \exp\left[a \cos \alpha(n-n_q)\right]\right\} \qquad (35)$$

K of (35) can be now introduced into the equaitons for calculating the horizontal or parallel flux. At the interfaces the K-values are related to each other by putting Ψ the same on both sides of the interface.

The coefficient of anisotropy $\left[(\bar{K}_x/\bar{q}_z) - 1\right]$ has been calculated as an example for two layers where

$$\frac{D_2}{D_1} = \frac{10}{6}; \quad \frac{K_{2s}}{K_{1s}} = 10 \qquad \frac{a_2}{a_1} = 6$$

with $\Psi_o = 0$. The absisa in Fig. 6 gives the rate of rain P. Two specific points are given: P^s where saturation appears at the surface and another where there is no latteral flow P = 0.63. This rate of rain equals exactly the point where the unsaturated conductivities of the two layers become identical. This unique point of seemingly zero anisotropy is unique for a two-layer problem. For multi-layer problems it disappears. The same method can be used to calculate a transition layer at the soil surface if it is represented by a sequence of layers, each uniform and isotropic and varying slightly from its neighbouring layers. Some of the details of such a study are interesting. For example, if the saturated hydraulic conductivity reduces monotonically with depth there must be a maximum in the pressure head distribution within the surface transition. On increasing gradually the rain, saturation will appear first within the surface transition at this point of maximum pressure. The place of saturation will move upward as the rain intensity will increase. These and others can be easily proven however, the details are beyond the scope of this report.

It is instructive that the coefficient of anisotropy increases with the rate of rain to a power higher than one (at least in the two-layer problem). Therefore the latteral flow increases with the rain to a power higher than two.

7. WATER CONCENTRATION AND OUTCROPPING

The latteral flow due to splashing of raindrops and due to surface transition layers was more nearly following the rain as if it is steady and its rate was proportional to the soil surface slope. In the present part of the report any layering is considered which does not have to be parallel to the soil surface. Therefore the moisture will concentrate not necessarily at concave parts of the landscape.

There are many cases where the soil layers are more or less parallel to the soil surface. This is the case in sand dune. Observation will discover such repeated layering sometimes few millimeters thick due to segregation of particle by size and due to slight chemical and physical stabilization of freshly deposited layers. Where the soil layers are parallel to the surface the moisture concentrations due to splashes surface transition and layering all add up.

There will be many more cases of other changes in the soil layering. The most common is that of truncated layers due to an excavation or an erosion cut. In an unsaturated soil the diagonal streamlines reach near the soil surface and cannot come out. They then bend downward and accumulate until saturation forms and seepage out of the soil starts. Geological faults have been known to produce water outcrops as springs. This was hard to explain when no saturated water table could be observed away from such faults. The latteral unsaturated flow as above renders an explanation.

In pollution studies one wishes to follow the actual flow path of the water. Thus one can predict that infiltrating polluting water can move laterally large distances before they reach groundwater. In one place more than one hundred meters of interlayering of clay lenses and sand was above the groundwater table. It was estimated that a point source of polluting water would spread materially so long as some ponded water will remain within the sandy layers. Then the coefficient of anisotropy will be of the order of the ratio between the sand and clay conductivity. The slope was measured in percents and the anisotropy in thousands. Thus the horizontal travel was expected to be several hundreds meters to several kilometers before the pollutant would reach the water table. Observations tended to confirm the qualitative prefunction of the elevation difference from a known point q and as a function of the conductivity at that point.

$$K = K_q + (P-K_q)\left\{1 - \exp\left[a \cos \alpha(n-n_q)\right]\right\} \tag{35}$$

K of (35) can be now introduced into the equaitons for calculating the horizontal or parallel flux. At the interfaces the K-values are related to each other by putting Ψ the same on both sides of the interface.

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An interesting application of the latteral flow with respect to pollution may be concieved as follows. A proper convex cover of soil layers can act as a 'straw roof'. Though highly permeable it can prevent the penetrating water from leaching through a source of pollution. Very often engineers look only for impermeable materials to prevent flow through a structural element. Here there is another possible way of obtaining such a protection.

REFERENCES

ZASLAVSKY, D. and G. SINAI. 1978. Surface Hydrology. I. Field observations that require explanation. II. Lateral flow due to rain drop splashes. III. Rainfall effective distribution. IV. Lateral flow in the soil surface - qualitative considerations.

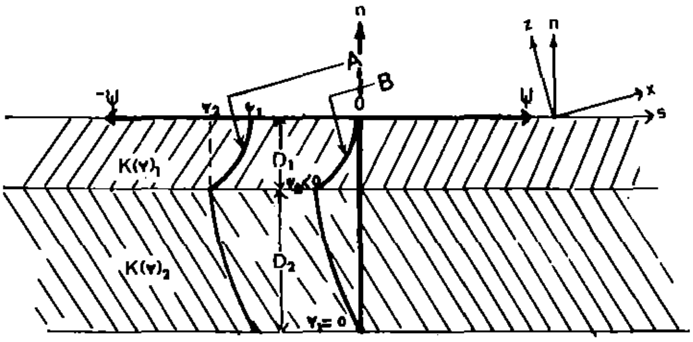


Fig. V.3

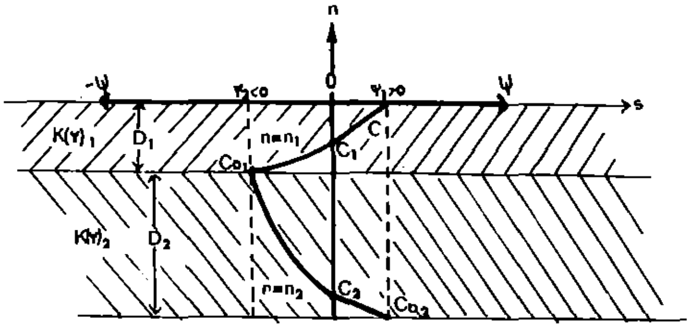


Fig. V.4

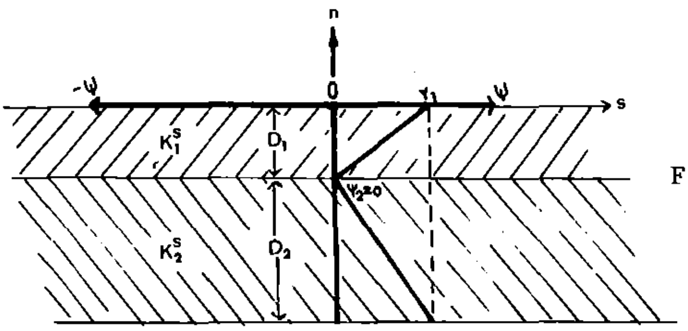


Fig. V.5

Fig. V.3. Two layers pressure head distribution, state A no saturation and state B initial saturation

Fig. V.4. Two layers pressure head distribution, state C - partial saturation

Fig. V.5. Two layers pressure head distribution, state D - total saturation

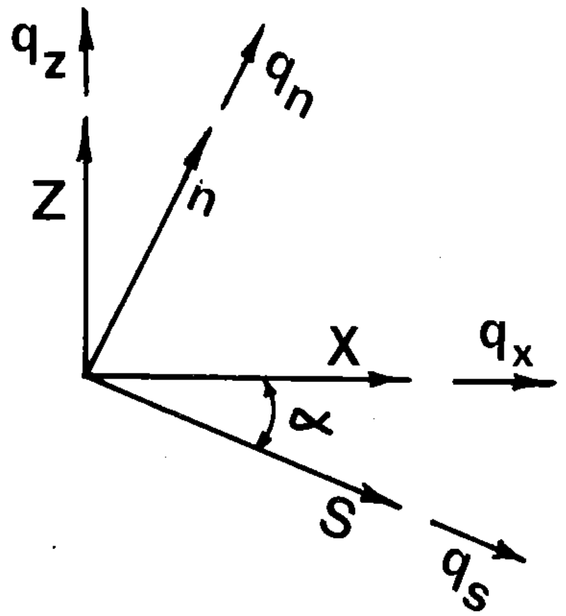


Fig. V.1. Upright and sloping coordinate systems and related flux components

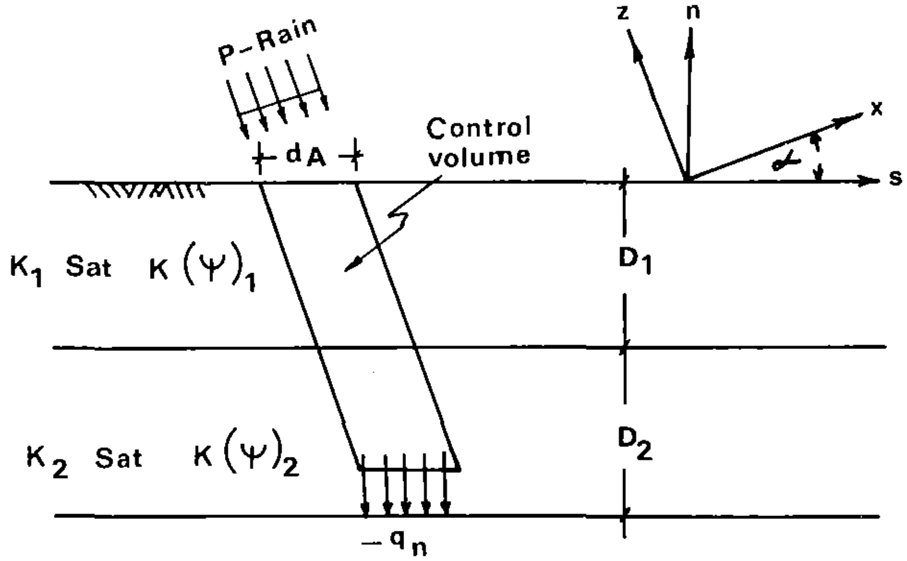


Fig. V.2. Control element in a two layer flow problem
P - rain intensity; K_{sat} - saturated hydraulic conductivity; $K(\psi)$ - hydraulic conductivity at negative pressure head ψ; 1, 2 two layers; slope α

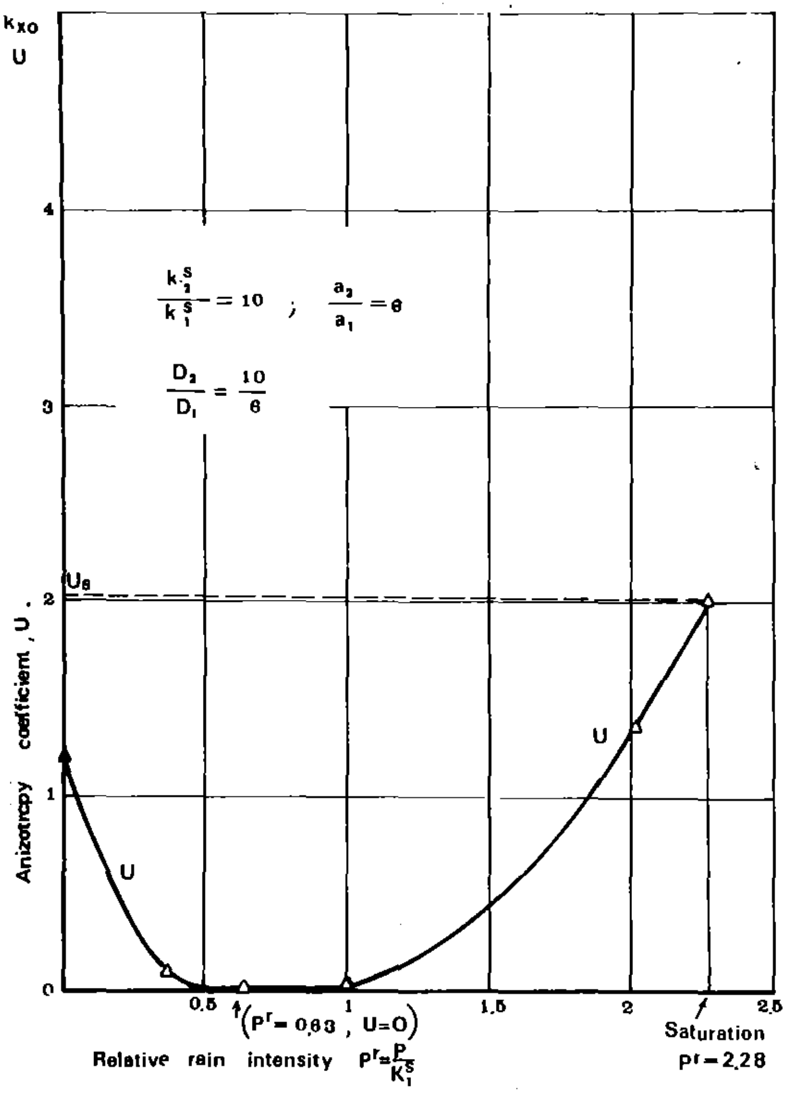


Fig. V.6. Anisotroby coefficient U as a function of rain intensity P for cyclic two layers problem

VI. NON-STEADY TRANSITION LAYER FLOW-NUMERICAL SOLUTION

ABSTRACT

A numerical solution by a mixed finite element and finite difference method is used for studying a two dimensional flow regime with a non uniform soil and rain. Specifically the soil has a sinusoidal surface and a transition layer from soil bulk to the air. The transition is by 8 layers each uniform and isotropic. The hydraulic conductivity changes stepwise from 10^{-5} cm/sec. to 100 cm/sec. The porosity changes stepwise from 0.4 to 0.9. A wide range of problems has been run with the thickness of the transition layer varying between 0.45 to 0 the half cycle length of the sinusoidal varying from 40 meters to 0.2. meter. The rain changed from 0.01 to 4 cm/hour.

The first group of conclusions to be drawn from the numerical result are following. Without a transition layer saturation will appear only if the rate of rain exceeds the hydraulic conductivity. It occurs after some small amount of rain and immediately associated with water flow above the soil. With a transition layer it takes more rain the thicker is the transition layer. However even rain which is much smaller than the hydraulic conductivity can produce saturation and seepage out of the soil.

The total rain necessary to produce saturation is almost the same for a range of rain intensities varying 400 fold.

1. INTRODUCTION

In previous parts of this report (ZASLAVSKY, SINAI, 1978 I - V) it has been established that rain is associated with a horizontal flow component at the soil surface. This component is proportional to the land's slope and to the rain itself to some power greater than one. This horizontal flow component in turn, causes an accumulation of moisture in concave parts of the landscape. Field observations have shown such accumulations. Contrary to existing notions the horizontal flow occurs with a low rate rain that does not necessarily exceeds the hydraulic conductivity of the soil. It happens in a non-saturated soil as well as in a saturated one and does not require an underlying impermeable layer or the vicinity of a watertable.

Three mechanisms have been shown to contribute towards the formation of a horizontal flow.

a. The splashing of raindrops over a sloping land
b. Infiltration into a soil surface transition layer, in which the hydraulic characteristics change gradually from the soil bulk to the air
c. Layered subsoil

The theory was advanced for steady state infiltration. The raindrop splashing mechanism follows by a split second the changes in the rain. Following the gradual changes of the soil conditions it can be taken as a quasi steady state process. The flow through a layered subsoil tends to have more moderate time changes with deeper soil and more uniform rain. The flow through the surface transition layer is probably the farthest away from the steady rate solution.

The analytical treatment in the previous parts of this report considered a uniform slope. The curved soil surface has been taken as first order changes only leaving the basic phenomenon as on a uniform infinit slope.

It is the purpose of the following to treat a more realistic problem of a non steady flow on a curved soil surface. A numerical method has been adapted for this purpose.

Powerful as the numerical method was it was hardly sufficient for a two dimensional non steady state problem. A sinusoidal soil surface has been treated with varying half cycle length L and amplitude A. Different rain intensities P have been tried. The thickness of transition layer δ was also changed. The purpose was to check some of the previously drawn conclusions (some well proved and others more tentative).

Among these conclusions were:

- The latteral rate of flow is roughly proportional to the slope
- There is moisture concentration proportional to the concavity of the landscape
- Saturation will first appear within the transition zone at its most concave part, regardless of the rain intensity
- Early during wetting of the soil by rain the hydraulic force tends to be orthogonal to the soil surface and so is the flow. There is always a downhill flow component parallel to the soil surface.

  However the horizontal component may be temporarily uphill
- As the wetting front moves deeper into the transition layer the net horizontal flow becomes downhill
- The accumulation of water in concave parts of the landscape continues after the rain has ceased (drainage time)
- The total excess accumulated moisture depends mostly on the total rain and less on its momentary intensity
- After saturation appears within the transition zone it can spread upward and sideways. Eventually it leads to seepage of water out of the soil in the form of runoff
- The fact that the rain gets first into the soil, even when it is of a high intensity, and then comes out is of a general significance but especially in accounting for interaction between runoff and erosion.

The present and next part of this report are meant to retest these conclusions. The numerical solution serves as a simulative experimental tool. The fundamental laws of this simulation are well tested throughout the literature of hydrology and soil physic. The only questionable part is the accuracy of the numerical solution.

2. DESCRIPTION OF THE SIMULATION MODEL

A sinusoidal soil surface was described as in fig. 1. A transition layer is underlied by a thick uniform soil (H). The shape of the soil surface is given by its amplitude z.

$$z = (H + A) + A \cos \left(\frac{x}{L} \pi \right) \qquad (1)$$

H is the sublayer depth which was in all problems 10 meters. The amplitude A varied in the different problems between 2×10^{-4} meter to 2 meters. However most problems were of A = 2 meters. These far apart values where chosen to produce a given range of slopes and curvatures while the half cycle L had the values 40, 20, 2 and 0.2 meters (see table).

The boundary conditions are as follows:

- A vertical line below the hill's top is a symmetry line and thus a streamline
- A vertical line below the valley's bottom is a symmetry line and thus a streamline
- The initial condition is an hydrostatic state throughout the profile with the phreatic surface $\psi = 0$ at $z = 0$.
  ψ - pressure head and z - elevation

A case could be argued to substitute the true state of hydrostatic equilibrium by a state of field capacity. However the latter state depends on the history of its attainment. It has therefore been postponed for the stage when a long term regime would be digestable by the computers.

fig. 1 The transition layer has been represented by 8 layers more or less parallel to the soil surface and of the same thickness. More exactly the grid points representing the separate layers were chosen in a way that will assure a uniform thickness of the layer in a direction normal to the soil surface. Thus the subsequent sublayers do not follow exactly eq. (1) less a fixed depth (fig. 2.).

fig. 2 Each layer was considered uniform and isotropic. There is a stepwise change in the hydraulic properties of the different layers.

The detailed values of porosity, retention curves and a saturated hydraulic conductivity are given in figures 3 and 4.

fig.3
fig.4

The data for layers 4-8 have been taken from actual soil data. The data for layers 1, 2 and 3 have been chosen artificially maintaining the following trends

a. The saturated hydraulic conductivity increases exponentially

layer number:	1	2	3	4	5	6	7	8
saturated hydraulic conductivity cm/sec.	100	10	1	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}

b. The rate of conductivity decreases with increased suction increase as the saturated conductivity increases.

c. The crossing of conductivity curves for the layer pairs 1 - 2, 2 - 3, 3 - 4 etc. follow a monotonic order.

d. The moisture content at saturation approaches 100% at the top layer

layer number	1	2	3	4	5	6	7	8
porosity	0.9	0.85	0.8	0.75	0.7	0.6	0.5	0.4

e. The rate of moisture decrease due to suction increases with porosity

f. The crossing of moisture curves for the layer pairs 1 - 2, 2 - 3, 3 - 4 etc. follow a monotonic order.

3. THE DIFFERENTIAL EQUATION

The differential equation is

$$\mathrm{div}\; q_i = - \frac{\partial \Theta}{\partial t} \qquad (2)$$

q_i - the ith flux component
Θ - volumetric moisture content
t - time

The flow equation is

$$q_i = - K_{ij} \nabla\phi_j \tag{3}$$

K_{ij} = the hydraulic conductivity capable of being a non-isotropic tensor

ϕ = the hydraulic head $\phi = z + \psi$

z = the elevation

ψ = the pressure head

Putting (3) in (2) one has explicitly

$$L(\psi) = \frac{\partial}{\partial x_i} K^R(\psi)\, K^s_{ij} \frac{\partial\psi}{\partial x_j} + K^R(\psi)\, K^s_{ij} -$$

$$\left[\frac{\partial\Theta}{\partial\psi} + \beta S_s\right]\frac{\partial\psi}{\partial t} = 0 \qquad X_j = x, z \tag{4}$$

In reality while the algorithm was prepared to take non-isotropic media the problems run here where all of isotropic layers. In the problem encountered here the anisotropic behaviour is only the result of the overall average behaviour of the layered soil. The outstanding difference between this model and some that have been claimed (although not computed) in the past is that the anisotropy is not a fixed property of the soil. The anisotropy changes with the moisture regime itself. This is contrary to the inference of eq. (4) where the whole change with moisture is attached to the isotropic term $K^R(\psi)$. The non isotropy is independent of the moisture regime.

The duality found between a layered soil and a non isotropic one on the average lends us to believe that any anisotropy in the hydraulic conductivity in the soil will be found to be moisture dependent or more generally flow dependent.

4. THE NUMERICAL METHOD - BRIEF DESCRIPTION

The solution was by finite element method. It has been based on a combination between a variational principle (RITZ Method) and a weighed residue (GALERKIN Method) as described by NEUMAN ET AL (1972-1975).

The method is applied by minimizing a proper functional letting the coefficient of $(\partial\psi/\partial t)$ vanish at saturation and $K^R(\psi) = 1$ at saturation (eq. 4). At any given moment the solution $\psi(x,z,t)$ may be described by a complete set of functions $\xi_n(x,z)$ and $\psi_n(t)$ time dependent coefficients.

$$\psi(x,z,t) = \sum_{nsi}^{\infty} \psi_n(t)\xi_n(x,z) \tag{5}$$

In reality a finite sum of N terms renders ψ^N which is the projection of ψ in a N dimensional space. Therefore $L(\psi)^N$ is orthogonal to any one of the vectors ξ_n making this space.

ψ^N converges to ψ if

$$\lim_{N \to \infty} //\psi - \psi^N// = 0 \tag{6}$$

where the norm of a function f, // f // is defined as

$$//f//^2 = \int f^2 \, dV \tag{7}$$

V being the volume over which f is defined. For a finite N $\lim //\psi - \psi^N// = \min.$ and in our case

$$//L(\psi)^N - 0// = \min \tag{8}$$

The problem is to find the coefficients ψ_n for ξ_n by the following

$$\frac{\partial}{\partial\psi} \int_V \left[L(V)^N\right]^2 dV = 0 \tag{9}$$

This GALERKIN method fits a given moment t. The derivative $\partial\psi/\partial t$ must therefore be determined independently. The more interested reader is directed to the original article by NEUMAN ET AL (1974).

The grid points for solution are given in fig. 5, the upper part giving the details of the transition layer.

fig. 5 There were three stages of the calculation

a. Preparatory program calculating the grid coordinates for each problem the boundary conditions and the material properties
b. The main multilayer calculation in terms of the pressure head, moisture content total head, incoming and outcoming discharges. It was based on previously written program (NEUMAN ET AL, 1974) and was adopted to the present problems
c. Data processing and plotting as may become more clear in the following.

5. ESTIMATE OF THE SOLUTION QUALITY

The semi implicit method of calculation with a system of linear equation enables a convergence of the solution. The choice of the time step t affects the manner of convergence. A stepwise change in the influx of rain or outflow of water is expected to introduce a monotonic gradual change inside the flow medium. Too large Δt steps cause fluctuations that gradual decay towards the true value. A stepwise gradual change in the rain over a small number of time steps can eliminate the problem. We shall not go into details of the analysis. Eventually the time step has been chosen by a trial and error method choosing a time step

$$\Delta t_I = t_2 - t_1 \tag{10}$$

and after the first itteration checking

$$\Delta t_{II} = \frac{\psi_2 - \psi_1}{K_2 - K_1} \tag{11}$$

and comparing t_{II} with t_I changing until $t_I < t_{II}$. There were of course sensitive points in the field that determined the necessary time steps.

The density of grid points must also be related to the rates of change, the typical time constant being $(\Delta x/K) \sim \Delta t$. The density of grid points was much lower than this criterion would call for. A series of tests was made on the influence of grid points placing on the solution. The largest deviation from a denser grid was found near a wetting front. Typical errors in head were 1 - 6 cm or a relative error of ½%. Taking even the limited head difference over the whole transition layer as little as 100 cm (from the hilltop to the valley bottom) the error makes some 1 - 6% at its maximum.

Where the rain flux q was locally larger than the hydraulic conductivity K the time step is determined by the flux $(\frac{\Delta x}{q} < \frac{\Delta x}{K})$. The difficult problems to solve were therefore those with thin transition layer (small Δx) and high rain intensity. In fact they take proportionally more time steps to reach the same total rain.

The part of a problem that gave most difficulties in computation was the seepage appearance at the soil surface.

The grid density was increased horizontally from 11 to 21 column with a maximum change of only 1/4% at the wetting front (not more than 3 cm of water). This is while the accuracy of the whole calculation was to 1 cm of water. The potential increase in accuracy (doubling the number of points indicated about 4 fold increase in price (from about 1000 U.S. dollars to 4000 U.S. dollars per run).

The accuracy tests were run on several problems with different geometrical scales and rates of rain.

Finally the moisture content profile at different times (fig. 6) and the surface moisture at different times (fig. 7) was plotted and would compare well with classical one dimensional solutions of infiltration. The run presented in fig. 6, 7 is from problem 438 with no transition layer and a very low rate rain of 0.01 an hour. This value can be related to the conductivity of the soil (10^{-5} cm/sec.)

and the final moisture content found from fig. 4. It compares well with figs. 6, 7. The transition is reasonable (BRAESTER 1973). However one cannot expect a perfect fit, at least because there is some latteral movement of moisture even in soil with no transition layer. This is also why in fig. 7 there are two different curves for uphill (x = 0) and the bottom (x = 20).

In conclusion there is no way to ascertain beyond a shadow of doubt that the numerical solutions are perfectly accurate. However after the computation of some 60 different problems with various changes in the parameters the solution seems to be well behaved and the resultsmake sense. Furthermore the present demand from the solution is far less than a perfect numerical accuracy. Rather it is required only that trends will be properly indicated. It is used as an experimental tool. The conclusions from these numerical experiments should compare with a number of analytical conjectures. made before and a large volume of observations (ZASLAVSKY and SINAI 1978 I - V).

Following are the lists of problems solved and some of the results.

tabel

6. COLUMNS' EXPLANATION AND NOTES ABOUT RESULTS

Column 1. The problems mentioned here are of wave length L 20 m 40 m 2 m and 0.2 m

Column 2. The amplitude of soil surface varied within each wave length. The first large group of L = 20 m has the same amplitude 2 meters.

Column 3. The thickness of transition layer δ. In the first group of L = 20 m it is 0.45 m and 0 (the last being no transition)

Column 4. The rain intensity P. In the first group of L = 20 m it is mainly P = 4 cm/hour, 1 cm/hour 0.01 cm/hour and an intermittent rain (problem 460). Note that the hydraulic conductivity of the subsoil is 10^{-5} cm/sec. = 0.036 cm/hour. Thus the lowest intensity is 3.6 times smaller and the highest is 270 times higher. In some problems the rain has been increased gradually stepwise thus P gives only the intensity of the main rain. Therefore, it can easily be checked that the product of the time T and intensity P does not produce the total rain D.

Columns 5,6 These give the average slope A/L and the curvature A/L^2. These have an important effect but not necessarily on the time and total depth to saturation, which are the only results recorded in this table.

Columns 7-9 The total rain Depth D_{sat} the Time T_{sat} and a number of calculation time steps TS_{sat} to the first appearance of saturation within the profile. In all cases of transition layers concentration was observed towards the concave part. The reason for no saturation shall be discussed in each case.

Columns 10-13 Total Rain Depth D_{seep} time T_{seep} and Calculation steps TS_{seep} for the spreading of saturation up to the soil surface and seepage in problems with transition layers. In problems with no transition layer this time and the time for saturation become identical. In thinner transition layer there is a tendency to decrease the time difference between first saturation and seepage.

Columns 16-18 Total rain D_f - D_d Time T_f and a number of calculation steps TS_f to the end of the computer run. In some problems the run was stopped too nearly for anything to happen. This can be easily identified. Still they were brought because they served to see some of the trends with no exception to the conclusions that will be drawn in the following.

7. SOME OF THE MAIN RESULTS AND CONCLUSIONS

The details of the flow regime as found by the calculation will be presented in the following part of the report. Here only some of the aspects that stand out from the problem table will be discussed.

The appearance of saturation within the transition layer is the first outstanding result. It appeared with the low rain (problem 522) the high rain (problem 388) with the highest rain (problem 815) and the intermittent rain (problem 460).

In a soil without a transition layer the saturation appeared when and only when the surface was flooded by a high rain. (problem 533 $D_{sat} = D_{seep}$). It has never appeared with the low rain (problem 438). This is just as predicted by the classical theory.

At intermediate transition thicknesses (15 cm in problems 752 A, 752 F and 5 cm in problem 536) the times for total rains for saturation are intermediate between the tick transition layer (45 cm) and no transition layer (533).

Some of the results for saturation appear in fig. 8.

There are several facts standing out.

a. There is hardly any difference in the total rain necessary for saturation over an extremely wide range of rains. (problems 522, 388, 812)
b. The main effect is of the thickness of transition layer. For high rain intensity there is proportionality between the thickness of layer and the total rain depth at first saturation.
c. At thinner layers and very low rates of rain no saturation will appear.

A similar conclusion is to be drawn for the appearance of seepage (saturation reaches the soil surface). Two trends stand out:

a. At lower rate of rain the total rain necessary to reach seepage is larger.
b. The effect of a thick transition layer is to increase considerably the total rain at seepage if the intensity is high enough. At low rain intensity, very thin layers will never reach the state of seepage.

These results are quite reasonable. There is a latteral movement of moisture that tends to increase the total duty of added water in the concave valley. However the question whether there will appear saturation and seepage depends on the interplay between several mechanisms. The added moisture spreads over a given soil depth. This depth increases with prolonged times. For a given total rain the wetting depth is larger with a low rate of rain. Thus with low rates of rain there is more time for latteral motion but also more time for deeper moisture distribution.

The latteral discharge depends on the total thickness of wetted non isotropic or layered soil. Thus with thin transition layers the discharge cannot be as high as with thick transition layers. This is true at least over long enough time where the wetting penetrates beyond the thin transition layer. In a tick transition layer the latteral discharge can grow over some time. Therefore thin layers may require less rain to reach a saturation point but with low enough rate of rain may not reach saturation at all.

Concentration of water in concave parts of the landscape is proved under non steady state flow regime. This concentration can reach saturation and produce seepage and runoff. The known experience of farmers is simulated here that deep permeable soil surface and a level field delay runoff. The solution also simulates the well known experience in many places that regardless of the detailed flow regime sizable runoff would start only after a certain amount of precipitation accumulated.

Contrary to this observation in areas of very thin transition layers (e.g. soils with smooth surface crust) the saturation and seepage will be produced only at high rates of rain. However such conditions do not exclude moisture concentration even at a very low rate of rain.

The above arguments and some more aspects of the flow regime will follow in the next part of this report.

REFERENCES

BRAESTER, V., 1973. Moisture variation of the soil surface and and the advance of the wetting front during infiltration of constant flux. Water Resour. Res. Vol. 9, no. 3: 687-694

NEUMAN, S.P., 1972a. Finite element computer program for flow in saturated - unsaturated media. Second annual report, part 3. Project no. A10 - SWC - 77. Hydraulic Eng. Lab., Technion, Haifa, Israel.

_____ 1974a. Galerkin approach to unsaturated flow in soils in finit element methods in flow problems (ed) Oden, J.T. at al. VAH Press the University of Alabama in Huntsville, Alabama: 517-522 197

_____ R.A. FEDDES and E. BRESLER, 1974b. Finite element simulation of flow in saturated - unsaturated soil considering water uptake by plants. Third annual report (part 1) Project no. A10 - SWC - 77. Hydraulic and Hydrodynamic Eng. Lab., Technion, Haifa, Israel

_____ R.A. FEDDES and E. BRESLER, 1975. Finite element analysis of two-dimensional flow in soils considering water uptake by roots. I. Theory Soil Science Am. Proc. Vol. 39 no. 2: 224-230

ZASLAVSKY, D. and G. SINAI, 1978. Surface Hydrology. I. Field observations that require explanation. II. Lateral flow due to rain drop splashes. III. Rainfall effective distribution. IV. Lateral flow in the soil surface - qualitative considerations. V. Steady lateral flow in a layered soil

Problem Number	Main Geometric Data			Rain	Geometric Ratios		Main Results: Saturation			Main Results: Seepage			Program Data: End of Rain (Drainage)			Program Data: End of Rain			Notes
							Total Rain	Time	Time Steps	Total Rain	Time	Time Steps	Total Rain	Time	Time Steps	Total Rain	Time	Time Steps	
Notation	L	A_m	δ	P	A/L	A/L^2	D_{sat}	T_{sat}	TS_{sat}	D_{seep}	T_{seep}	TS_{seep}	D_d	T_d	TS_d	D_f	T_f	TS_f	
Units	M	M	M	cm/hour			cm	hours	-	cm	hours	-	cm	hours	-	cm	hours	-	
Notes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
388	20	2	0.45	1.0	0.1	0.005	14.9	14.9	107	21.3	21.3	153	206	206	602	206	1388	1140	
538	20	2	0.45	1.0	0.1	0.005	13.8	13.8	85	19.1	19.1	102	207	207	845	207	1390	1597	
539	20	2	0.45	1.0	0.1	0.005	13.8	13.8	85	18.75	18.75	100	207	207	899	207	1390	1815	
522	20	2	0.45	0.01	0.1	0.005	15.5	1550	163	60	6000	15.5	120	18000	894	120	20000	986	
815	20	2	0.45	4.0	0.1	0.005	17.2	5.8	60	23.2	7.3	109	70	19	297	70	130.5	520	
460	20	2	0.45	(A)	0.1	0.005	15.0	31.5	142	-	-	-	-	-	-	20	34	152	intermittent rain
752 A	20	2	0.15	1.0	0.1	0.005	7.1	7.1	22	7.7	7.7	24	-	-	-	17.2	17.2	102	
752 F	20	2	0.15	0.01	0.1	0.005	8.0	800	45	12	1200	50	50	15075	470	150	100000	741	
536	20	2	0.05	1.0	0.1	0.005	2.6	5.4	22	3.4	6.2	24	-	-	-	17.2	20	293	stepwise rain
438	20	2	0	0.01	0.1	0.005	-	-	-	-	-	-	130	20000.	810	130	100000	970	
533	20	2	0	1.0	0.1	0.005	0.97	6.7	80	0.97	6.7	80	2	14	81	2	100	24	
535	40	12	0.45	1.0	0.3	0.0075	14.3	14.3	101	19	19	195	40	40	483	40	150	630	
535 L	40	12	0.45	0.01	0.3	0.0075	-	-	-	-	-	-	-	-	-	0.02	2	3	too short rain
521	2	0.2	0.45	1.0	0.1	0.05	10.4	10.4	33	24.4	24.4	88	100	102	202	100	1000	569	
521 K	2	0.2	0.45	0.01	0.1	0.05	-	-	-	-	-	-	-	-	-	0.02	2	8	too short rain
379	2	0.02	0.45	1.0	0.01	0.005	16	16	65	38	38	155	120	121	341	120	1000	659	
379 I	2	0.02	0.45	0.01	0.01	0.005	-	-	-	-	-	-	-	-	-	100	10000	348	
378 A	2	0.02	0.15	1.0	0.01	0.005	7.5	7.5	13	14	14	26	50	52	75	50.	600	194	
378 H	2	0.02	0.15	0.01	0.01	0.005	-	-	-	-	-	-	-	-	-	600	60000	757	
520 A	2	0.2	0.15	1.0	0.1	0.05	5.6	5.6	12	8.4	8.4	15	100	105	82	100	600	181	
520 J	2	0.2	0.15	0.01	0.1	0.05	-	-	-	-	-	-	160	16000	286	160	30000	403	
523	2	0.2	0.05	1.0	0.1	0.05	2.4	4.4	49	2.9	4.9	54	38	40	157	38	200	221	
537	2	0.02	0.05	1.0	0.01	0.005	3.2	6	24	5.2	8	29	42.2	50	123	42.2	80	129	
537 B	2.	0.02	0.05	0.01	0.01	0.005	-	-	-	-	-	-	300	30200	529	300	50000	628	
524	2	0.02	0.005	0.5	0.01	0.005	2	8.2	40	2.3	8.8	46	3.9	13	79	3.9	100	102	
525	0.2	0.02	0.15	1.0	0.1	0.5	7	7	25	15	15	45	100	105	188	100	500	167	
525 P	0.2	0.02	0.15	0.01	0.1	0.5	-	-	-	-	-	-	500	50000	623	500	150000	706	
526	0.2	$2x10^{-4}$	0.15	1.0	10^{-3}	$5x10^{-3}$	7.6	8.8	25	-	-	-	18.3	18.5	61	8.3	50	99	
527	0.2	0.1	0.025	1.0	0.5	2.5	4.2	4.2	19	4.6	4.6	21	-	-	-	520	1304	444	
528	0.2	0.02	0.025	1.0	0.1	0.5	2.5	6.3	30	3.8	7.1	34	524.2	932	342	524.2	3000	474	
529	0.2	$2x10^{-4}$	0.025	0.5	10^{-3}	$5x10^{-3}$	8.1	19.6	53	9.35	22.11	85	9.35	22	85	9.35	100	207	
530	0.2	0.1	0.005	0.5	0.5	2.5	2	8.6	42	2	8.6	42	5.7	17	80	5.7	100	107	
531	0.2	0.02	0.005	0.5	0.1	0.5	2	8.6	42	2.3	9.2	45	6.2	18	81	6.02	3000	252	
531 U	0.2	0.02	0.005	0.01	0.1	0.5	-	-	-	-	-	-	300	3000	619	300	150000	799	
532	0.2	$2x10^{-4}$	0.005	0.5	10^{-3}	$5x10^{-3}$	2.3	7.8	46	2.3	7.8	46	2.4	8	48	2.4	50	108	
532 W	0.2	$2x10^{-4}$	0.005	0.01	10^{-3}	$5x10^{-3}$	-	-	-	-	-	-	300	30000	547	300	800000	726	
534	0.2	0.1	0	0.2	0.5	2.5	0.62	4.1	19	-	-	-	1.4	9	40	1.4	100	189	

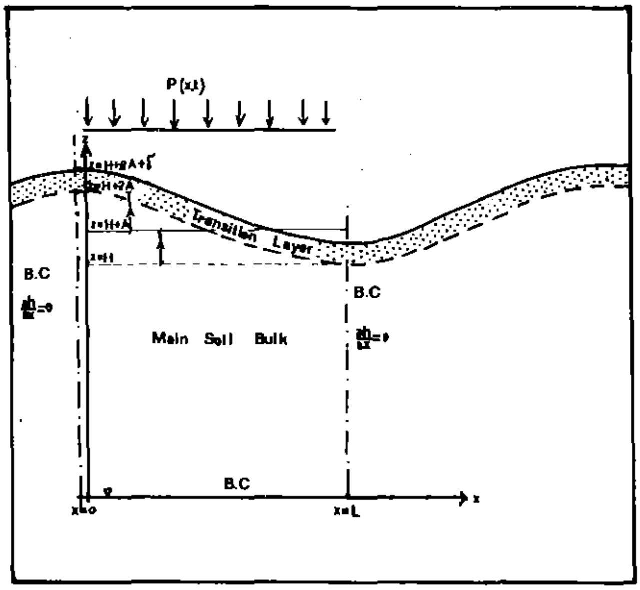


Fig. VI.1. Geometry and boundary conditions of the flow problem

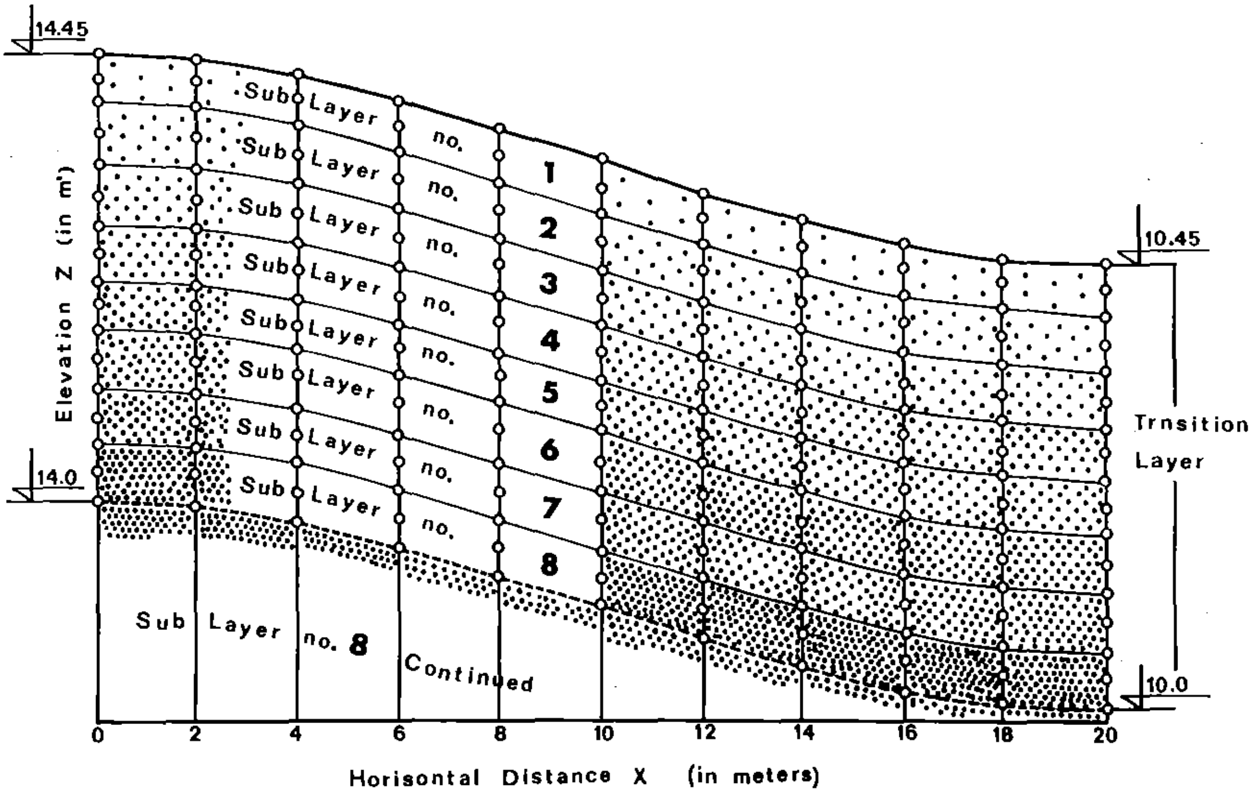


Fig. VI.2. Setup of sub-layers to simulate the surface transition

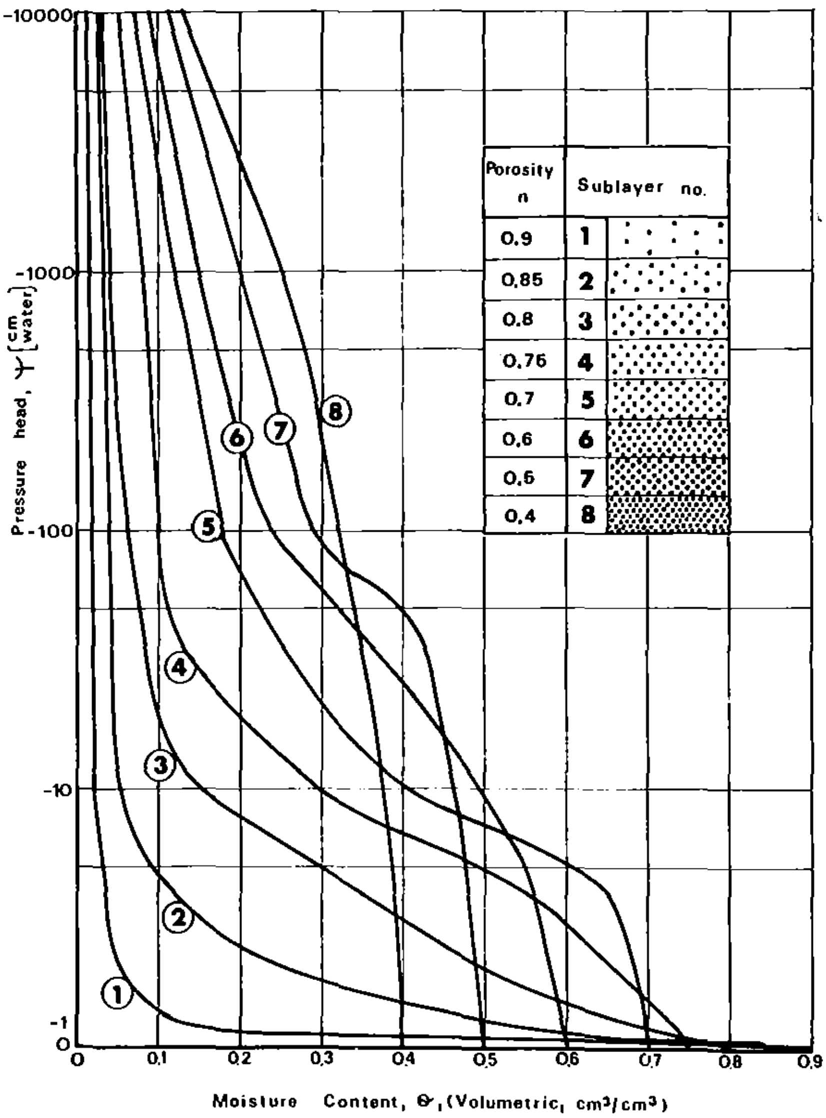


Fig. VI.3. Moisture retention curves of 8 sub-layers constituting the surface transition

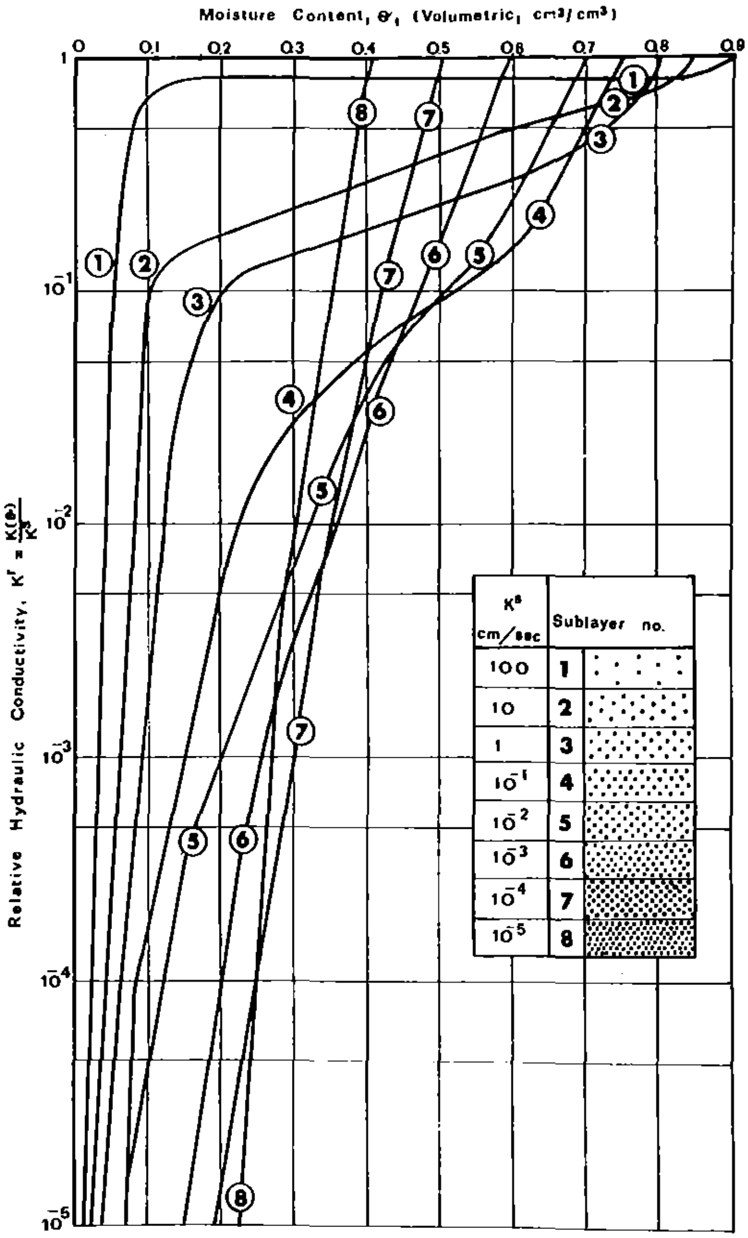


Fig. VI.4. Hydraulic conductivity as a function of the moisture content K(θ) in the 8 sub-layers constituting the surface transition

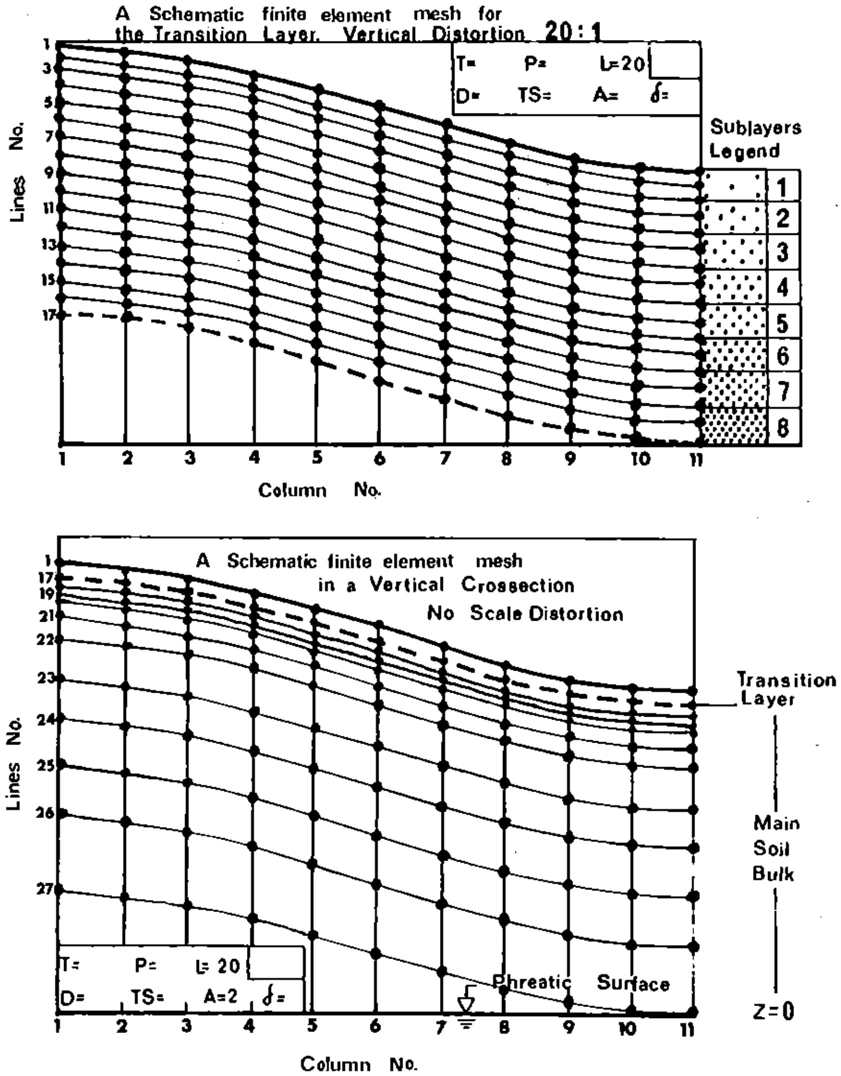


Fig. VI.5. Details of the numerical calculation mesh nodes

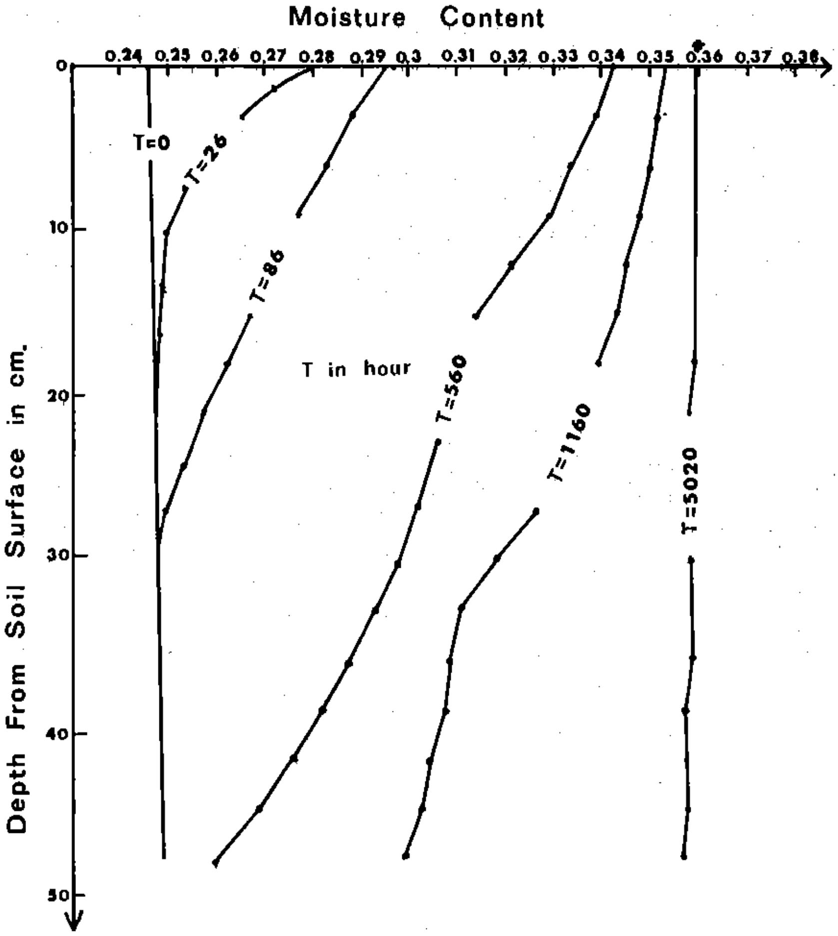


Fig. VI.6. Moisture - depth distribution at different times problem (438) with no transition layer and low rate of rain

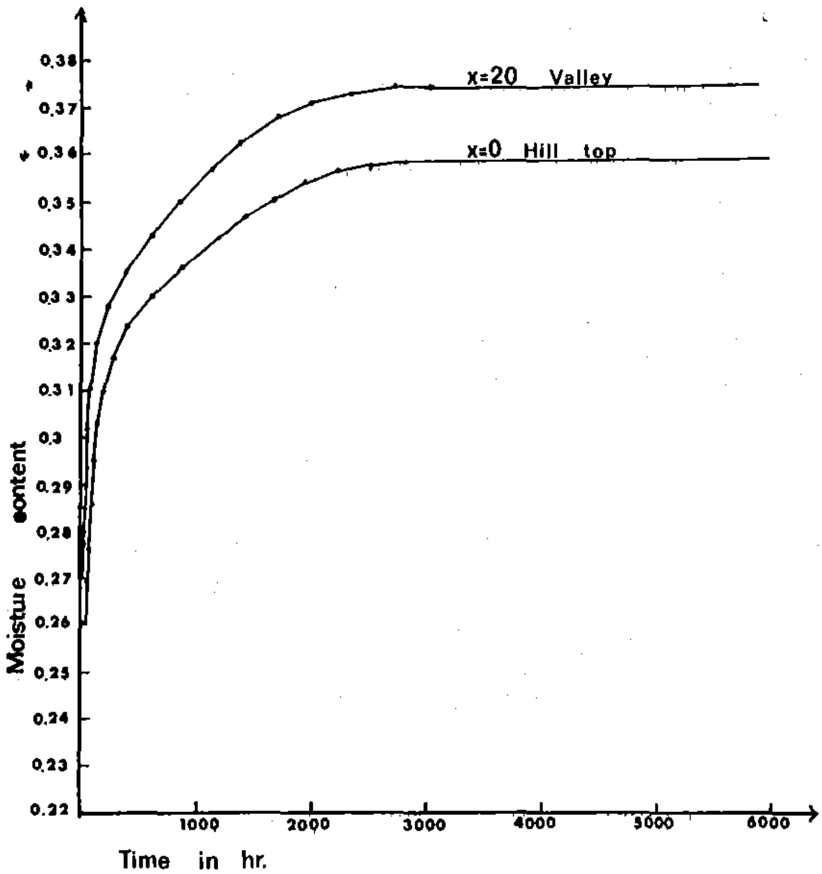


Fig. VI, 7. Surface moisture with time at hill top and valley bottom. Problem 438 with no transition layer and a low rate of rain

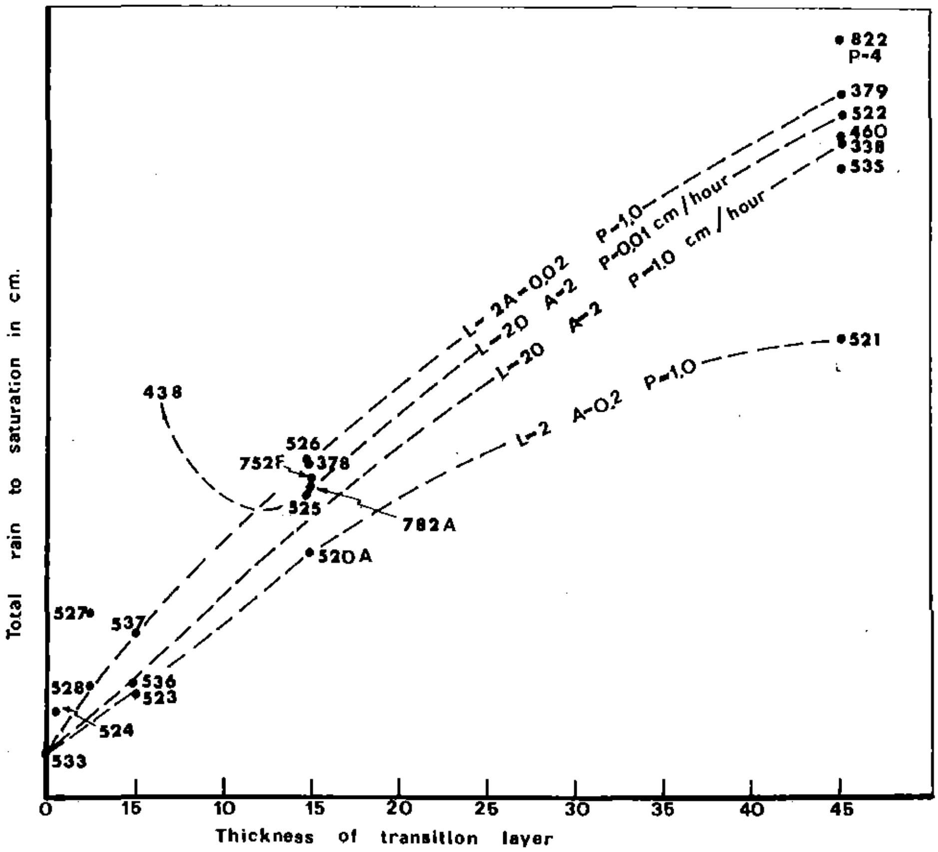


Fig. VI.8. Cummulative rain depth to first appearance of saturation or initial seepage in a gullied soil. Dots with numbers represent problems with different parameters

P - rate of rain L - half cycle length

A - amplitude of landscape wave

VII. LATTERAL FLOW IN A TRANSITION LAYER - RESULTS OF NUMERICAL SOLUTION

ABSTRACT

This report closes a series of 7 parts. Through this series classical surface hydrology has been seriously questioned as a matter of principle and due to experimental observations. In this last part the details of non steady flow regime are demonstrated by a numerical solution. It showed that the existance of a transition layer at the soil surface produces accumulation of rain water at the concave part of the landscape. This saturation may be spread and form at some stage seepage face and outflow from the soil that leads to runoff. This flow can cause erosion. Eroded gullies in turn render an earlier runoff and stronger erosion. The excess rain accumulation can be several times the average rain. Still saturation can be formed only if the accumulation is over a thin enough soil layer.

1. INTRODUCTION

In parts I - V of this report various observations have been made and analytic deduction brought as to the latteral flow near the soil surface associated with vertical infiltration. In part VI the problem of latteral flow in a soil surface transition layer has been set for a numerical solution. The numerical solution serves as an experimental tool to study the effect of various factors on the latteral flow, on the concentration of moisture in concave parts of the landscape and eventually on runoff.

The table of solutions has been presented in the previous part of the report. A partial one will be reproduced here for the present discussion only.

Several conclusions have been drawn in the previous part of this report with the help of the numerical results. Among them:

- Without a more permeable transition layer at the soil surface saturation appears if and only if the rain exceeds the hydraulic conductivity of the soil and after a certain amount of rain came down.
- With a more permeable transition layer at the surface, moisture concentrates and produces saturation. This saturation appears even under rain which is less than the hydraulic conductivity of the subsoil.
- In a thick transition layer the appearance of saturation is at some total rain with only small changes due to the rain intensity or whether it is continuous or intermittent. This has been true over rain intensities changing 1 : 400
- After saturation at some point it spreads to the surface and produces seepage and possibly runoff
- The total rain necessary for saturation increases with the thickness of the transition layer. The total rain necessary for seepage greatly increase with the thickness of the transition layer. This is true at long enough rain intensity.

In the following some of the results will be shown in more details to learn about the actual flow regime in the soil.

2. PRESENTATION OF RESULTS

The reader may be reminded that the soil surface had a sinussoidal form with half cycle L and amplitude A and a transtion layer of thickness δ changing its conductivity from the soil bulk (K_{sat} = 10^{-5} cm/sec.) to the surface (K_{sat} = 100 cm/sec.) and the porosity from the soil bulk (n = 0.4) to the surface (n = 0.9). The water was at hydrostatic equilibrium initially (T.S = 0 T = 0 D = 0) with water table (ψ = 0) at a depth of some 10 meters (z = 0).

Following is a partial table of problems whose solutions will be used in the present discussion.

Problem Number	half cycle	amplitude	thickness of transition	main rain intensity
388	20	2	0.45	1
522	20	2	0.45	0.01
815	20	2	0.45	4
438	20	2	0	1
533	20	2	0	0.01
752 A	20	2	0.15	1
752 F	20	2	0.15	0.01

Each drawing will have an identification table in the form of

fig. 1 fig. 1.

The results will be presented along with the development of the discussion. They will include moisture distribution, hydraulic head distribution integral moisture accumulation etc.

3. THE FLOW REGIME - INDICATION OF LATTERAL FLOW

fig. 2-6

To show the details of the head distribution 3 sections have been magnified (fig. 2)(A) at the top of the hill (B) at its maximum slope (C) at its bottom.

Fig. 3 shows the lines of equal head at high rate of rain and after some 112 mm of rain entered the soil. It is clear that at the wetting front the flow is normal to the transition layer. Behind the wetting front steamlines (that would be normal to the equipotential) clearly slope downstream. It is mostly pronounced in the high slope section B.

In fig. 4 one can observe slanting flow in the upper drawing which is of soil with a transition layer $\delta = 0.45$ (problem 522) and practically vertical flow in the lower drawing in a soil with no transition layer $\delta = 0$ (problem 438). Both have the same very small rains, about one third of the subsoil hydraulic conductivity. Both are taken at the steepest slope after the precipitation of 256 mm of rain.

Fig. 5 shows the evolvement of the flow regime through the equipotential in a soil with a very high rain intensity (problem 815). It is interesting to note the marking of elevation on the left and right side of the drawings. If the hydraulic head registered on the equipotentials is lower than the elevation, the flow is under suction and maybe not saturated. One can study fig. 5 as well as figures 2 - 4 and identify again the high suction gradients at the wetting front and ascertain that in all the examples the flow is under suction. Certainly there is no flooding or perched water formation anywhere. Still the latteral flow component forms both at rains which are far higher and far lower than the hydraulic conductivity. In fig. 5 one can see that in the beginning the flow tends to be normal to the soil surface as suction gradients are predominant. Later the normal flow is maintained only within the wetting front. The latteral flow continues quite significantly after the rain has stopped (the lowest part of fig. 5).

Fig. 6 clearly shows how a soil with no transition layer at its surface differs from that with a transition layer. The flow starts being more or less normal to the soil surface. After prolonged rain the flow becomes vertical.

4. RELATION BETWEEN SLOPE AND THE LATTERAL FLOW COMPONENT

In previous parts of the report it has been shown that under steady state infiltration the horizontal flow component is proportional to the slope.

The question is whether one can draw a simple rule like that for the complex non steady flow being considered here. To study this problem one has to integrate the moisture content Θ over a vertical column to get $w(xt) = \int \Theta\ (xzt)\ dz$. Then by conservation

$$\frac{\partial}{\partial t}\ w(xt)\ - P = \frac{\partial Q(x)}{\partial x} \tag{1}$$

P - being the rate of rain. We assume the lower bound of z over which Θ is integrated to exclude any deep leakage of water. The values of w have been found in the computation and so where $\frac{\partial}{\partial t}$ w(xt) . This allows in principle to find (Q(x) by integration over x from a point where Q(x) = 0 at x = 0. One can now divide this calculated discharge at every point by the local slope. The following is such a table for problem 522. Instead of the discharge the weighed average of the horizontal hydraulic gradient has been expressed, exactly proportional to the discharge.

Place	Point	Ratio of horizontal gradient to local slope
hill top	0	
	1	1.02
	2	1.37
	3	1.48
	4	1.46
steepest slope	5	1.45
	6	1.35
	7	1.35
	8	1.34
	9	1.20
bottom valley	10	0.82

Very clearly the horizontal hydraulic gradient follows very closely the local slope. This conclusion is far from general. At most it is a hint that such a rule of a thumb is reasonably taken. At the bottom, where more water accumulates the gradient decreases. At the hill top, moisture depletion also will tend to limit the gradient.

5. FORMATION OF SATURATION AND SEEPAGE

figs.7,8

Out of the many results two sets have been chosen to illustrate the main findings(fig. 7, 8).

In these drawings 4 sequences of pictures from top to bottom show four stages of flow within the top soil transition layer. This layer has been blown up in the vertical scale 20 : 1 to be able to show the details. Lines of equal moisture have been drawn with the moisture content indicated on a volume basis. Zones of saturation have been shaded. Due to vertical scale blow up lines of equal moisture should not be used to deduce about flow directions.

The first sequence of problem 522 in fig. 7 shows early precipitation stages with the beginning of moisture built up at the bottom valley. At some stage saturation appears. As rain continues the saturated area spreads upward and sideways until it reaches the soil surface. After the cessation of the rain (drainage period) the zone of higher moisture is still maintained for a long period despite respreading by downward and sideways flow of the excess water. These saturation and seepage occur despite the fact that the rain is about 1/3 of the hydraulic conductivity of the subsoil. Accumulation in the concave part produces local precipitation duty which is far higher than the average.

Problem 438 in fig. 7 has the same low rate rain but no surface transition layer. There occurs no saturation, no seepage and no runoff.

In fig. 8 the comparison between soils with and without transition layers is repeated. Problems 388 and 533 differ from those in fig. 7 by their high rain intensity (some 30 times larger than the hydraulic conductivity. Qualitatively the same phenomenon occurs at high and low rate of rain. With the transition layer in both cases the rain enters first the soil. It then produces horizontal flow components followed by moisture accumulation in concave places. Saturation appears first in such concave places within the transition layer. Later saturation reaches the surface in one place only, where runoff could start in the usual sense.

There is a new and important conclusion from the above. It has been argued before that surface saturation can be produced by low intensity rains that enter first the ground. Here it is shown that with the surface transition layer there is no other mechanism. Every drop of rain penetrates first into the soil. The classical model that predicts runoff only when the rain exceeds the local infiltration capacity fails not only for low rates of rain. It is incorrect under any circumstances as long as the surface transition is a universal phenomenon.

The details of the flow at the bottom valley can not be seen in fig. 7-8. However it has been found that upward hydraulic gradient actually forms and there is a flow out of the soil in addition to the inhability of any additional rain to penetrate the soil at this point.

The seepage area is capable of producing runoff. Furthermore it can start an erosive process by piping (seepage forces).

In one example the outflow has been actually calculated. It
fig.9 only serves as an illustration (fig. 9). The high rate of rain (about 110 times higher than the hydraulic conductivity) should have produced by the traditional model water excess very soon and interrupted very shortly after the rain has stopped. Note the type of hydrograph actually obtained by excess flow in what would be considered a point in the field where runoff is produced.

6. GULLIES AND THE SEEPAGE MECHANISM

Consider the sinusoidal landscape as in all the above solutions with one difference. There is an erosive vertical cut at the bottom valley (e.g. on the right of all the profiles in drawings 7, 8). The solution remains unchanged up to the time when saturation appears. The exposed boundary under suction acts as an impermeable one i.e. a streamline. In that it is not different than the symmetry line assumed in the original solution.

Seepage flow out of the soil, will start as soon as saturation will occur. This is much earlier than the initiation of seepage when saturation has to reach the uneroded soil surface. Recall typical times for saturation and seepage or the total rain necessary before first saturation and first seepage. For the present discussion this may be considered as total rain for seepage out of the soil with undisturbed smooth and continuous transition layer and with truncated transition layer.

Problem number	Half cycle length	Amplitude	Transition layer thickness	Main rain intensity	Total rain for seepage: truncated transition layer (Gullies)	Total rain for seepage: No truncation (smooth surface)	Notes
	L	A	δ	P	D_s	D_{seep}	
	m	m	m	cm/hour	cm	cm	
388	20	2	0.45	1	14.9	21.3	
522	20	2	0.45	0.01	15.9	60	
815	20	2	0.45	4.00	17.2	23.2	
752A	20	2	0.15	1.00	7.1	7.7	
752F	20	2	0.15	0.01	8.0	12.0	
536	20	2	0.05	1.00	2.6	3.4	
535	40	12	0.45	1.0	14.3	19.0	
521	2	0.2	0.45	1.0	10.4	24.4	
379	2	0.02	0.45	1.0	16.0	38.0	
378A	2	0.02	0.15	1.0	7.5	14.0	
379I	2	0.02	0.45	0.01	-	-	
378A	2	0.02	0.15	0.01	-	-	
520A	2	0.2	0.15	1.0	5.6	8.4	
520J	2	0.2	0.15	0.01	-	-	
523	2	0.2	0.05	1.0	2.4	2.9	
537	2	0.02	0.05	1.0	3.2	5.2	
537B	2	0.02	0.05	0.01	-	-	
524	2	0.02	0.005	0.5	2	2.3	
525	0.2	0.02	0.15	1.0	7	15	
526	0.2	0.0002	0.15	13.0	8.8	-	not run
527	0.2	0.1	0.025	1.0	4.8	4.6	long enough
528	0.2	0.02	0.025	1.0	2.5	3.3	
529	0.2	0.0002	0.025	0.5	8.1	9.35	
530	0.2	0.1	0.005	0.5	2.0	2.0	
531	0.2	0.02	0.005	0.5	2.0	2.3	
532	0.2	0.0002	0.005	0.5	2.3	2.3	
525P	0.2	0.02	0.15	0.01	-	-	
531V	0.2	0.02	0.005	0.01	-	-	
532W	0.2	0.0002	0.005	0.01	-	-	

The conclusion is that gullies shorten considerably the time and total rain for the beginning of seepage outflow. Smooth deep plough layers will increase the amount of rain that can penetrate the soil before seepage can start. Step by step in the above table one can observe.

a. The lower the slope (and the concavity) the more rain it takes to form saturation and seepage
b. The deeper the layer δ the more rain it takes for saturation and seepage.

If the seepage forces are the main erosive mechanism then erosion can start as soon as seepage starts. The upward and outward gradient as found in the numerical solution may seem small. However, note that a slight and local dent in the soil surface can cause a local concentration of streamlines and highly increased seepage forces. Once, erosion has been initiated the seepage gradients and outflow on the gully sidewall are much higher and they cause elongation, widening and branching of the gully.

A very interesting explanation is suggested here for the formation of tunnel erosion which is very typical to many areas. In Israel it is pronounced especially in the wind blown loess soils of the south. It is a well established fact there that the initial bulk density is low. A delicate cohesive structure of the loess can be collapsed on saturation. The appearance of such saturation in the profile has been demonstrated well in the analysis above. It is exactly where we find underground tunnels in the loess area. On collapse of the structure a free space is produced. Water seepsout on the upper end of this space and enters back to the soil on the lower end. By transporting eroded soil more and more free space is formed (the difference between the initial soil and the hydraulically redeposited soil). The tunnel is elongated and widened until its top falls in. Often it finds an outlet to a larger truncation.

Evidently erosion will progress at an increasing rate once it has been started. Moisture concentration will start erosion and runoff and the erosion in turn will produce a more efficient system of moisture accumulation seepage and eventual runoff.

7. HORIZONTAL REDISTRIBUTION OF RAIN WATER

figs. 10-14

In search for a more economic expression of the results, the vertical integral of moisture content has been performed at 11 grid lines of the numerical solution.

$$w(xt) = \int \theta(xzt)\, dz \qquad (2)$$

From it the intial value W_o at $t = 0$ has been substracted to give $\Delta w(xt)$ as a result of the rain. The average $\overline{\Delta w}(t)$ is calculated and finally the relative value $\Delta w/\overline{\Delta w}$ is expressed. Where it is a unit the local addition equals the average added water. Where there is accumulation the ratio is higher than one. Figures 10-14 show the results for five different problems. Consider first fig. 10 for problem 388 (24 cm rain per day). The curves in the lower part are during the rain. Initially at 2.5 hours and 2.5 cm of rain there is some uphill accumulation as the flow is normal to the soil surface and the gravity force is still negligible, compared with the pressure gradient. Later the local water duty in the concave part (on the right) increases and reaches even twice the average. After the end of the rain (upper part of the drawing - drained) there is still a build up of the moisture at the lower concave part and depletion in other parts. Fig. is of a very low rate of rain 0.24 cm rain per day). The relative excess is very significant in the concave part and reaches more than three times the average rain. This higher concentration builds up further for some time after the end of the rain. Despite deep drainage it is maintained for a relatively long period. After more than a year it is still twice the average added rain. In all the above problems evaporation was not included.

Problem 438 in fig. 12 is with no transition layer and serves for comparison. There is a very little moisture accumulation and it is to a great extent uphill. Fig. 13 exhibits the behaviour under a very high rate of rain (40 mm per hour - 96 cm/day). It becomes clear that during the rain itself there is less time to move laterally and the accumulation is less than in problem 388. Both are less than in 522. Later on after the rain has stopped there is still time for the accumulation to develop.

Several outstanding conclusions may be taken tentatively:

a. The relative accumulation depends first and foremost on the total rain
b. The lower the rate the more latteral accumularion occurs
c. The accumulation due to latteral flow continues long after the rain has stopped
d. Intermittent rain may be taken on the average as a low rate continuous rain.

One has to remember that latteral migration of rain and accumulation in concave parts of the landscape may produce saturation seepage and runoff but they do not have to. The accumulation can occur over a long column of soil when the moisture is so distributed as to have low local values. High intensity rains while causing somewhat less moisture accumulation have less time to penetrate deep into the soil. Thus they are more liable to produce saturation and seepage.

The last figure of this group (14) shows a low rate of rain over a layer 15 cm thick (a third of that given in previous drawings). The degree of moisture accumulations is impressive. During the rain it reaches four times the average rain. It should be mentioned again that despite this impressive accumulation the saturation and seepage will be formed only if the moisture concentrates at a thin enough layer.

The accumulation has been further condensed by calculating a concentration coefficient $\Delta\sigma$ as follows:

$$\Delta\sigma(t) = \frac{1}{L} \int_{\sigma}^{L} |\Delta W(xt) - \overline{\Delta w}(t)| dx \qquad (3)$$

where $\Delta w(xt)$ is the added moisture integrated over a column. $\overline{\Delta w}(t)$ is the average addition (without accumulation).

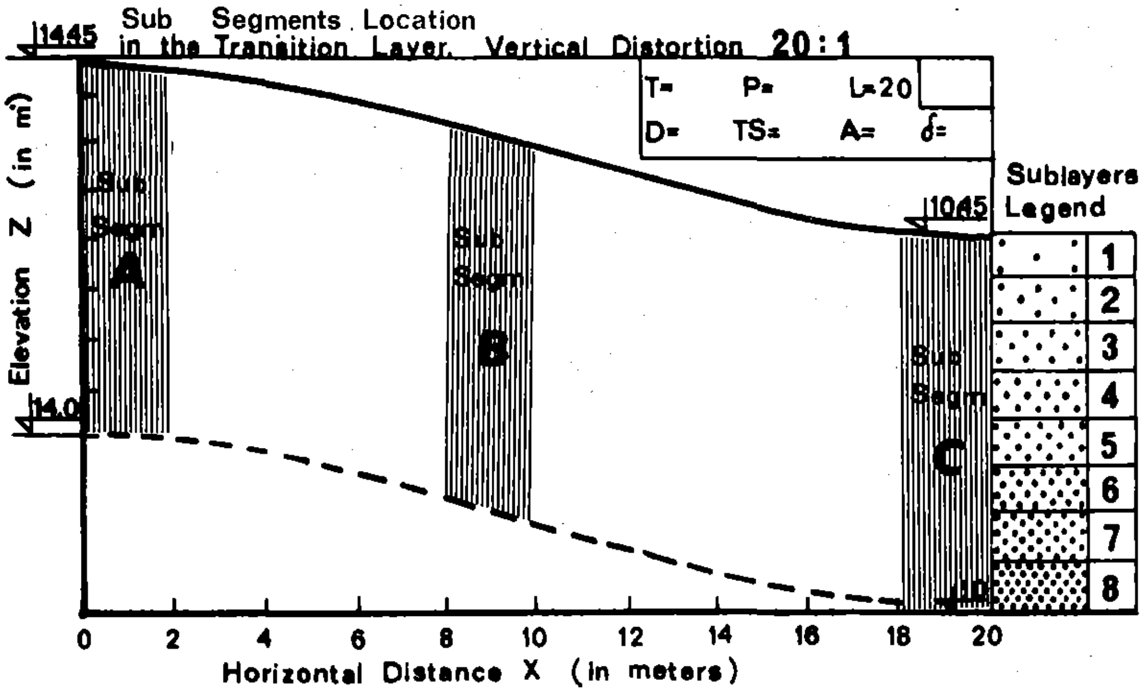
In fig. 15 one can observe on the left a comparison between the problems 522, 388 and 815 varying only by the rate of rain. As in 522 the rain is low it has ample time to concentrate. On the right one can compare low rates with no transition layer with 15 cm one on 45 cm one. All at low rate of rain. The same at somewhat different scale can be seen in figure 16. The conclusions are repetitive of those that have already been mentioned.

In the numerical solution the rain has been taken to be uniformly distributed over the surface. Note that in reality splashing rain drops and slanting rain may cause higher local concentration over which the act of the surface transition layer is superimposed. It can be by far the more important process. Especially, that it is tied with erosion by seepage. Furthermore it has been shown to be valid over a wide range of slopes and curvatures.

The distribution of $\Delta w(xt)$ figs. 10 - 14 reminds a negative of the surface sinusoid giving the higher moisture accumulation in the more concave parts of the landscape. It is clearer why moisture will give a high correlation with the concavity even though it is not a perfect one. Note also that in the above a reasonable proportionality (but not a perfect one) was found between the horizontal discharge and the slope. The relatively limited range of our calculations cannot serve as a perfect proof to that effect. Strictly speaking the slope of moisture distribution may be a coincidence. To be certain a large number of threedimensional problems should be run. However recall that under steady state conditions it has been analytically proved. In the field it has been observed. The individual mechanisms have been demonstrated beyond any doubt. It therefore seems safe enough to consider the present deductions at least as a correct trend that calls for a complete revision of out approach to surface hydrology.

T= Time(hr.) זמן מתחילת התהליך	**P**= Rain intensity (cm/hr.) עצמת הגשם	**L**= Half wave Length (m) of the soil contour חצי אורך גל של פני הקרקע	Computer Problem No. מספר בעית המחשב
D= Cumulative Rain from T = 0 (cm) גשם מצטבר מתחילת התהליך	**TS**= Time step of computer solution, from t = 0 צעד זמן של פתרון המחשב	**A**= Amplitude of wave of the soil contour (m) אמפליטודה גל של פני הקרקע	**δ**= transition layer thickness (m) עובי שכבת המעבר

Fig. VII.1. Problems parameters legend for figures of solution



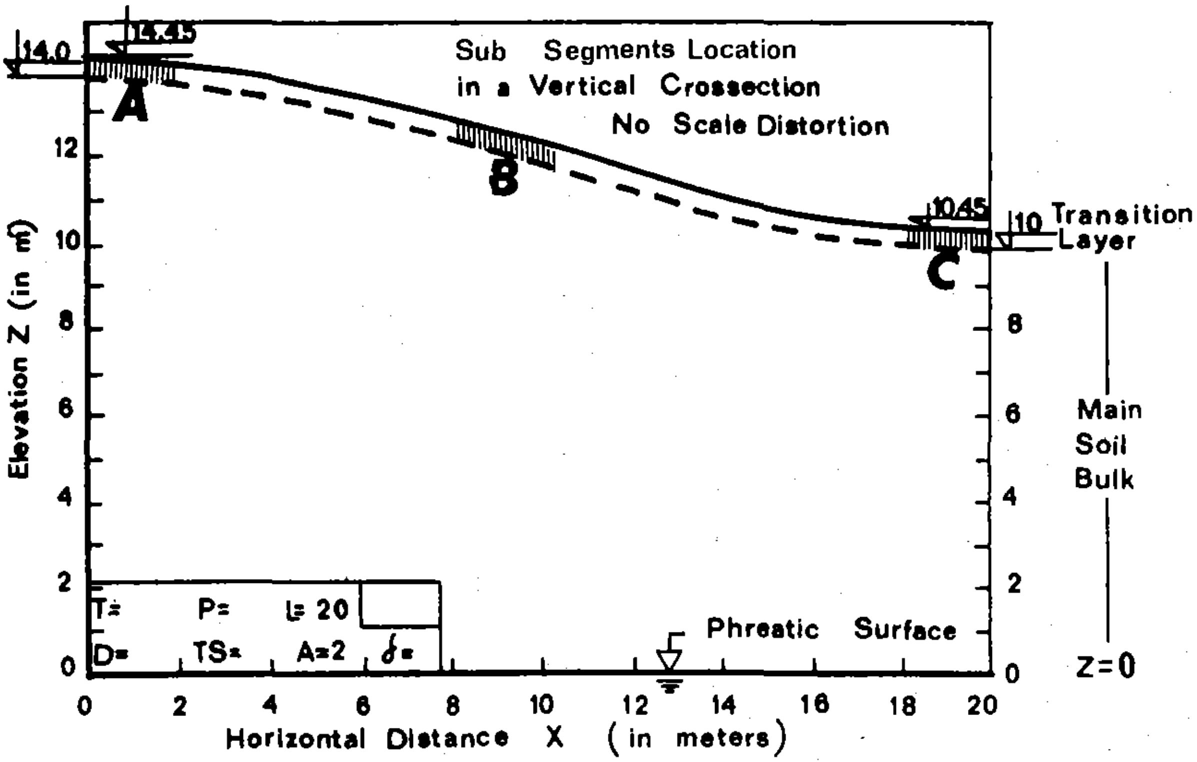


Fig. VII.2. Segments ABC in the transition layer enlarged for discussion on flow direction

Lines of Equal Head in the Transition Layer (in cm)
No Scale Distortion

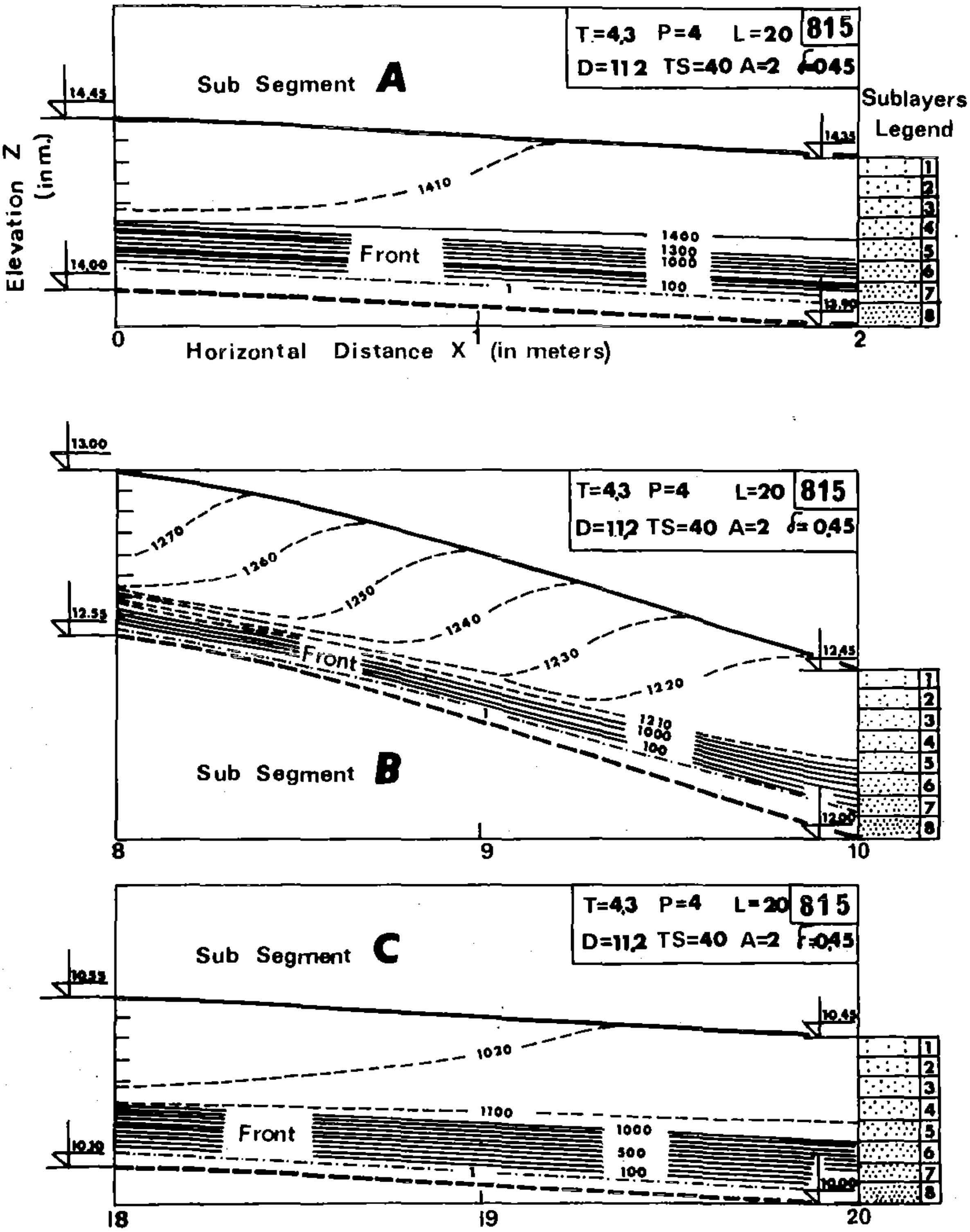
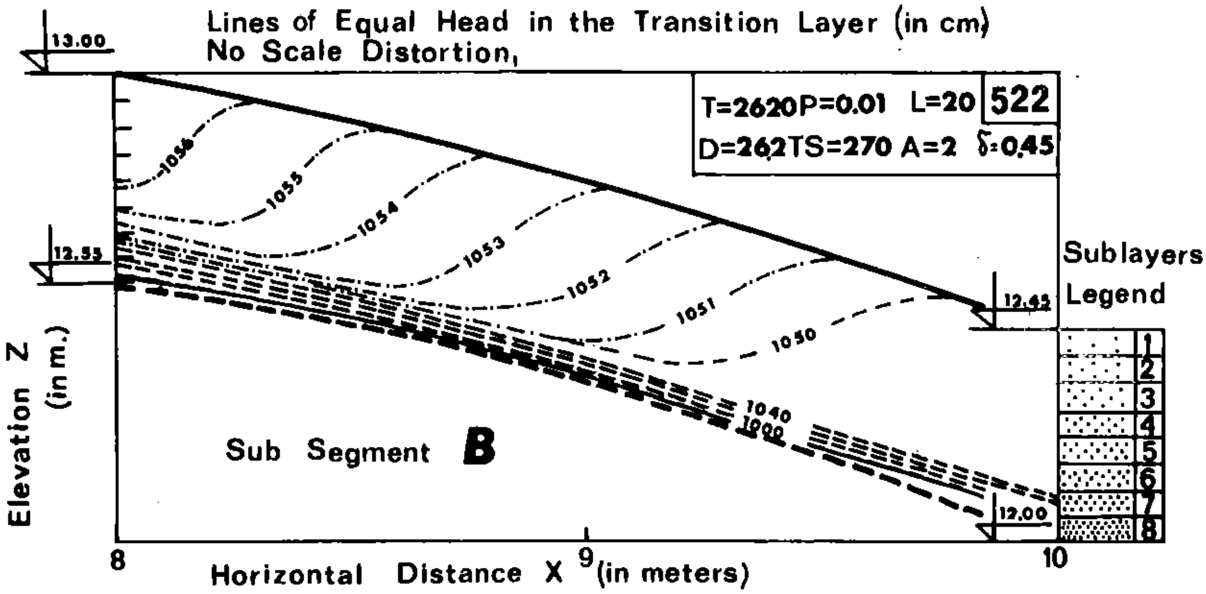


Fig. VII.3. Sections ABC. Head distribution of prob. 815. Note almost vertical flow at the top (A), latteral flow in the middle (B) and vertical at the bottom of the hill (C)



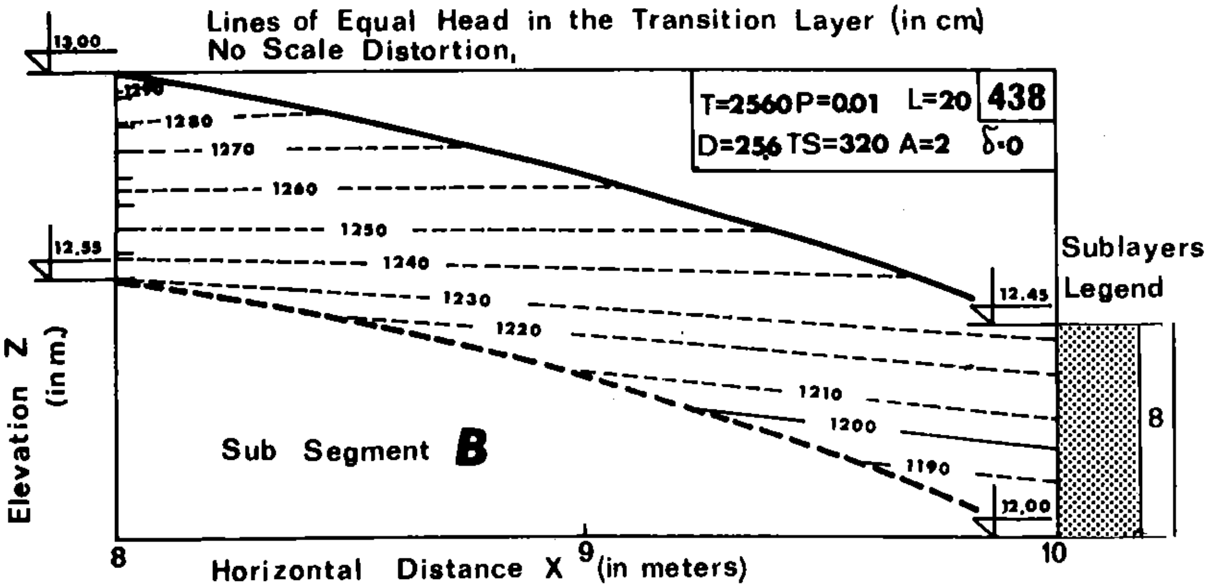


Fig. VII.4. The middle section B of highest slope. Head distribution in a uniform soil (438) and a soil with a transition layer (522). A significant latteral flow occurs in the transition layer (522)

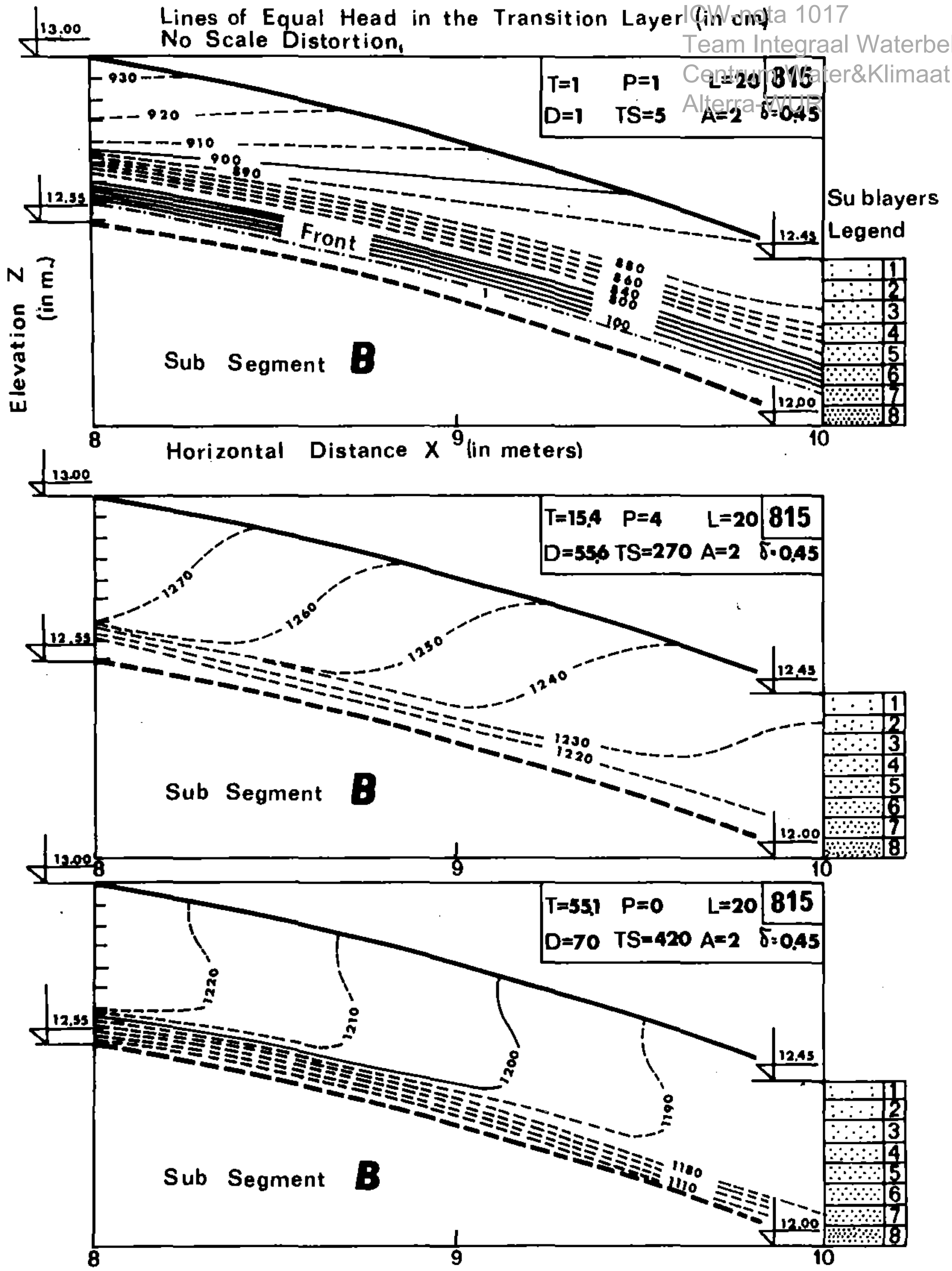


Fig. VII.5. Middle section B of highest slope. Head distribution. Prob. 815 with a transition layer and high intensity rain. Flow direction changes with time. During wetting (top frame) direction mostly normal to soil surface. At longer times there is a significant horizontal component. During drainage there is a very large horizontal component. In the wetting front flow continues to be normal to soil surface

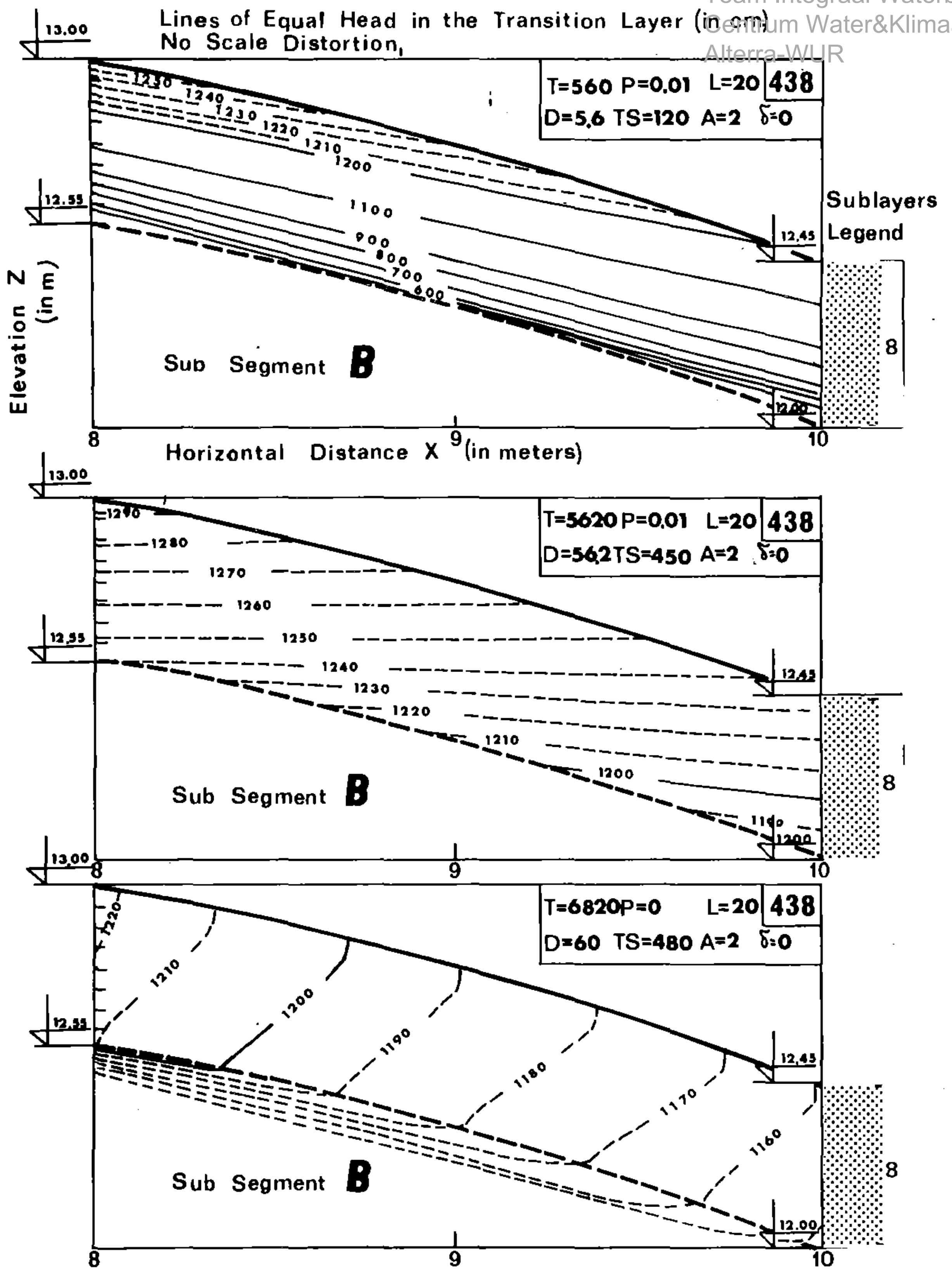


Fig. VII.6. Middle section B of highest slope. Head distribution. Soil without a transition layer, prob. 438. During wetting (top frame) flow is perpendicular to the surface. In steady state (middle frame) flow is vertical. During drainage (bottom frame) flow is mostly downwards at the wetting front and somewhat laterally at the top soil (compare with fig. 5)

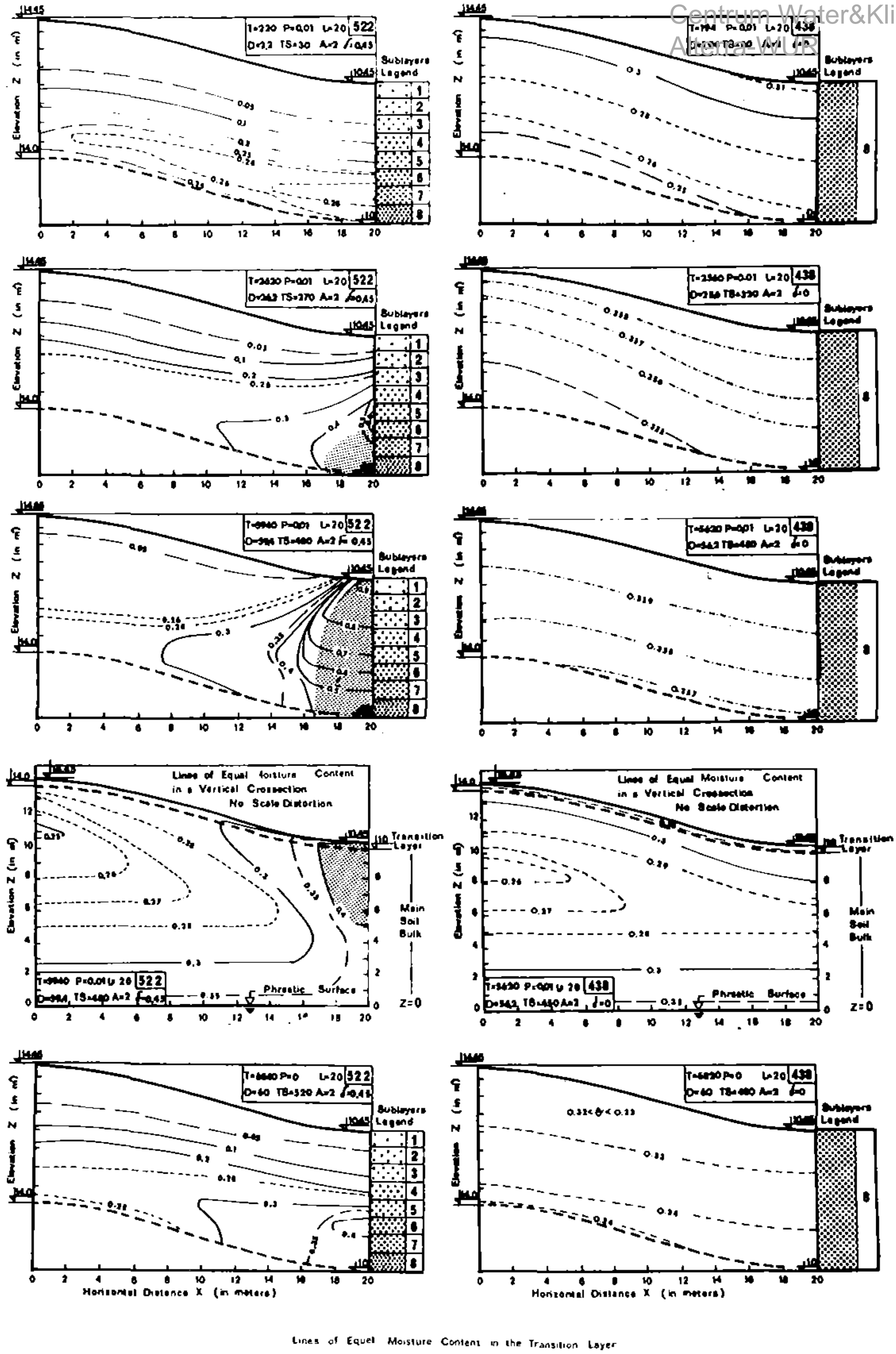
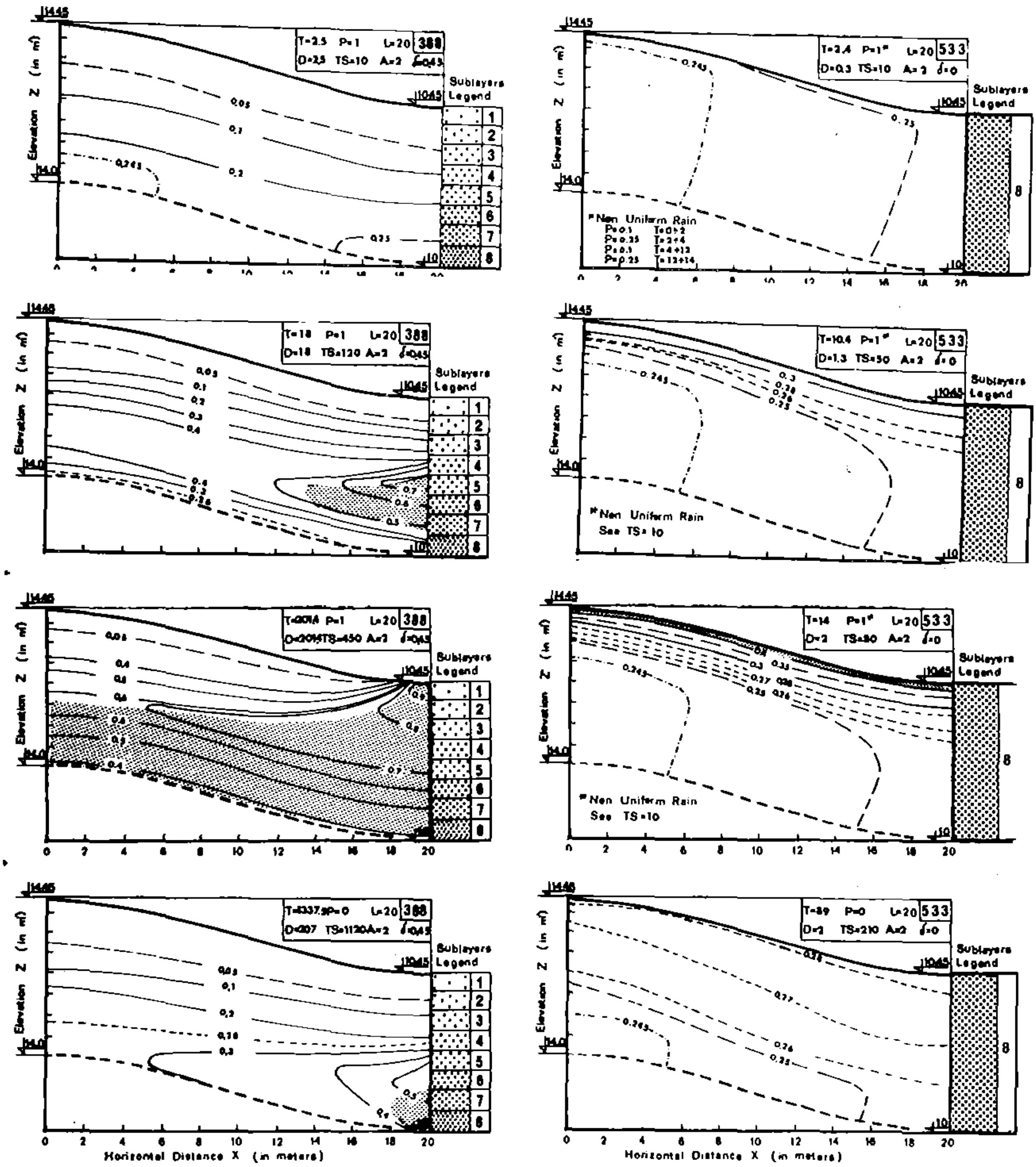


Fig. VII.7. Moisture field for problems (522) and without a transition (438) both of low intensity rain. From top to bottom four flow stages: Initial wetting; first saturation; appearance of surface seepage and drainage



Lines of Equal Moisture Content in the Transition Layer
Vertical Distortion 20:1

Fig. VII.8. Moisture field for problems (388) with transition and (533) without a surface transition layer. From top to bottom four stages: Initial wetting; first saturation; appearance of surface seepage and drainage

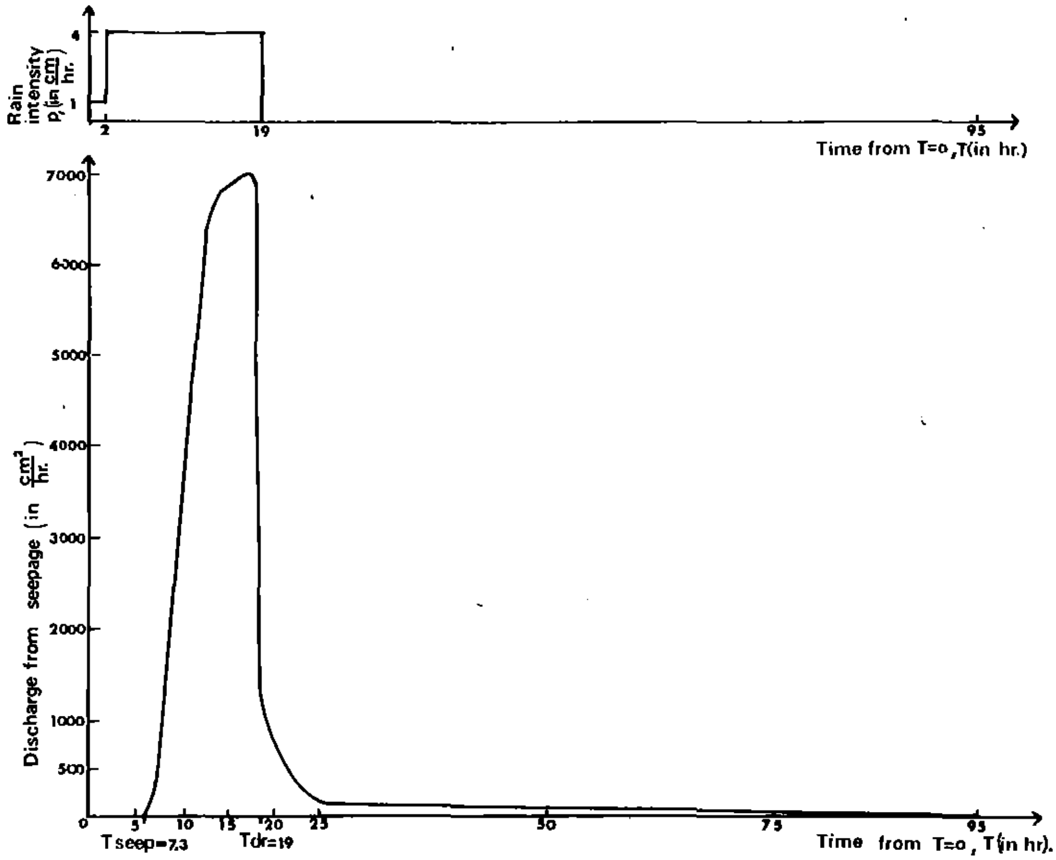


Fig. VII.9. Overland discharge by seepage due to concentration of moisture in concave landscape

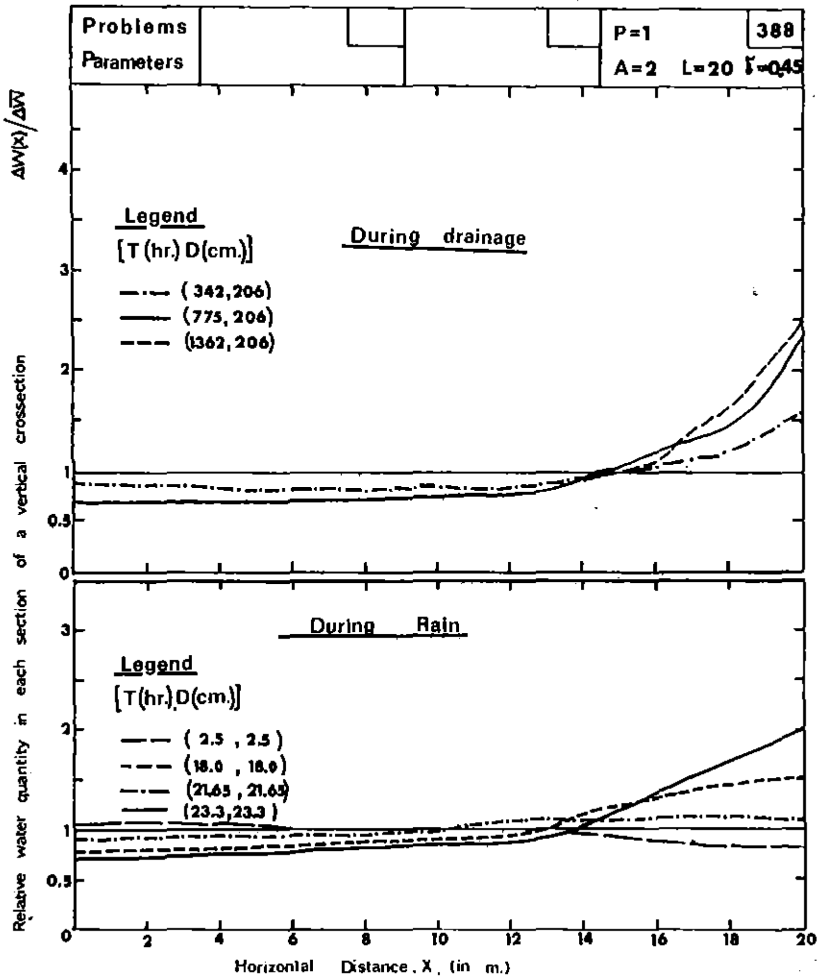


Fig. VII.10. Horizontal distribution of relative added moisture depth at different times. Prob. 388

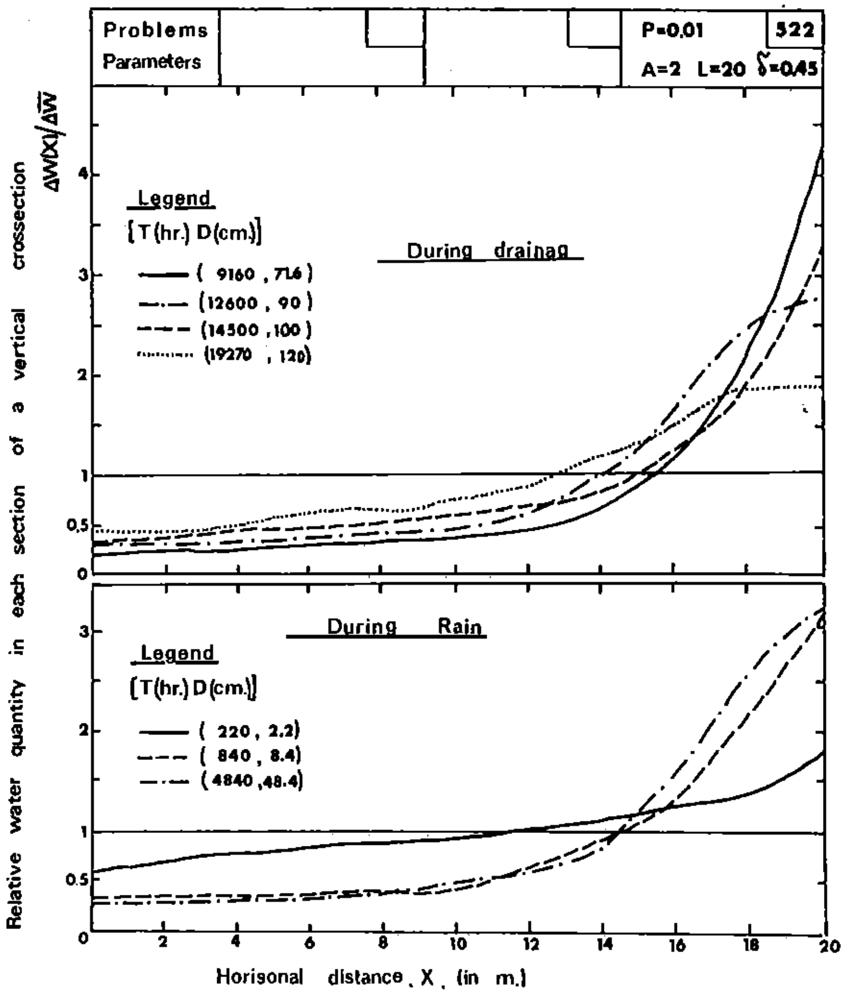


Fig. VII.11. Horizontal distribution of relative added moisture depth at different times. Prob. 522

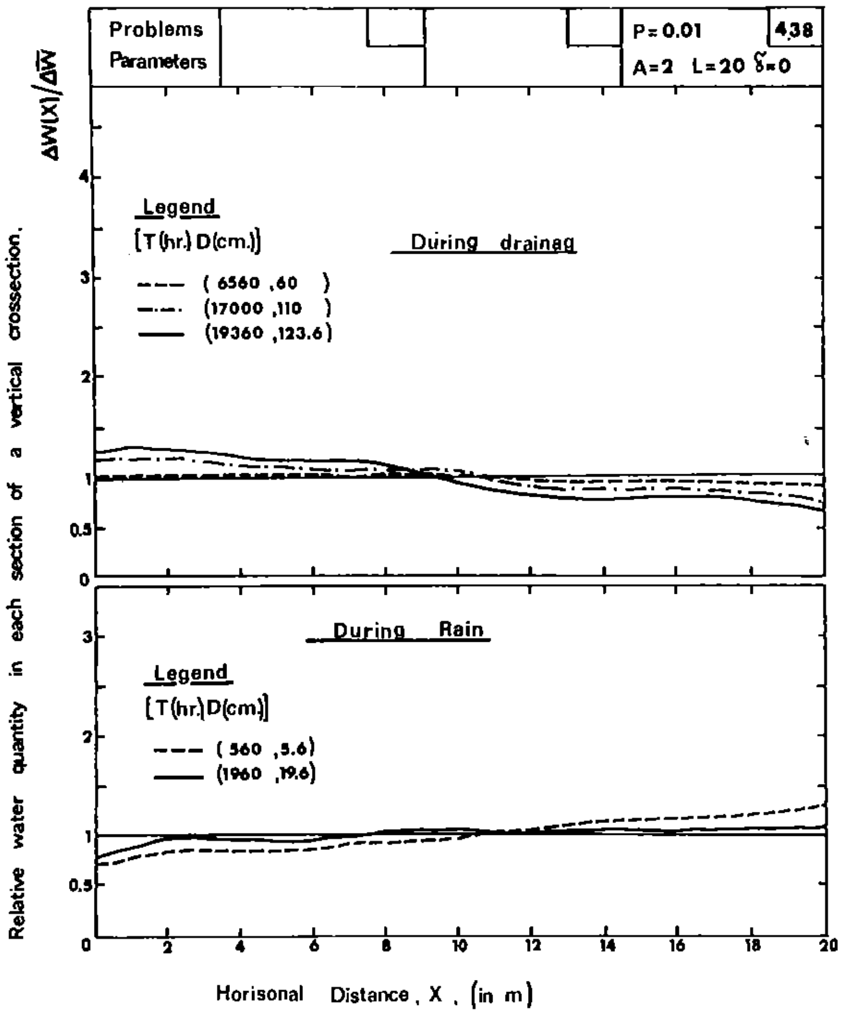


Fig. VII.12. Horizontal distribution of relative added moisture depth at different times. Prob. 438

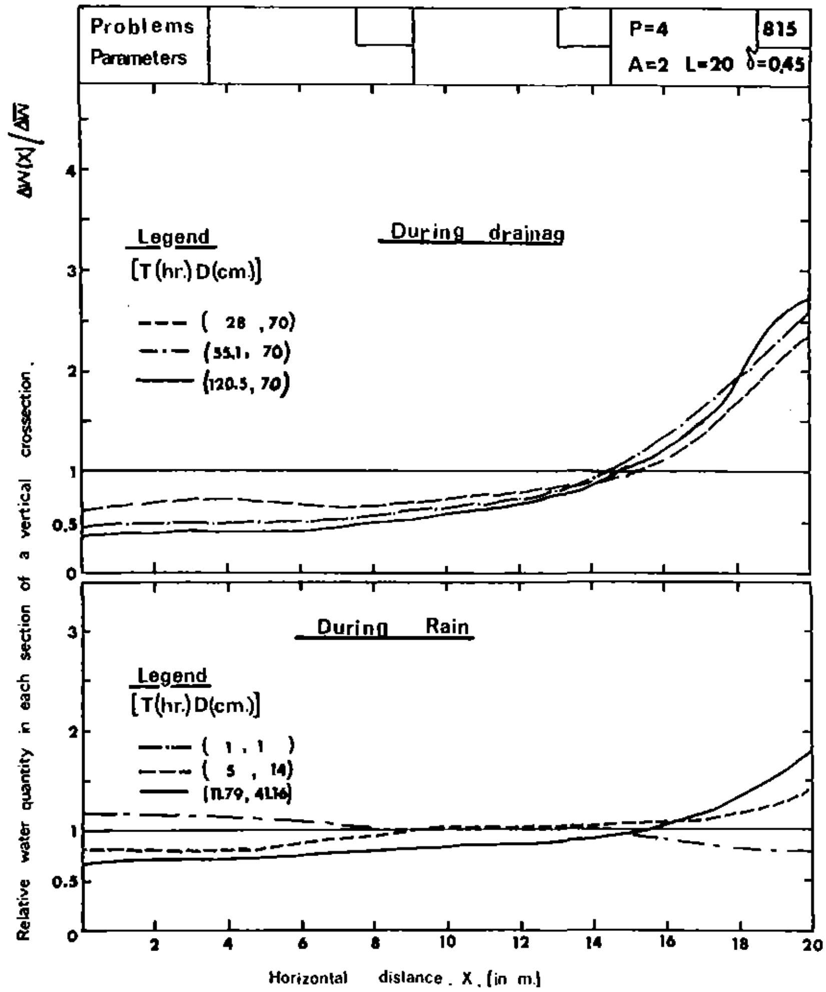


Fig. VII.13. Horizontal distribution of relative added moisture depth at different times. Prob. 815

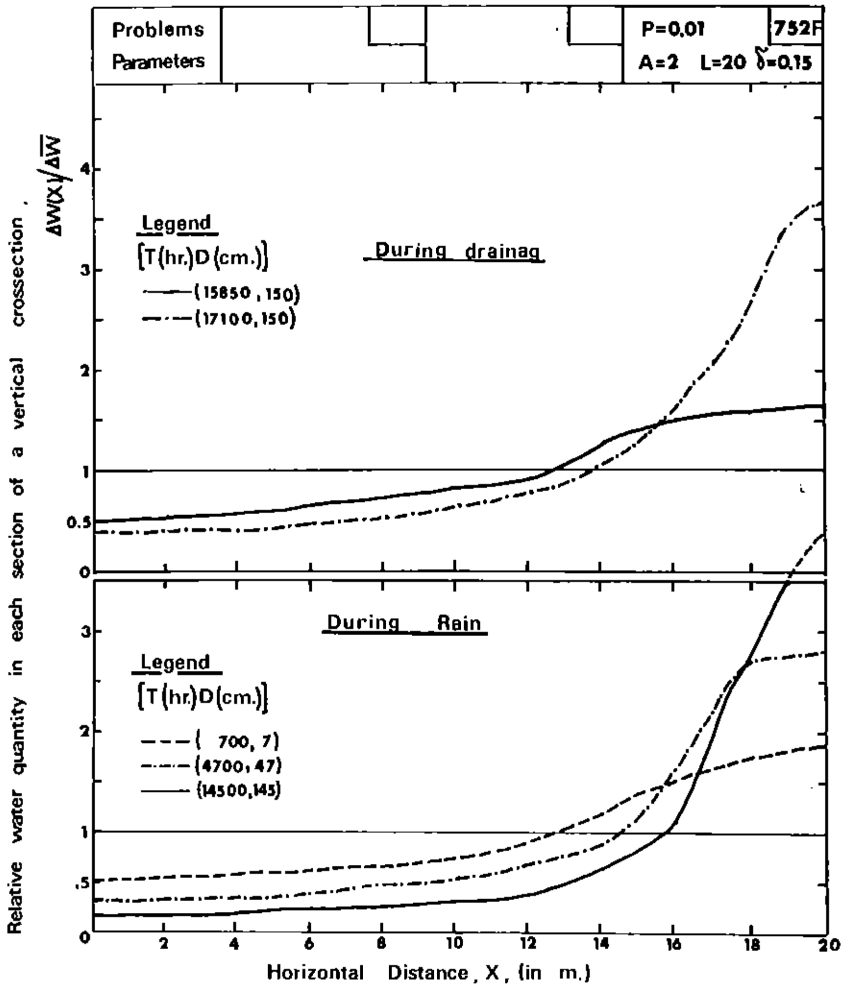


Fig. VII.14. Horizontal distribution of relative added moisture depth at different times. Prob. 752F

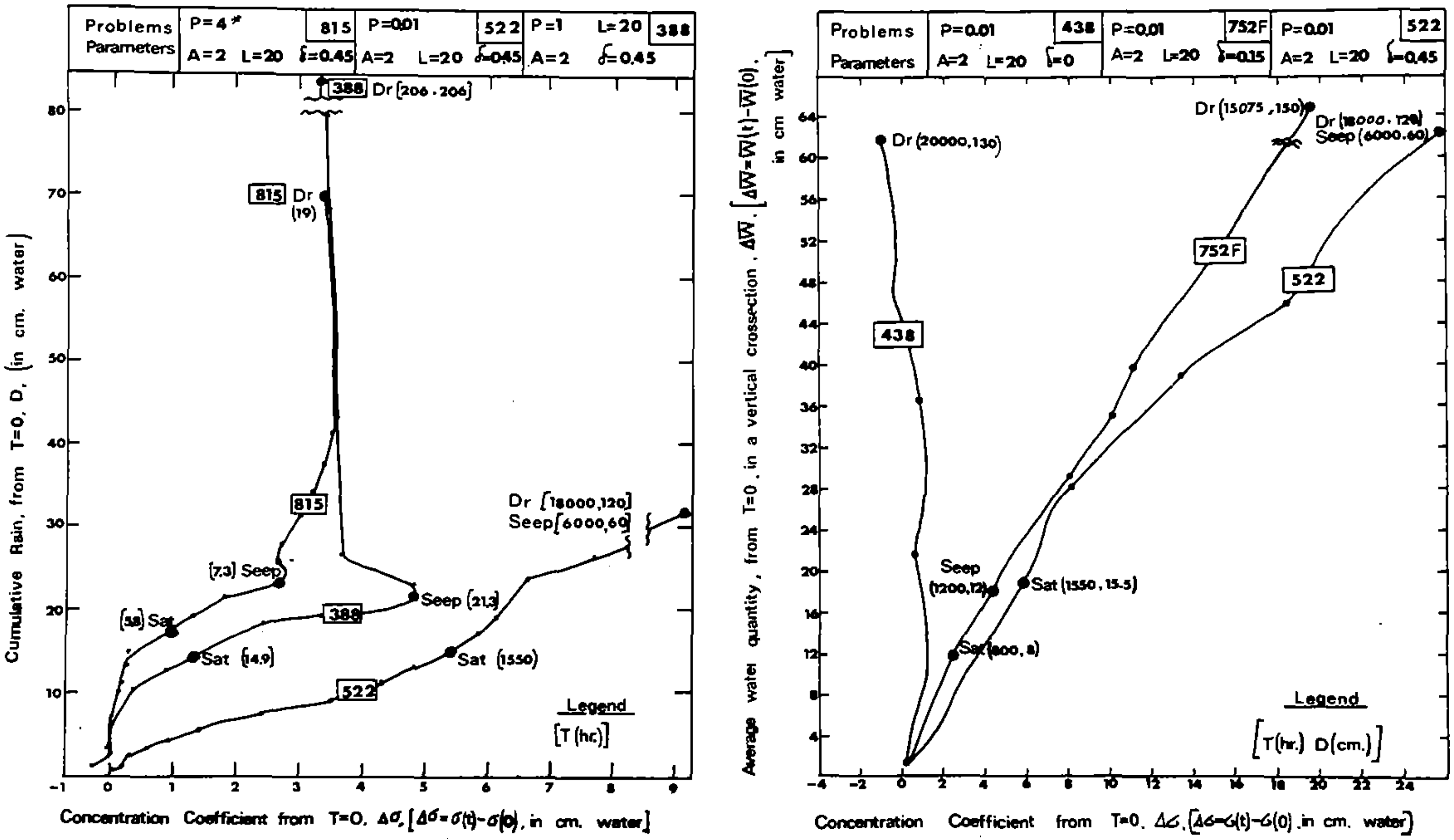


Fig. VII.15. Concentration coefficient or average amplitude of moisture concentration as a function of added precipitation

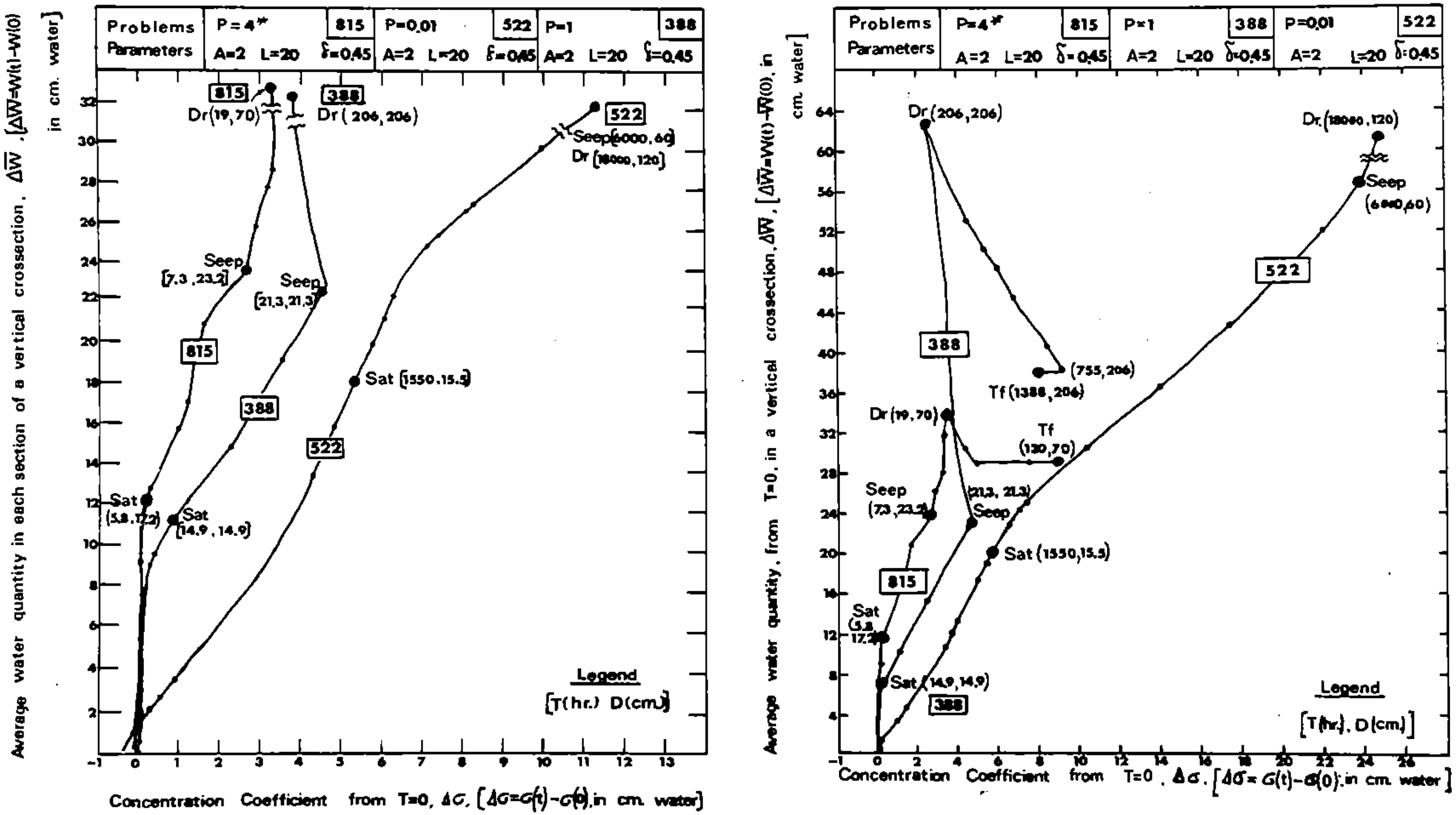


Fig. VII.16. Concentration coefficient or average amplitude of moisture concentration as a function of added precipitation