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CALCULATION OF CAPILLARY CONDUCTIVITY AND CAPILLARY RISE FROM GRAIN SIZE DISTRIBUTION

II. ASSESSMENT OF THE n-VALUES IN A FORMULA OF BROOKS AND COREY FOR THE CALCULATION OF HYDRAULIC CONDUCTIVITY FROM GRAIN SIZE DISTRIBUTION

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1. INTRODUCTION

In a former paper evidence was drawn from data in literature that the value of the exponent in a formula of BROOKS and COREY (1964) to calculate hydraulic conductivity may be overrated by the theoretical deduction (BLOEMEN 1977). It was suggested that as far as undisturbed soil samples are concerned, the real values of the exponent may even fall below a theoretical minimum when heavier soils are involved.

The deduction of the BROOKS and COREY formula is based on several key assumptions. It will be very difficult to decide when these assumptions fail and how this would effect the value of the exponent in the formula. It was therefore suggested that the empirical approach to assess the value of the exponent for a large variety of soils would be the most practical and direct way. Establishing a relationship between the exponent and some adequate characteristic of the soil would make the result of a finite number of determinations transferable to all soils. A more complete soil characteristic than clay percentage to correlate with the value of the exponent was considered necessary for reasons of accuracy. Because with respect to hydraulic conductivity the agricultural interest will be mainly with subsoils it was felt that this characteristic should bear on textural features.

According to a review of techniques for the statistical summary of textural data presented by FOLK (1966) there is one technique whereby the entire grain size distribution enters into the determination of one parameter. This so called method of moments multiplies the weight percentage within each size class of a grain size analysis by some power of the deviation of the mean size of that class from a general mean size. From the sum of these products a standard deviation is calculated.

Some of the objections brought up against the moment of methods do not apply to a method to obtain one comprehensive parameter from a grain size analysis which is practised in this paper. This method calculates what is called a grain size distribution index between a lower and an upper limit of grain size. It is shown in this and in a following paper that this index is an efficient parameter in a procedure to calculate hydraulic conductivities from grain size analyses.

2. CALCULATION OF A GRAIN SIZE DISTRIBUTION INDEX

In a grain size analysis there is generally a percentage of unanalysed fine material and sometimes of unanalysed coarse material too. The cumulative distribution curve refers to the proportion of soil between a lower and an upper grain size limit. The basic data are a succession of weight percentages measured at a number of size intervals.

To decide of what nature the one comprehensive grain size parameter which is to be correlated with the exponent in BROOKS and COREY's formula should be, it should be considered that this exponent is a pore size distribution index. It is determined by the slope of a derivative of the moisture retention curve (BROOKS and COREY, 1964; App. II). The moisture retention curve is essentially a cumulative pore size distribution curve and has a great similarity of form with the cumulative grain size distribution curve. In fact this phenomenon is marked enough to inspire several authors to construct pore size distributions from i.a. textural data (HUSZ, 1969; HARTGE, 1969; RENGER, 1971). Therefore, it was felt that from grain size distribution curves some parameter for slope should be calculated.

The slope of a distribution curve between two size intervals can easily be calculated as the tangent to the abscissa of a straight line through the two data points, if ordinate and abscissa have the same scale. This is possible by using log scales. When the logs of the size intervals S_i are plotted on the abscissa and the logs of the cumulative weight percentages P_i are plotted on the ordinate the slope between two size intervals is

$$tg_i = \frac{\log \frac{P_{i+1}}{P_i}}{\log \frac{S_{i+1}}{S_i}} \qquad (1)$$

The slope between two size intervals should enter a final value for the entire distribution in proportion of the weight percentage per size class, so

$$f_i = (P_{i+1} - P_i) \frac{\log \frac{P_{i+1}}{P_i}}{\log \frac{S_{i+1}}{S_i}} \qquad (2)$$

The mean f-value between the cumulative weight percentages of lower and upper size limit of the analysis (P_1 and P_n) would be

$$f = \frac{\Sigma_1^n (P_{i+1} - P_i) \frac{\log \frac{P_{i+1}}{P_i}}{\log \frac{S_{i+1}}{S_i}}}{P_n - P_1} \qquad (3)$$

This mean value is a dimensionless parameter which hereafter will be called the grain size distribution index

In table 1 an example of calculation is given.

Table 1. Calculation of grain size distribution index f from a grain size analysis in GIESEL a.o. (1972, table 1, nr 4)

i	S_i	P_i	$P_{i+1}-P_i$	$\log \frac{S_{i+1}}{S_i}$	$\log \frac{P_{i+1}}{P_i}$	$P_{i+1}-P_i \frac{\log \frac{P_{i+1}}{P_i}}{\log \frac{S_{i+1}}{S_i}}$
1	2 µm	7.1				
2	20 µm	12.8	5.7	1	0.256	1.459
3	60 µm	22.8	10.0	0.477	0.250	5.241
4	200 µm	42.5	19.7	0.523	0.270	~~2.780~~ 10,170
5	600 µm	95.5	53.0	0.477	0.352	39.110
6	2000 µm	100.0	4.5	0.523	0.020	0,172 +
						48.762

$$f = \frac{48.762}{92.9} = 0,525$$

The calculation in table 1 can easily be computerized. As long as the lower and upper size limit of the analysis is the same, f is not appreciably dependent on the size intervals between, providing their number suffices. The scale in table 1 is rather coarse and gives values of f which may vary from those calculated with scales using more size intervals.

The calculation in table 1 is not, like the method of moments, making use of the assumption that the centre of gravity of size classes is at the half way mark of that class. It is using the measured data points. It accepts the consequences of the unanalysed open ends of the size distribution and is a comparative measure for the size distribution between two size limits. It is clear that all features of this distribution are integrated into the index f, but single features like mean, sorting, skewness and peakedness are not recognizable in it.

3. SOME REMARKS ON THE GRAIN SIZE DISTRIBUTION INDEX F

In fig. 1 examples are given of grain size distributions with their index f. They are taken from table 3 which gathers the data dealt with in this paper.

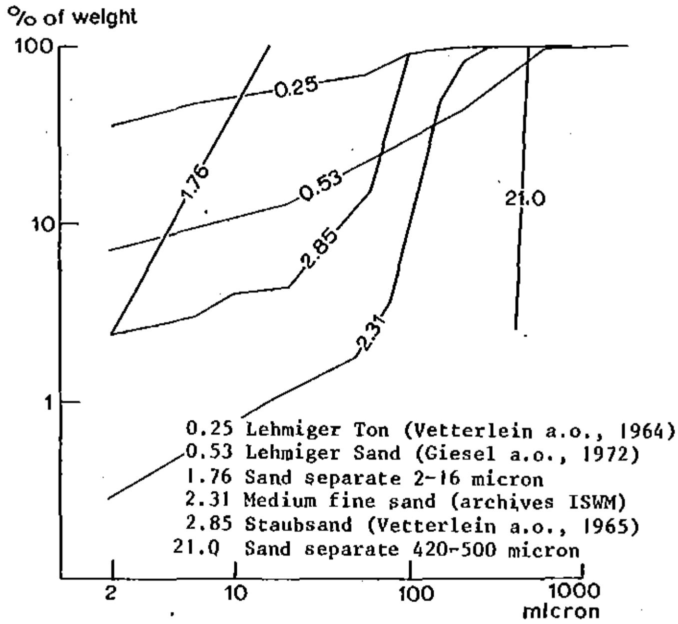

Fig. 1. Grain size distributions with their index f

It is clear from fig. 1 that there will be a strong influence of the clay percentage on the value of f. This is obvious, because the index f refers to particles > 2 micron. According to granular analyses of 113 soil layers, however, the variety of the index f increases as the clay percentage decreases. With low clay percentages this variety is very large. This is shown in fig. 2. The data will be provided in a following paper in another connection.

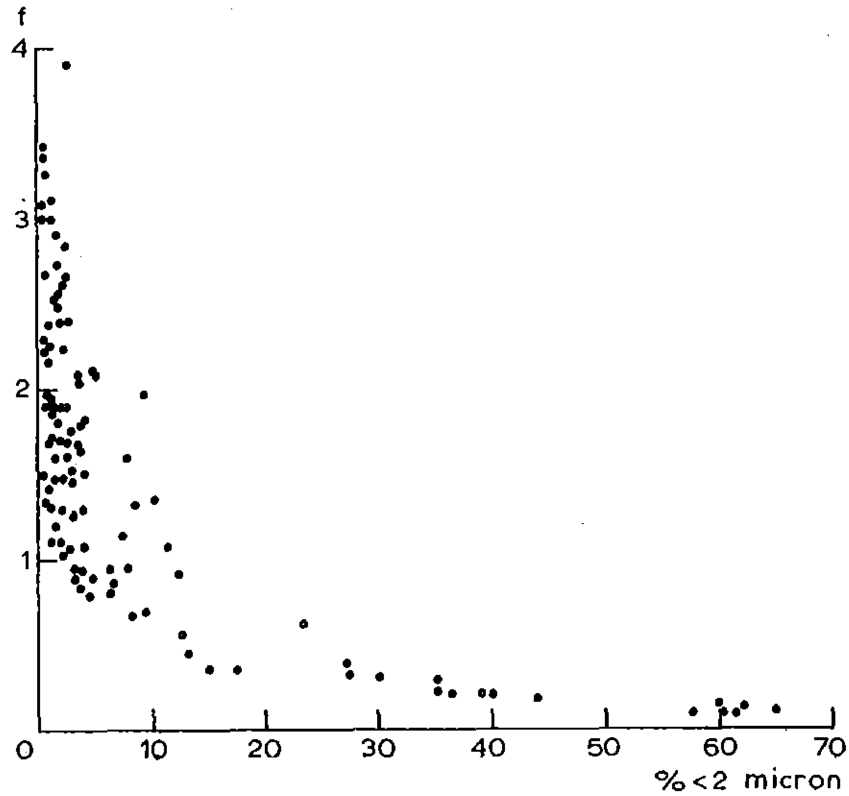

Fig. 2. Clay percentage plotted against index f

The scatter in fig. 2 can be taken as a justification for the calculation of the index f in which more textural properties than clay percentage obviously appear. The mean grain size certainly does not belong to these for there is no correlation between the index f and the median, being that diameter which has half the grains (by weight) finer, and half coarser.

For a distinction between the influences of clay percentage and other textural features on the index f the value of f for the soils involved in this paper are also determined for the imaginary condition that all soils are essentially the same as far as distribution is concerned. As, according to FOLK (1966) the majority of sedimentologists agree that the size distribution of sediments approaches log normality, the index f for simulated log normal distribution seems to be a useful comparitive measure. It can be calculated for any soil that is adequately described. Besides clay percentage one additional point on the given distribution curve has to be chosen as a parameter because a lognormal distribution shows as a straight line when plotted on a logarithmic size scale and a probability scale for weight percentages. The median was chosen because this parameter will in a following paper appear to be also efficient with respect to the calculation of hydraulic conductivities from grain size analyses. Now respective lognormal distributions can be drawn on probability paper. In fig. 3 examples are given. Then weight percentages can be read for the subsequent particle sizes and the index f can be calculated. For the soils in table 3 (referred to in chapter 4) this procedure was performed. The values of f for lognormality are entered in table 3 as f_{logn}. It appears that the real values of f show a wider variety than originates from the differences in clay percentages. This results from deviations of the actual distributions

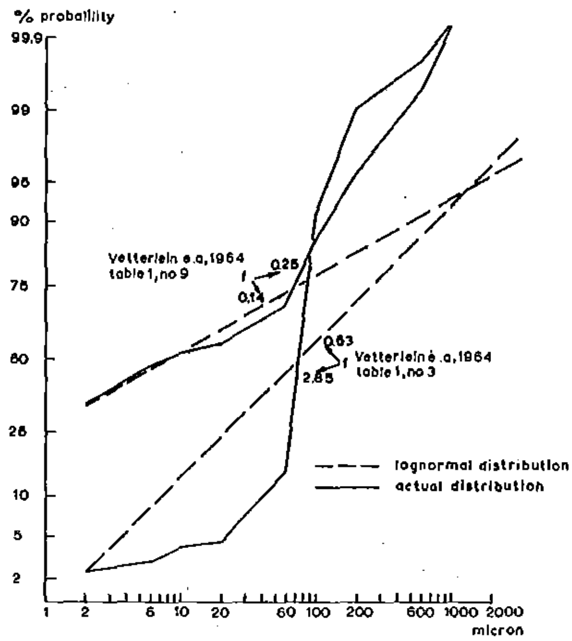

Fig. 3. Differences between actual and lognormal grain size distributions with the same median and clay percentage

from log normality. It is also obvious that these deviations are larger in soils with higher values of f, which are generally the soils with low clay percentages. This is also very clear in fig. 4, where f and f_{logn} are plotted against clay percentages. The data are from table 3. So in the value of f both clay percentage and size distribution between 2 micron and an upper limit are represented. If for some reason it would be necessary to represent the deviations

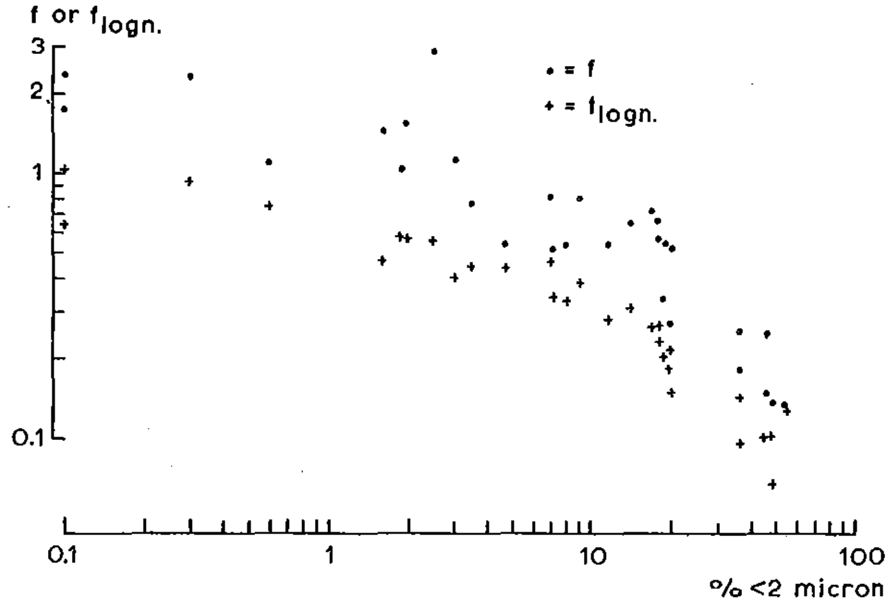

Fig. 4. Indexes f and f_{logn} from table 3 plotted against %<2 micron

from lognormality in a parameter of their own, the fraction f/f_{logn} could be taken. It is shown in fig. 2 that, if clay percentage and median are the same but grain size distribution between 2 micron and the upper limit of the analysis is not lognormal f/f_{logn} will exceed unity. The value of f/f_{logn} does not tell anything about the nature of the deviation from lognormality so sedimentologists will have no interest in it. It did not seem to have any special significance with respect to the calculation of hydraulic conductivities from grain size analyses either. This will be due to the fact that the index f is not only representing clay percentage by correlation but the ratio f/f_{logn} as well, as follows from inspection of fig. 3.

For sand separates the index f can also be computed. In table 2 they are given for some separates. In the calculation it is assumed that 2½% of the sieved separate is finer than the lower size limit and 2½%coarser than the upper limit (according STAKMAN, personal communication).

Table 2. Grain size distribution index f for current separates

size limits	f	size limits	f
2 - 16 µm	1.76	210 - 300 µm	10.3
16 - 50 µm	3.2	300 - 420 µm	10.9
50 - 75 µm	9.0	420 - 500 µm	21.0
75 - 105 µm	10.9	420 - 600 µm	10.3
105 - 150 µm	10.3	600 - 1200 µm	5.3
150 - 210 µm	10.9	1200 - 1700 µm	10.5

4. A RELATIONSHIP BETWEEN THE VALUE OF THE EXPONENT N AND OF THE GRAIN SIZE DISTRIBUTION INDEX F

In table 3 those data from literature, presented in a former paper (BLOEMEN 1977), are entered which remain when the grain size distribution index f is to be correlated with the exponent n in BROOKS and COREY's formula which is a measure of pore size distribution. This is such a small number of useful data because most authors unfortunately do not provide sufficient data on texture. Most authors only give clay, silt and sand percentages. The German authors in table 3 give complete distributions, but BECHER and GIESEL a.o. in a scale which is perhaps not specific enough to allow minor differences in distribution to show in f. Table 3 is supplemented with usable data from the archives of the laboratory of the I.S.W.M. Hydraulic conductivity was determined at moisture tensions increasing from zero to a maximum of about 200 cm of water in undisturbed

Table 3. Clay percentage, grain size distribution index f and f_{logn} median Md and exponent n in BROOKS and COREY's formula

Reference	%>2 µm	f	Md	f_{logn}	n
VETTERLEIN, E. en R. KOITZSCH (1964)					
3 Staubsand	2.5	2.85	75	0.53	12.40
4 Staub	1.9	1.02	40	0.57	14.40
5 Anlehmiger Sand	4.7	0.54	113	0.43	3.80
6 Schwach lehmiger Sand	8.0	0.53	130	0.33	3.13
7 Lehmiger Sand	11.6	0.53	122	0.28	2.83
8 Sandiger Lehm	18.9	0.34	90	0.20	2.51
9 Lehmiger Ton	36.4	0.25	8	0.14	2.41
10 Siltiger Ton	47.4	0.25	2	0.10	2.10
11 Ton	57.9	0.2	<2		1.96
6-10 µm	0	7.16			21.5
60-100 µm	0	7.16			27.0
BECHER, H.H. (1971)					
1 Sand Bv	3	1.12	365	0.40	3.05
2 Parabraunerde Al	7	0.80	29	0.46	2.54
3 " A/g/l	9	0.80	30	0.38	2.20
4 " Bt	18	0.57	25	0.26	1.65
5 " Bv	14	0.64	27	0.31	1.74
6 Marsch	54	0.13	<2	0.128	1.00
GIESEL, W., M. RENGER und O. STREBEL (1972)					
1 Kiesiger Sand	0	1.71	470	0.64	4.80
2 Mittelsand	1.6	1.44	340	0.46	3.20
3 Feinsand	3.5	0.77	170	0.43	2.85
4 Lehmiger Sand	7.1	0.51	255	0.34	2.30
5 Sandiger Lehm	20.0	0.27	48	0.21	2.00
6 Sandig-toniger Lehm	36.5	0.18	15	0.095	1.70
7 Schluffiger Ton (hohe Lagerungsdichte)	47.8	0.135	4	0.065	1.40
8 Idem (mittlere Lagerungsdichte)	46.0	0.143	5	0.10	1.30
9 Toniger Schluff (hohe Lagerungsdichte)	20.0	0.52	25	0.245	2.60
10 Idem (mittlere Lagerungsdichte)	19.5	0.54	31	0.18	2.55

(continued on the next page)

Table 3 (continued)

Reference	%>2 µm	f	Md	f_{logn}	n
Archives of the I.S.W.M. laboratory					
1. Loessloam	17	0.71	23	0.26	2.43
2. Sandy clay	18	0.66	44	0.23	1.99
3. Medium fine sand	2	1.53	94	0.57	4.49
4. id.	0.6	1.10	80	0.75	3.90
5. id.	0.1	2.36	169	1.02	6.4
6. id.	0.3	2.31	155	[illegible]	5.5

soil cores (WIT, not published). In identical samples grain size distributions were determined.

In fig. 5 the index f is plotted against the exponent n. The disturbed samples of VETTERLEIN a.o. (1964) distinguish themselves from the undisturbed samples of other authors owing to a higher level of n at the same value of f. In a preceeding paper this distinction was

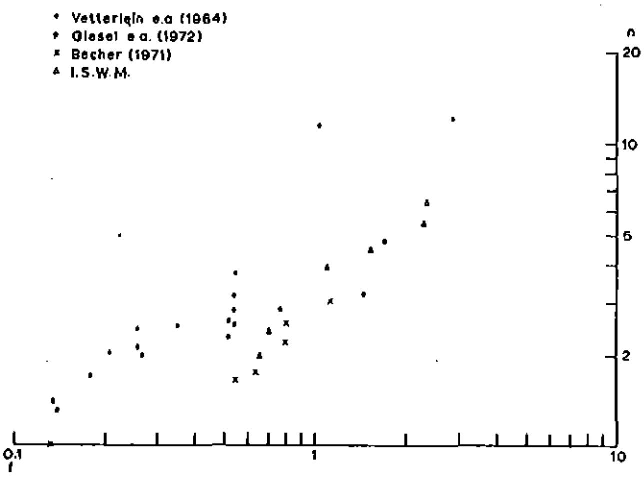

Fig. 5. The grain size distribution index f plotted against the exponent n in BROOKS and COREY's formula. Data points marked + are disturbed samples

shown to be in good agreement with the result of comparative research on the effect of sample treatment by LALIBERTE and COREY (1967).

The data of VETTERLEIN a.o. are not plotted in fig. 6 which is dedicated to undisturbed samples, with the exception of some sand separates which are added for clearness' sake. In fig. 6 a curve is drawn which is considered to be an empirical estimation of the mean relationship between index f and exponent n for undisturbed soils. The curve is extrapolated towards the data points for the sand separates on the assumption that soil samples with a very small variety of grain size would not differ much, disturbed or not, as far as pore size distribution is concerned.

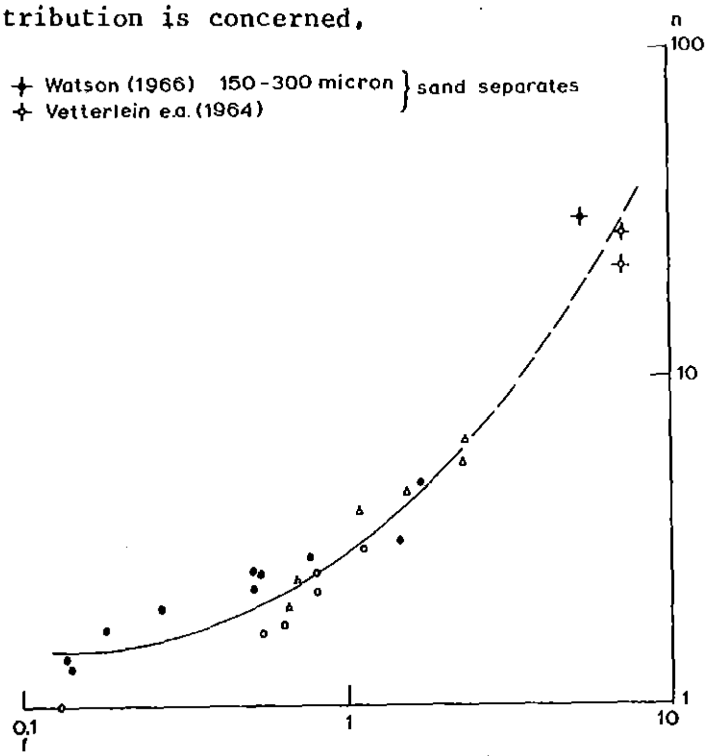

Fig. 6. As fig. 5 but without disturbed samples and three sand separates added

It may be asked whether the grain size distribution index f is indeed a more complete characteristic of soil texture to correlate with the pore size distribution index n than clay percentage. To verify the confirmative answer which may follow from fig. 4, the data in fig. 6 were repeated in fig. 7, but now the exponent n was plotted against clay percentage.

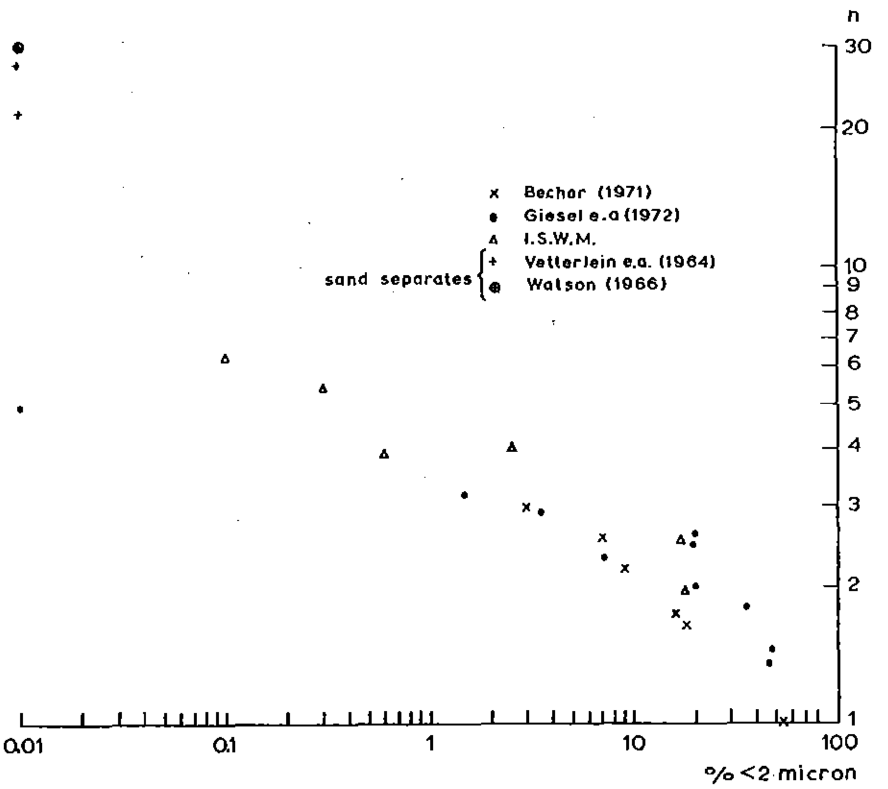

Fig. 7. The clay percentage plotted against the exponent n in BROOKS and COREY's formula. Data points are the same as in fig. 6

The scatter in fig. 2, which is very large when clay percentages are low, reflects in fig. 7. When the clay percentage is low, the index f can have largely different values. These are correlating well with the exponent n, the clay percentages are not. With a much wider scale on the abscissa in fig. 6 than in fig. 7 there is still a considerable difference in scatter between figs. 6 and 7. This is convincing evidence that the index f is a better measure for pore size distribution than clay percentage.

The reliability of the curve in fig. 6 suffers undoubtedly from some causes. In the first place there will be a lack of accuracy in the data points from literature because the values of n were computed from values of conductivity and moisture tension which are mostly read from diagrams (BLOEMEN, 1977) . This lack of accuracy is increased owing to the large size intervals in the scale which is used by BECHER (1971) and GIESEL a.o. (1972). This may obscure part of a relationship between f and n. In the second place the incompatibility of the data of different authors will impair the reliability of the curve. This incompatibility shows in the higher values of n given by GIESEL a.o. compared with those of BECHER. This gives reason to assume that differences in laboratory conditions, methods, apparatus and perhaps even qualities of the investigators can cause differences in the results of investigation where there should be none.

In spite of the criticism the curve in fig. 6 leaves little doubt as to the validity of the essential issue that real values of the pore size distribution index n will be lower than theoretical values. More data are required to make sure of the correct relationship between grain size distribution index and pore size distribution index. In the meantime the curve in fig. 6 may be valuable for practical purposes and serve provisionally to assess the value of the exponent n as a function of grain size distribution.

5. SUMMARY AND CONCLUSIONS

It is shown in this paper that a more complete characteristic of soil texture than clay percentage can be computed from an adequate grain size analysis. This socalled grain size distribution index f gathers in one parameter all properties of grain size distribution between a lower and an upper limit. The lower limit is generally 2 micron. The clay percentage will have a great influence on the value of f. By introducing the index f of lognormal distribution curves with the same clay percentages and medians it could be shown, however, that though the influence of separate properties of texture like sorting, skewness and peakedness can not be distinguished, they have a collective independent influence on the value of the index f.

The grain size distribution index f can be considered as a convenient measure for the pore size distribution. The determination of the moisture retention curve which as a matter of routine had to preceed the calculation of hydraulic conductivities with the BROOKS and

COREY formula is rendered unnecessary through this. It can be replaced by a grain size analysis. The practical advantage of this is increased because there is a definite deviation between real pore size distribution indexes and those calculated from moisture retention curves according to BROOKS and COREY's method.

The pore size distribution index is only one of three parameters in the BROOKS and COREY formula. The others are air entry pressure and saturated conductivity. In a following paper it will be shown that the index f can also serve as a parameter for the indirect determination of these quantities.

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