Extension to 3D of ''The Effect of Line Averaging on Scalar Flux Measurements with a Sonic Anemometer near the Surface'' by Kristensen and Fitzjarrald

ARJAN VAN DIJK*

Department of Meteorology and Air Quality, Wageningen University and Research, Wageningen, Netherlands

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ABSTRACT

Here, the theory of Kristensen and Fitzjarrald, for the transfer function of sonic anemometers for vertical scalar fluxes, is extended from anemometers that use one vertically oriented acoustic path to measure the vertical velocity, to anemometers where the signals measured along three independent paths are involved in the estimation of the vertical velocity. It is found that sonics involving three axes give a significantly stronger attenuation of the small-scale contributions to the flux, than sonics with a vertical acoustic path. This effect should therefore be taken into account when a three-axis sonic anemometer is used to estimate vertical fluxes.

1. Introduction

Kristensen and Fitzjarrald (1984, hereafter KF) have developed a method to estimate the transfer function of a (1D or 3D) sonic anemometer with a vertical acoustic path with respect to vertical scalar flux measurements. To get a nonzero flux, their model involves anisotropic turbulence and is therefore more complex than studies by, for example, Kaimal et al. (1968) and Oncley (1989). In this article we will extend the analysis of KF from a sonic with one axis vertically aligned to a sonic that uses three acoustic paths for the vertical velocity and show the resulting transfer functions for the most popular sonic anemometer configurations. To distinguish between the type of sonic studied by KF and the model presented in this study we will refer to the former as 1D and to the latter as 3D.

2. Model for a 3D sonic

In their relation 8, KF give the relation between fully resolved spectral contribution $\phi_{3S}(\mathbf{k})$ to the vertical flux (subscript 3) of scalar S (subscript S) and the estimate $\tilde{\phi}_{3S}(\mathbf{k})$ for this component as seen by the array of a 1D sonic anemometer, which averages over a vertical pathlength l, (see Fig. 1) and a scalar sampler for S, which takes point-measurements in the center of the sonic-volume:

$$\tilde{\phi}_{3S}(\mathbf{k}) = \mathrm{sinc}\left(\frac{k_3 l}{2}\right) \cdot \phi_{3S}(\mathbf{k}), \tag{1}$$

for a sonic with a vertical path,

$$\mathrm{sinc}(x) \equiv \frac{\sin(x)}{x}. \tag{2}$$

In these relations wavevector $\mathbf{k}$ has components (k_1, k_2, k_3), which we will also write in the following cylindrical coordinates: (k, $K\sin\theta$, $K\cos\theta$). The sinc function gives the Fourier-transform of the step function over which the sonic integrates (no integration outside the anemometer, uniformly weighted integration from transducer to transducer). For anemometers that use signals along three different axes to estimate the vertical velocity, this relation becomes somewhat more complicated. We will assume that the sonic has its acoustic paths in a configuration as shown in Fig. 2, that is, along three axes $\mathbf{l}_a$, $\mathbf{l}_b$, and $\mathbf{l}_c$, which all have angle α with the vertical axis ($\alpha = 54.7°$ gives three perpendicular axes):

$$\mathbf{l}_a = l\begin{pmatrix} -\sin\alpha \\ 0 \\ \cos\alpha \end{pmatrix}, \qquad \mathbf{l}_b = \frac{l}{2}\begin{pmatrix} \sin\alpha \\ \sqrt{3}\,\sin\alpha \\ 2\cos\alpha \end{pmatrix}, \quad \text{and}$$

$$\mathbf{l}_c = \frac{l}{2}\begin{pmatrix} \sin\alpha \\ -\sqrt{3}\,\sin\alpha \\ 2\cos\alpha \end{pmatrix}. \tag{3}$$

For such sonics the three acoustic paths equally contribute to the measured spectrum of the vertical flux. Consequently, the relation between the observed spectrum and exact spectrum is given by the average of the transfer functions over the three paths:

* Current affiliation: Institute for Marine and Atmospheric Research Utrecht, Utrecht University, Netherlands

Corresponding author address: Arjan van Dijk, IMAU, Princetonplein 5, NL-3584 CC Utrecht, Netherlands.
E-mail: A.vanDijk@phys.uu.nl

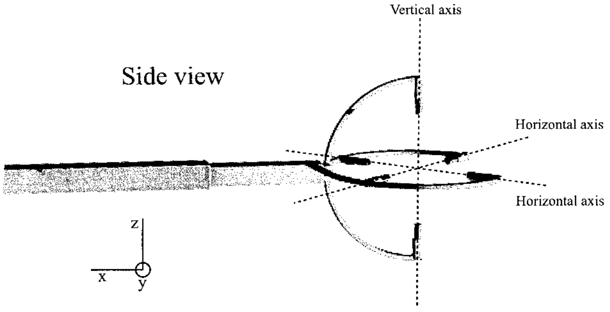

FIG. 1. Configuration of the anemometer studied by KF.

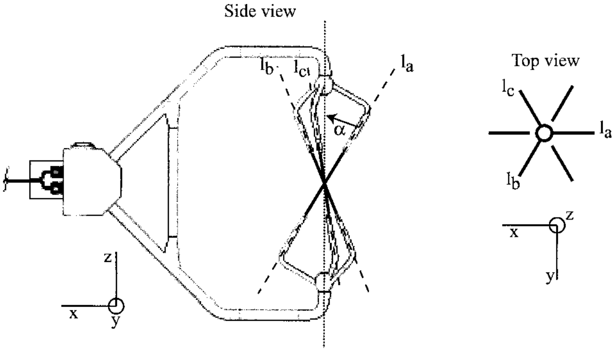

FIG. 2. The acoustic paths of the 3D sonic anemometer considered in this study. Angle α gives the deviation from the vertical of *all three* acoustic paths.

$$\tilde{\phi}_{3S}(\mathbf{k}) = \frac{1}{3}\left[\operatorname{sinc}\left(\frac{\mathbf{k}\cdot\mathbf{l}_a}{2}\right) + \operatorname{sinc}\left(\frac{\mathbf{k}\cdot\mathbf{l}_b}{2}\right) + \operatorname{sinc}\left(\frac{\mathbf{k}\cdot\mathbf{l}_c}{2}\right)\right] \times \phi_{3S}(\mathbf{k}), \tag{4}$$

for a 3D sonic.

We substitute this relation for the sinc function in KF's relation 32 and find for the one-point spectrum $\tilde{C}o(k)$ of vertical flux $\overline{wS}$ of scalar quantity S, estimated from measurements taken with a sonic anemometer, of which the first coordinate axis is aligned with the mean flow:

$$\tilde{C}o(k) = \int_0^\infty \int_0^{2\pi} K\,A(\sqrt{k^2 + K^2})\frac{k^2 + K^2\sin^2\theta}{k^2 + K^2} \times \frac{1}{3}\left[\operatorname{sinc}\left(\frac{\mathbf{k}\cdot\mathbf{l}_a}{2}\right) + \operatorname{sinc}\left(\frac{\mathbf{k}\cdot\mathbf{l}_b}{2}\right) + \operatorname{sinc}\left(\frac{\mathbf{k}\cdot\mathbf{l}_c}{2}\right)\right] d\theta\, dK. \tag{5}$$

For the statistical model we will use KF's relations 40, 42, and 47 for the cospectrum at height z. This model has a $k^{-7/3}$ behavior for small waves (β is a stability dependent parameter):

$$Co(k) = \beta z\overline{wS}(|k|z)^{-7/3}, \tag{6}$$

$$A(\kappa) = \frac{1}{3\pi\kappa^2}\left(\frac{2}{3\beta}\right)^{0.75}\overline{wS}zF\left[\left(\frac{2}{3\beta}\right)^{0.75}\kappa z\right], \quad \text{and} \tag{7}$$

$$F(s) = \frac{91}{30}s^{-7/3}. \tag{8}$$

With relations 5, 7, and 8 we can estimate transfer function $T(k)$ for a 3D sonic anemometer, provided that the axes of the acoustic paths are given. The transfer function is the ratio between the path-averaged cospectrum and the underlying true cospectrum, relation 6.

3. Numerical results for popular sonics

The Campbell 3D sonic, when frontally aligned with the mean flow direction, has its acoustic paths of length l along axes as specified by relation 3 with $\alpha = 30°$ (see Fig. 2). The axes of the Gill–Solent model are found from the Campbell axes by rotation over 30° around the z-axis. We solve relations 5, 7, and 8 for the transfer function of a 3D sonic for the same set of dimensionless wavenumbers as was done by KF. This is done via numerical approximation of the double integral with the Numerical Algorithms Group library's routine D01FCF on a domain of $(\theta, K) \in [0..2\pi] \times [0..5000]$ with an accuracy of 0.00005. We tested our integration program on the 1D case and reproduced the findings in Fig. 1 and Table 1 in the study by KF. Both the Campbell and the Gill configurations are studied. In addition we studied a sonic of which the configuration is found from that of the Campbell by increasing the angle α in relation 3 for the sonic paths to $\alpha = 45°$.

Our results are plotted in Fig. 3 and listed in Table 1, where they are compared with the 1D transfer function. The transfer functions of the Campbell and of the Solent sonics collapsed onto a single curve, which is why only one of these transfer functions is presented. We see that, for the most commonly used types of 3D sonic anemometer (Campbell and Gill–Solent), the transfer function drops faster with increasing wavenumber than the transfer function of a 1D sonic. This difference is significant. As could be expected, the difference between 1D and 3D transfer functions increases with the angle between the vertical axis and the acoustic paths: the transfer function for the 45° configuration is lower than that of the 30° model. For a model with orthogonal axes ($\alpha = 54.7°$) we expect an even stronger attenuation of the small scales.

4. Conclusions

We have developed a 3D generalization of the theory by KF for the transfer function of a sonic anemometer

TABLE 1. Transfer functions for 1D and for 3D sonic anemometers.

kl	1D	3D–30°	3D–45°
0.01	1.0000	1.0000	1.0000
0.1	0.9998	0.9992	0.9990
0.2	0.9980	0.9976	0.9972
0.5	0.9914	0.9900	0.9886
1	0.9720	0.9670	0.9619
1.2	0.9620	0.9550	0.9479
1.4	0.9509	0.9417	0.9325
1.6	0.9390	0.9274	0.9157
1.8	0.9263	0.9122	0.8979
2	0.9130	0.8962	0.8792
2.2	0.8993	0.8797	0.8597
2.4	0.8851	0.8626	0.8397
2.6	0.8707	0.8452	0.8192
2.8	0.8560	0.8274	0.7983
3	0.8412	0.8096	0.7773
4	0.7668	0.7201	0.6720
5	0.6954	0.6353	0.5729
6	0.6296	0.5588	0.4855
7	0.5706	0.4922	0.4117
8	0.5185	0.4355	0.3515
9	0.4728	0.3879	0.3038
10	0.4330	0.3481	0.2664
14	0.3181	0.2445	0.1822
20	0.2241	0.1700	0.1288
30	0.1495	0.1134	0.8580E-01
40	0.1121	0.8503E-01	0.6436E-01
50	0.8968E-01	0.6802E-01	0.5149E-01
60	0.7473E-01	0.5668E-01	0.4291E-01
70	0.6406E-01	0.4859E-01	0.3678E-01
80	0.5605E-01	0.4251E-01	0.3218E-01
90	0.4982E-01	0.3779E-01	0.2860E-01
100	0.4484E-01	0.3401E-01	0.2574E-01

for the vertical turbulent flux of a scalar. From a numerical approximation of this transfer function for the configurations of two commonly used types of anemometer, we have seen that the 3D function decays significantly faster with wavenumber than the 1D function. We therefore conclude that, to achieve maximum accuracy, the 3D character of the anemometer must be taken into account when one corrects measured fluxes for path-averaging by the sonic. With the procedure outlined in this note one can estimate the transfer function involved in the correction.

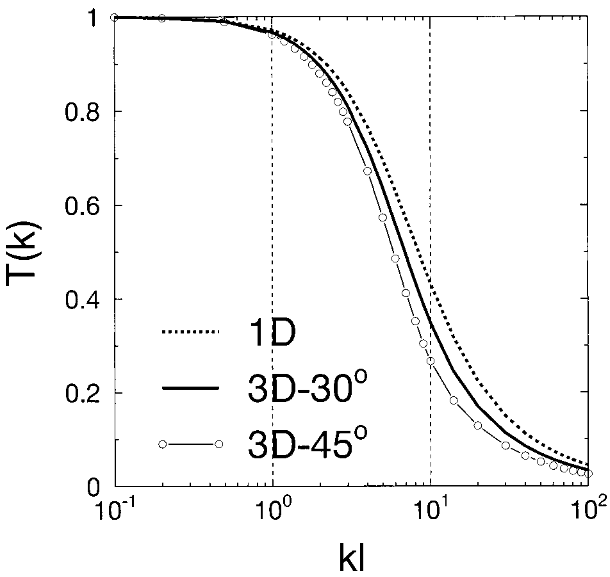

FIG. 3. Transfer functions for sonic anemometers: the dashed line gives the 1D case, the continuous line is for the 3D Campbell–Solent type of sonic and the circles represent a configuration where the acoustic paths make an angle of 45° with the vertical axis. The quantity on the horizontal axis is wavenumber $k \equiv 2\pi/\lambda$ of a flux contribution made dimensionless with acoustic pathlength *l*.

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