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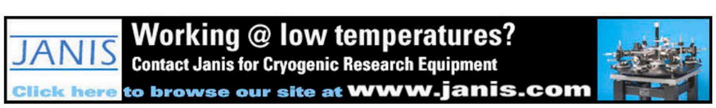

Thermal runaway in microwave heated isothermal slabs, cylinders, and spheres

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The absorption of electromagnetic energy within a microwave heated isothermal slab, cylinder, and sphere is analyzed and compared to each other. It is shown that the absorbed heat oscillates as a function of temperature, regardless of the geometry of the irradiated object. It is possible to formulate this behavior in a simple mathematical equation, which proves that the oscillation is basically caused by resonance of the electromagnetic waves within the object. This oscillation, combined with the heat loss, is found to be responsible for thermal runaway phenomenon in isothermal objects. Based on such an observation, a general rule to prevent thermal runaway has been developed. © *1998 American Institute of Physics.* [S0021-8979(98)08201-2]

I. INTRODUCTION

One of the difficulties associated with the application of microwave heating (frequency range 2450 MHz) is the catastrophic phenomenon of thermal runaway, in which a slight change of microwave power causes the temperature of the irradiated object to increase rapidly. The microwave sintering of ceramics especially is seriously hampered by this phenomenon. The aim of this investigation is to find the physical origin of the runaway process and hope that it leads to a general rule in preventing runaway. The simplest possible system is conceived in order to achieve this. In the first place it has been assumed that the irradiated object is situated in free space without any reflections from the ambient. The object was irradiated by a plane harmonic wave from one side. Deliberately the choice has been not to describe actual experiments, because then the characteristics of the oven combined with the irradiated object might veil the physical origin of the runaway. In the second place it has been assumed that the objects are isothermal. This does not look very realistic, but in the case of small Biot numbers it is a good approximation. In the small Biot number limit many studies have been performed on ceramics. Another category of materials which behaves almost isothermally is the liquids. The temperature gradient within the liquid causes strong convection, which diminishes the temperature differences in the liquid. The study of isothermal systems might be regarded as an initial step towards a more complete understanding of the runaway phenomenon. Nonisothermal objects will be taken into account in the second stage of this investigation. The idea is to regard such a system as a multilayer of isothermal layers with different temperatures.

The principles developed are applied to demineralized water. Compared to other materials water has a number of advantages. Its physical parameters are well known;[1] as a liquid it behaves almost isothermally, and the theory can been verified experimentally with little effort. A potential application of the study of water in relation to thermal runaway is the improvement of the quality of microwave dried foodstuffs. The mathematical formulas developed below are generally true and also applicable to ceramics. However one should be very careful in generalizing the results of the calculations. When compared to ceramics the dielectric constant of water hardly depends on temperature. The dielectric loss factor in many ceramics especially is a strong function of temperature, which is a major factor in thermal runaway in these materials. On the other hand, there are some remarkable similarities between the thermal behavior of water and ceramics.

The specific aim of this article is to investigate how the geometry of the irradiated object influences the runaway process. This has been done by comparing a slab, a cylinder, and a sphere to each other. In an earlier article[2] based on the work of Kriegsmann[3] and of Stuerga *et al.*,[4] it was shown that the runaway effect in a slab of water is caused by resonance of the electromagnetic waves inside the irradiated object. It is a matter of standing waves. It is obvious that the same thing will happen to (infinitely long) cylinders and spheres, irradiated from all sides, with the condition that the waves at the surface are in phase. This is analogous to a slab irradiated from two sides with coherent waves. The only difference is that the ''standing waves'' are described by cylindrical functions (Bessel and Hankel functions) or spherical functions.

In the case of cylinders and spheres irradiated from one side there is neither cylindrical nor spherical symmetry. It is impossible to apply the basic idea of standing waves, as developed for the slab, to cylindrical or spherical waves, because these waves are not present in an elementary form. It will be demonstrated that also in this case the phenomenon of thermal runaway is still present.

Attention has to be paid to the notation. In contravention of the usual notation[5] the symbol α will be used as the phase constant and β as the attenuation constant. Both are real numbers. This notation reads easier and it fits in better with the theory of wave propagation as described in classic books. The runaway phenomenon, as described in this article, is a result of the application of the wave propagation theory.

II. THE ISOTHERMAL SLAB

Let us consider a layer of material specimen, irradiated from one side by microwave radiation with a frequency of

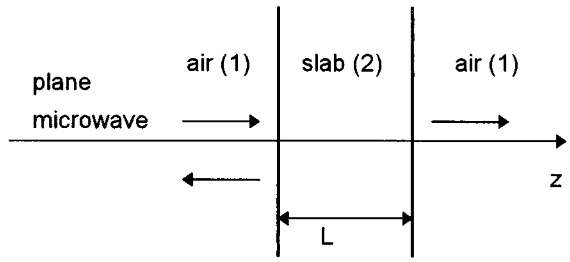

FIG. 1. A layer, being irradiated from one side, in an echo-free cavity.

2450 MHz. The wave is a plane, harmonic one and impinges normally upon the material (Fig. 1). The differential equation describing the relationship between temperature T and time t (Fourier's law) reads

$$L\rho C_p \frac{dT}{dt}=D-2h(T-T_0), \quad (1)$$

where ρ is the density, C_p is the thermal capacity, and L stands for the layer thickness. D is the heat production (total amount of power per square meter generated along the z axis, extending from $z=0$ to $z=L$). The total heat loss is described by an effective heat transfer coefficient h, multiplied by the temperature difference $(T-T_0)$ between slab and ambient. The expression for the heat loss is an approximation, only valid in the case of small temperature differences. The absorbed energy D follows from Maxwell's equations, together with the appropriate boundary conditions at the surface of the slab. Approximately one obtains

$$D\approx P\frac{4\alpha_1}{\alpha_2}[1+2|R_{12}|^2 e^{-2\beta_2 L}\cos(2\alpha_2 L)], \quad (2)$$

where P is the microwave power and R_{12} the reflection coefficient. This approximation is based on two requirements; first, the phase constant α_1 of air is much smaller than the phase constant α_2 of the irradiated medium. This is a very general demand and most solids and liquids answer this demand; second, α_2 should be much smaller than the attenuation factor β_2 of the medium. This is true for water.

The special thing about water as compared to ceramics is the fact that α_2 hardly depends on temperature. This means that in the first approximation the transient temperature as a function of heating time will be represented as a straight line. Superimposed on this line is an oscillation of which the amplitude increases with temperature, but decreases with layer thickness (Fig. 2). The oscillation is caused by resonance within the medium. With respect to wave propagation the slab behaves like a violin string. The heat production has a maximum if the wavelength λ_2 of the medium equals $2L/n$ for $n=1,2,3...$ The steady state temperature follows from

$$D-2h(T-T_0)=0. \quad (3)$$

In order to have runaway three intersecting points of heat production and heat loss are necessary. This will be the case if the cosine has a maximum around 80 °C, which results in a thickness L of 1.6 cm for $n=2$. This is the smallest thickness for which runaway might occur (Fig. 3). A plot of the steady state temperature versus the microwave power for $L=1.6$ cm shows the familiar S shape (Fig. 4).

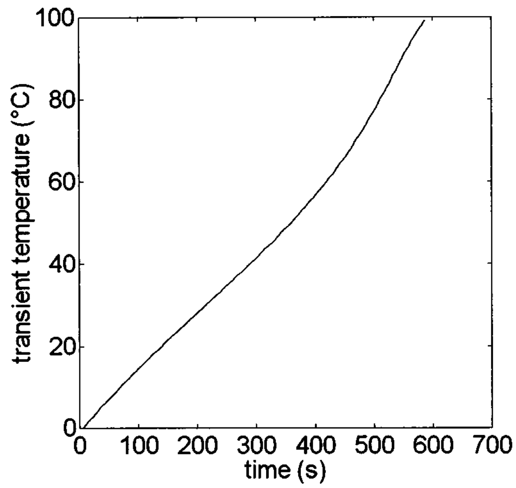

FIG. 2. Transient temperature of a slab without heat loss as a function of heating time, showing that the absorbed microwave power is almost temperature independent. $L=1.6$ cm, $P=40$ kW/m^2, water.

III. THE ISOTHERMAL CYLINDER

A cylinder is irradiated by a plane harmonic wave from one side in the same way as the slab. The electric field vector of the incident field is parallel to the central symmetry axis of the cylinder (Fig. 5). The equation of the heat balance differs a little from Eq. (1), because the surface is larger.

$$\frac{1}{2}L\rho C_P\frac{dT}{dt}=\frac{2D}{\pi L}-2h(T-T_0). \quad (4)$$

In this case L is the diameter of the cylinder.

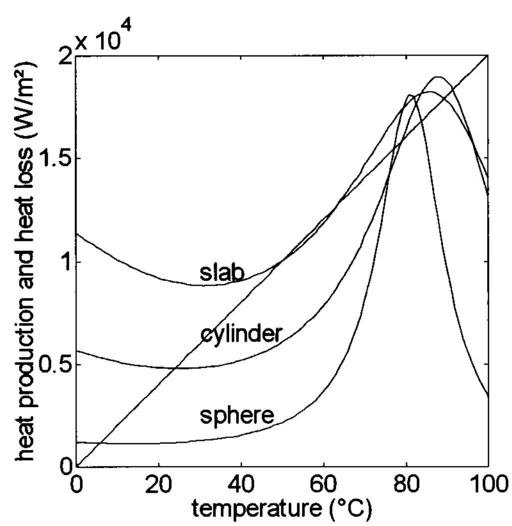

FIG. 3. The absorbed microwave power oscillates as a function of temperature. This oscillation, combined with the heat loss (the straight line), is responsible for thermal runaway. Slab $L=1.6$ cm, $P=40$ kW/m^2; cylinder $L=1.2$ cm, $P=9$ kW/m^2; sphere $L=1.4$ cm, $P=150$ kW/m^2; $h=100$ W/m^2, ambient temperature is 0 °C; the medium is water.

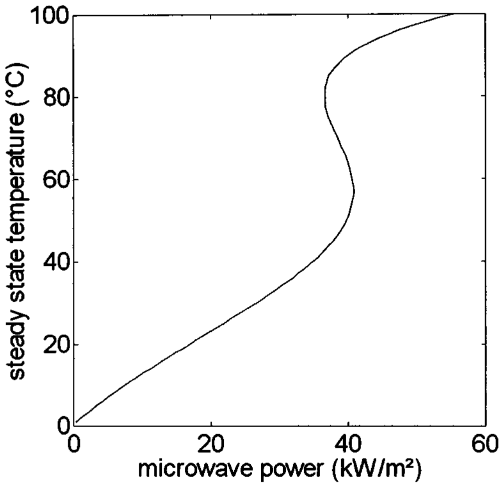

FIG. 4. The S-shaped curve of a slab (water, $L=1.6$ cm). Increasing the microwave power from 40 to 45 kW/m^2 results in a temperature jump of about 40 °C.

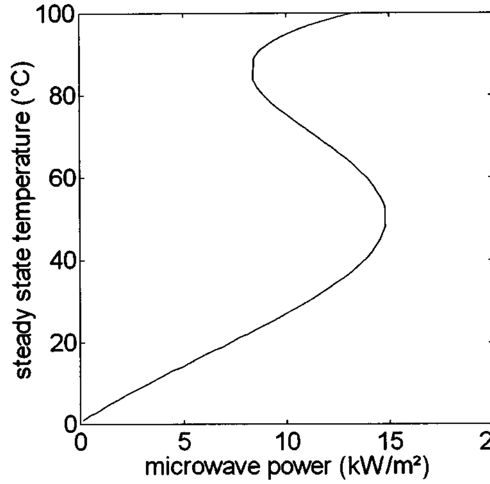

FIG. 6. The S-shaped curve of a cylinder (water, $L=1.2$ cm). Increasing the microwave power from 14 to 15 kW/m^2 results in a temperature jump of about 50 °C.

The heat production D is directly proportional to the absolute square of the electric field E_2 within the cylinder. In polar coordinates r and ϕ the field is[6]

$$E_2=A\sum_{n=-\infty}^{n=+\infty} c_n J_n(k_2 r)e^{in\phi}, \tag{5}$$

where A is the amplitude of the incident electric field, J_n is the Bessel function of the first kind of order n, and k_2 is the complex wave number ($k_2=\alpha_2+i\beta_2$). The coefficients c_n determine the character of the field. They can be found by applying the boundary conditions at the surface of the cylinder.

$$c_n=\frac{k_1[J_n'(k_1R)H_n(k_1R)-J_n(k_1R)H_n'(k_1R)](i)^n}{k_2J_n'(k_2R)H_n(k_1R)-k_1J_n(k_2R)H_n'(k_1R)}, \tag{6}$$

here H_n are Hankel functions of first kind of order n, R is the radius of the cylinder, and $k_1=\alpha_1$. To achieve the total absorbed energy D of the isothermal cylinder the absolute square of the electric field has to be integrated over the cross section of the cylinder. This can be done analytically, but the result is a very complicated meaningless equation. A much better result is achieved by using the asymptotic expansion of the Bessel function.

$$J_n(x)\approx\sqrt{\frac{2}{\pi x}}\cos\left(x-\frac{n\pi}{2}-\frac{\pi}{4}\right), \tag{7}$$

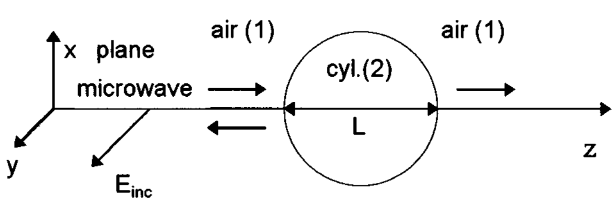

FIG. 5. A cylinder, being irradiated from one side, in an echo-free cavity.

where x is large.

Replacing Bessel functions by cosines and integrating them over the cross section yields

$$D\sim[1-2|R_{12}|^2e^{-2\beta_2L}\cos(2\alpha_2L)]. \tag{8}$$

Except for the sign this equation is identical to Eq. (2) of the slab. A maximum heat production is expected for $L=\lambda_2(1+2n)/4$, where $n=0,1,2,\ldots$. This means that the diameter L should equal 1.2 cm in order to have a maximum at 80 °C (Fig. 3) for $n=1$ (water). The consequence is thermal runaway (Fig. 6). The same kind of S shape has been calculated for ceramic cylinders in the small Biot number limit.[7] Numerical calculations demonstrate that, in the first approximation, the heat production is temperature independent. The runaway process for water is caused solely by the temperature dependence of the phase constant.

The phenomenon of thermal runaway has also been studied in the case of an isothermal cylinder irradiated from one side by a plane harmonic wave with the electric field vector lying in the plane of incidence (perpendicular to the vector drawn in Fig. 5). This leads to the same behavior as described above. Especially Eq. (8), the most important one, was exactly the same.

IV. THE ISOTHERMAL SPHERE

A sphere is irradiated from one side by a plane harmonic wave in free space. In this case Fourier's law reads

$$\frac{1}{3}L\rho C_p\frac{dT}{dt}=\frac{2PD}{\pi L^2}-2h(T-T_0), \tag{9}$$

where L is the diameter of the sphere. The mathematical description of the electromagnetic field is based on the so-called Mie theory.[8] In terms of spherical coordinates r, ϕ, and θ, the spherical components of the electric field within the sphere, are

$$E_r = A\sum_{n=1}^{\infty} i^{n-1}\,\frac{2n+1}{k_2 r}\,\sqrt{\frac{\pi}{2k_2 r}}\,b_n J_{n+1/2}(k_2 r)\times\frac{dP_n(\cos\,\theta)}{d(\cos\,\theta)}\cos\,\phi\,\sin\,\theta, \tag{10}$$

$$E_\theta = A\sum_{n=1}^{\infty} i^{n}\,\frac{2n+1}{n(n+1)}\,\sqrt{\frac{\pi}{2k_2 r}}\left[a_n J_{n+1/2}(k_2 r)\,\frac{dP_n(\cos\,\theta)}{d(\cos\,\theta)} - ib_n\left(\frac{n+1}{k_2 r}\,J_{n+1/2}(k_2 r) - J_{n+3/2}(k_2 r)\right)\times\frac{\partial P_n^1(\cos\,\theta)}{\delta\theta}\right]\cos\,\phi, \tag{11}$$

$$E_\phi = A\sum_{n=1}^{\infty} i^{n}\,\frac{2n+1}{n(n+1)}\,\sqrt{\frac{\pi}{2k_2 r}}\left[-a_n J_{n+1/2}(k_2 r)\times\frac{\partial P_n^1(\cos\,\theta)}{\partial\theta} + ib_n\left(\frac{n+1}{k_2 r}\,J_{n+1/2}(k_2 r) - J_{n+3/2}(k_2 r)\right)\times\frac{dP_n(\cos\,\theta)}{d(\cos\,\theta)}\right]\sin\,\phi, \tag{12}$$

where P_n are the Legendre polynomials and P_n^1 correspond to the associated Legendre functions of the first kind. The coefficients a_n and b_n can be found by applying the electromagnetic boundary conditions at the surface of the sphere. For a_n, respectively, b_n, one yields

$$a_n = \frac{2i\sqrt{k_1 k_2}/\pi R}{\frac{(k_2-k_1)(n+1)}{R}\,J_{n+1/2}H_{n+1/2} - k_2^2 J_{n+3/2}H_{n+1/2} + k_1^2 J_{n+1/2}H_{n+3/2}}, \tag{13}$$

$$b_n = \frac{2i\sqrt{k_1 k_2}/\pi R}{\frac{(k_2-k_1)(1-n)}{R}\,J_{n+1/2}H_{n+1/2} - k_1 k_2 J_{n+3/2}H_{n+1/2} + k_1 k_2 J_{n+1/2}H_{n+3/2}}. \tag{14}$$

The Bessel functions have the argument k_2R, while the Hankel functions always depend on k_1R. Here R is the radius of the sphere. The total amount of absorbed heat D of the isothermal sphere is proportional to the integral over the volume of the sphere of the absolute square of the total electric field. The last one equals the sum of the absolute squares of the spherical field components. This (very elaborate) integration is analytically possible, but it also results in a meaningless equation. The problem is that the equation does not converge in such a way that the first order terms of the Bessel and Hankel functions are the most dominant ones. The most dominant term is determined by the diameter of the sphere. However, numerical analysis (in the case of water) shows that a simplification of the exact solution is possible. For relatively small L ($L<3$ cm) the oscillating behavior of D is mainly described by $|a_2|^2$. If the diameter lies somewhere between 3 and 6 cm, then Eq. (8) creates a perfect fit. The minimum diameter for which runaway occurs is 1.4 cm (Fig. 3). The high and sharp peak causes a large jump of the steady state temperature (Fig. 7). On the other hand, this first order ($n=1$) runaway effect is very sensitive to small changes of the diameter. Increasing or decreasing the diameter of 1.4 cm by 0.1 cm is enough to destroy the runaway phenomenon. Numerical analysis proves that also in the case of the sphere the absorbed heat D is temperature independent in the first approximation, resulting in the same kind of graph as in Fig. 2. Studies of a ceramic sphere inside a rectangular cavity[9] also show how the absorbed microwave power oscillates as a function of the radius.

V. DISCUSSION AND CONCLUSION

The isothermal slab, cylinder, and sphere, irradiated from one side by a plane harmonic microwave, behave in the same way in relation to thermal runaway. In the first approximation the dissipated microwave power is independent of temperature. The phenomenon of thermal runaway is basically caused by the temperature dependence of the phase constant of the irradiated medium in combination with heat

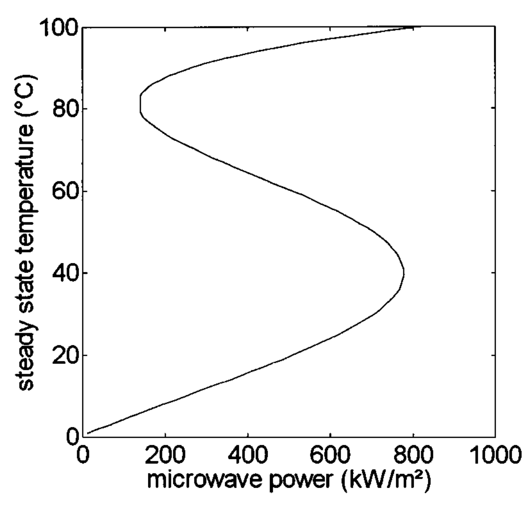

FIG. 7. The S-shaped curve of a sphere (water, $L=1.4$ cm). Increasing the microwave power from 78 to 80 kW/m^2 results in a temperature jump of 60 °C.

loss. For watery objects the absorbed energy D is reflected by a simple approximation:

$$D \sim 1 \pm 2|R_{12}|^2 e^{-2\beta_2 L} \cos(2\alpha_2 L), \tag{15}$$

where the + sign refers to the slab. L is the thickness of the slab or the diameter of the cylinder or sphere. In all three cases the oscillation is caused by resonance within the objects. This suggests that the geometry of an irradiated isothermal object is irrelevant in relation to thermal runaway. If there is a possibility for resonance of the electromagnetic waves within the object, thermal runaway will occur.

The calculations demonstrate that the characteristic dimension must be at least equal to a complete wavelength (slab) or to 3/4 of a wavelength (cylinder and sphere) at a relatively high temperature, in order to have runaway. At that specific temperature the heat production has a maximum. This maximum has to precede a minimum at low temperatures. This might already be the fact if a quarter of a period of oscillation is present. Thus the conclusion is that any isothermal object with characteristic dimension L, irradiated from one side by microwaves, will never be overtreated or damaged by the phenomenon of thermal runaway if L is smaller than $\pi/4\Delta\alpha_2$, where $\Delta\alpha_2$ is the difference between the maximum and the minimum value of the phase constant α_2. For water this results in a dimension of about 1 cm.

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