Solving the Newsvendor Problem under Partial Demand Information

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Mots-clés : *stochastic inventory control, newsvendor, partial demand information.*

1 Abstract

We consider the problem of controlling the inventory of a single item with stochastic demand over a single period. This problem is known as the "Newsvendor" problem [10], since it is the prototype of the problem faced by a newsvendor trying to decide how many newspaper to stock on his stand before observing demand. The newsvendor faces costs if he orders too much or if he orders too little. This problem therefore consists in deciding the size of a single order that must be placed before observing demand when there are overage and underage costs. A thorough literature review on the Newsvendor problem is presented by [7]. Most of the research on single-period inventory models has focused on the case in which demand distribution parameters are known. Nevertheless, it is clear that the applicability of these models directly depends on the accuracy of demand parameters estimation.

The importance of parameter estimation has been pointed out in several works [3, 2]. According to [2], there are two main approaches to demand estimation in the inventory theory literature : the frequentist and the Bayesian approach. In the frequentist approach [8, 1] a parametric demand distribution is empirically selected and point estimates (e.g. maximum likelihood or moment estimators) for the unknown parameters are obtained according to the observed data. In the Bayesian approach [4, 5] a "prior" distribution is selected for the demand parameter, based on collateral data and/or subjective assessment. From the prior distribution a "posterior" distribution is derived as new demand data become available. The posterior distribution of the parameter is then used to construct, first, the posterior distribution of the demand, and then to derive the optimal order quantity and objective function value. [4] points out that the literature on the application of Bayesian methods to demand estimation in the context of inventory modeling is considerable in volume. Furthermore, most of the above mentioned works focus on the critical issue of unobserved lost sales and on the ability to incorporate in the estimation procedures suitable adjustments to account for the unobserved component of the demand [8, 1]. Nevertheless, as discussed by [6], little research exists, to date, on the effect of demand estimation on optimal inventory policies for single-period inventory models.

In this work, we introduce a novel strategy to address the issue of demand estimation in single-period inventory optimization problems. Our strategy is based on the theory of statistical estimation introduced by [9] and it complements existing strategies based on maximum likelihood estimators or on Bayesian analysis. Consider a possibly very limited set of past demand observations. Classical strategies based on maximum likelihood estimators or on Bayesian analysis would analyze these data and provide a single most-promising order quantity and an estimated cost associated with it ; the latter being either a point estimate [4] or a confidence interval obtained via a normal approximation [6]. These strategies are typically justified on the basis of some asymptotic convergence properties they feature. In contrast, our approach employs exact confidence intervals in order to identify a range of candidate order quantities that, with prescribed confidence probability, includes the real optimal order quantity for the underlying stochastic demand process with unknown parameter(s). In addition, for each candidate order quantity that is identified, our approach computes an upper and a lower bound for the associated cost. This range covers, according to the prescribed confidence probability, the actual cost the decision maker will incur if he selects that particular quantity. Our approach is therefore particularly appealing in all those situations in which the decision maker aims to assess, with suitable confidence level and from a (possibly limited) set of available data, the risk of exceeding a certain cost when a given candidate quantity is selected. Once the set of candidate optimal order quantities has been identified, it may comprise one or more elements. If it comprises more than a single value, expert assessment or any existing approach based on classical estimation strategies [4, 5, 6] may be employed to select the most promising of these values. Therefore, our approach complements existing strategies in the literature. However, we stress the fact that, in contrast to other approaches in the literature, our approach does not simply provide a point estimation, it provides instead complete information to the decision maker about the set of potentially optimal order quantities — according to the available data and to the chosen confidence level — and about the confidence interval for the expected cost associated with each of these quantities. We discuss our novel approach for three demand distribution in the exponential family : Binomial, Poisson, and Exponential.

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