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ON THE EVALUATION OF PARAMETER VALUES IN MODELS OF THE WATER BALANCE

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CONTENTS

1. INTRODUCTION

Data of groundwater levels and run-off values of a watershed may occasionally be available together and in sufficiently large amounts that it is worthwhile to try to get as much information on the hydrology of the watershed out of these data as possible. These possibilities are being studied with models of the water balance equation, because run-off is one item on the water balance and groundwater level fluctuations are the result of interplay of all items. In this paper results are discussed of the application of a model of the water balance on data from watersheds in the district of Salland in the province of Overijssel, collected in the years 1970, 1971 and 1972.

2. MODELLING THE WATER BALANCE

a. A very simple model

With a model of the water balance daily values of run-off, actual evapotranspiration and storage coefficient are calculated. For the day k runoff R_k and evapotranspiration E_k are settled with precipitation P_k. This gives the change in storage. Divided by the storage coefficient μ_k this is converted into the change in groundwater level ΔG_k. This is settled with the groundwater level G_k and gives the groundwater level for the next day. So the principle is

$$\frac{P_k - R_k - E_k}{\mu_k} = \Delta G_k \qquad (1)$$

and

$$G_k = G_k + G_k \qquad (2)$$

The items on the model of the water balance are described as functions in which the parameters are combined with variables expressed in terms of open water evaporation, precipitation and depth or head of the phreatic water. The saimplest example of such a deterministic model has 6 parameters and rests on a study on the determination of storage, discharge and evapotranspiration from groundwater level data (BLOEMEN, 1970).

The drainage process is described with the equation:

$$R_k = a(S-G_k)^2 \tag{3}$$

in which S represents an unknown drainage level and a lumps together all resistances to groundwater flow. G is the depth of the phreatic level below soil surface.

Actual evapotranspiration E is calculated as the lowest value of two. One is:

$$E_k = g\ E_{ok} \tag{4}$$

in which E_o is measured or calculated open water evaporation and g is a reduction factor according to Penman's theory. The other is:

$$E_k' = d_1\ G_k^{-d_2} \tag{5}$$

in which d_1 and d_2 are parameters expressing how E is limited by the depth of the groundwater table G.

Finally the storage coefficient is calculated as a function of the groundwater depth with parameters f and m as:

$$\mu_k = f\ G_k^m \tag{6}$$

In this model the amounts B of water involved with changes of the groundwater level from G_1 to G_2 must be found by integration because (6) is not linear. The solution is:

$$B = \int_{G_1}^{G_2} f\, G^m\, dG = \left[f\, \frac{G^{m+1}}{m+1} \right]_{G_1}^{G_2} \qquad (7)$$

$$B = \frac{f}{m+1} \left(G_2^{m+1} - G_1^{m+1} \right) \qquad (8)$$

This implies that the groundwater level G_k after a 24-hours change in water storage B, calculated as B = P-R-E can be calculated from the groundwater level G_{k-1} at zero hours as:

$$G_k = \sqrt[m+1]{\frac{-(m+1)(P-R-E)_k}{f} + G_{k-1}^{m+1}} \qquad (9)$$

Equation (9) represents the simplest model to calculate successive groundwater levels with. Application on data from some regions will make it necessary to allow for positive or negative infiltration. This is proportional to the difference between phreatic and piezometric heads h and inversely proportional to the hydraulic resistance in vertical direction, or:

$$I_k = \frac{h_{phrk} - h_{piezk}}{c} \qquad (10)$$

When only phreatic heads are measured, infiltration may be calculated from the differences Δh_i between the phreatic head in the watershed and those describing the grade of the phreatic level outside the catchment toward it over a length L. It is assessed in $m^3.day^{-1}$ for the area of the catchment as:

$$I_k = \sum_{i=1}^{n} \Delta h_{ik} \frac{W_i kD_i}{L_i} \qquad (11)$$

in which W_i is the width of streamflow in the direction of L_i.

In fig. 1 a flow diagram of the simple model in eq. (9) is given. There are 6 parameters. All magnitudes are expressed in meters. G is the measured groundwater depth below soil surface.

b. A better representation of physical reality

Of course a better representation of the physical aspects of runoff, evapotranspiration, etc. can be given in a model. There are three main points:

i It is obvious that the calculation of evapotranspiration and storage coefficient is depending on the assumption that there is a continuous equilibrium between suction and the distance to the phreatic level. Of course, a better conception is that the moisture tension Ψ at a fixed depth z under soil surface is calculated as a function of groundwater depth G and capillary conductivity K_c. The equations are derived from the function (RIJTEMA, 1965):

$$K_c = K_o \; e^{-\alpha\Psi} \tag{12}$$

in which K_o is capillary conductivity at zero suction, Ψ is suction and α is a soil constant. The equations are:

$$\Psi_k = \frac{1}{-\alpha} \ln T_k \tag{13}$$

and

$$T_k = 1 - (1-e^{-\alpha z})(1 + \frac{V_{ck}}{K_o}) \tag{14}$$

V_c is the upward capillary flow and as a simplification it is assumed to equal actual evaporation. A test on the value of T_k decides whether eq. (4) is valud or the equation:

$$E_k = d_1 \; \Psi_k^{-d_2} \tag{15}$$

If eq. (4) does not hold Ψ will be imaginary ($T_k < 0$) and eq. (15) is substituted for V_c. E and Ψ are then solved by iteration. If, however, on the particular day precipitation exceeded E according eq. (4) then this is yet applied in the water balance instead of E according eq. (15), but Ψ calculated as if eq. (15) was valid is upheld.

Consequently the storage coefficient   is calculated as:

$$_k = c_1 \; _k^{c_2} \qquad (16)$$

ii It is clear that a part of precipitation which is measured with 24 hours intervals, may have its effect on the rising groundwater level in the next interval. So it is necessary to calculate the amount F of rainfall P which percolates to the groundwater in the same interval in dependence on the depth of the phreatic level. The equation is:

$$F_k = (P_{k-1} - F_{k-1} + P_k)\, e^{-q\, G_{k-1}} \qquad (17)$$

The parameter is q and its physical meaning is somewhat obscured because as a rule there are time lags between 24 hours intervals of measuring precipitation and measuring the groundwater level. Besides, precipitation is a 24 hours total and groundwater level is a 24 hours average at best. Details like these are not accounted for in the model.

iii The drainage process will have to be described in a more complicated form. In the medium high grounds on which the model of the water balance was to be applied a rising phreatic level means an increase of drainage flow contributing to the depletion of the groundwater reservoir, because it rises over the beds of an increasing number of channels of different order. Integration of groundwater flow from an observation point with a high phreatic level will show a smooth curve in a diagram like fig. 2, where runoff of a watershed and phreatic head at same point in it are plotted. Such a relationship can only be described on a physically acceptable basis by schematizing arbitrarily to for instance three drainage levels, as in fig. 2. Further simplification is that deeper channels have bigger mutual distances. As impermeable layers appear in this area only at a considerable depth it is assumed that when applying the Hooghoudt formula for steady flow the quadratic term in it can be neglected as far as flow to channels

Fig. 2. Runoff of a watershed plotted against phreatic head at same point in it

at large mutual distances is concerned.

So the drainage of these medium high grounds is described in the model of the water balance as:

$$R_k = b_o(S_1-G_k) + b_1(S_1-G_k)^2 + b_2(S_2-G_k) + b_3(S_3-G_k) \qquad (18)$$

The drainage levels S_i are solved as parameters, the parameters

b_i stand for the resistance to groundwater flow to the water levels S_i in the channels.

In fig. 3 a flow diagram is shown of the model of the water balance, after the preceeding remarks have been applied in the formula. There are 16 parameters. Evapotranspiration and runoff are calculated in millimeters.day^{-1}, moisture tension and groundwater levels are calculated in centimeters.

c. Accounting for non-steady outflow

A further development of the model is attractive because in eq. (18) only steady groundwater flow is accounted for. In 24 hours intervals there will be a fair chance that non-steady flow will be of considerable importance. As a matter of fact the scatter in fig. 2 emphasizes the probability that non-steady flow occurs when groundwater levels are high. It would complicate things too much to describe both steady and non-steady flow to different levels. A simplification is justified by the triviality of the quantities of water which are drained by channels at large mutual distances. When these are ignored runoff can better be represented by a description of steady and non-steady flow to the most important group of channels.

Steady flow is calculated with the Hooghoudt formula as a function of the mean phreatic head h_k, which is equal to $S-G_k$. This mean is of course the average of two observations with a 24 hours interval. To prevent the calculation of too much runoff when big changes in phreatic head occur, there has to be integrated between h_k and h_{k+1}. The equation is:

$$R_k = \frac{\int_{h_k}^{h_{k+1}} (ah + bh^2)dh}{h_k - h_{k-1}} \qquad (19)$$

a and b are the well known constants in the Hooghoudt formula.

Non-steady flow can be described as a function of the change in groundwater level during the 24 hours intervals. This conception is based on a well known theory that after rainfall groundwater outflow

Fig. 4. A rise of the groundwater level from h_1 to h_2 gives an increased water storage A and an extra runoff a. A fall from h_1 to h_2' gives a decreased water storage B and a reduction b of runoff

increases more than harmonizes with the relatively slow rise of the groundwater table. Groundwater levels are only constant when outflow and percolation are equal. In that case an equation for steady flow like eq. (19) is valid. When no percolation occurs there is also a fixed relation between outflow and phreatic head. This tail recession outflow is in the ratio of $1 : \frac{12}{\pi^2} = 0.82 : 1$ to steady outflow (KRAAYENHOFF VAN DE LEUR, 1958).

In fig. 4 is schematically shown how deviations from steady outflow are related to changes in groundwater level. As an explanation it may be accepted that only when outflow is steady at every point between two channels the phreatic head and the distance to the channels are balanced. It is assumed that the groundwater table is elliptical in cross section. It is not when outflow is not steady.

When the phreatic head changes from h_1 to h_2 the area A of the vertical cross section of the groundwater reservoir above the water level in the channel, perpendicular on it and between the channel and the middle between two channels has changed with:

$$\Delta A_k = \frac{\pi \cdot l(h_{k-1} - h_k)}{4} \qquad (20)$$

l is half the distance between two channels.

On the assumptions that the phreatic level would rise parallel to itself if no discharge occurred, rainfall was equally falling and the storage coefficient was homogeneous then the cross sectional area of the groundwater reservoir above the water level in the channel would change:

$$\Delta A'_k = 1(h_{k-1} - h_k) \qquad (21)$$

The deviations from steady outflow will be proportional to:

$$\Delta R_k :: 1(h_{k-1}-h_k) - \frac{\P \cdot 1(h_{k-1}-h_k)}{4} \qquad (22)$$

or

$$\Delta R_k :: (1 - \frac{\P}{4}) \cdot 1(h_{k-1}-h_k) \qquad (23)$$

Introducing the storage coefficient gives the identity:

$$\Delta R_k = (1 - \frac{\P}{4}) \cdot \mu \cdot 1(h_{k-1}-h_k) \qquad (24)$$

which is expressed in millimeters of water

$$\Delta R_k = (1 - \frac{R}{4}) \cdot \mu(h_{k-1}-h_k) \qquad (25)$$

Finally it is assumed that fluctuations of the water level in the channels may be neglected, so $h_{k-1}-h = w_k-w_{k-1}$ if w is groundwater depth and that ΔR is non-linear dependent on groundwater depth. Now eq. (25) can be written:

$$\Delta R = (1 - \frac{\P}{4}) \cdot e^{-\beta(\bar{G}-x)} \cdot \mu(G_k-G_{k-1}) \qquad (26)$$

β and x are parameters and $\bar{G} = \frac{1}{2}(G_k+G_{k-1})$. The term $(1-\frac{\P}{4})$ is called p, it may have other values, for instance $p = (1 - \frac{2}{3})$ or $p = (1-\frac{2}{\P})$ when the phreatic level is parabolic or sinusoidal in cross section.

Now a substitute for eq. (18) is:

$$R = \frac{{}_{h_{k-1}}^{h_k}(ah+bh^2)dh}{h_k - h_{k-1}} + p \cdot e^{-\beta(G-x)} \mu(G_{k-1}-G_k) \qquad (27)$$

Fig. 5. The head responsible for runoff can be written as:

$$h = S - G - V_{1+2} - Z$$

V = fall in water level between water level S and the water level at groundwater outflow

Z = drop of water level at weir or overflow between measuring flume and point of groundwater outflow

As G is groundwater depth below soil surface M a rise of the groundwater will make the second term in eq. (27) positive. A fall will make it negative, however, to the limit of 18% of the first term, as stated before.

d. A c c o u n t i n g   f o r   t h e   c h a n n e l

In the first term of eq. (27) stationary flow is calculated as a function of the head of phreatic water over the water level S in the channel. Often somewhere downstreams in the channel there is a water level gauge. These data can be used as given S, but then there has to be allowed for a fall in the water level between the gauge and the point in the channel where the outflow is of the region where the groundwater level is measured. If there are weirs or overflows between the two points there has to be allowed for a drop of the water level at weirs or overflows too. In fig. 5 this is schematized. It is clear that the head h which is responsible for runoff is written as:

$$h = S - G - V - Z \tag{28}$$

in which V is a fall and Z is a drop of the water level. Both are dependent on the amount of runoff, but for Z this is ignored for convenience sake and to keep the model within bounds.

The slope V is calculated with Manning's formula, reduced to (acc. VISSER, 1971):

$$Q = K_m(0.49 + 0.8\ W/D)\ D^{2.67}\ I^{0.5} \tag{29}$$

Q is runoff in $m^3.sec^{-1}$, it is computed from runoff R_k in mm. day^{-1} as 0.000116 A.R_k in which A is the area of the watershed. K_m is Manning's factor for bed roughness, B is width of channel bed, D is water depth in the channel at the point where S is measured and I is hydraulic gradient. Now when H is introduced as the elevation of the channel bed, D is written as S-H and it follows that:

$$I^{0.5} = \frac{Q}{K_m\{0.49 + 0.8W/(S-H)\}(S-H)^{2.67}} \tag{30}$$

and

$$V_k = L\left[\frac{Q_k}{K_m\{0.49 + 0.8W/(S-H)\}(S-H)^{2.67}}\right]^2 \tag{31}$$

where L is the distance along the channel between water level gauge and test well. L must be estimated, K_m and Z are parameters.

In fig. 6 a flow chart is given of a model in which eq. (27) for runoff and eq. (31) for channel flow are incorporated. It is noted that the calculation of the change in groundwater level ΔG_k only takes into account the first term of the equation for runoff because the second term actually is a change in storage. It is assumed that groundwater levels are not measured near water courses where the phenomenon occurs referred to in par. 2c. Tests are provided for to

decide if the second term of eq. (27) should be fixed on its limit of 18% of the first term, because no percolation occurs as explained in par. 2c.

The model has 15 parameters. Levels and elevations are in meters above sea level. Amounts of precipitation are in $mm.day^{-1}$.

3. APPLYING THE MODELS TO WATERSHED DATA

a. Parameter evaluation

The parameters in the models are adjusted to their optimum position by an automatic technique. This requires a criterion of fit. Though in principle every variable magnitude, which is measured and also calculated with the model can be used, there generally are only data available on runoff of watersheds and on groundwater levels in the watersheds. For the models in fig. 1 and 3 the criterion of fit was the sum of squares of differences between observed and computed groundwater levels at corresponding days. The model in fig. 6 is clearly more suitable as a model for the prediction of runoff. The criterion of fit was an error function in which the runoff measured at the point where the water level S was gauged was compared with runoff computed with the model. In both cases the error function was:

$$F^2 = \sum_{1}^{n} (obs_i - comp_i)^2 \tag{32}$$

Though there is a diversity of techniques to minimise the error function a simple uni-variate technique was used. It proved its usefulness in a comparative test with the Simplex method for function minimisation (NELDER and MEAD, 1965) by scoring a higher averaged proportion of the initial variance of the observed groundwater levels that is included in the groundwater levels calculated with the same model and the same data of ten test cases, and with less trials.

Uni-variate methods have in common that for only one coordinate at a time a minimum is sought (SPANG, 1962). As a rule the rotation of coordinates is determined objectively, for instance by calculation of first derivatives. Minima are found with second derivatives or po-

Fig. 7. Example with two parameters of how the uni-variate minimising routine, used in the study, operates

lynomials. Iterations are continued until no further decrease of F^2 is acquired.

The technique that was used on the models of the water balance started from the idea that in the beginning of the process of adjustment minima for separate parameters do not have much significance for the location of the space minimum. The technique can be demonstrated on a problem with only two parameters x and y. The error function (f(x,y) is the third dimension. In the coordinates x and y in fig. 7 estimates of the parameter values give the point $x_o y_o$. Now the value of one parameter, for instance x, is reduced with a fixed amount Δx. If $f(x_o - \Delta x, y_o) > f(x_o, y_o)$ this change is rejected and the value of x is increased with the same amount. If $f(x_o + \Delta x, y_o) < f(x_o, y_o)$ this change is accepted and $x_1 = x_o + \Delta x$. In that case and also when the change had been rejected the other parameters' value is reduced with Δy. If $f(x_1, y_o - \Delta y) > f(x_1, y_o)$ the change is rejected and $y_o + \Delta y$ is tried. After that the first parameter is tried again and in rotation the parameters are adjusted to their values which give the minimum value of F^2 when for instance $F^2 = f(x_{16}, y_{13})$. The rotation of the parameters, the size and direction of the changes are arbitrarily but fixed at the beginning so the minimisation of F^2 can be made automatic. The number of parameters is not of importance. Some further details are:

i Trial changes in parameter values can be programmed to have the same direction as the preceeding succesful one. If accepted, it is the next parameters turn again, if not the opposite direction is tried. This can save a lot of trials.

ii When with initial stepsizes no further progress is made, a second stage begins with stepsizes which are reduced in a fixed ratio. A third and fourth stage can be programmed.

iii Iterations can be stopped when some criterion is introduced.

In fig. 7 is shown how the adjustment of two parameters may work out.

b. Performance of the models

The performance of the models can be studied by defining their efficiency with the coefficient R^2 as the proportion of the initial variance of the observed magnitude accounted for by the model (MURRAY, 1970).

$$R^2 = \frac{F_o^{\,2} - F^2}{F_o^{\,2}} \tag{33}$$

in which $F_o^{\,2}$ is the initial variance defined by the sum of the squared deviations of observed magnitudes from their mean as:

$$F_o^{\,2} = \Sigma\ (\overline{obs} - obs)^2 \tag{34}$$

The coefficient R^2 for the efficiency of the model has its principal signification as a relative measure when comparing results of different models on the same data or of the same model on different data. For a more definite appreciation the standard error between computed and observed quantities should also be known. It can be computed as:

$$S_a = \frac{\Sigma(\text{obs} - \text{comp})^2}{n-p} \qquad (35)$$

Here n is the number of observations and p the number of parameters in the model.

Table 1. Performance of the water balance models with 6 and 16 parameters when minimising on an error function of groundwater depth

Watershed	Number of parameters					
	six			sixteen		
	R^2	S_a	n	R^2	S_a	n
Mb	0.947	8.9	183	0.965	8.3	655
B	0.917	9.2	673	0.934	8.8	510
F	0.727	24.6	370	0.910	15.5	1220
II	0.608	18.5	475	0.786	14.0	1265
JJ	0.828	16.5	182	0.911	14.7	1494
Ha	0.572	16.3	283	0.878	14.3	820
Ib	0.719	13.2	862	0.864	12.7	1056
01	0.908	11.4	253	0.933	10.8	996
00	0.947	11.5	456	0.942	11.4	958
04	0.933	10.5	126	0.954	10.2	719

R^2 = coefficient for the efficiency of the model; S_a = standard error between computed and observed groundwater levels in cm; n = number of trials

In table 1 the coefficient R^2 and the standard error S_a are given for the output of the models in figures 1 and 3. The average number of groundwater levels used for parameter evaluation was 62. The number of trials is also listed. From 57 to 96 per cent of the initial variance are accounted for. The model with 15 parameters averaged 91 per cent, the model with 6 parameters 81 per cent. So the model with two and a half as much parameters gave rather a better output.

The standard error S_a ranged from 8.3 cm to 24.6 cm. S_a can be high when R^2 is low and vice versa. The standard errors are relatively large because the big initial variances of groundwater levels in the type of soils under consideration.

Table 2. Performance on the water balance model with 15 parameters when minimising on an error function of runoff

Watershed	R^2	S_a	n	Watershed	R^2	S_a	n
01	0.83	0.43	163	Ha	0.61	0.21	285
3	0.70	0.45	202	II	0.69	0.34	242
4	0.72	0.41	198	LL	0.68	0.46	198
5	0.78	0.24	260	Mc	0.85	0.40	172
6	0.88	0.43	173	OO	0.80	0.34	115
7	0.89	0.26	103	B	0.76	0.50	273
CC	0.80	0.39	175	F	0.47	0.59	366
FF	0.71	0.33	170	G	0.68	0.66	200
GG	0.62	0.60	267				

R^2 = proportion of variation in observed runoff that is included in calculated runoff; S_a = standard error between calculated and observed runoff in $mm.etm^{-1}$; n = number of trials

In table 2 R^2 and S_a are listed for the output of the model in fig. 6. R^2 is between 47 en 89% and S_a is between 0.21 $mm.day^{-1}$ and 0.66 $mm.day^{-1}$. There is no correlation between R^2 and S_a. The result of the operations on some of the watersheds are definitely insufficient but as a whole they are satisfactory. The low number of trials needed with this model is striking.

Tables 1 and 2 show that the results of minimisation can be very different. It is not very probable that this is because of a restricted appropriateness of the models for the area in which the data are collected is homogeneous enough. Accuracy of input data will have a major effect. Water level recording, calibration of weirs, assessment of areal precipitation and open water evaporation are sources of error which will result in more or less accuracy in the models output. A special instance is the factor for bed roughness K_m in the model in fig. 6. This coefficient will not be the constant that it is supposed to be in the model because of variations in the density of aquatic vegetation.

c. Parameter values

The optimal values of the parameters in the models in fig. 3 and 6 are listed in tables 3 and 4. As parameter q in eq. (17) was experienced to have exceedingly small values it was assumed that q = 0. Also p in eq. (27) was fixed at a value of 0.3. As no data were available to evaluate parameter c in eq. (10) or r in eq. (11) it was evaluated as a free constant and so lost its physical signification.

The difference between tables 3 and 4 is that in table 3 parameter values are listed which were obtained by minimizing the sum of squares of the differences between observed and computed groundwater levels in a watershed. The parameter values in table 4 were obtained with the agreement between observed and computed runoff of the same watersheds as a criterion of fit.

There is hardly any conformity in the comparable values of the same parameter. This could be expected for some reasons:

i Though the physical interpretation of the parameter may be the same in both models, when groundwater levels served as a criterion of fit the parameter values will relate to a very local situation viz. the near surroundings of the test well. In case runoff served as a criterion of fit the parameter values will relate to the watershed as a whole.

ii It is a question whether parametric values obtained by minimisation still have a physical meaning. The adjustment of the parameter to its optimum values by fitting the model to some criterion forces the parameters to assume values which are statistically the best. This need not be the same as physically correct. This explains why a small model with 6 parameters can give such a good fit.

iii If there is only one criterion of fit a statistical interdependence between parameters in a model of the water balance equation is inevitable. Runoff, storage coefficient and evapotranspiration (for a part) are computed as functions of groundwater levels which again are calculated from a water balance upon which computed runoff, evapotranspiration appear etc. A calculation like that is inductive and different combinations of values of the parameters in such a model can have the same error function. To what extent a minimum error function will possibly go together with different combinations of parametric values is discussed in paragraph 4b.

iiii Parameter evaluation has its own inaccuracy as a determination.

Parametric values in the same model but for different watersheds also show a considerable variation. All factors mentioned before also have their effects here. Especially the points under ii and iii demonstrate their validity in some spurious parameter values. These are very evidently physically not correct. Yet the consequence is not a bad fit, because less notable deviations in other parameters give a necessary correction.

For parameters g in eq. (4) and K_m in eq. (3) it can easily be judged if a parameter value is acceptable from a physical point of view. It is also obvious according to eq. (12) that a very low value of the parameter α gives a very low gradient of capillary conductivity and a very high capillary rise and must consequently be mistrusted. The combination of a very high value of d_1 and a very low value of d_2, as in table 3, must as well be regarded as a result of parameter interdependence.

4. TESTING OF THE MODELS

a. Synthetic test data

To what extent parameter interdependence and the capacity of the uni-variate method, described in paragraph 3a, to find the minimum error function, are decisive for the stability of the optimum values of parameters was tested with synthetic data, as recommended for the testing of different optimising methods (O'DONNELL, 1966). These synthetic data are obtained by feeding the model with a set of input data and letting it generate an output record with parametric values which are evaluated by minimisation from the same input data. The groundwater level or runoff record that forms the output is error free and is compatible with a set of known parameter values and the initial sets of input data on precipitation and open water evaporation. Therefore the error function F^2 has the value zero at the minimum for coincidental and systematic errors in the input data or a lack of adaptability of the model cannot be restrictive anymore.

The test on the stability of parameter values was carried out for the model in fig. 3 with records of groundwater levels in a test well in watershed nr. 01 and for the model in fig. 6 with records of runoff in watershed nr. 6. These examples were chosen because of their very good fit, according tables 1 and 2.

The test consisted of repeating the evaluation of parameter values with six different combinations of six different initial parameter values and synthetic groundwater levels or runoff as a criterion of fit. For the generating of these sets of starting values the view was adopted that starting values will always be best estimates and will be randomly scattered about the optimum values (IBBITT, 1970). Therefore six possible initial values were perturbed from the good values by fixed percentages, three being smaller and three larger than the correct value. With a die six combinations of these initial values were arranged. These sets were starting points in the parameter space for the minimising routine. So six values for each parameter were evaluated. These values deviate from the known correct value. To know if these deviations are statistically significant they are

submitted to the test of Student. Calculated were:

1) the mean deviation from the correct value p'. This is Δp, computed as

$$\Delta p = \frac{\Sigma(p_i - p')}{n} \qquad (36)$$

2) the standard deviation S_d of the six values from their mean $\bar{p}$

$$S_d = \frac{\Sigma(p_i - \bar{p})^2}{n-1} \qquad (37)$$

3) the test criterion according to Student

$$t_o = \frac{\Delta p \sqrt{6}}{S_d} \qquad (38)$$

b. T e s t s o n s i g n i f i c a n c e o f p a r a m e t e r v a l u e s

In table 5 the results of six parameter evaluations, the correct parameter values and Δp, S_d and t_o according eqs. (30), (37) and (38) are listed for the two models with different criteria of fit. When testing two-sided with 5 degrees of freedom and 90 per cent confidence limits it holds that $t_{0.90}$ = 1.476. So for half of the parameter values in both models the zero hypothesis exceeds the test value and the values are statistically not significant. This must be regarded upon as a consequence of the models mechanisms. In par. 3c under iii is explained how these are inductive and will inevitably lead to dependences between parameters. For instance parameter evaluation on an error function of runoff may stop with parameter values that calculate groundwater potentials too high, which difference with physical reality is corrected by drainage constants being underestimated. Repeating the minimisation will perhaps give groundwater potentials to low and overestimation of drainage constants. The fit between observed and computed runoff may be just as good in both cases. The minimising routine being inefficient is no explanation in view of the very high values of R^2 obtained with synthetic data. It is obvious that combi-

nations of very different parameter values can compute practically identical runoff records.

The difference between inductive and not-inductive model mechanisms can be shown with a test on the significance of the parameter values in eq. (27) when runoff is calculated as a function of measured instead of computed groundwater levels. The storage coefficient has now to be computed directly as a function of groundwater depth and eq. (8) has to be incorporated in eq. (27). Eq. (27) is now written as:

$$R = \frac{\int_{h_{k-1}}^{h_k} (ah+bh^2)dh}{h_k - h_{k-1}} \cdot p \cdot e^{-\beta(\bar{G} - x)} \cdot \frac{f}{m+1} (G_k^{m+1} - G_{k-1}^{m+1}) \qquad (39)$$

in which h = S - W - Z and a, b, β, x, f and m are parameters.

Now records of daily runoff, groundwater level, precipitation and open water evaporation are input data. The parameters are evaluated on the error function of runoff. Then the record of calculated runoff was again used as synthetic test data and a test on the significance of the parameter values was conducted as before. All information is listed in table 6. The parameter values are significant.

The conclusion at this stage is that evaluation of parameters in models with inductive mechanism on only one criterion of fit will not give significant values for all parameters. A study of literature proves it not to be likely that available and more sophisticated minimising routines than the univariate method would give better results with one criterion of fit. This has nothing to do with the number of parameters. The model in fig. 1 is also inductive. It will only be possible to improve the accuracy of determination of parameters when different criteria of fit for separate parts of the model could be used. The models construction as well as the uni-variate minimising routine would have to be adapted to this purpose. Perhaps the introduction of some new principles in the technique of minimisation would make a more efficient evaluation of parameter values possible.

It has now to be recognized that lack of significance of parameter values means in principle that they loose their applicability when a single parameter is teared from its context in the model. So for instance simulating a runoff record with a lower value of some drainage constant than was evaluated cannot confidently be done, nor can a new model part be added when previously optimised parameter values are maintained. Of course these restrictions are relative and dependent on which confidence limits one uses. This is a matter that can be decided by personal preference and experience in a particular line of work.

c. Checking the predicting ability of a model

A most interesting aspect of hydrological models is, apart from the stability of separate parameters, the predicting ability of the model as a whole. The most severe test on this would be for instance to check a sequence of computed daily runoff, generated with the model, on the same sequence of measured runoff. If the sequence would be very long a comparison between distributions of measured and computed runoff in the same period would be a convenient summary. The test should not include the data records used for parameter evaluation, unless they are only a small part of the test data.

The opportunity for a test presents itself in the area where all data are collected, upon which the discussion on previous pages is based. The Dienst voor de Waterhuishouding of Rijkswaterstaat started runoff measurements in the area in 1951 and presented frequency distributions of runoff for 5 watersheds, based on observations in seven winter seasons from 1 November untill 1 April in the years 1951 up to and including 1958 (TROMP, 1958).

For three of these watersheds the same frequency distribution is derived from runoff records generated with the model in fig. 6. For that reason parameter values in the model were evaluated from data records between september 25, 1952 and June 30, 1953. In this period of about the same length as those used for the evaluation of the parameter values in table 2, runoff did not show the peaks that occurred in the years 1951 up to and including 1958.

Now the parameter values were applied to records of precipitation and open water evaporation in these years. For the first day of the sequence best estimates of groundwater level, runoff and water level in the channel has to be made. Effects of errors in these estimates disappear after a few days and do not have a perceptable influence on the frequency distribution of computed runoff.

To have the calculation continue from day to day the missing record of the water level S in the channel has to be generated as well. S is calculated as:

$$\log S_i = a + b \log R_i \qquad (40)$$

a and b are derived from the data records of S and R that are used for parameter evaluation when S and R are plotted on logarithmic scales; a is the intercept on the S-scale for $R = 1\ mm.etm^{-1}$ and b is the difference between a and the intercept for $R = 10\ mm.etm^{-1}$.

In fig. 8 the three exceedance frequencies of calculated and measured runoff are compared. There is good agreement which shows still better when calculated and measured runoff, exceeded with the same frequency, are plotted against each other. There is clearly some systematic deviation between measured and calculated runoff which only in the watershed C^* becomes rather large.

When the standard deviation between calculated and measured runoff with the same exceedance frequencies are computed (S^* in table 7) it shows a relationship with the coefficient R^2 for the fitting of the model according eq. (33) to the data records used for parameter evaluation.

Table 7. The better the model fits the data records used for evaluation of parameter values the better the predicting ability of the model

Watershed	S^*	R^2
E_d	0.025 l/sec/ha	0.95
C^*	0.055 "	0.75
D	0.027 "	0.98

5. CONCLUSIONS

The gauging of runoff will generally have the object to deduce from date records some characteristics about runoff distribution. This may require quite a lot of time because runoff occurring with a time interval of 10 years may very well show up in a gauging record of one year but just as well once in a gauging record of 15 years. Therefore gauging has to be kept up during a considerable longer time than the time interval one is interested in. This will easily give organizing and financial problems and methods to generate synthetic sequences of daily runoff which can be used to estimate the T-year runoff from a short period of record, must be favoured. It has been shown that a model of the water balance provides the possibilities for predicting exceedance frequencies of daily runoff with reasonable accuracy, which probably is depending on data quality. In fig. 9 examples are given of frequency distributions of calculated daily runoff for 12 watersheds involved in this study. In the same way it will be possible to model the water balance with the purpose of constructing frequency distributions of runoff peaks, which are perhaps of greater practical interest than daily values.

As there must be reservations with respect to parameter values obtained by the minimisation of one error function it should be considered whether the best policy for the study of water balance modelling would not be a separation of purposes, for instance:

i the construction of the smallest possible models for certain purposes as for instance the prediction of runoff, without much concern about interpretation of parameter values;

ii the construction of models which reflect the physical reality as closely as possible with adequate criteria of fit and minimising routines that match the demand for accurate values of parameters with physical significancy.

6. LITERATURE

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