

Table 11.2 Calculation of the drainable porosity in a silty clay for a drop in watertable depth from 0.50 m to 1.20 m

Depth below soil surface, z (cm)	Height above watertable 1 $h_1 = -z$ (cm)	$pF =$ $\log(h_1)$	$\theta_1(z)$ (-)	Height above watertable 2 $h_2 = -z$ (cm)	$pF =$ $\log(h_2)$	$\theta_2(z)$ (-)	$\Delta\theta =$ $\theta_1 - \theta_2$ (-)	Average $\Delta\theta$ (-)	Average $\Delta\theta$ x100 (mm)
0	50	1.70	<u>0.476</u>	120	2.08	<u>0.459</u>	0.017		
10	40	1.60	0.479	110	2.04	0.461	0.018	0.0175	1.75
20	30	1.48	0.483	100	2.00	0.463	0.020	0.0190	1.90
30	20	1.30	0.847	90	1.95	0.466	0.021	0.0205	2.05
40	10	1.00	0.492	80	1.90	0.468	0.024	0.0225	2.25
50	0	$-\infty$	<u>0.507</u>	70	1.85	0.470	0.307	0.0305	3.05
60			0.507	60	1.78	0.473	0.034	0.0355	3.55
70			0.507	50	1.70	0.476	0.031	0.0325	3.25
80			0.507	40	1.60	0.479	0.028	0.0295	2.95
90			0.507	30	1.48	0.483	0.024	0.0260	2.60
100			0.507	20	1.30	0.487	0.020	0.0220	2.20
110			0.507	10	1.00	0.492	0.015	0.0175	1.75
120			0.507	0	$-\infty$	<u>0.507</u>	<u>0.000</u>	<u>0.0075</u>	<u>0.75</u>
						Total	0.289	0.2805	28.05

$$\mu = \frac{28.05}{700} = 0.04$$

## 11.4 Unsaturated Flow of Water

The flow of soil water is caused by differences in hydraulic head, as was explained in Chapter 7, where water flow in saturated soil (i.e. groundwater flow) was discussed. The following sections deal with the basic relationships that govern soil-water flow in the unsaturated zone, the most important properties that influence soil-water dynamics, and some methods of measuring those properties.

### 11.4.1 Basic Relationships

#### *Kinetics of Flow: Darcy's Law*

For the one-dimensional flow of water in both saturated and unsaturated soil, Darcy's Law applies, which can be written as

$$q = -K\nabla H \quad (11.23)$$

where

$q$  = discharge per unit area or flux density (m/d)

$K$  = hydraulic conductivity (m/d)

$H$  = hydraulic head (m)

$\nabla$  = differential operator ( $\nabla = \partial/\partial x + \partial/\partial y + \partial/\partial z$ ) (see also Chapter 7)

It was only in 1927 that Israelsen noticed that the equation for flow in unsaturated media presented by Buckingham in 1907 is equivalent to Darcy's Law, the only difference being that the hydraulic conductivity is dependent on the soil-water content, which we denote as  $K(\theta)$ . With the hydraulic head defined as in Equation 11.19, Darcy's Law for unsaturated soils may be written as

$$q_x = -K(\theta) \frac{\partial H}{\partial x} = -K(\theta) \frac{\partial h}{\partial x} \quad (11.24)$$

$$q_y = -K(\theta) \frac{\partial H}{\partial y} = -K(\theta) \frac{\partial h}{\partial y} \quad (11.25)$$

$$q_z = -K(\theta) \frac{\partial H}{\partial z} = -K(\theta) \frac{\partial h}{\partial z} \quad (11.26)$$

where  $q_x$ ,  $q_y$ , and  $q_z$  are the components of soil-water flux in the  $x$ -,  $y$ - and  $z$ -directions.

#### *Conservation of Mass: Continuity Equation*

In Chapter 7 (Section 7.3.3), a general form of the continuity equation was derived for water flow independent of time, considering the mass balance of an elementary volume that could not gain or lose water. In unsaturated soil, however, a similar elementary volume can gain water at the expense of air. If we state that this happens at a rate  $\partial\theta/\partial t$ , we can write Equation 7.9 in the following form

$$\frac{\partial\theta}{\partial t} = -\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} - \frac{\partial q_z}{\partial z} = -\nabla \cdot \mathbf{q} \quad (11.27)$$

#### *General Unsaturated-Flow Equation*

The general equation of water flow in isotropic media (i.e. media for which the hydraulic conductivity is the same in every direction) is obtained by substituting Darcy's Law (Equations 11.24, 11.25, and 11.26) into the continuity equation (Equation 11.27), which yields

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x} \left( K(\theta) \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial y} \left( K(\theta) \frac{\partial H}{\partial y} \right) + \frac{\partial}{\partial z} \left( K(\theta) \frac{\partial H}{\partial z} \right) \quad (11.28)$$

or

$$\frac{\partial\theta}{\partial t} = \nabla \cdot \mathbf{K}(\theta) \nabla H \quad (11.29)$$

For saturated flow, the water content does not change with time (ignoring the compressibility of water and soil), so that  $\partial\theta/\partial t = 0$ , and hence

$$\nabla \cdot \mathbf{K} \nabla H = 0 \quad (11.30)$$

If  $\mathbf{K}$  is constant in space, the Laplace Equation for steady-state saturated flow in a homogeneous, isotropic porous medium follows from Equation 11.30

$$\nabla^2 H = 0 \quad (11.31)$$

where

$$\nabla^2 = \text{Laplace Operator (see also Chapter 7, Section 7.6.5)}$$

Substituting  $H = z + h$  into Equation 11.28 yields

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left( K(\theta) \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K(\theta) \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K(\theta) \frac{\partial h}{\partial z} \right) + \frac{\partial K(\theta)}{\partial z} \quad (11.32)$$

Since  $\theta$  is related to  $h$  via the soil-water retention curve, we can also express  $K(\theta)$  as  $K(h)$  (see following section). Through the introduction of the specific water capacity,  $C(h)$ , Equation 11.32 may be converted into an equation with one dependent variable

$$\frac{\partial \theta}{\partial t} = \frac{d\theta}{dh} \frac{\partial h}{\partial t} = C(h) \frac{\partial h}{\partial t} \quad (11.33)$$

where

$C(h)$  = specific water capacity, equalling  $d\theta/dh$  (i.e. the slope of soil-water retention curve) ( $m^{-1}$ )

Replacing  $K(\theta)$  by  $K(h)$  and substituting Equation 11.33 into Equation 11.32 yields

$$C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left( K(h) \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K(h) \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K(h) \frac{\partial h}{\partial z} \right) + \frac{\partial K(h)}{\partial z} \quad (11.34)$$

Equation 11.34 is known as Richards' Equation. With  $p/pg$  substituted for  $h$ , this equation applies to saturated as well as to unsaturated flow (hysteresis excluded). To solve Equation 11.34, we need to specify the hydraulic-conductivity relationship,  $K(h)$ , and the soil-water characteristic,  $\theta(h)$ . When we consider flow in a horizontal direction only ( $x$ ), Equation 11.34 reduces to an equation for unsteady horizontal flow

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[ K(h) \frac{\partial h}{\partial x} \right] \quad (11.35)$$

Similarly, the equation for unsteady vertical flow is

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[ K(h) \left( \frac{\partial h}{\partial z} + 1 \right) \right] \quad (11.36)$$

For steady-state flow,  $\frac{\partial \theta}{\partial t} = 0$  and  $h$  is only a function of  $z$ . Hence Equation 11.36 reduces to

$$\frac{d}{dz} \left[ K(h) \left( \frac{dh}{dz} + 1 \right) \right] = 0 \quad (11.37)$$

(Section 11.4.2 will deal with steady-state flow in more detail.)

For transient (i.e. unsteady) flow, we find the commonly used one-dimensional equation by substituting Equation 11.33 into Equation 11.36, which yields

$$\frac{\partial h}{\partial t} = \frac{1}{C(h)} \frac{\partial}{\partial z} \left[ K(h) \left( \frac{\partial h}{\partial z} + 1 \right) \right] \quad (11.38)$$

Equation 11.38 provides the basis for predicting transient soil-water movement in layered soils, each layer of which may have different physical properties.

### 11.4.2 Steady-State Flow

The most simple flow case is that of steady-state vertical flow (Equation 11.37). Integration of this equation yields

$$K(h) \left( \frac{dh}{dz} + 1 \right) = c \quad (11.39)$$

where  $c$  is the integration constant, with  $q_z = -c$ . Rewriting yields

$$q = -K(h) \left( \frac{dh}{dz} + 1 \right) \quad (11.40)$$

where

- $q$  = vertical flux density (m/d)
- $K(h)$  = hydraulic conductivity as a function of  $h$  (m/d)
- $h$  = pressure head (m)
- $z$  = gravitational potential, positive in upward direction (m)

Rearranging Equation 11.40 yields

$$\frac{dz}{dh} = \frac{-1}{1 + \frac{q}{K(h)}} \quad (11.41)$$

To calculate the pressure-head distribution (i.e. the relationship between  $z$  and  $h$  for a certain  $K(h)$ -relationship and a specified flux  $q$ ), Equation 11.41 should be integrated

$$\int_0^{z_r} dz = - \int_0^{h_r} \frac{dh}{1 + \frac{q}{K(h)}} \quad (11.42)$$

where

- $h_r$  = the pressure head (m); the upper boundary condition
- $z_r$  = the height of capillary rise for flux  $q$  (m)

To calculate at what height above the watertable pressure head  $h_r$  occurs, integration should be performed from  $h = 0$ , at the groundwater level, to  $h_r$ . When the soil profile concerned is heterogeneous, integration is performed for each layer separately

$$\int_0^{z_n} dz = - \int_0^{h_1} \frac{dh}{1 + \frac{q}{K_1(h)}} - \int_{h_1}^{h_2} \frac{dh}{1 + \frac{q}{K_2(h)}} - \dots - \int_{h_{N-1}}^{h_n} \frac{dh}{1 + \frac{q}{K_N(h)}} \quad (11.43)$$

where

- $N$  = the number of layers in the soil profile
- $h_1, h_2, \dots, h_N$  = the pressure heads at the top of Layer 1, 2, ...,  $N$

Solving Equation 11.43 yields the height of capillary rise,  $z$ , for given flux densities.

The  $h$ -values at the boundaries between the layers are unknown initially, and must be determined during the integration procedure. Thus, starting from  $h = 0$  and  $z = 0$  at the watertable, we steadily decrease  $h$  until  $z$  reaches  $z_r$ , the known position

of the  $i$ -th boundary. Since pressure head is continuous across the boundary (as opposed to water content), the value  $h_i$  may be used as the lower limit of the next integration term. In this way, the integration proceeds until either the last value of  $h$  ( $h_N$ ) is reached or until  $z$  reaches the soil surface.

Equations 11.42 and 11.43 may be solved analytically for some simple  $K(h)$ -relationships. For more complicated  $K(h)$ -relationships, it would be very laborious, if not impossible, to find an analytical solution. Therefore, integration as described by Equations 11.42 and 11.43 is usually performed numerically, as, for example, in the computer program CAPSEV (Wesseling 1991).

For a marine clay soil in The Netherlands, the results of calculations with CAPSEV are shown in Figures 11.13A and 11.13B. Figure 11.13A shows the height of capillary rise and the pressure-head profile for different vertical-flux densities. Figure 11.13B shows the pressure-head profile during infiltration for several values of the vertical-flux density. The height of capillary rise was calculated for a watertable at a depth of 2 m. The soil profile consisted of five layers with differing soil-physical parameters.

**Example 11.2**

For drainage purposes, it can be useful to know the maximum flux for a given watertable pressure head and a certain watertable depth. Suppose that we have a crop

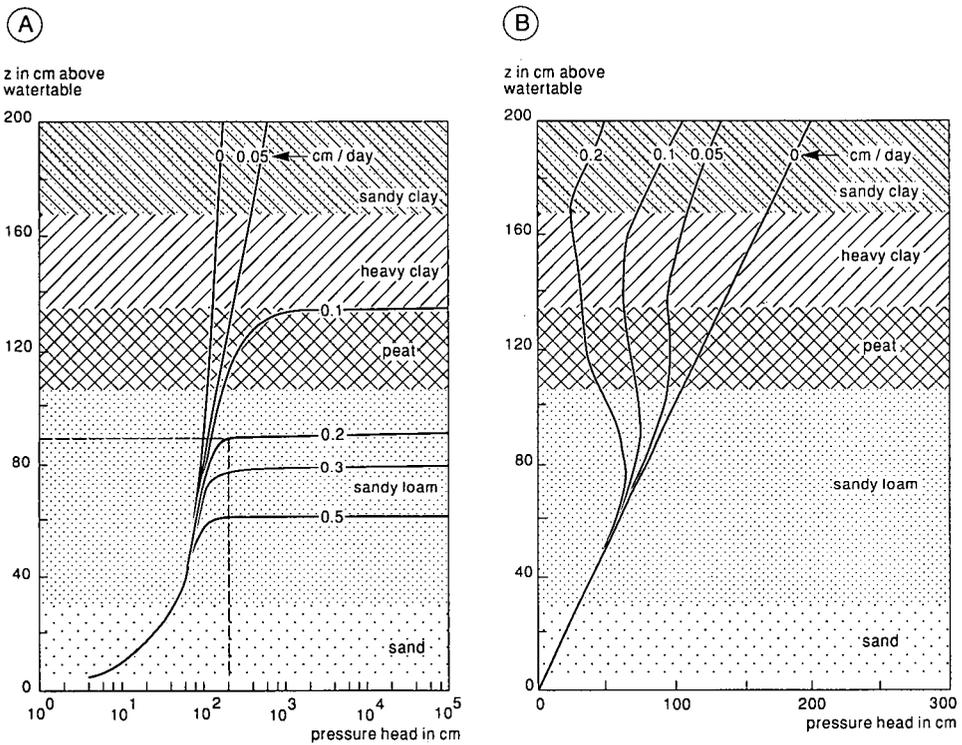


Figure 11.13 Calculations with computer program CAPSEV for a 5-layered soil profile (Wesseling 1991)

- A. Height of capillary rise
- B. Pressure-head profiles in case of infiltration

which, on the average, is transpiring at a rate of 2 mm/d. This water is withdrawn from the rootzone, say from the top 0.20 m of the soil profile. We assume that the crop would suffer from drought if the pressure head in the centre of the rootzone were to fall below  $-200$  cm. We further assume that the groundwater level can be fully controlled by drainage. To prevent drought stress for the crop under any condition, the controlled groundwater depth should be such that, under steady-state conditions with a pressure head of  $-200$  cm at 0.10 m depth (i.e. the average depth of the rootzone), the water delivery from the groundwater by capillary rise would equal water uptake by the roots, equalling 2 mm/d.

We can find the required groundwater depth from Figure 11.13A. We start on the horizontal axis at a pressure head of  $-200$  cm, draw an imaginary upward line until it crosses the 2 mm/d flux-density curve, then go horizontally to the vertical axis and find a depth of 0.86 m. This means that the desired watertable depth is  $0.86 + 0.10 = 0.96$  m below the soil surface.

### 11.4.3 Unsteady-State Flow

To obtain a solution for the unsteady-state equation (Equation 11.38), appropriate initial and boundary conditions need to be specified. As initial condition (at  $t = 0$ ), the pressure head or the soil-water content must be specified as a function of depth

$$h(z,t=0) = h_0 \quad (11.44)$$

The boundary conditions at the soil surface ( $z = 0$ ) and at the bottom of the soil profile ( $z = -z_N$ ) can be of three types (see also section 11.8.2):

- Dirichlet condition: specification of the pressure head;
- Neumann condition: specification of the derivative of the pressure head, in combination with the hydraulic conductivity  $K$ , which means a specification of the flux through the boundaries;
- Cauchy condition: the bottom flux is dependent on other conditions (e.g. an external drainage system).

This list is not exhaustive, while also, depending on the type of problem to be solved, boundary conditions can be defined by combinations of the above options. Equation 11.38 is a non-linear partial differential equation because the parameters  $K(h)$  and  $C(h)$  depend on the actual solution of  $h(z,t)$ . The non-linearity causes problems in its solution. Analytical solutions are known for special cases only (Lomen and Warrick 1978). Most practical field problems can only be solved by numerical methods (Feddes et al. 1988). (Numerical methods used in the modelling of soil-water dynamics will be discussed in Section 11.8.)

## 11.5 Unsaturated Hydraulic Conductivity

The single most important parameter affecting water movement in the unsaturated zone is the unsaturated hydraulic conductivity,  $K$ , which appears in the unsaturated flow equation (Equation 11.38). In the case of saturated flow in soil, the total pore

space is filled with water and is thus available for flow. During unsaturated flow, however, part of the pores are filled with air and do not participate in the flow. The unsaturated hydraulic conductivity,  $K(\theta)$  or  $K(h)$ , is therefore lower than the saturated conductivity. Thus, with decreasing soil-water content, the area available for flow decreases and, consequently, the unsaturated hydraulic conductivity decreases. The  $K$  in unsaturated soils depends on the soil-water content,  $\theta$ , and, because  $\theta = f(h)$ , on the pressure head,  $h$ . Figure 11.14 shows examples of  $K(\theta)$ -relationships for four layers of a sandy soil with a humic topsoil, together with the soil-water retention characteristics (De Jong and Kabat 1990).

Over the years, many laboratory and field methods have been developed to measure  $K$  as a function of  $h$  or  $\theta$ . These methods can be divided into direct and indirect methods (Van Genuchten et al. 1989). Direct methods are, almost without exception, difficult to implement, especially under field conditions. Despite a number of improvements, direct-measurement technology has only marginally advanced over the last decades.

Nevertheless, indirect methods, which predict the hydraulic properties from more easily measured data (e.g. soil-water retention and particle-size distribution), have received comparatively little attention. This is unfortunate because these indirect methods, which we call 'predictive estimating methods', can provide reasonable estimates of hydraulic soil properties with considerably less effort and expense. Hydraulic conductivities determined with estimating methods may well be accurate enough for a variety of applications (Wösten and Van Genuchten 1988). Other important indirect methods are inverse methods of parameter estimates with analytical models that describe water retention and hydraulic conductivity (Kabat and Hackten Broeke 1989).

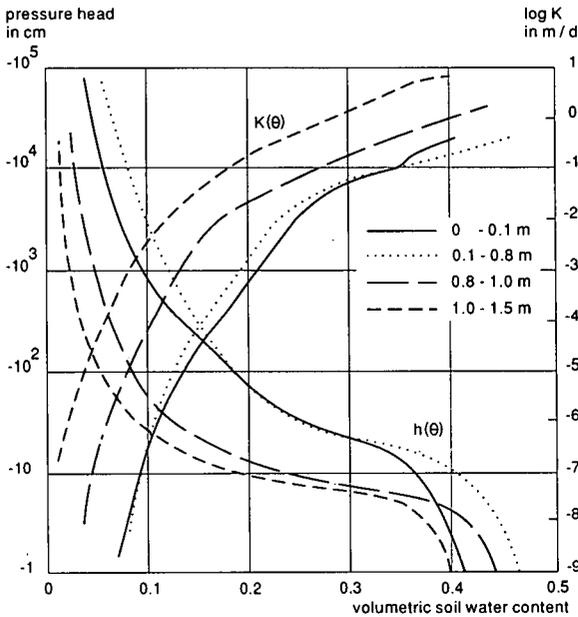


Figure 11.14 Soil-water retention,  $h(\theta)$ , and hydraulic conductivity,  $K(\theta)$ , curves for four layers of a sandy soil (after De Jong and Kabat 1990)

### 11.5.1 Direct Methods

Comprehensive overviews of direct methods of measuring the unsaturated hydraulic conductivity,  $K$ , and the soil-water diffusivity ( $D$ ),  $D = K(\theta)/(d\theta/dh)$ , are given by Klute and Dirksen (1986) for laboratory methods, and by Green et al. (1986) for field methods.

In the steady-state methods, the flux,  $q$ , and hydraulic gradient,  $dH/dz$ , are measured in a system of time-invariant one-dimensional flow, and the Darcy Equation (Equation 11.40) is used to calculate  $K$ . The value of  $K$  obtained is then related to a measured  $h$  or  $\theta$ . The procedure can be applied to a series of steady-state flow situations.

Transient laboratory methods include the method developed by Bruce and Klute (1956), in which the diffusivity is estimated from horizontal water content distributions, and the sorptivity method of Dirksen (1975).

The most common field methods include the 'instantaneous profile method', a good example of which is described by Hillel et al. (1972). In this method, an isolated, free-draining field is saturated and subsequently drained by gravity, while the field is covered to prevent evaporation. The hydraulic conductivity is calculated by applying Darcy's Law to frequent measurements of pressure head and water content during the drying phase. Various simplifications of this instantaneous profile concept, based on unit-gradient ( $dH/dz = 1$ ,  $H = h + z$ , so  $h = \text{constant}$ , see Equation 11.37) approaches, have been developed (e.g. Libardi et al. 1980). This unit gradient does not require pressure-head measurements. These methods provide the hydraulic soil properties between saturation and field capacity, since gravity drainage becomes negligible at water contents below field capacity.

Clothier and White (1981) developed a method to determine  $K(\theta)$ ,  $\theta(h)$ , and  $D(\theta)$  from sorptivity measurements. 'Sorptivity' is the initial infiltration rate during the infiltration process. It can be measured quickly and is therefore a practical method of determining the hydraulic soil properties.

The 'crust method' of Bouma et al. (1971) is a field variant of steady state laboratory approaches. A soil column is isolated from the surrounding soil, covered with a crust, and a constant head is maintained on the crust. Because the hydraulic conductivity of the crust is relatively small compared with that of the soil, the pressure head in the soil will be lower than zero. Because a constant head is maintained above the crust, a steady-state flow will develop in the crust and a steady-state flux, lower than the saturated flux of the soil, will enter the soil and create a steady-state unsaturated flow. Hydraulic conductivities for different pressure heads can be determined with crusts of different material and thickness. The method allows us to determine hydraulic conductivities in the  $h$ -range of 0 to  $-100$  cm.

All the above methods of measuring  $K(\theta)$ ,  $K(h)$ , or  $D(\theta)$  are typically based on Darcy's Law, or on various numerical approximations or simplifications of Equation 11.38. This enables us to express  $K$  or  $D$  in terms of directly observable parameters. These direct methods are relatively simple in concept, but they also have a number of limitations which restrict their practical use. Most methods are time-consuming because restrictive initial and boundary conditions need to be imposed (e.g. free drainage of an initially saturated soil profile). This is especially problematic under field conditions where, because of the natural variability in properties and the

uncontrolled conditions, accurate implementation of boundary conditions may be difficult.

Even more difficult are the methods requiring repeated steady-state flow or other equilibrium conditions. Many of the simplified methods require the governing flow equations to be linearized or otherwise approximated to allow their direct inversion, which may introduce errors. A final shortcoming of the direct methods is that they usually lack information about uncertainty in estimated soil hydraulic conductivity, because it is impractical to repeat the measurements a number of times.

A laboratory method which is more or less a transition between direct and indirect methods is the 'evaporation method' (Boels et al. 1978). In this method, an initially wet soil sample is subjected to free evaporation. The sample, 80 mm high and 100 mm in diameter, is equipped with four tensiometers. The sample is weighed at brief time intervals and, at the same time, the pressure head is recorded. From these weight and pressure-head data, the average soil-water retention at each time interval can be determined. An iterative procedure is now used to derive the soil-water retention, and the instantaneous profile method to derive the hydraulic conductivity for each depth interval of 20 mm around a tensiometer. In this way, the method yields soil-water retention and hydraulic conductivity for  $h = -100$  to  $-800$  cm for sandy soils and for  $h = -20$  to  $-800$  cm for clay soils.

The advantages of the evaporation method are that it simultaneously yields both soil-water retention and hydraulic conductivity over a relatively wide  $h$ -range. The experimental conditions, in terms of boundary conditions, are close to natural conditions, because water is removed by evaporation. Disadvantages are that the procedure takes a considerable time (approximately 1 month per series of samples), and that the soil-water retention and hydraulic conductivity are based on an iterative procedure.

### 11.5.2 Indirect Estimating Techniques

Many of the disadvantages of the direct techniques do not apply to the indirect techniques. The indirect methods can basically be divided into two categories: 'predictive estimates' and 'parameter estimates'. The advantage of both methods is that neither depends on the created ideal experimental conditions.

The usefulness of predictive estimates depends on the reliability of the correlation or transfer function, and on the availability and accuracy of the easily measured soil data. The estimate functions are often called 'pedo-transfer functions' because they transfer measured soil data from one soil to another, using pedological characteristics.

The parameter-estimate approach for soil hydraulic properties is based on inverse modelling of soil water flow. This approach is very flexible in boundary and initial conditions. The inverse approach was developed parallel with advances in computer and software engineering (Feddes et al. 1993a).

#### *Prediction of the $K(h)$ Function from Soil Texture and Additional Soil Properties*

The methods discussed in this section are referred to as 'pedo-transfer functions'. Pedo-transfer functions are usually based on statistical correlations between hydraulic soil properties, particle-size distribution, and other soil data, such as bulk density, clay

mineralogy, cation exchange capacity, and organic carbon content. Other pedo-transfer functions relate parameters of the Van Genuchten model (see the following section) in a multiple regression analysis to, for example, bulk density, texture, and organic-matter content (e.g. Wösten and Van Genuchten 1988).

The development of pedo-transfer functions offers promising prospects for estimating soil hydraulic properties over large areas without extensive measuring programs. Such pedo-transfer functions are only applicable to areas with roughly the same parent material and with comparable soil-forming processes. Developing and testing these methods is as yet far from complete. Vereecken et al. (1992), for example, concluded that errors in estimated soil-water flow were more affected by inaccuracies in the pedo-transfer functions than by errors in the easily measured soil characteristics.

Another approach to identifying hydraulic soil properties on a regional scale is by identifying 'functional soil physical horizons'. This approach was followed by Wösten (1987), who used measured values of soil-water retention and hydraulic conductivity of representative Dutch soils, and classified these in groups according to texture and position in the soil profile. These groups were called functional soil physical horizons. Another example is the Catalogue of Hydraulic Properties of the Soil by Mualem (1976a).

#### *Predicting the K(h) Function from Soil-Water Retention Data*

The most simple form of parameter estimating concerns the prediction of K(h) from soil-water retention data. Water retention is more easily measured than hydraulic conductivity, and the estimating methods are usually based on statistical pore-size distribution models (Mualem 1976b). The most frequently applied predictive conductivity models are those of Mualem and Burdine (Van Genuchten et al. 1989). Van Genuchten (1980) combined Mualem's model with an empirical S-shaped curve for the soil-water retention function to derive a closed-form analytical expression for the unsaturated hydraulic conductivity curve.

The empirical Van Genuchten Equation for the soil-water retention curve reads

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{(1 + |\alpha h|^n)^m} \quad (11.45)$$

where

$\theta_r$  = residual soil-water content (i.e. the soil water that is not bound by capillary forces, when the pressure head becomes indefinitely small) (-)

$\theta_s$  = saturated soil-water content (-)

$\alpha$  = shape parameter, approximately equal to the reciprocal of the air-entry value ( $m^{-1}$ )

$n$  = dimensionless shape parameter (-)

$m = 1 - 1/n$

After combining Equation 11.45 with the Mualem model, we find the Van Genuchten-Mualem analytical function, which describes the unsaturated hydraulic conductivity as a function of soil-water pressure head

$$K(h) = K_s \frac{[1 - |\alpha h|^{n-1} (1 + |\alpha h|^n)^{-m}]^2}{[1 + |\alpha h|^n]^{m\lambda}} \quad (11.46)$$

where

- $K_s$  = saturated hydraulic conductivity (m/d)
- $\lambda$  = a shape parameter depending on  $dK/dh$

The shape parameters in Equations 11.45 and 11.46 can be fitted to measured water-retention data. The Van Genuchten model in its most free form contains six unknowns:  $\theta_r$ ,  $\theta_s$ ,  $\alpha$ ,  $n$ ,  $\lambda$ , and  $K_s$ . Although, with specially developed computer programs, the mathematical fitting procedure enables us to find these unknowns for measured data, its use as a predictive model in this form is difficult. The fitting procedure is improved when some measurable parameters are known approximately, so that they can be optimized in a narrow range around these measured values.

To predict  $K(h)$  from the water-retention curve, we need to measure  $K_s$ . However, if a few  $K(h)$ -values are known in combination with soil-water-retention values,  $K_s$  can be found with an iteration procedure and need not be measured. Computer software has been developed (Van Genuchten et al. 1991) to fit the analytical functions of the model to some measured  $\theta(h)$  and  $K(h)$ -data. The same program allows the  $K(h)$ -function to be predicted from observed water-retention data.

Yates et al. (1992) recently evaluated parameter estimates with different data sets and for various combinations of known and unknown parameters. They concluded that predicting the unsaturated hydraulic conductivity from soil-water retention data and measured  $K_s$ -values yielded poor results. Using a simultaneous fitting of  $K(h)$  and  $\theta(h)$  data, while treating  $\lambda$  as an unknown parameter, improved results significantly. Apparently, water-retention data combined with a measured  $K_s$  are not always sufficient to describe the  $K(h)$ -function with Equation 11.46.

The Van Genuchten model in its original form is inadequate for very detailed simulation studies, because it is only valid for monotonic wetting or drying. By adding only one parameter, Kool and Parker (1987) extended the model so as to include hysteresis in  $\theta(h)$  and  $K(h)$  functions.

#### *Inverse Problem combined with Parameter Optimization Techniques*

In this approach, the direct flow problem can be formulated for any set of initial and boundary conditions and solved with an analytical or numerical method. Input data are measured soil-water contents, measured pressure-head profiles, or measured discharge under known boundary conditions, or any combination thereof, always as a function of time. Certain constitutive functions for the hydraulic properties are assumed, and unknown parameter values in those functions are estimated with the use of an optimization procedure. This optimization minimizes the objective function (e.g. the sum of the squared differences between observed and calculated values of either water content or pressure head) until a desired accuracy is reached.

The inverse method can be applied to both laboratory experiments and field experiments. A disadvantage of the laboratory procedure is that we cannot explore the full potential of this method, because of the necessarily limited size of the soil sample. Moreover, the collection of soil samples always introduces some disturbance that may affect flow properties. Thus, applying the method in-situ seems to offer the best prospects. The capabilities of this technique have been shown by Feddes et al. (1993a and b) and Kool et al. (1987).

## 11.6 Water Extraction by Plant Roots

Under steady-state conditions, water flow through the soil-root-stem-leaf pathway can be described by an analogue of Ohm's Law with the following widely accepted expression

$$T = \frac{h_m - h_r}{R_s} = \frac{h_r - h_l}{R_p} \quad (11.47)$$

where

- $T$  = transpiration rate (mm/d)  
 $h_m, h_r, h_l$  = matric heads in the soil, at the root surface, and in the leaves, respectively (mm)  
 $R_s, R_p$  = liquid flow resistances in soil and plant, respectively (d)

If we consider the diffusion of water towards a single root, we can see that  $R_s$  is dependent on root geometry, rooting length, and the hydraulic conductivity of the soil. This so-called microscopic approach is often used when evaluating the influence of complex soil-root geometries on water and nutrient uptake under steady-state laboratory conditions. In the field, the components of this microscopic approach are difficult to quantify for a number of reasons. Steady-state conditions hardly exist in the field. The living root system is dynamic, roots grow and die, soil-root geometry is time-dependent, and water permeability varies with position along the root and with time. Root water uptake is most effective in young root material, but the length of young roots is not directly related to the total root length. The experimental evaluation of root properties is difficult, and often impossible.

Although detailed studies can be relevant for a better understanding of plant physiological processes, they are not yet usable in describing soil-water flow. Thus, instead of considering water flow to single roots, we follow a macroscopic approach. In this approach, a sink term,  $S$ , is introduced, which represents water extraction by a homogeneous and isotropic element of the root system, and added to the continuity equation (Equation 11.27) for vertical flow (Feddes et al. 1988)

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z} - S \quad (11.48)$$

Consequently, the one-dimensional equation for transient soil-water movement (Equation 11.38) can be rewritten as

$$\frac{\partial h}{\partial t} = \frac{1}{C(h)} \frac{\partial}{\partial z} \left[ K(h) \left( \frac{\partial h}{\partial z} + 1 \right) \right] - \frac{S(h)}{C(h)} \quad (11.49)$$

where

$$S = \text{sink term (d}^{-1}\text{)}$$

The sink term,  $S$ , is quantitatively important since the water uptake can easily be more than half of the total change in water storage in the rootzone over a growing season.

Feddes et al. (1988), in the interest of practicality, assumed a homogeneous root distribution over the soil profile and defined  $S_{\max}$  according to

$$S_{\max} = \frac{T_p}{|Z_r|} \quad (11.50)$$

where

- $S_{\max}$  = the maximum possible water extraction by roots ( $d^{-1}$ )
- $T_p$  = the potential transpiration rate (mm/d)
- $|Z_r|$  = the depth of the rootzone (mm)

Prasad (1988) introduced an equation to take care of the fact that, in a moist soil, the roots principally extract water from the upper soil layers, leaving the deeper layers relatively untouched. He assumed that root water uptake at the bottom of the rootzone equals zero and derived the following equation

$$S_{\max}(Z) = \frac{2T_p}{|Z_r|} \left( 1 - \frac{|z|}{|z_r|} \right) \quad (11.51)$$

Both root water-uptake functions are shown in Figure 11.15.

So far, we have considered root water uptake under optimum soil-water conditions (i.e.  $S_{\max}$ ). Under non-optimum conditions, when the rootzone is either too dry or too wet,  $S_{\max}$  is dependent on ( $h$ ) and can be described as (Feddes et al. 1988).

$$S(h) = \alpha(h)S_{\max} \quad (11.52)$$

where

$\alpha(h)$  = dimensionless, plant-specific prescribed function of the pressure head

The shape of this function is shown in Figure 11.16. Water uptake below  $|h_1|$  (oxygen deficiency) and above  $|h_4|$  (wilting point) is set equal to zero. Between  $|h_2|$  and  $|h_3|$  (reduction point), water uptake is at its maximum. Between  $|h_1|$  and  $|h_2|$ , a linear relation is assumed, and between  $|h_3|$  and  $|h_4|$ , a linear or hyperbolic relation between  $\alpha$  and  $h$  is assumed. The value of  $|h_3|$  depends on the evaporative demand of the atmosphere and thus varies with  $T_p$ .

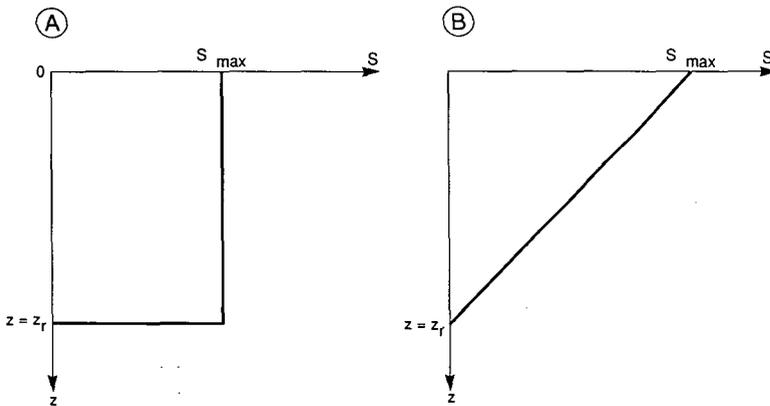


Figure 11.15 Different water-uptake functions under optimum soil-water conditions,  $S_{\max}$ , as a function of depth,  $z$ , over the depth of the rootzone,  $z_r$ , as proposed by A: Feddes et al. (1988) and B: Prasad (1988)

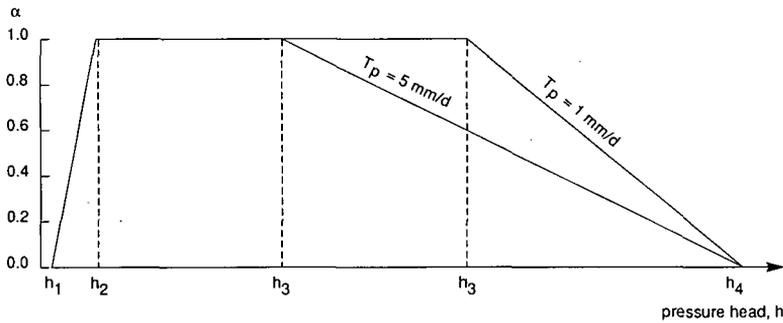


Figure 11.16 Dimensionless sink-term variable,  $\alpha$ , as a function of the absolute value of the pressure head,  $h$  (after Feddes et al. 1988)

## 11.7 Preferential Flow

In the previous sections, we described unsaturated-zone dynamics for isotropic and homogeneous soils. The fact that most soils are neither was already recognized in the 19th century. In natural soils, the transport of water is often heterogeneous, with part of the infiltrating water travelling faster than the average wetting front. This has important theoretical and practical consequences. Theoretical calculations of the field water balance, the derived crop water use, and the estimated crop yield are incorrect if preferential flow occurs but is not incorporated. Practically, preferential flow has a strong impact on solute transport and on the pollution of groundwater and subsoil (e.g. Bouma 1992). Preferential flow varies considerably from soil to soil, in both quantity and intensity. In some soils, preferential flow occurs through large pores in an unsaturated soil matrix, a process known as 'by-pass flow' or 'short-circuiting' (Hoogmoed and Bouma 1980). In other soils, flow rates vary more gradually, making it difficult to distinguish matrix and preferential pathways.

Preferential flow of water through unsaturated soil can have different causes, one of them being the occurrence of non-capillary-sized macropores (Beven and Germann 1982). This type of macroporosity can be caused by shrinking and cracking of the soil, by plant roots, by soil fauna, or by tillage operations. Wetting-front instability, as caused by air entrapment ahead of the wetting front or by water repellency of the soil (Hendrickx et al. 1988) can also be viewed as an expression of preferential flow. Whatever the cause, the result is that the basic partial differential equations (Equations 11.38 and 11.49) describing water flow in the soil need to be adapted.

Hoogmoed and Bouma (1980) developed a method of calculating infiltration, including preferential flow, into clay soils with shrinkage cracks. The method combines vertical and horizontal infiltration. It is physically based, but was only applied to soil cores of 200 mm height and was not tested in the field. Bronswijk (1991) introduced a method in which preferential flow through shrinkage cracks is calculated as a function of both the area of cracks at the soil surface, and the maximum infiltration rate of the soil matrix between the cracks.

The division of soil water over the soil matrix and the macropores, and the fate of water flowing downward through the macropores, is handled differently in the