Day 2:

$$h_2 = h_1 e^{-\alpha\Delta t} + \frac{R_2}{0.8\mu\alpha}(1 - e^{-\alpha\Delta t})$$

$$= 0.69 \times e^{-0.113} + \frac{0.007}{0.8 \times 0.05 \times 0.113}(1 - e^{-0.113})$$

$$= 0.78 \text{ m}$$

and $q_2 = q_1 e^{-\alpha\Delta t} + R_2(1 - e^{-\alpha\Delta t})$
$= 0.003 e^{-0.113} + 0.007(1 - e^{-0.113})$
$= 0.004 \text{ m/d}$

Day 3: etc.

It can be seen from Table 8.3 that, for 7 days (i.e. from Day 3 to Day 10), the watertable is above its maximum permissible height (1 m below soil surface or 0.80 m above drain level). This is not surprising because the total recharge after the rainstorms (69 mm in 5 days) is much more than the field application losses (25 mm in 10 days) on which the design of the drainage system was based. If these high levels of the watertable restrict plant growth, and thus crop production, the design criteria will have to be adjusted accordingly.

8.4 Comparison between Steady-State and Unsteady-State Equations

The question of whether to use the steady-state or the unsteady-state approach to calculate the required drain spacing depends mainly on the availability of data. Table 8.4 summarizes the input requirements for the steady-state and the unsteady-state.

Table 8.4 Input requirements for steady-state and unsteady-state equations

	Steady-state equations	Unsteady-state equations
Soil data:		
- Profile description	Yes	Yes
- Hydraulic conductivity	Yes	Yes
- Drainable pore space	No	Yes
Agricultural criteria:		
- q/h ratio	Yes	No
- h_0/h_t ratio	No	Yes
Technical criteria:		
- Drain depth, pipe size, etc.	Yes	Yes

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To apply the equations, we have to simplify the soil profile. We have already mentioned that unsteady-state equations can only be applied if a homogeneous soil profile is assumed; for a layered soil profile, steady-state equations have to be used. In both cases, the hydraulic conductivity, which is considered to be constant within each soil layer, should be known. For unsteady-state equations the drainable pore space is also required. As it is even more complicated to measure the drainable pore space than, say, the hydraulic conductivity, the applicability of unsteady-state equations is limited.

In the unsteady-state approach, the agricultural criterion is based on the rate of watertable drawdown h_0/h_t (Section 8.3.3) instead of a watertable-discharge criterion q/h as in steady-state equations (Section 8.2.4). The agricultural criteria are often based on relationships that only take the (variation in) depth of the watertable into consideration (Chapter 17). Thus it can be concluded that, on the one hand, steady-state equations are preferred because fewer soil data are required, but, on the other hand, the agricultural criteria are often based on a variation in the depth of the watertable. Fortunately, it is possible to combine the two approaches because the corresponding criteria can be converted into one another.

Consider the Hooghoudt Equation and assume flow only below drain level (Equation 8.6)

$$q = \frac{8 \, K \, d \, h}{L^2}$$

In the unsteady-state equations, the design criteria are expressed in the reaction factor α (Equation 8.29)

$$\alpha = \frac{\pi^2 K d}{\mu L^2}$$

Combining these two equations by eliminating L yields

$$\frac{h}{q} = \frac{\pi^2}{8\mu\alpha} \tag{8.40}$$

With the use of Equation 8.40, it is possible to establish the unsteady state criteria (i.e. the required drawdown of the watertable in a certain period of time) in experiments on a pilot scale. These unsteady-state criteria can be converted into steady-state criteria, which can be applied on a project scale. In this way, it is not necessary to measure the drainable pore space on a project scale, which, in practice, is virtually impossible.

Example 8.7
We can also solve Example 8.5 in an indirect way by converting the unsteady-state drainage criterion h_0/h_t into a steady-state criterion q/h. We know the rate of drawdown of the watertable over a period of 10 days, so we can calculate the reaction factor α with Equation 8.32

$$h_t = 1.16 \, h_0 \, e^{-\alpha t} \rightarrow \alpha t = - \ln \frac{h_t}{1.16 h_0}$$

$$\alpha t = - \ln \frac{0.3}{1.16 \times 0.8} = 1.13$$

$$\alpha = \frac{1.13}{10} = 0.113$$

By applying Equation 8.40, we can convert this unsteady-state criterion into a steady-state criterion.

$$\frac{h}{q} = \frac{\pi^2}{8\mu\alpha} = \frac{\pi^2}{8 \times 0.05 \times 0.113} = 218 \text{ d}^{-1}$$

Neglecting the flow above drain level, we can now calculate the drain spacing by applying the simplified version of Hooghoudt (Equation 8.6)

$$L^2 = 8 \text{ K d} \frac{h}{q} = 8 \times 1.0 \times d \times 218 = 1744 \times d \text{ m}^2$$

First estimate: L = 90 m, D = 7.7 m, d = 4.63 m (Example 8.5). Thus

$$L^2 = 1744 \times 4.63 = 8075 \text{ m}^2$$
$$L = 90 \text{ m, so okay}$$

Note: The reaction factor, α, is a function of the parameters L, K, d, and μ (Equation 8.29). Except for the spacing L, these parameters are hard to establish. This example shows that an alternative way of obtaining α is by monitoring the drawdown of the watertable after a sudden rise (e.g. caused by irrigation or heavy rainfall). In this example, α was calculated only from the watertable level at t = 0 and t = 10 days. If more data are available, α can be found by an exponential regression (Chapter 6).

8.5   Special Drainage Situations

8.5.1   Drainage of Sloping Lands

Up to now, we have only considered drainage in flat lands. Many agricultural areas, however, are sloping, so the question arises: 'In how far can equations for flat lands be applied to sloping lands?' When a hillside is drained by a series of parallel drains, the situation is as depicted in Figure 8.15. The highest watertable height, h, above drain level is now not midway between the drains but is closer to the downslope drain.

Schmidt and Luthin (1964) solved the hillside seepage problem of steady vertical recharge to parallel ditches penetrating to a sloping impervious layer. The resulting drain spacings for gently sloping areas (slope < 0.1) do not differ much from the spacings for flat lands. This is in agreement with the results of Bouwer (1955), who
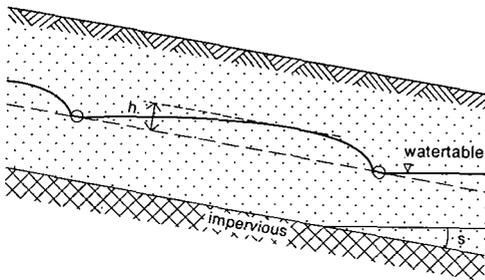

Figure 8.15 Flow to parallel drains in a homogenous soil overlying a sloping impervious layer ·

conducted a large series of tests in sand-tank models, and numerical simulations done by Fipps and Skaggs (1989). Because the vast majority of agricultural land will not have slopes in excess of 0.1, our conclusion is that we can apply solutions for flat land to sloping land without any alteration as long as the slopes are not steeper than 0.1.

This conclusion implies that we assume no difference in efficiency between drains laid parallel or perpendicular to the slope. Where the hydraulic conductivity of the soil is low, it could be advisable to lay the drains parallel to the contour lines, hence perpendicular to the slope. As the backfilled trenches have, and retain, a higher permeability than the original soil profile, any surface runoff may possibly be intercepted by the trenches. (Further considerations on the layout of subsurface drainage systems in sloping areas are presented in Chapter 21.)

8.5.2 Open Drains with Different Water Levels and of Different Sizes

Open drains in slightly sloping or undulating areas often have different drain levels. This problem has already been discussed in Chapter 7, Section 7.8.2. By applying the Dupuit-Forchheimer theory, we could find the general equation for the watertable (Equation 7.71).

In practice, open drains, besides having different water levels, are often of different sizes. Figure 8.16 schematically depicts this situation for a two-layered soil profile. We assume that the wet perimeter and the water levels in the drains are known and that there is no recharge (e.g. from precipitation); so $R = 0$. For these conditions, Ernst (1956) proposed the following solution, which is based on the fact that groundwater flow is fully equivalent to the flow of electricity, heat, or fluid between two parallel glass plates, provided that the boundary conditions are appropriately chosen.

For the Dupuit-Forchheimer conditions to be valid, Ernst set a preliminary condition that $L > 2(D_1 + D_2)$. For the sake of simplicity, D_1 is considered constant, which, in fact, it is not. As a result, the proposed method is only valid if $\Delta h < \frac{1}{2} D_1$, where Δh is the fall of the watertable. The water flow in the vicinity of
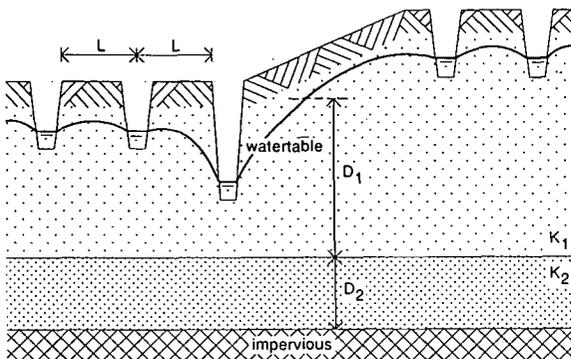

Figure 8.16 Flow to parallel open drains of different sizes and with different water levels in a two-layered soil profile

a drain is schematically depicted in Figure 8.17, in which we see that the flow is partially drained by the shallow drain, while another part of the flow continues towards a deeper drain.

To obtain a solution for the flow in the vicinity of such a drain, we replace the groundwater flow in Figure 8.17 with a resistance network as in Figure 8.18A. In this figure, we see the following variables:

$h_{0,i}$ = the water level in the drain (m)
$h_{1,i}$ = the piezometric head of the groundwater (m)
$q_{1,i-1}$ = the horizontal flow upstream of the drain (m/d)
$q_{0,i}$ = the radial flow towards the drain (m/d)
$q_{1,i}$ = the horizontal flow downstream of the drain (m/d)

In analogy to this network, the flow problem in Figure 8.16 is replaced by the resistance network (Figure 8.18B). In this figure, we recognize

$h_{0,1}, h_{0,2}, ...$ = the water level in the drain (m)
$h_{1,1}, h_{1,2}, ...$ = the piezometric head of the groundwater (m)
$w_1, w_2, ...$ = the radial resistances to flow (d)
$L_1/KD, L_2/KD, ...$ = the horizontal resistances to flow (d/m)

If $R = 0$ (as was assumed), we have, according to the principle of continuity, for any arbitrary drain i

$$q_{1,i-1} = q_{0,i} - q_{1,i} \tag{8.41}$$

Following Ernst's concept (Section 8.2.2) and assuming flow from left to right as well as upward flow to be positive, we find the radial flow towards a drain to be (compare Equation 8.20)

$$q_{0,i} = \frac{1}{w_i} (h_{1,i} - h_{0,i}) \tag{8.42}$$

where

$$w_i = \frac{L_i}{\pi K_r} \ln \frac{a D_r}{u}$$
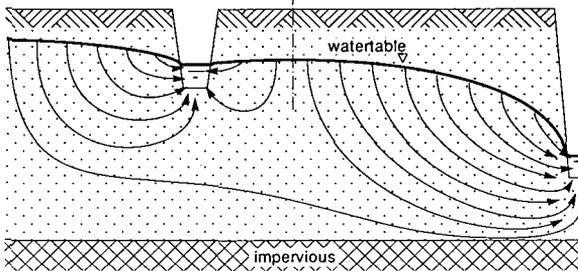
where
w_i = radial flow resistance (d)

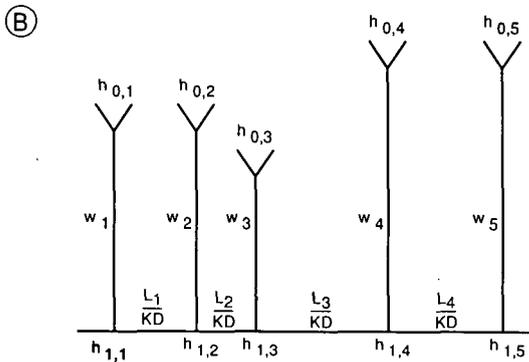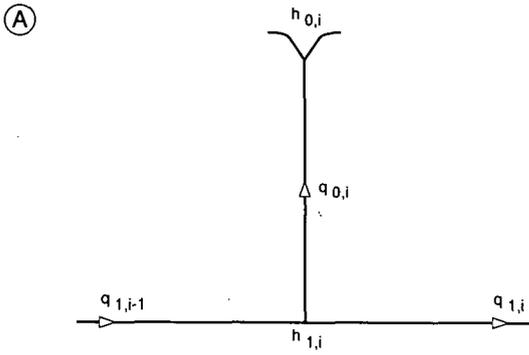Figure 8.17 Detail of Figure 8.16: flow pattern near a drain

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Figure 8.18 Schematic representation of a resistance network (A), simulating the flow problem of Figure 8.16 (B)

and the horizontal flow rate between two drains (compare Equation 8.19)

$$q_{1,i} = \frac{8KD}{L_i^2} (h_{1,i} - h_{1,i+1})$$ (8.43)

Eliminating the flow rates $q_{0,i}$, $q_{1,i}$, and $q_{1,i-1}$ from Equations 8.41 – 8.43 yields

$$\frac{8KD}{L_{i-1}^2} (h_{1,i-1} - h_{1,i}) = \frac{1}{w_i} (h_{1,i} - h_{0,i}) + \frac{8KD}{L_i^2} (h_{1,i} - h_{1,i+1})$$ (8.44)

With n open drains, we obtain n first-degree equations with n unknowns $h_{1,i}$. Further, we have to know the conditions at the boundaries: in the case of Figure 8.18, $h_{1,1}$ or $h_{1,5}$ or a given value of the horizontal flow at the left-hand side of the first drain.

If there is recharge or precipitation, we consider the case of a steady state (i.e. we consider R to be constant with time.) In fact, we assume that, along each 1 m section of the watertable, a quantity of water, R, enters the groundwater. Therefore, for the horizontal flow, we have

$$\frac{dq_1}{dx} = R$$ (8.45)

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Again, Darcy's Law holds

$$q_l = -KD\frac{dh_l}{dx} \tag{8.46}$$

Eliminating q from the latter two equations yields

$$\frac{d^2h_l}{dx^2} = -\frac{R}{KD} \tag{8.47}$$

Integrating Equation 8.47 yields

$$h_l = \left(\frac{R}{2KD}\right)x^2 + Ax + B \tag{8.48}$$

Differentiating Equation 8.48 with respect to x and substituting the result into Equation 8.46 gives

$$q_{l,i} = Rx - KDA \tag{8.49}$$

Further, for the radial flow towards the drains, we have

$$q_{0,i} = \frac{h_{l,i}}{w_i} \tag{8.50}$$

Using the latter two equations, we can apply the same procedure as for $R = 0$, hence eliminating the q-values, which again yields n equations with n unknown h_l-values.

8.5.3 Interceptor Drainage

In general, interceptor drains are used for two different purposes, i.e.:
− to intercept seepage water from neighbouring irrigation canals;
− to intercept foreign water that seeps down a hill.

The first type of interceptor drains are often installed in irrigated areas parallel to, and a short distance away from, conveyance canals. The flow towards such a drain is similar to the flow between drains with different water levels. If we assume that
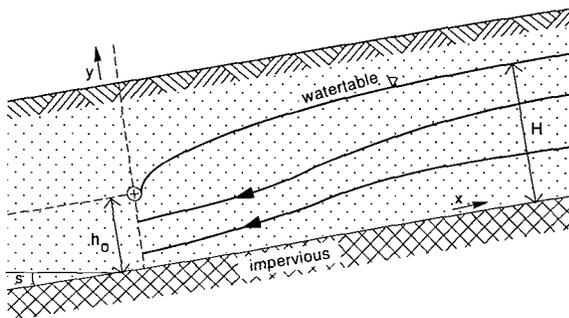

Figure 8.19 Flow towards an interceptor drain through a homogenous soil overlying a uniformly sloping layer

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there is no recharge from precipitation, we can use the Dupuit Equation to calculate the flow per unit length (Section 7.8.2).

The second type of interceptor (or hill-side) drainage is shown in Figure 8.19. Donnan (1959) presented a solution for this type of drainage. He assumed a homogeneous uniform soil layer on top of an impervious layer with a slope s. Without an interceptor drain, the slope of the watertable will be parallel to the slope of the impervious layer, so the amount of seepage water flowing downhill can be calculated with Darcy's Equation

$$q = K H s \qquad (8.51)$$

where

$\quad$ q $\;=\;$ flow rate per unit width (m^2/d)
$\quad$ K $\;=\;$ hydraulic conductivity of the top layer (m/d)
$\quad$ H $\;=\;$ height of the watertable above the impervious layer before the installation of the interceptor drain (m)
$\quad$ s $\;=\;$ slope of the impervious layer (–)

If an interceptor drain is constructed at the bottom of the hill at a height h_0 above the impervious layer, the slope of the watertable in the vicinity of the drain will no longer be parallel to the impervious layer, but will curve towards the drain. With a coordinate system as in Figure 8.19, we can assume that the slope is approximately s + dh/dx, so the amount of seepage flow through a cross-section at a distance x uphill from the drain will be

$$q = K y \left(s + \frac{dy}{dx} \right) \qquad (8.52)$$

where
$\quad$ y $\;=\;$ height of the watertable above the impervious layer at distance x (m)
$\quad$ $\dfrac{dy}{dx}$ $\;=\;$ hydraulic gradient at x (–)

Because of continuity, the flow with or without the interceptor drain must be equal, so

$$K H s = K y \left(s + \frac{dy}{dx} \right) \qquad (8.53)$$

Integrating with $y = h_0$ at $x = 0$ gives (Donnan 1959)

$$x = \frac{1}{s} \left[2.3 \, H \log \frac{H - h_0}{H - y} - (y - h_0) \right] \qquad (8.54)$$

where
$\quad$ x $\;=\;$ distance uphill from the interceptor drain (m)

Equation 8.54 can be used to calculate the height of the watertable at any distance x uphill from the interceptor drain. Theoretically, $y = H$ is only reached at $x = \infty$.

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Example 8.8

An irrigation scheme (500 × 1000 m) is located in a sloping area (Figure 8.20). The deep percolation losses are 1 mm/d. The soil consists of a permeable layer, 6 m thick and with a hydraulic conductivity of 2.5 m/d, on top of an impervious layer with a slope of s = 0.04. To control the watertable in the area downhill from the irrigated area at a level of 2 m below the soil surface, an interceptor drain will be constructed. We have to calculate the required depth and capacity of the interceptor drain, and the uphill elevation of the watertable after the construction of the interceptor drain.

To control the watertable at 2 m below the soil surface, the height of the interceptor drain above the impervious layer has to be

$$h_0 = 6.0 - 2.0 = 4.0 \text{ m}$$

The percolation losses result in a seepage flow, per metre width, of

$$q_s = 500 \times 0.001 = 0.5 \text{ m}^2/\text{d}$$

The elevation of the watertable above the impervious layer before the construction of the interceptor drain can be calculated with Equation 8.51

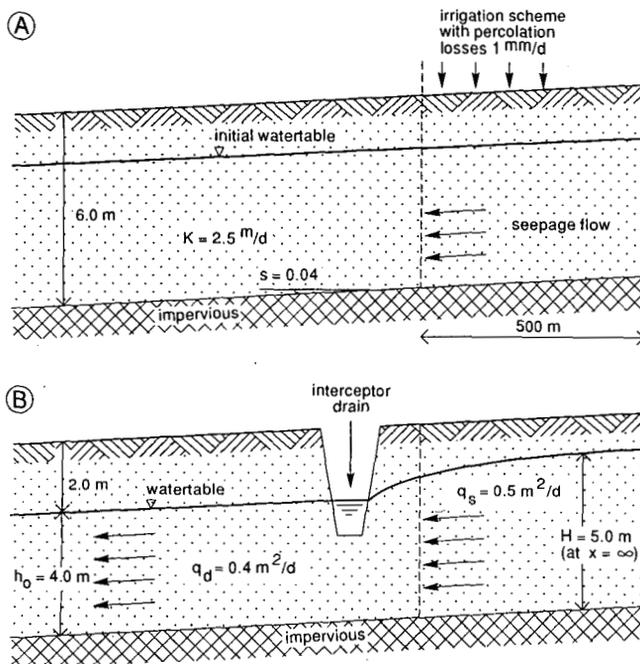$$H = \frac{q_s}{Ks} = \frac{0.5}{2.5 \times 0.04} = 5.0 \text{ m}$$

Figure 8.20 The calculation of an interceptor drain in a sloping area: (A) before construction and (B) after construction (Example 8.8)

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After the interceptor drain has been constructed, the seepage flow downhill from the drain will be

$$q_d = K\,h_0\,s = 2.5 \times 4.0 \times 0.04 = 0.4\,\text{m}^2/\text{d}$$

Thus the required capacity of the interceptor drain per metre length is

$$q_i = q_s - q_d = 0.5 - 0.4 = 0.1\,\text{m}^2/\text{d}$$

In other words 20% of the percolating water will be intercepted. As the scheme is 1000 m long, the discharge at the outlet of the interceptor drain will be

$$Q_i = 1000 \times q_i = 1000 \times 0.1 = 100\,\text{m}^3/\text{d} = 1.16\,\text{l/s}$$

The elevation of the watertable uphill from the interceptor drain can be described by Equation 8.54

$$x = \frac{1}{s}[2.3\,H\log\frac{H - h_0}{H - y} - (y - h_0)]$$

$$x = \frac{1}{0.04}[2.3 \times 5.0\log\frac{5.0 - 4.0}{5.0 - y} - (y - 4.0)]$$

$$x = 287.5\log\frac{1.0}{5.0 - y} - 25 \times (y - 4.0)$$

Thus:

$$y = 4.0\,\text{m} \quad \text{at} \quad x = \quad 0\,\text{m}$$
$$y = 4.2\,\text{m} \quad \text{at} \quad x = \quad 23\,\text{m}$$
$$y = 4.4\,\text{m} \quad \text{at} \quad x = \quad 54\,\text{m}$$
$$y = 4.6\,\text{m} \quad \text{at} \quad x = \quad 99\,\text{m}$$
$$y = 4.8\,\text{m} \quad \text{at} \quad x = 181\,\text{m}$$

and

$$y = 4.9\,\text{m} \quad \text{at} \quad x = 265\,\text{m}$$

8.5.4 Drainage of Heavy Clay Soils

Heavy clay soils often have such a low hydraulic conductivity that they require very narrow drain spacings. The narrowest spacing applicable in practice is a matter of economics (e.g. crops to be grown, prices of products). The hydraulic conductivity may be so low that no subsurface drainage with economically justifiable spacing is possible. One should then use a surface drainage system of furrows and small ditches, possibly combined with bedding of the soil (Chapter 20).

For moderate hydraulic conductivity, it may happen that the infiltration rate is too low for the water to enter the soil, so that frequent surface ponding will occur. A suggested limit for the installation of a subsurface drainage system is that the infiltration rate of the soil must be such that the rainfall to be expected in two or three subsequent days must easily infiltrate during that time. If not, a subsurface drainage system will not work satisfactorily and one has to resort to a surface drainage system.
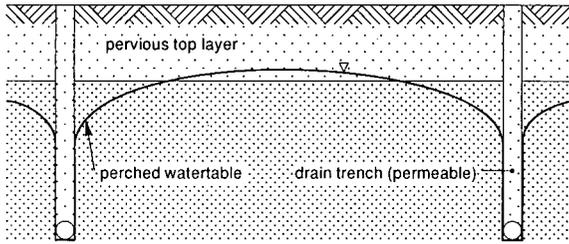
Figure 8.21 Perched watertable built up in heavy clay soil, just below the top layer with a higher hydraulic conductivity

Heavy clay soils of low hydraulic conductivity often have a top layer with a surprisingly high hydraulic conductivity because of the activity of plant roots or the presence of a tilled layer. In such cases, rainfall will build up a perched watertable on the layer just below the top layer (Figure 8.21). Under these conditions, a subsurface drainage system can be effective because of the interflow in the permeable top layer, but it will only work as long as the backfilled trench remains more permeable than the original soil.

Unless one can expect the hydraulic conductivity of the subsoil to increase with time (e.g. because of the soil ripening process; Chapter 13), it makes no sense to install a drainage system at a great depth. In fact, a system reaching just below the top layer should be sufficient. The system can be improved by filling the trench with coarse material or adding material like lime. Further improvement can be sought in mole drainage, perpendicular to the subsurface lines. (More information on this topic will be provided in Chapter 21.)

In none of the cases discussed above is it possible to apply a drainage theory, since the exact flow paths of the water are not known. Besides, these heavy soils often have a seasonal variation in hydraulic conductivity because of swelling and shrinking.

Heavy clay soils of the type found in the Dutch basin clay areas often have relatively high permeable layers in the subsoil (Van Hoorn 1960). If a subsurface drainage system can be placed in such a layer, these soils can often be drained quite well (Figure 8.22). In these circumstances, the Ernst theory discussed earlier can be applied. As a matter of fact, this theory was developed mainly for such applications. Complications can arise due to two circumstances:

- The layer with the higher hydraulic conductivity is too deep for a normal subsurface system to be installed in it. If so, the drainage problem could possibly be solved
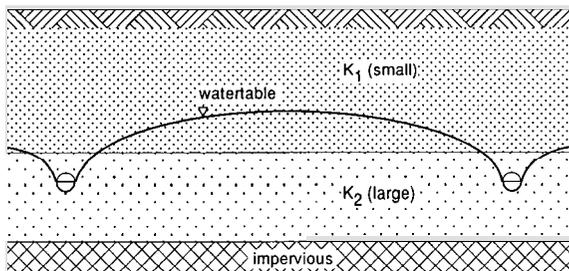

Figure 8.22 Drains in a layer with high hydraulic conductivity under less permeable heavy clay topsoil

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by installing tubewells. It is obvious that, for the wells to be effective, the permeable layer must satisfy certain conditions with respect to the transmissivity, KD, and that the piezometric pressure in this layer must be low. (This will be further discussed in Chapter 22.);

– The upper layer has such a low hydraulic conductivity that the groundwater does not percolate to the deeper layer at a reasonable rate. Now we are back to the beginning of this section, and the only solution is to apply surface drainage.

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