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On a specific method to solve semi-obnoxious continuous facility location problems*

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Abstract Semi-obnoxious continuous location problems are mostly modeled in literature combining a convex objective representing minimum cost and a multiextremal objective representing the nondesirable part of a facility. Deterministic methods have been designed to solve such problems and generic one or bi-objective heuristic methods have been applied. This paper describes a dedicated method to solve semi-obnoxious location problems making use of its specific structure.

Keywords: facility location, obnoxious, pareto, heuristic, global optimization, metaheuristic

1. Introduction

Many models have been introduced in literature to describe the location of a facility in the plane. Objectives vary from minimum transportation cost in the Weber problem, maximising market share in competitive Huff-like models, centre problems maximising cover to finally obnoxious objectives to describe that a facility is nondesirable. Obnoxious means according to Erkut and Neuman (1989) [3], that the facility generates a disservice to the people nearby while producing an intended product or service. An intriguing aspect of objective functions describing the nondesirable effect of the facility is that it leads to multiextremal optimisation problems that are hard to solve.

Semi-obnoxious models typically combine a convex objective (e.g. Figure 1) to describe the attraction aspect of the facility with the obnoxious objective (e.g. Figure 2) and thus inherit its multimodal character leading to new challenges for optimisation methods. One can either combine both objectives in a multi-objective fashion or try to represent the efficient solutions that generate the Pareto front. The latter is comprehensible when locating only one facility for decision makers, as one has one graph representing the trade-off of the objectives and one graph representing te corresponding efficient locations on a map.

One of the research questions is how to generate the efficient locations given that we are dealing with a nonconvex objective function. An elaborate overview of literature on the topic is given by Yapicioglu et al. (2006) who approachs the problem by generic bi-objective Particle Swarm algorithms. Another way to approach the problem is to use deterministic branch-and-bound like algorithms that guarantee the quality of found locations, e.g. [6]. The argumentation for using heuristic stochastic or deterministic GO algorithms is that the objective function

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is nonconvex. However, seen from the bi-objective perspective, one of the objectives is convex and the other is not. In Section 2 we describe an approach using the model of [1] as an example problem.

In Section 2, we describe a specific metaheuristic to generate efficient solutions of the semi-obnoxious continuous single facility model using the presented method. Illustrations are showed in Section 3. Finally we conclude in Section 4.

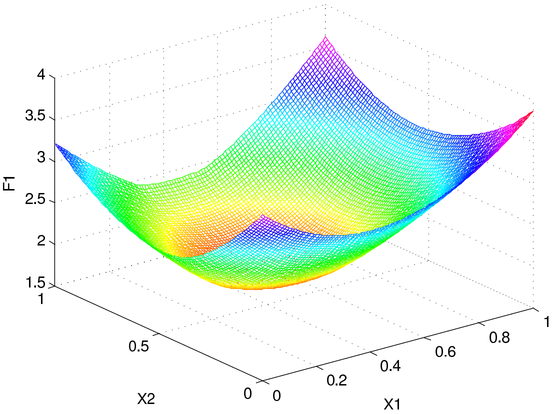

Figure 1. Graph minsum (Weber) function f_1

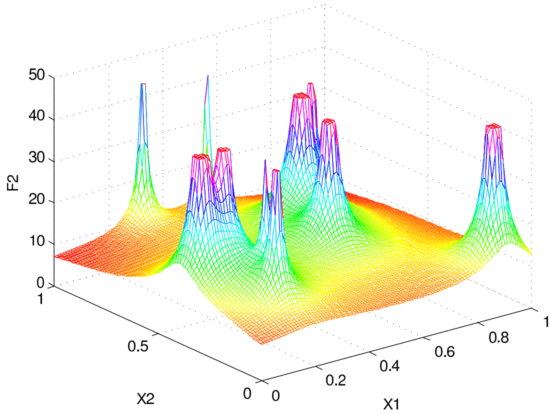

Figure 2. Graph obnoxious function f_2

2. A bi-objective approach

To describe the approach, we use as an example problem the semi-obnoxious model described in Brimberg and Juel [1] where the problem is notated as Bicriterion Semi-obnoxious Location Problem (BSLP). In this problem a desirable and an obnoxious objective function must be minimised. The desirable or convex objective is the classic minsum transportation cost.

$$f_1(x) = \sum_i w_i d_i(x), \tag{1}$$

where w_i are weights and d_i the (Euclidean or rectangular) distance from facility location x to fixed (demand) point p_i, $i = 1, \ldots, m$. Minimising (1) is called the median problem.

The obnoxious function minimises the overall obnoxiousness when far from a demand-point, but also it reflects the local effects when close to a demand point [7].

$$f_2(x) = \sum_i v_i d_i(x)^{-b}, \tag{2}$$

where $b > 0$ takes on a specified value depending on the type of facility being considered and v_i is again a weight like the population size [2].

Figures 1 and 2 give an impression of the two objective functions for 10 randomly generated fixed points and weights. One can observe that minsum objective f_1 is convex whereas obnoxious objective f_2 is multiextremal. Notice that f_2 function does not permit getting too close to an existing facility as $d_i(x)$ tends to zero.

A decision maker is usually interested in the efficient points over the feasible set X of such a problem where f_1 as well as f_2 is minimised. An efficient (nondominated) location x^* is defined in multiobjective sense such that there does not exist another location $x \in X$ with $f_1(x) < f_1(x^*)$ and $f_2(x) \leq f_2(x^*)$ or alternatively $f_2(x) < f_2(x^*)$ and $f_1(x) \leq f_1(x^*)$. One is usually interested in the set of efficient locations X^* and the so called Pareto front $\{(f_1(x), f_2(x)) | x \in X^*\}$ that sketches the trade-off between the two objective values.

There are several ways to approach the generation of efficient solutions. One can combine the objectives in one weighted function or alternatively restrict iteratively one objective like $f_1(x) \leq tc$ and minimise the other. Let

$$R(tc) = \{x \in X | f_1(x) \leq tc\} \tag{3}$$

denote a level set of the convex objective f_1. Notice that in our case $R(tc)$ is a convex set. One can follow the last approach by using Algorithm 1.

Algorithm 1 Eff(X, f_1, f_2, δ)

Determine $S := \arg\min_{x \in X} f_2(x)$
$tc := \min_{x \in S} f_1(x)$
while ($S \neq \emptyset$)
  $tc := tc - \delta$
  $S := \arg\min_{x \in R(tc)} f_2(x)$
  $tc := \min_{x \in S} f_1(x)$
endwhile

First of all, if at a certain iteration S is completely interior with respect to $R(tc)$, we know that after reduction of the level tc of the second objective we only have to check the boundary of $R(tc)$ on the appearance of better function values of f_2. Secondly, we are interested in this approach from the perspective of meta-heuristics. Given that we found the solution $x \in S$ on the boundary of $R(tx)$ one can use the information of convexity to restrict new generation of points only in the direction d with $d^T \nabla f_1(x) \leq 0$. Moroever, we use developed a method to generate points uniformly over an ellipsoidal set approximating the current contour of f_1 by fitting a quadratic function through the current population of sample points.

In this work a metaheuristic method is implemented following the different steps described in Algorithm 1. For optimising the f_1 function a gradient based local optimiser is applied (Weiszfeld-like method) and for optimising f_2 a metaheuristic global optimization algorithm based on subpopulation has been implemented. The method will be specified further in the full paper.

3. Illustration

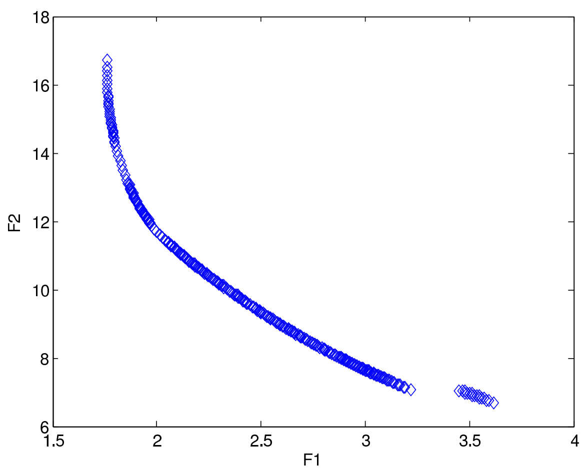

Figure 3. Pareto Front

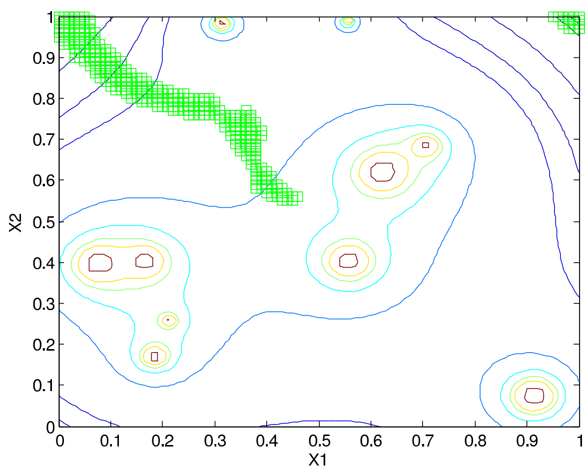

Figure 4. Efficient points and contours of f_2

To illustrate the behaviour of the algorithm, the steps of Algorithm 1 have been followed and hence a Pareto set of efficient points was generated. A case has been used where 10 demand points have been generated randomly together with the weights. Figure 3 shows the Pareto Front obtained for the two objective functions. In Figure 4, contour lines of f_2 have been drwan to give an impression of the optimum points. It can be see that the areas with low objective function are typically in the corners, as is usual in obnoxious objective functions, also called the "mother in law effect". The green squares in this figure represent the set of Pareto efficient points. They tend to the low values of f_2 as well as to the middle of the figure where typically the optimum of the minsum objective f_1 can be found.

4. Conclusions

In this work the new metaheuristic algorithm is developed and tested on four different semiobnoxious problems solved in [8] by using different particle swarm optimisers (PSO). These problems were previously defined and solved in [1], [2], [4], [5] and [7]. Comparison between the new method and PSO methods will be provided. We designed and evaluated specific methods for generating efficient solutions for the semi-obnoxious one facility problems in the plane making use of the idea that one of the objectives is convex approximating its contour by an ellipsoidal region.

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