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# Technological change and tropical deforestation: a perspective at the household level

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ABSTRACT. We analyse the effects of technological change in agriculture on forest clearing by households in developing countries. The possible effects are found to be many and diverse, depending on the type of change and the institutional context. We conclude that agricultural intensification is certainly not the panacea that some believe it to be.

## 1. Introduction

Tropical deforestation is a prominent environmental issue, caused mainly by agricultural conversion by either smallholders or large commercial farms and companies (for recent overviews see Brown and Pearce, 1994; Angelsen and Kaimowitz, 1999; van Kooten, Sedjo, and Bulte, 1999). To understand deforestation and thus promote forest conservation, it is important to analyse what happens at the extensive margin, or the agriculture–forestry interface. It is often claimed that agricultural intensification, defined as yield-increasing technological change, will promote forest conservation because it relieves pressure on the remaining

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forest. This is sometimes referred to as the 'intensification myth', because, by making agriculture more profitable, production is just as likely to expand at the extensive as at the intensive margin (see Angelsen and Kaimowitz, 1999).

In this paper, we explore the relationship between technology that affects the agricultural sector and deforestation by considering land-use decisions of rural households. For this purpose we treat technological change as an exogenous process ('manna from heaven') and consider its effect on the area of land that individual households and the agricultural sector as a whole choose to hold as cropland. Throughout we assume that using more agricultural land comes at the expense of area available for natural forest, so the relation between intensification and deforestation is the logical 'complement' of the analysis of intensification and land used for cropping that is modelled here.

We develop a model that capture some of the relevant trade-offs facing rural households in developing countries. The objectives are twofold: (1) to analyse the effect of different types of technological change (Hicks neutral versus labour- and capital-augmenting innovations) on land-use decisions by households, and (2) to explore the importance of the institutional setting on agricultural innovations. More specifically, we study household decision-making in situations where households can sell or hire labour (and other inputs) in 'perfect markets' and where labour markets do not exist, which is likely the case for many developing countries (see De Janvry, Fafchamps, and Sadoulet, 1991; Bulte and van Soest, 1999). We demonstrate that contextual considerations matter a great deal, and the effects of technical change are to be assessed at the case study level. Empirical evidence from previous research indeed confirms that the impact of technical progress can go both ways (for example, Godoy *et al*. 1997, versus Foster, Rosenzweig, and Behrman, 1997).

## 2. A household model of forest clearing

We begin by developing a simple base-case model where a rural household maximizes utility

$$U\,(c^{\mathrm{M}},c^{\mathrm{L}}) \tag{1}$$

where $U$ is a continuously differentiable, concave, and additively separable utility function, with consumption goods ($c^{\mathrm{M}}$) and leisure ($c^{\mathrm{L}}$) as arguments. The term consumption goods includes not only manufactured goods but also food. Therefore, the underlying assumption is that food purchased at markets and own output are perfect substitutes. Leisure is a catch all for representing any household activities other than work (for example, childbearing and rearing).

Maximization takes place subject to a number of constraints. First, the household faces the budget constraint

$$P^{\mathrm{M}}c^{\mathrm{M}} = P^{\mathrm{A}}q^{\mathrm{A}} - wL^{\mathrm{H}} \tag{2}$$

where $P^{\mathrm{M}}$ is the price of (purchased) consumption goods, $P^{\mathrm{A}}$ is the price of agricultural output, $q^{\mathrm{A}}$ is production realized by the household, and $L^{\mathrm{H}}$ is the labour that the household hires (sells) at the prevailing wage rate w.

Without lost of generality, $P^{M}$ is set equal to one. The left-hand side (LHS) of (2) then represents expenditures on consumables, while the right-hand side (RHS) represents available income. Equation (2) is consistent with the interpretation that the household 'purchases' some food off-farm (in which case $q$ is consumption and takes a negative value), or, equivalently but more likely, that it retains part of its production for household consumption (the shadow price is still $P^{A}$).

The agricultural production function is

$$q^{A} = \theta f(\mu A, \nu L^{D}) \tag{3}$$

where $A$ is cultivated area and $L^{D}$ is labour dedicated to agricultural production, both measured in physical units. Technical change increases the performance of physical units over time, with parameters θ, μ, and ν representing technology indices that translate physical into efficiency units. Parameters, θ, μ, and ν are used to represent the 'state of technology'; θ, μ, and ν denote Hicks-neutral (that is, non-input specific), land-augmenting, and labour-augmenting technological progress, respectively. By increasing θ, the productive of land and labour go up, while increasing μ (ν) raises production per unit of land (labour). The function $f(\cdot)$ is assumed to be continuous and concave in its arguments, with $f_{j} > 0$, $f_{ij} > 0$ and $f_{jj} < 0$ ($i,j = A,L^{D}$), where subscripts indicate partial derivatives. (The matrix of second derivatives is negative semidefinite at every point.) Thus, $f_{AA} < 0$ implies declining marginal productive of land if the agricultural land base is expanded, and $f_{AL} > 0$ suggests that the marginal productivity of labour increases when more land is used for crop production.

We do not include capital as a separate input. While land and labour are likely the most important inputs at the extensive margin, it is clear that capital may also play a role. One way to incorporate capital in the analysis is by assuming that it is implicit in the parameters θ, μ, and/or ν. For example, if we are interested in the effect of capital inputs, such as machinery, this may be captured in the parameter measuring labour-augmenting technical change, while as increased fertilizer use is a form of land-augmenting technical change. Alternatively, capital is readily included as a separate input ($K$) in the production function: $q = \theta f(\mu A,\nu L^{D},\sigma K)$. The analysis for this production function is a straightforward extension of the model below (albeit analytically much less tractable).

In regions of tropical rainforest, maintaining the quality of agricultural land and increasing its area are labour-intensive activities (for example, forest needs to be cleared). Therefore, we assume agricultural land is produced using labour

$$A = a(L^{C}) \tag{4}$$

where $L^{C}$ is the amount of time devoted to land clearing and maintenance. More time spent working translates into better results, albeit at a diminishing rate; hence, $A_{L} > 0$ and $A_{LL} < 0$.

The household's (physical) labour constraint is

$$\bar{L} + L^{H} = L^{D} + L^{C} + c^{L} \tag{5}$$

where $\bar{L}$ is the household's fixed time endowment that can be augmented with hired labour $L^{H}$; alternatively, $L^{H}$ may be negative, which implies the selling of household labour (that is, engaging in off-farm employment). Available labour can be devoted to agricultural production $L^{D}$, forest clearing $L^{C}$, and leisure $c^{L}$ (see below). In the following section we assume that household and hired labour are prefect substitutes.

In theory, we could introduce an additional constraint reflecting the potential scarcity of land; households may not be able to freely choose the quality of land that they would like to hold. Specifically, desired land holdings should never exceed available land. However, in what follows we model the behaviour of a representative household at the extensive margin where land is not scarce, and the household can therefore freely choose the quantity of land.

The general framework sketched above allows us to analyse the impact of the institutional context in which the household operates. So far we have assumed that the household participates in the markets for labour, agricultural outputs, and consumption goods. This is a bold assumption as many peasant households are unable to participate in markets because of prohibitive transaction costs (transportation costs, mark-ups by merchants or search costs), the thinness of local markets or the existence of price risk and risk aversion (Ellis, 1998; Sadoulet and De Janvry, 1995; Singh, Squire, and Strauss, 1986). By varying the set of markets facing the household, we explore the importance of this institutional context, but we begin by considering the ideal case.

**3. The economist's dream: perfect markets**

The rural household model is readily solved by substituting equations (2), (3), (4), and (5) into (1)

$$U = U\{P^{A}\theta f[\mu A(L^{C}),\nu L^{A}] - \omega L^{H},\bar{L} + L^{H} - L^{D} - L^{C}\} \tag{6}$$

Taking derivatives with respect to decision variables $L^{D}$, $L^{C}$, and $L^{H}$ and assuming additive separability of the utility function ($U_{ML} = 0$), the necessary conditions for an optimal solution are given by

$$P^{A}\theta\nu f_{L} - \omega = 0 \tag{7}$$

$$\mu f_{A} A_{L} - \nu f_{L} = 0 \tag{8}$$

$$\omega U_{M} - U_{L} = 0 \tag{9}$$

The interpretation of these conditions is as follows. From (7), the marginal value product of agricultural labour should equal the prevailing (exogenous) wage. From (8), the marginal product of labour in forest clearing should equal that in crop production. Equation (9) states that the ratio of marginal utilities should equal the price ratio (recall that $P^{M} = 1$). The optimal agricultural land base (and, hence, forest stock) is thus determined by the optimal allocation of labour to land clearing $L^{C^*}$.

To derive the comparative static results for forest stocks, we totally differentiate the system (7)–(9) (see appendix 1). In particular, we explore what happens to $A^*$ (the agricultural land base) when the values of the technological change parameters $\theta$, $\mu$, and $\nu$ increase, effectively

Table 1. *Comparative static results for agricultural land: perfect markets*

| | $dA^*$ |
|---|---|
| $d\theta$ | + |
| $d\mu$ | $\pm$: $s(\epsilon_A)$ |
| $d\nu$ | + |
| $dP$ | + |
| $d\omega$ | − |

mimicking technological innovation. For completeness, we also consider the effect of changing the relative prices for outputs and labour on $A^*$. The results are summarized in table 1.

The assumption of perfect markets implies that the household can solve production (profit) and consumption (utility) decisions separably. Hence, the household first determines the optimal allocation of labour and land, allocating labour until the marginal value product of labour equals the wage rate. Next, it decides how to spend income on commodities and leisure. The results from table 1 provide no support for the 'intensification myth'; indeed, in an economist's dream world, it will achieve just the opposite. Innovations in agriculture tend to promote deforestation, although in the case of land-augmenting technical change this result is ambiguous. The case of relative prices appears more mixed. The intuition for the results is as follows.

From the first (fourth) row of table 1, Hicks neutral technological change (higher prices for output) unambiguously increases the amount of land that households choose to hold. It increases the value of the marginal product of all inputs, including agricultural land, at the extensive margin or frontier. *The cost per efficiency unit of land goes down*, hence the area demanded will expand. At the margin, more cropland comes at the expense of natural forest.

The case of land-augmenting technical change is more complex, and the net effect depends on the elasticity of marginal productivity [$\epsilon_A = \mu A \times (f_{AA}/f_A)$], which measures the curvature of the production function in the input land. First, households may decide to allocate more time to the clearing of land (thereby increasing $A$) because this type of technical progress lowers the cost per efficiency unit of land. The cost of producing an additional hectare of agricultural land in terms of labour is the same, yet the benefits of having an additional hectare are greater. However, there is an opposite effect: To produce the same level of output, less land is needed. This effect depresses the demand for land. To make the analysis more confusing, land-augmenting technical change also affects the marginal productivity of labour ($L^D$) as $f_{LA} > 0$, thus rendering agricultural production more profitable (increasing demand for land). The net effect is determined by the magnitude of $\epsilon_A$, and it is an empirical matter as to which possible case represents the 'actual' situation.

Labour-augmenting technical change reduces the price per efficiency unit of labour. Depending on the elasticity of marginal productivity [$\epsilon_L = L^D \times (f_{LL}/f_L)$], more or less labour may be employed to work on the field (recall the previous discussion regarding land). At the margin, however, land becomes more profitable ($f_{LA} > 0$), so deforestation is unambiguously promoted.

Finally, we consider the implications of technological change elsewhere in the economy that raises the wage level $\omega$. (This comparative static result is clearly not applicable in the case where rural households are unable to participate in labour markets, as discussed in the next section.) We find—consistent with intuition—that increasing the opportunity cost of labour in forest clearing and cropping will divert labour to alternative uses in the economy, thus promoting forest conservation.

**4. Complex reality: incomplete markets**

In this section, we relax the heroic assumption of perfect markets, and assume that the household cannot hire and sell labour, although it can still trade agricultural outputs for consumption goods. When rural households do not have access to the labour market, constraints (2) and (5) need to be modified as follows

$$P^{M}c^{M} = P^{A}q^{A} \tag{2'}$$

and

$$\bar{L} = L^{A} + L^{D} + c^{L} \tag{4'}$$

Upon solving the revised model, conditions (7)–(9) are replaced by

$$P^{A}U_{M}\theta\nu f_{L} = U_{L} \tag{10}$$

and

$$\mu f_{A}A_{L} = \nu f_{L} \tag{11}$$

Since there is one less choice variable ($L^{H}$), only two conditions remain. Equation (10) indicates that for an optimal solution the marginal utility of leisure should be equal to the value of the marginal product of labour in production. Equation (11) prescribes that the marginal product of labour should be equal in forest clearing and crop production. The relevant comparative static results for this situation are provided in table 2 (see appendix 2 for details). The comparative static results for the equilibrium stock of land are significantly different (and more complex) than in the case of perfect markets. Relaxing the assumption of perfect markets means that the production and consumption problems are no longer 'separable', but have to be solved jointly (Singh, Squire, and Strauss, 1986).

Table 2. *Comparative static results for agricultural land: missing labour market*

| | $dA^*$ |
|---|---|
| $d\theta$ | $\pm$: $[1 + \eta_M]$ |
| $d\mu$ | $\pm$: $s(\epsilon_A, \eta_{ML}, \eta_{MA})$ |
| $d\nu$ | $\pm$: $g(\epsilon_L, \eta_{ML})$ |
| $dP$ | $\pm$: $[1 + \eta_M]$ |

*Notes*: Where the following elasticities are distinguished: $\eta_M = c^M \times (U_{MM}/U_M)$, $\epsilon_A = \mu A(f_{AA}/f_A)$, $\epsilon_L = \nu L^D(f_{LL}/f_L)$, $\eta_{ML} = Pf_L\theta\nu(U_{MM}/U_M)$ and $\eta_{MA} = P\theta\mu f_A A_L(U_{MM}/U_M)$.

The most important observation from table 2 is that all comparative static results are ambiguous if the household faces an imperfect set of markets. Table 2 encompasses all the outcomes of table 1, but all the comparative statics results could possibly change sign. This suggests that the institutional context is an important determinant of the effect of new technologies on forest stocks.

First consider the result for $dA/d\theta$ (and $dA/dP$), where the sign is determined by the elasticity of the marginal utility of consumption of purchased goods [$\eta_M = c^M \times (U_{MM}/U_M)$]. The effect can be decomposed into an income and substitution effect. As profitability of agricultural production increases, the household has an incentive to expand area under cultivation (analogous to the perfect markets case) and allocate more time to crop production. This is the substitution effect. However, increasing production translates into higher income, increasing demand for consumption goods and leisure. This income effect works in the opposite direction of the substitution effect, because the household now cannot hire extra labour (that is, increased leisure is at the expense of forest clearing $L^D$ and crop production $L^A$). The net effect is ambiguous: labour allocated to forest clearing and crop production may on balance increase or decrease, as measured both in physical and efficiency units, and agricultural area $A$ may be expanded or contracted. It may be inferred from appendix 2 that $L^D$ and $L^C$ will increase or decrease together, that is, the household will never decide to spend more hours in crop production and simultaneously reduce hours spent clearing forestland (or vice versa).

The effects of land- and labour-augmenting technical change on deforestation are more complex because now $L^D$ and $L^C$ may move in opposite directions, depending on the elasticities of marginal productivity ($\epsilon_A$ and $\epsilon_L$). Total time spent working is endogenous as well, and jointly determined by the income and substitution effects ($\eta_{MA}$, $\eta_{ML}$) resulting from the technology-induced increase in marginal productivity (see appendix 2). The general lesson from this highly simplified and stylized model is that imperfect markets may trigger complex and counterintuitive responses.

The incomplete markets case may actually be much simpler than indicated by our rather complex model. In particular, some households may be located in isolated areas where transaction costs are prohibitively high, such that not trading in any market is optimal. In a way, we have now returned to the simplest conceivable model: the household combines agricultural land and its own labour to feed itself. While it is certainly possible to use a utility function describing the pleasure from consuming food and leisure, it is perhaps more natural to think of such a household as trying to meet a certain consumption target at the lowest feasible cost (in terms of labour input). This is referred to as a subsistence or 'full belly' model.[1]

The comparative statics of full belly households are much simpler than the responses of the utility-maximizing households considered here. The household will bring into production as much cropland as needed to meet

[1] It is also possible to imagine a slightly more elaborate 'full belly' model, in which the household trades part of its agricultural output for a fixed (or predetermined) quantity of consumption goods.

the subsistence requirement. Hicks neutral technical change (increasing $\theta$) and land-augmenting technical change (increasing $\mu$) have the same effect on the household's decision with respect to clearing land. Deforestation is reduced because the household requires less land to produce the needed amount of food. Finally, labour-augmenting technical change (increasing $\nu$) also tends to reduce deforestation. Given that labour and land are substitutes in the agricultural production function, labour-augmenting technological progress implies that more output is produced using the same (physical) labour input. Since the household only aims to produce a certain amount of agricultural produce, less labour and land may be used.

It appears that many conservationists and development workers implicitly assume that the 'full belly' description of agricultural households is not apt (Angelsen, 1999). At least one is inclined to think so, as the comparative statics of subsistence models are roughly consistent with the 'intensification myth'. It should be realized, however, that the conditions under which the simple full belly model holds are rather specific.

## 5. Conclusions

The main comparative static results of the household level model indicate that it makes little sense to talk in general terms about the effect of technological change in agriculture on deforestation. We find that the effects of technological change in agriculture on long-run forest stocks can be positive or negative, but often they are ambiguous. Specifically, whether and how technical change in agriculture affects forest stocks depends on the form of technological change (Hicks neutral, capital augmenting, or labour augmenting), the institutional setting, the specifications of the production and utility functions. This implies that the effects will vary with the local situation.

There are some caveats, however. First, we have ignored capital as a separate input, which may be an important omission if farmers need to invest in certain capital inputs to deforest additional land, as this requires households to either save or borrow money. Since many rural households face constraints of all sorts when they turn to capital markets, they are often forced to invest their own accumulated surpluses. If farmers are rationed in credit market and incomes (and savings) are low, they may forego profitable investment opportunities, and use less land for crop production than they would if credit were readily available to them. If technological progress allows such a household to increase its savings, it facilitates financing further deforestation. (Similar considerations come into play if there are improved opportunities to earn an off-farm income.) In this sense, the interaction of technical change and imperfect capital markets might add an extra dimension to the discussion.

Next, we have also ignored the role of risk and risk aversion, although likely very important for rural households (Angelsen, 1999). Even if we are willing to assume that technical change does not affect the level of risk that the household faces (an unlikely assumption), it will likely have an effect on cropping decisions. The reason is that technical change will increase the household's income, which will change its attitude towards risk. The common assumption is that households display decreasing absolute risk aversion, which implies that the household becomes less risk averse as income (wealth) increases. In other words, as the household gets richer it is willing to gamble a bit more. The effect of deforestation is obviously dependent on the 'source' of risk. If investments in further deforestation are risky, households are more inclined to clear more forest if they have higher incomes. Technical progress and deforestation may therefore go hand in hand. But the reverse is also possible. If activities other than deforestation that compete for labour, say, are risky, richer households are expected to opt more often for these possibilities, thus reducing pressure on the natural forest.

More often, however, we can expect that technical change in agriculture *will* change the level of risk that rural households bear. Assume that new technologies do not change the expected production level, but reduce the variance of output. Clearly, depending on the cost of these technologies, risk averse households may adopt them as lower variance is strictly preferred. The effect on deforestation of such innovations crucially depends on the institutional setting. In a 'full belly' type of world, reducing the agricultural output variance implies that households will have to deforest *less* to ensure a certain consumption target. Technical change may therefore reduce deforestation, but this is certainly not a robust result. Assuming that households aim to maximize utility (as in our models), the effect is just the opposite. Technological change that reduces the risk of cropping will make cropping more attractive, and therefore provide an impetus for further deforestation.

The complex story presented in this paper may still be too simple if we are interested in the economy-wide effects of technical change. In addition to the feedback triggered by the income and substitution effects at the household level, technological progress may cause general equilibrium effects at the level of the economy. This implies that it is not appropriate to treat prices of factors and commodities as 'exogenous'. Rather, they will adjust as new technologies affect demand and supply, thus altering land-clearing decisions by households, and adding further to the ambiguity. In addition, new technologies will often affect migration, spurred by regional differences in potential household income. Depending on the type of technological change (and the region that is suitable for adopting it), migration may occur towards or away from the extensive margin. We are forced to conclude that technological change in agriculture is not a panacea that will solve the perceived deforestation problem in tropical countries. Depending on the market context, preference, type of technical change, and the nature of the crop, the net effect could go in any direction.

A recent attempt to systematize the empirical evidence concludes that technical change in frontier agriculture generally tends to promote forest conversion (Angelsen and Kaimowitz, 2001). An exception might be labour-intensive technological change (land-augmenting in our model), but farmers are generally reluctant to adopt such technologies, since labour, rather than land, is the scarce factor. New technologies outside frontier agriculture, represented by the increase in the wage rate in our model, should attract labour and promote forest conservation, unless the investment effect just discussed is strong.

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## Appendix 1

Taking the total differential of the equilibrium for the perfect markets case gives the following system

$$\begin{bmatrix} \nu\mu f_{LA} f_L - \nu^2 f_{LL} & \begin{matrix}[\mu^2 A_L^2 f_{AA} + \mu f_A A_{LL} \\ - \nu\mu f_{LA} A_L\end{matrix} & 0 \\ \nu^2 P\theta f_{LL} & \nu\mu P\theta f_{LA} A_L & 0 \\ w\nu P\theta f_L U_{MM} + U_{LL} & wP\theta\mu A_L f_A U_{MM} + U_{LL} & -w^2 U_{MM} - U_{LL} \end{bmatrix} \times \begin{bmatrix} dL^D \\ dL^C \\ dL^H \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & -A_L f_A\left[1+\epsilon_A - \frac{\nu A f_{AL}}{A_L f_A}\right] & f_L\left[1+\epsilon_L - \frac{\mu A_L f_{AL} L^D}{f_L}\right] & 0 \\ -\nu\theta f_L & -\nu P f_L & -\nu A P \theta f_{AL} & -P\theta f_L[1+\epsilon_L] & 1 \\ -w\theta f\, U_{MM} & -wPf\, U_{MM} & -wP\theta A\, f_A U_{MM} & -wP\theta f_L\, L^D U_{MM} & -U_M + wL_H U_{MM} \end{bmatrix} \times \begin{bmatrix} dP \\ d\theta \\ d\mu \\ dv \\ dw \end{bmatrix}$$

The determinant of the Hessian matrix reads as

$$D^{PM} = -[w^2 U_{MM} + U_{LL}] \times [\nu^2 \mu^2 P\theta A_L^2 (f_{LA}^2 - f_{AA} f_{LL}) - \mu\nu^2 P\theta f_{LL} f_A A_{LL}] < 0$$

Since the production function is concave, the matrix of second derivatives with respect to both inputs in the production function is negative semidefinite at every point (Varian, 1992, p. 496). Hence, $f_{AA} f_{LL} > f_{LA}^2$ and thus the determinant is (strictly) negative.

Comparative status results are obtained with Cramers rule. For that purpose, (1) replace the concerning column of the Hessian matrix with the relevant column of the RHS matrix, (2) determine the determinant of the 'new' matrix, and (3) determine the comparative statics by dividing the 'new' determinant by the 'true' determinant of the Hessian matrix $D^{PM}$. For example, consider the case of $dL^D/d\theta$. For this purpose replace the *first* column of the Hessian by the *second* column of the RHS matrix.

The comparative statics are as follows

$$\mathrm{Sign}\left(\frac{dA}{dP}\right) = \mathrm{Sign}(-\nu\theta f_L[w^2 U_{MM} + U_{LL}] \times [\nu\mu f_{LA} A_L - \nu^2 f_{LL}]) > 0$$

$$\mathrm{Sign}\left(\frac{dA}{d\theta}\right) = \mathrm{Sign}(-\nu P f_L[w^2 U_{MM} + U_{LL}] \times [\nu\mu f_{LA} A_L - \nu^2 f_{LL}]) > 0$$

$$\mathrm{Sign}\left(\frac{dA}{d\mu}\right) = \mathrm{Sign}\left(-\nu^2 P\theta f_{LL} A_L f_A\left[1 + \epsilon_{AL} - \frac{\mu A f_{AL}^2}{A_L f_A f_{LL}}\right]\right)$$

where $\epsilon_{AL} = \dfrac{\mu A_L L^C f_{AA}}{f_A} < 0$, and

$$\mathrm{Sign}\left(\frac{dA}{d\nu}\right) = \mathrm{Sign}(\nu\mu P\theta f_{AL} A_L f_L) > 0$$

$$\mathrm{Sign}\left(\frac{dA}{dw}\right) = \mathrm{Sign}([w^2 U_{MM} + U_{LL}] \times [\nu\mu f_{LA} A_L - \nu^2 f_{LL}]) < 0$$

**Appendix 2**

The system for the missing labour market case is as follows

$$\begin{bmatrix} \nu^2 P\theta U_M f_{LL} + U_{LL} + \nu^2 P^2\theta^2 U_{MM} f_L^2 & \nu\mu P^2\theta^2 f_A f_L A_L U_{MM} + \mu P\theta f_{LA} A_L U_M + U_{LL} \\ \nu\mu f_{AL} A_L - \nu^2 f_{LL} & \mu f_A A_{LL} + \mu^2 A_L^2 f_{AA} - \nu\mu f_{AL} A_L \end{bmatrix} \times \begin{bmatrix} dL^D \\ dL^C \end{bmatrix} =$$

$$\begin{bmatrix} -\nu\theta f_L U_M [1+\eta_D] & -\nu P f_L U_M [1+\eta_D] & -\nu P^2\theta^2 f_L f_A A U_{MM} - P\theta f_{AL} A U_M & -P\theta f_L U_M [1 + \eta_L + \epsilon_L] \\ 0 & 0 & -f_A A_L\left[1+\epsilon_A - \dfrac{\nu A f_{AL}}{f_A A_L}\right] & f_L\left[1+\epsilon_L - \dfrac{\mu f_{AL} A_L L^D}{f_L}\right] \end{bmatrix} \times \begin{bmatrix} dP \\ d\theta \\ d\mu \\ d\nu \end{bmatrix}$$

The determinant reads as

$$D^{MM} = -\nu\mu A_L f_{LA} U_{LL} - \mu\nu^3 P^2\theta^2 f_L^2 U_{MM} A_L f_{LA} + \mu^2\nu^2 A_L^2 P\theta U_M [f_{AA} f_{LL} - f_{AL}^2] > 0$$

The comparative statics are as follows

$$\frac{dA}{d\theta} = (1 / D^{MM}) \times (-\nu P f_L U_M [1 + \eta_M])$$

where $\eta_M = \dfrac{P\theta f U_{MM}}{U_M} < 0.$

$$\frac{dA}{d\mu} = \frac{1}{D^{MM}} \begin{pmatrix} -[U_{LL} f_A A_L + \nu^2 P^2\theta^2 f_A A_L U_{MM} f_L^2] \times \left(1 + \epsilon_A - \dfrac{\nu A f_{AL}}{f_A A_L}\right) \\ -\nu^2 P\theta U_M f_{LL} f_A A_L (1 + \eta_{ML} + \epsilon_A) \\ + \nu^3 P\theta A_L f_{AL} U_M \left(\dfrac{\mu f_{AL} A}{\nu f_L} + \eta_{MA}\right) \end{pmatrix},$$

$$\frac{dA}{d\nu} = \frac{1}{D^{MM}} \begin{pmatrix} [\nu^2 P\theta f_L U_M f_{LL} U_{LL} + U_{LL} f_L + \nu^2 P^2\theta^2 f_L^3 U_{MM}] \times [1 + \epsilon_L] + \\ P\theta f_L U_M [\nu\mu A_L f_{AL} - \nu^2 f_{LL}] \times [1 + \epsilon_L + \eta_{ML}] \end{pmatrix}$$

where $\eta_{ML} = \dfrac{P\theta\nu f_L U_{MM}}{U_M}$, or the change in marginal utility (of consuming goods) resulting from a marginal increase in labour allocated to agricultural production.