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Stone Age Equilibrium*

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Abstract

We introduce the notion of a stone age equilibrium to study societies in which property rights are absent, bilateral exchange is either coercive or voluntary, and relative strength governs power relations in coercive exchange. We stress the importance of free disposal of goods which allows for excess holdings larger than consumption, thereby modelling the power to withhold goods from others. Under complete, transitive, continuous and strictly-convex preferences, stone age equilibria exist. The maximum of the lexicographic welfare function in which agents are ranked by descending strength always corresponds to a stone age equilibrium. Every stone age equilibrium is weakly Pareto efficient.

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So Isaac left that place and encamped in the valley of Gerar, and stayed there. Then Isaac's slaves dug in the valley and found a spring of running water, but the shepherds of Gerar quarreled with Isaac's shepherds, claiming the water as theirs. He called the well Esek, because they made difficulties for him. His men then dug another well, but the others quarreled with him over that also, so he called it Sitnah. He moved on from there and dug another well, but there was no quarrel over that one, so he called it Rehoboth, saying, 'Now the Lord has given us plenty of room and we shall be fruitful in the land.'

Genesis 26, verse 17,19-22, The New English Bible (1970)

1 Introduction

Economics is about scarcity and conflict. It analyses institutions to mitigate scarcity and to settle conflicts. The aim of this paper is to broaden the Walrasian idea of exchange through voluntary exchange and to include coercive exchange based on power relations. Coercive exchange is more fundamental than voluntary exchange. The former can be envisaged without established institutions, the latter requires generally acknowledged property rights.

This paper develops the notion of a *stone age equilibrium*, an equilibrium concept for an economy with voluntary as well as coercive exchange. An allocation is a stone age equilibrium if no agent can force a preferred exchange and no pair of agents prefers a voluntary exchange. Our work is inspired by but goes beyond work by Piccione and Rubinstein (2007) who introduce the notion of a jungle equilibrium, an equilibrium that reflects coercive exchange only. We provide a general unifying framework where the jungle equilibrium and the Walrasian equilibrium emerge as special cases. In fact the jungle and the Walrasian economy both adopt extreme assumptions about the power relations in the economy. On one side Piccione and Rubinstein's jungle is extreme because a stronger agent has complete power over a weaker agent and can take whatever he wants. The Walrasian economy is the other extreme as no agent has any coercive power over any other agent and all exchange is strictly voluntary.

To fix ideas, consider a society in anarchy where no social rules exist and power is related to relative strength. Agents meet each other bilaterally and exchange their holdings. Exchange can be either coercive or voluntary depending upon holdings and relative strength. During an encounter, the stronger agent may take away some goods from the weaker without consent. We call this coercive exchange. Coercive exchange is subject to limits of taking which are specified later in Section 2.3. Furthermore, agents may voluntarily trade. The economy is in a stone age equilibrium if no one prefers to take from another weaker agent and there are no bilateral gains from trade. The latter means that for every two-person economy with initial endowments $\omega^i = z^i$ and $\omega^j = z^j$ individual holdings (z^i, z^j) are on the contract curve of this economy and, consequently, that the lens of individually rational allocations with respect to (z^i, z^j) in the two-person Edgeworth box is empty.

This paper is not the first to notice that coercive exchange is an important economic activity. An early model of coercive exchange has been developed by Haavelmo (1954) noticing that economic development may be governed just as much by "grabbing" as by voluntary exchange. Haavelmo's model, it seems, was largely ignored and did not impact later work.[1] Only with the rising interest in institutional economics models of anarchy have received some attention. Bush (1972) and Bush and Mayer (1974) developed a model of anarchy where agents allocate their resources to production or arms. In this model an equilibrium establishes "proto" property rights which form the basis of a constitutional contract; cf. Buchanan (1975). Skaperdas (1992) examines a similar model to spell out the conditions for (partial) cooperation. More recent work by Bös and Kolmar (2003), Muthoo (2004) and Hafer (2006) and others has modified and extended this approach using a repeated game framework. Another strand of literature explores models of conflict not to explain the emergence of property rights but shifts in property rights. Examples are Grossman (1995) who studies a model where redistribution prevents crime and Baker (2003) and Ansink and

[1] Haavelmo's work has not been cited in the literature on anarchy and contests with the exception of the survey paper by Garfinkel and Skaperdas (2007).

Weikard (2009) who examine land tenure and water rights contests, respectively.

Bowles and Gintis (2000) note that costless enforcement of agreements such as voluntary exchange is one of the more implausible assumptions of the Walrasian account of exchange. They conclude that power relations matter for understanding the economic process, institutions (why capital hires labour), and resulting allocations. Our notion of a stone age equilibrium brings together the idea of equilibrium in conflict with the idea of a Walrasian general equilibrium. An observed allocation (provided it is an equilibrium) is not just explained as an exchange equilibrium but it is an exchange equilibrium that reflects underlying power relations. Although there is now a body of work on interaction in anarchy which helps to better understand the impact of power relations on resource and income distributions, the connection between general (Walrasian) equilibrium and interaction in anarchy has not been studied. Our notion of a stone age equilibrium is trying to establish this missing link.

The next section provides a formal account of the stone age economy, describes the agents and the limits of taking and defines the stone age equilibrium. Section 3 provides various examples. Section 4 proves the existence of a stone age equilibrium. Section 5 concludes.

2 The stone age economy

2.1 Agents, resources, free disposal and preferences

A stone age economy consists of n agents, numbered $i = 1, \ldots, n$, $n \geq 2$, m goods, numbered $k = 1, \ldots, m$, $m \geq 1$, and the economy's total endowments $\omega \in \mathbb{R}^m_{++}$. Unlike in the classical exchange economy there are no individual entitlements. The set of all agents is $N = \{1, \ldots, n\}$. An allocation $z = (z^0, z^1, \ldots, z^n)$ assigns a bundle z^i to each agent i, $i \in N$, and $z^0 \in \mathbb{R}^m_+$ expresses the endowments not allocated to any agent. We will refer to z^i also as agent i's holdings. An allocation is called feasible if $z^0 + \sum_{i \in N} z^i = \omega$, where z^0 indicates the slack available in the economy.

At this point, it is important to realize that an individual agent is not forced to consume all his possessions and may voluntarily dispose or waste some of the resources available to him. This property is called *free disposal*. For example, a person who successfully defends a small well may consume only a limited amount of the well's capacity without letting others access the water.[2] So, we must distinguish between agent i's possessions z^i and this agent's consumption $x^i \in \mathbb{R}^m_+$. Free disposal then means that $x^i \leq z^i$. Of course, monotonic preferences imply $x^i = z^i$ meaning no-spillage is optimal, but such preferences assume away some interesting issues such as satiation and the power to withhold goods from other agents, which we discuss further in Section 3. Similarly, $z^0 \geq 0$ defines free disposal at the level of the economy.

Allowing for free disposal implies that agent i's preference relation over holdings on $\mathbb{R}^m_+$ that compares pairs of holdings z^i and $\hat{z}^i$ must be distinguished from the preference relation over consumption bundles on $\mathbb{R}^m_+$ that compares pairs of consumption x^i and $\hat{x}^i$. Of course, these preference relations are related and should be defined in a consistent manner. We take the preference relation over consumption, denoted $\succsim_i^C$, as the primitive of the stone age economy and use it to define the preference relation over holdings, denoted $\succsim_i$. We assume that $\succsim_i^C$ is complete, transitive, continuous and strictly convex. Note that we do not impose monotonicity and, thus, allow for satiation. Given the holdings $z^i \in \mathbb{R}^m_+$, agent i's consumption x^i is a best or maximal element of $\succsim_i^C$ on the subset $x^i \leq z^i$. For each z^i, the assumptions on $\succsim_i^C$ guarantee a unique maximal element that we denote as $M^i(z^i)$, which is a continuous function of z^i. The function $M^i\left(z^i\right)$ defines agent i's preference relation $\succsim_i$ on $\mathbb{R}^m_+$ over holdings: Whenever agent i compares the pair of holdings z^i and $\hat{z}^i$, he actually considers the maximal consumption bundle attainable from z^i to the maximal consumption bundle from $\hat{z}^i$. Formally, $z^i \succsim_i \hat{z}^i$ if and only if $M^i\left(z^i\right) \succsim_i^C M^i\left(\hat{z}^i\right)$. In case $\succsim_i^C$ is also monotonic, then $M^i\left(z^i\right) = z^i$ for all $z^i \in \mathbb{R}^m_+$ and, therefore, $\succsim_i$ coincides with $\succsim_i^C$. We have the following properties for $\succsim_i$.

Lemma 1 *For each agent i, the preference relation $\succsim_i$ is complete, transitive and continuous.*

[2] Access to freshwater always has been a matter of great controversy over mankind's history in arid areas.

Proof. Since $M : \mathbb{R}_+^m \to \mathbb{R}_+^m$ is a function, any two z^i and $\hat{z}^i$ in $\mathbb{R}_+^m$ can be compared through $\succsim_i^C$, which implies $\succsim_i$ is complete. Furthermore, $z^i \succsim_i \hat{z}^i \succsim_i \tilde{z}^i$ if and only if $M^i(z^i) \succsim_i^C M^i(\hat{z}^i) \succsim_i^C M^i(\tilde{z}^i)$, which implies $\succsim_i$ is transitive. Next, consider the convergent sequence $\{z^i(k)\}_{k=1}^{\infty}$ such that $z^i(k) \to \bar{z}^i$ as $k \to \infty$. Then, $\succsim_i$ is said to be continuous if $z^i(k) \succsim_i \hat{z}^i$ for all k implies that $\bar{z}^i \succsim_i \hat{z}^i$. Suppose $z^i(k) \succsim_i \hat{z}^i$ for all k. By definition, $z^i(k) \succsim_i \hat{z}^i$ if and only if $M^i(z^i(k)) \succsim_i^C M^i(\hat{z}^i)$. Since the function M is continuous, we have that $M^i(z^i(k)) \to M^i(\bar{z}^i)$ as $k \to \infty$. Then, by continuity of $\succsim_i^C$, $M^i(z^i(k)) \succsim_i^C M^i(\hat{z}^i)$ for all k implies $M^i(\bar{z}^i) \succsim_i^C M^i(\hat{z}^i)$. Then also, $\bar{z}^i \succsim_i \hat{z}^i$, which establishes continuity. ■

2.2 Relative strength

Exchange in the stone age economy is either voluntary or coercive. Coercive exchange is driven by an allocation of goods representing control over resources, by the agents' preferences and, most importantly, by the relative strength between any pair of agents. Coercive exchange means that one agent gains control of resources from another agent without consent of the latter. We model power relations to capture the idea that relative strength affects the ability to take goods from others. The larger the difference in strength, the easier it is for the stronger to take goods away from the weaker.

To quantify relative strength, we introduce a vector $s = (s_1, \ldots, s_n) \in \mathbb{R}_{++}^n$, $\sum_{i \in N} s_i = 1$, with the interpretation that agent i is stronger than agent j if $s_i > s_j$. Players i and j are equally strong if $s_i = s_j$. Without loss of generality we may renumber the agents such that $s_1 \geq s_2 \geq \ldots \geq s_n$, which we do from here on. In what follows, we think of agent i as the stronger agent in the arbitrary pair of agents i and j meaning $j \geq i$ or $s_i \geq s_j$. Similarly, $i \leq j \leq k$ where $k \in N$ is a third agent. Vector s defines relative strength coefficients $s_{ij} = \frac{s_i}{s_j}$ for any pair of agents and our normalization implies $s_{ij} \geq 1$. Then, $s_{ji} = s_{ij}^{-1} \leq 1$ and, in particular, $s_{ii} = 1$. Furthermore, any multiplication $\lambda s = (\lambda s_1, \ldots, \lambda s_n)$, $\lambda > 0$, produces the same relative coefficients s_{ij}, which justifies our normalization of s to the unit simplex. Finally, it also implies some universal measure of strength such that the relative strength measure between i and k is related to their relative strength measures with any third agent in a multiplicative manner: $s_{ik} = s_{ij}s_{jk} \geq 1$, meaning relative strength induces a transitive relation among the agents in N. Then, the matrix $S = [s_{ij}]$ of all relative strength coefficients has diagonal elements equal to 1, upper-triangular elements that are greater than or equal to 1 and lower-triangular elements smaller than or equal to 1.

Relative strength is employed in specifying the set of consumption bundles the stronger agent can take away from the weaker agent. Before we discuss these issues, we first show how relative strength is more general than the notion of power in the jungle economy in Piccione and Rubinstein (2007).

Remark 2 *In the jungle economy in Piccione and Rubinstein (2007), the possibility of two or more equally strong agents is ruled out, while our approach can easily accommodate this situation when* $s_{i,i+1} = 1$*. Moreover, the power structure in the jungle equilibrium is extreme: The strongest in any set of agents has all the power. Such absolute power can be obtained as a special limit case of our specification. To see this, let* $\varepsilon > 0$ *be small and consider the vector* $s(\varepsilon)$ *with* $s_i(\varepsilon) = \varepsilon^{i-1} - \varepsilon^i$, $1 \leq i \leq n$, *and* $s_n(\varepsilon) = \varepsilon^{n-1}$*. So,* $s_i(\varepsilon) > s_{i+1}(\varepsilon) > 0$ *and* $\sum_{i=1}^{n} s_i(\varepsilon) = 1$*. Then we obtain the relative strength coefficients* $s_{i,i+1}(\varepsilon)$, $1 \leq i \leq n-1$, *and its limit as* ε *tends to* 0

$$s_{i,i+1}(\varepsilon) = \frac{1}{\varepsilon} \to \infty, \qquad 1 \leq i \leq n-1.$$

Taking this particular limit implies that the relative strength $s_{i+1}(\varepsilon)$ *of agent* $i+1$ *goes faster to* 0 *than the relative strength* $s_i(\varepsilon)$ *of the adjacent relative stronger agent* i*. If* i *is weaker than* j $(i > j)$*, then in the limit agent* j*'s relative strength is infinitely many times larger than* i*'s.*

2.3 Coercive exchange

In the stone age economy, we picture agents roaming around as individuals and occasionally encountering each other. During such bilateral encounter, it is natural to assume that there are limits in what one agent can take from the other agent since taking from another agent is usually not a costless event in that it involves preparation and utilization of goods (e.g. weapons).

We combine two robust ideas: First, an agent cannot take anything from a stronger or an equally strong agent. So, when agents i and j meet (recall $s_{ij} \geq 1$) agent j cannot take from i. Second, the bundle of takings $y^{ij} \in \mathbb{R}_+^m$ agent i can expropriate from the weaker agent j depends, of course, upon the initial holdings z^j and upon the relative strength $s_{ij} \geq 1$. A larger relative strength has a nondecreasing influence upon the maximum takings from a weaker agent. The set of feasible takings is defined by the correspondence $\Phi : \mathbb{R}_+^{m+1} \to \mathbb{R}_+^m$ that satisfies

$$0 \in \Phi\left(z^j, s_{ij}\right) \subseteq \left\{ y^{ij} \in \mathbb{R}_+^m \middle| \, y^{ij} \leq z^j \right\}$$

We assume that Φ is non-decreasing in z^j and $s_{ij} \geq 1$.

Recall that the distribution of strength is exogenously given and, in particular, independent of the holdings. We are aware that in a more general set up components of holdings z^i, z^j might represent tools like axes or knifes for hunting that can also be employed as weapons that ease either takings or defence. Thus, relative strength s_{ij} would depend, in such setting, on both z^i and z^j.

In our setting, instead of motivating Φ from the perspective of taking, we might as well consider the set of bundles $\Theta\left(z^j, s_{ij}\right)$ that agent j is able to defend against agent i. Then, it is natural to assume $0 \in \Theta\left(z^j, s_{ij}\right) \subseteq \left\{ \hat{z}^j \in \mathbb{R}_+^m \middle| \, \hat{z}^j \leq z^j \right\}$ and to write $\Phi\left(z^j, s_{ij}\right) = \left\{ y^{ij} \in \mathbb{R}_+^m \middle| \, y^{ij} \leq z^j \right\} \backslash \Theta\left(z^j, s_{ij}\right)$. Hence, Φ can be specified to reflect the power of every agent to take everything that cannot be defended. As we assume that Φ is non-decreasing in s_{ij} it holds that some bundle $z_{\min}^j \equiv \Theta\left(z^j, s_{ij}\right)$ is a minimum set $D^j \subset \mathbb{R}_+^m$ that j can defend against every other agent.

Before continuing, we first relate the stone age economy to the barter economy and the jungle economy.

Remark 3 *A barter economy has fully secure property rights and bilateral exchange in quan-*

tities. It can be obtained as the special case, first, when all agents have equal strength, that is if $s_{ij} = 1$ for all $i, j \in N$. Then, every agent is strong enough to defend all his holdings against any other agent such that $z^j \in \Theta(z^j, s_{ij}) = \left\{ \hat{z}^j \in \mathbb{R}^m_+ \middle| \hat{z}^j \leq z^j \right\}$ for all $i, j \in N$. By definition, $\Phi(z^j, s_{ij}) = \{0\}$ for all $i, j \in N$.

Remark 4 *The jungle economy in Piccione and Rubinstein (2007) can be obtained as a special case. First, preferences in the jungle economy are monotonic and, therefore, there is no need to distinguish consumption x^i from holdings z^i since these always coincide in the jungle equilibrium, i.e., $x^i = z^i$. So, under monotonic preferences $\succsim_i$ is equivalent to $\succsim_i^C$. The absence of free disposal in the Jungle Economy, however, implicitly rules out the strategic withholding of resources from other agents without consuming. Second, in the jungle economy agent i's consumption is restricted by the consumption set $X^i \subseteq \mathbb{R}^m_+$, which can also be interpreted as boundaries to individual holdings. For $z^i \in X^i$, the jungle economy then corresponds to the special case $\Phi(z^j, s_{ij}) = \left\{ y^{ij} \in \mathbb{R}^m_+ \middle| y^{ij} \leq z^j, z^i + y^{ij} \in X^i \right\}$ whenever $s_{ij} > 1$. This means only a slight difference in strength between two agents suffices for agent i to take whatever he wants from the weaker agent j, i.e. the weaker agent cannot defend any of her holdings whenever $s_{ij} > 1$.*

2.4 Stone Age Equilibrium

In this subsection, we introduce the stone age economy and the stone age equilibrium.

The stone age economy is implicitly defined in the previous three subsections. For ease of exposition, we summarize this economy in the following definition.

Definition 5 *The stone age economy is defined by the set of agents N, the vector of resources $\omega \in \mathbb{R}^m_{++}$, each agent's preferences $\succsim_i$, $i \in N$, over holdings, the normalized vector of strength coefficients $s \in \mathbb{R}^n_+$, and for each pair of agents a set of feasible takings $\Phi(z^j, s_{ij})$ that agent i can take from agent j.*

The stone age economy is an exchange economy where every interaction between agents is confined to bilateral exchange. Hence, we envisage that two individuals with different holdings, strengths, and preferences will meet. Such encounter may lead to coercive or voluntary exchange or both.

Voluntary exchange is based upon the economic principle of quid pro quo that excludes gifts. A gift is a valuable good given to another agent without an explicit agreement to receive something in return. Excluding or allowing gifts matters in case a satiated agent with excess holdings meets another agent who would prefer the excess holdings but, since the other is satiated, has nothing to offer in return. Quid pro quo[3] implies that in this case the agents fail to exchange altogether, whereas allowing for gifts would imply that excess holdings are simply given as a gift. In terms of defining voluntary exchange, we interpret quid pro quo as corresponding to bilateral exchange that makes both agents strictly better off, whereas gifts correspond to bilateral exchange that makes at least one agent strictly better off and does not make the other agent worse off. Our definition of voluntary trade in a stone age equilibrium is based upon quid pro quo because we think that it better fits a society based upon power relations where holdings may be taken from you and nothing is given away for free. In terms of Pareto improving exchange, only voluntary exchanges that are weakly Pareto improving are considered under quid pro quo.[4] In Section 3.5, we briefly discuss the case of allowing for gifts.

Having defined the stone age economy, we can now define its equilibrium concept. The stone age economy is in equilibrium if no pair of agents prefers further exchange. This concept combines both coercive and voluntary exchange into one equilibrium concept.

Definition 6 *A stone age equilibrium is a feasible allocation $z = (z^0, z^1, \ldots, z^n)$, $z^i \in \mathbb{R}^m_+$, for the stone age economy such that*

1. *Coercive exchange: There does not exist an alternative bundle $z^i + y^{i0} + y^{ij} \succ_i z^i$ such that $y^{i0} \leq z^0$ and $y^{ij} \in \Phi(z^j, s_{ij})$.*

[3] Quid pro quo is Latin for the English expression that a good turn deserves another.

[4] Weak Pareto efficiency of allocation z is defined as: there does not exist a $\hat{z}$ such that for all $i = 1, \ldots, n$ it holds that $\hat{z} \succ_i^C z$. Strong Pareto efficiency has $\succeq_i^C$ instead of $\succ_i^C$ with at least one strict preference. Strong Pareto efficiency implies weak Pareto efficiency, but not the other way around.

2. *Voluntary exchange: There does not exist a bilateral reallocation* $(\tilde{z}^i, \tilde{z}^j)$ *such that* $\tilde{z}^i \succ_i z^i$ *and* $\tilde{z}_i^j \succ_j z^j$ *satisfying* $\tilde{z}^i + \tilde{z}^j \leq z^i + z^j$.

3. *Utility maximization under free disposal: Agent i's consumption* x^i *is a maximal element of* $\succsim_i^C$ *on the subset* $x^i \leq z^i$.

Before we analyze the equilibrium we argue that the stone age equilibrium generalizes both the Walrasian and the jungle equilibrium in Piccione and Rubinstein (2007).

Remark 7 *A Walrasian equilibrium* $(p, x^1, \ldots, x^n)$ *emerges as a special case when* $\Phi(z^j, s_{ij}) = \{0\}$ *for all* $i, j \in N$, *and agents exchange on markets against uniform market prices. This happens if agents have equal strength, that is if* $s_{ij} = 1$, *or if every agent is strong enough to defend all his holdings against any other agent such that* $\Theta(z^j, s_{ij}) = \left\{ \hat{z}^j \in \mathbb{R}_+^m \middle| \hat{z}^j \leq z^j \right\}$ *for all* $i, j \in N$. *Or equivalently,* $\Phi(z^j, s_{ij}) = \{0\}$. *Hence, property rights are fully secure, and only voluntary exchange takes place.*

Remark 8 *The stone age equilibrium extends the jungle equilibrium in Piccione and Rubinstein (2007). To see this, the jungle equilibrium assumes* $\Phi(z^j, s_{ij}) = \{y^{ij} \in \mathbb{R}_+^m | y^{ij} \leq z^j, z^i + y^{ij} \in X^i\}$ *for all* $z^i \in X^i$, $\succsim_i$ *equivalent to* $\succsim_i^C$ *and utility maximizing agents. So, the stone age economy equilibrium allows for voluntary exchange and free disposal, while both activities are excluded from the jungle economy. The conditions for coercive exchange and utility maximization extend the conditions under the jungle equilibrium to accommodate for free disposal.*

3 Stone age equilibrium

In this section, we first establish existence, and then, we characterize the stone age equilibrium.

3.1 Existence

The jungle equilibrium in Piccione and Rubinstein (2007) coincides with the maximum of the lexicographic welfare function, where the order of lexicographic maxima is the order of agents in the jungle economy ranked from strongest to weakest. The next result shows that this lexicographic maximum always is a stone age equilibrium. Since this maximum always exists, we automatically establish the existence of the stone age equilibrium.

Proposition 9 *Let* $(x^1, \ldots, x^n)$, $x^i \in \mathbb{R}_+^m$, *be the maximum of the lexicographic welfare function in which the order of maximizing this preference relation is* $\succsim_1^C, \succsim_2^C, \ldots, \succsim_n^C$, $x^j \leq \omega - \sum_{i=1}^{j-1} x^i$, $j = 1, \ldots, n$, *and let* $x^0 = \omega - \sum_{i=1}^{n} x^i$. *Then,* $z = (x^0, x^1, \ldots, x^n)$ *exists, is unique and is a stone age equilibrium in which* $M^i(z^i) = x^i$ *for all* $i = 1, \ldots, n$. *Hence, every stone age economy has a stone age equilibrium.*

Proof. The proof consists of verifying that the conditions of Definition 6 hold for $z = (x^0, x^1, \ldots, x^n)$. We start with Condition 1. Recall the definition of $M^j(z^j)$ and that M^j is a continuous function. Then, in iteration $j = 1, \ldots, n$, the lexicographic maximum exists, is unique and satisfies $x^j = M^j\left(\omega - \sum_{i=1}^{j-1} x^i\right)$. For any $j \leq n-1$ and $k > j$, we have that

$$x^j + y^{jk} \leq x^j + x^k \leq \omega - \sum_{i=1}^{j-1} x^i$$

and, by weak monotonicity of M^j, this implies

$$x^j = M^j\left(\omega - \sum_{i=1}^{j-1} x^i\right) \succsim_j^C M^j\left(x^j + x^k\right) \succsim_j^C M^j\left(x^j + y^{jk}\right).$$

So, j prefers not to take from agent $k > j$. Similar, for any j, we have that

$$x^j + x^0 \leq \omega - \sum_{i=1}^{j-1} x^i \Longrightarrow x^j = M^j\left(\omega - \sum_{i=1}^{j-1} x^i\right) \succsim_j^C M^j\left(x^j + x^0\right).$$

So, agent i does not prefer to utilize the economy's slack $z^0 = x^0$. Therefore, z satisfies Condition 2. Next, since $z = (x^0, x^1, \ldots, x^n)$ is the maximum of the lexicographic welfare function, it is strongly Pareto efficient. Hence, no strongly (hence, no weakly either) Pareto improving bilateral exchange exists and Condition 2 also holds. Finally, $z^i = x^i$ and $x^i = M^i(x^i)$ implies $x^i = M^i(z^i)$ and, thus, condition 3 holds. To summarize, $z = (x^0, x^1, \ldots, x^n)$ satisfies the conditions of Definition 6 and is a stone age equilibrium. Recall each x^j, $j = 1, \ldots, n$, exists and is unique. Hence, the stone age equilibrium exists. ■

In Piccione and Rubinstein (2007), the jungle equilibrium coincides with the maximum of the lexicographic welfare function. So, although the relative strength between any pair of agents in a jungle economy is extreme, the jungle equilibrium allocation survives under less extreme strength relations, more restrictive correspondences Φ of feasible takings, and the possibility of voluntary exchange. The rationale is that the underlying fundamental asymmetry, namely agent i might take from agent j but never vice versa, is preserved in the stone age economy even in case the difference in relative strength is small, i.e., $s_j \approx s_i$ or even vanishes by taking the limit $s \to \left(\frac{1}{n}, \ldots, \frac{1}{n}\right)$. This result is independent of the correspondence Φ.

3.2 Characterization

In this subsection, we provide a characterization of stone age equilibria.

For the following result, we define agent i's set of coercive takings (T) that are strictly preferred to no-taking as

$$T^i\left(z^i\right) = \left\{y^{ij} \in \mathbb{R}^m_+ | M^i\left(z^i + y^{ij}\right) \succ_i^C M^i\left(z^i\right)\right\}.$$

Proposition 10 *If $z = (z^0, z^1, \ldots, z^n)$ is a stone age equilibrium, then*

1. *for each agent $i \in \{1, \ldots, n\}$ and good $k \in \{1, \ldots, m\}$ such that $M^i_k(z^i) < z^i_k$: agent i is satiated in good k.*

2. *for each good $k \in \{1, \ldots, m\}$ such that $z^0_k > 0$: all agents are satiated in good k.*

3. *for each agent i and agent j, $i, \in \{1, \ldots, n\}$: $T^i(z^i) \cap \Phi(z^j, s_{ij}) = \emptyset$.*

4. *for each pair i and i', $i, i' \in \{1, \ldots, n\}$: $(M^i(z^i), M^{i'}(z^{i'}))$ is weakly Pareto efficient in a two-agent barter economy with total endowments $z^i + z^{i'}$ and preference relations $\succsim_i^C$ and $\succsim_{i'}^C$.*

Proof. First, consider $M_k^i(z^i) < z_k^i$. Since agent i's consumption $M^i(z^i)$ is the maximal element of $\succsim_i^C$, this agent prefers to dispose $z_k^i - M_k^i(z^i) > 0$ units of good k. Hence, agent i is satiated in good k.

Second, consider the slack $z_k^0 > 0$ for good k. For $0 < \varepsilon \leq z_k^0$, consider agent i's increased holdings $\hat{z}^i = z^i + \varepsilon e_k \geq z^i$, where e_k denotes the k-th unit vector. Since $\hat{z}^i \geq z^i \geq M^i(z^i)$, the bundle $M^i(z^i)$ is also feasible under $\hat{z}^i$ and, therefore, $M^i(\hat{z}^i) \succsim_i^C M^i(z^i)$. Then, by definition of $\succsim_i$ and z is a stone age equilibrium, we also have that

$$z^i \succsim_i \hat{z}^i \Longleftrightarrow M^i\left(z^i\right) \succsim_i^C M^i\left(\hat{z}^i\right).$$

So, $M^i(\hat{z}^i) \sim_i^C M^i(z^i)$. Since the only difference between $\hat{z}^i$ and z^i is the increased amount of good k, agent i is indifferent between whether or not to have holdings over this extra amount of good k. Hence, agent i is satiated in good k. This must hold for all agents i.

Third, consider a bilateral encounter by agents i and j. z is a stone age equilibrium implies $z^i \succsim_i z^i + y^{ij}$ for all $y^{ij} \in \Phi(z^j, s_{ij})$. So, $y^{ij} \notin T^i(z^i)$. Hence, the intersection of $\Phi(z^j, s_{ij})$ and $y^{ij} \in T^i(z^i)$ must be empty.

Fourth, by definition of voluntary exchange: There does not exist a bilateral reallocation $(\tilde{z}^i, \tilde{z}^{i'})$ such that $\tilde{z}^i \succ_i z^i$ and $\tilde{z}^{i'} \succ_{i'} z^{i'}$ satisfying $\tilde{z}^i + \tilde{z}^{i'} \leq z^i + z^{i'}$. So, $(z^i, z^{i'})$ is weakly Pareto efficient in the two-agent exchange economy with total endowments $z^i + z^{i'}$ and preference relations $\succsim_i$ and $\succsim_j$. Next, $\tilde{z}^i \succ_i z^i$ and $\tilde{z}_{i'}^{i'} \succ z^{i'}$ translate as $M^i(\tilde{z}^i) \succ_i^C M^i(z^i)$, respectively, $M^{i'}(\tilde{z}^{i'}) \succ_{i'}^C M^{i'}(z^{i'})$. In other words, there does not exist a bilateral reallocation $(\tilde{z}^i, \tilde{z}^{i'})$ such that $M^i(\tilde{z}^i) \succ_i^C M^i(z^i)$ and $M^{i'}(\tilde{z}^{i'}) \succ_j^C M^{i'}(z^{i'})$ satisfying $\tilde{z}^i + \tilde{z}^{i'} \leq z^i + z^{i'}$. So, $(M^i(z^i), M^{i'}(z^{i'}))$ is weakly Pareto efficient in the two-agent exchange economy with total endowments $z^i + z^{i'}$ and preference relations $\succsim_i^C$ and $\succsim_{i'}^C$. ■

In case of monotonic preference relations $\succsim_i^C$, we obtain sharper results. Since monotonic preferences rule out satiation, it is impossible to have unallocated goods, i.e., $z^0 = 0$. Furthermore, every increase in at least one of the commodities improves agent i's utility and, therefore, $T^i(z^i)$ is equal to $\mathbb{R}^m_+ \setminus \{0\}$. Condition 2 simplifies to agent i failing positive net expansions at allocation z, i.e., $\Phi(z^j, s_{ij}) = \{0\}$. Finally, condition 3 reduces to $(z^i, z^{i'})$ lies on the contract curve of a two-agent barter economy with total endowments $z^i + z^{i'}$ and preference relations $\succsim_i^C$ and $\succsim_{i'}^C$.

Corollary 11 *If all preference relations are monotonic, then $z = (z^0, z^1, \ldots, z^n)$ is a stone age equilibrium if and only if $z^0 = 0$, $\Phi(z^j, s_{ij}) = \{0\}$, and for each pair i and i', $i, i' \in \{1, \ldots, n\}$: $(z^i, z^{i'})$ is weakly Pareto efficient in a two-agent barter economy with total endowments $z^i + z^{i'}$ and preference relations $\succsim_i^C$ and $\succsim_{i'}^C$.*

We conclude this section with examples. The first examples shows that, depending upon the parameter values, the set of stone age equilibria can either coincide with the entire contract curve of a two-agent barter economy or the jungle equilibrium. The second example shows that intermediate results can also be obtained.

Example 12 *Consider a two-agent stone age economy with two goods, monotonic preferences, and resources ω. Let*

$$\Phi\left(z^2, s_{12}\right) = \left\{ y^{12} \in \mathbb{R}^2_+ \middle| \, y^{12} \leq c(s_{12}) \cdot z^2 \right\},$$

where $c(s_{12}) = \max\{0, a - \frac{1}{s_{12}}\}$ and $a < 1$. Then, $\Phi(z^2, s_{12}) = \{0\}$ whenever $s_{12} \leq a^{-1}$ and otherwise $\Phi(z^2, s_{12})$ is a rectangular with lower-left corner 0 and upper-right corner the fraction $c(s_{12})$ of z^2. In case $s_{12} \leq a^{-1}$, direct application of our results for monotonic preferences implies that $z^0 = 0$. $\Phi(z^2, s_{12}) = \{0\}$ implies that there will be no coercive exchange and that the set of stone age equilibria is fully determined by condition 3. This yields the entire contract curve. In the remaining case, $s_{12} > a^{-1}$, $\Phi(z^2, s_{12}) \neq \{0\}$ with the exception of $z^2 = 0$. So, only $z^1 = \omega$ and $z^2 = 0$ can be part of a stone age equilibrium. In *this case, the stone age equilibrium coincides with the lexicographic maximum, i.e. the jungle equilibrium.*

Example 13 *Consider a three-agent stone age economy with two goods, monotonic preferences, $s_1 : s_2 : s_3 = 3 : 2 : 1$ and resources ω. Let*

$$\Phi\left(z^j, s_{ij}\right) = \left\{ y^{ij} \in \mathbb{R}^2_+ \middle| \, y^{ij} \leq c\left(s_{ij}\right) \cdot z^j \right\},$$

where $c\left(s_{ij}\right) = \max\{0, \frac{1}{2} - \frac{1}{s_{ij}}\}$. Then, $\Phi\left(z^2, s_{12}\right) = \Phi\left(z^3, s_{23}\right) = \{0\}$ and $\Phi\left(z^3, s_{13}\right) = \left\{ y^{ij} \in \mathbb{R}^2_+ \middle| \, y^{ij} \leq \frac{1}{6} z^j \right\}$. Direct application of Corollary 11 implies: $z^0 = 0$; $\Phi\left(z^2, s_{12}\right) = \{0\}$ rules out coercive trade between agents 2 and 3; $\Phi\left(z^3, s_{23}\right) = \{0\}$ rules out coercive trade between 2 and 3; and $\Phi\left(z^3, s_{13}\right) \neq \{0\}$ unless $z^3 = 0$ combined with monotonic preferences for agent 1 implies $z^3 = 0$. The traditional contract curve for an exchange economy with agents 1 and 2 and total endowments ω determines the set of stone age equilibria. Denote the latter contract curve as $C\left(\omega\right)$, then $z = \left(0, z^1, z^2, 0\right)$, $\left(z^1, z^2\right) \in C\left(\omega\right)$, is a stone age equilibrium. The stone age equilibrium $z = \left(0, \omega, 0, 0\right)$ coincides with the lexicographic maximum, i.e. the jungle equilibrium. This example illustrates that coercive exchange may reduce the set of weakly Pareto efficient allocations. It also shows that stone age equilibria may be non-unique.

3.3 Pareto efficiency

An important issue is Pareto efficiency of stone age equilibria. In this subsection we address this issue.

Recall that voluntary trade based upon quid pro quo is Pareto improving in the weak Pareto sense. For that reason, we investigate weakly Pareto efficiency of stone age equilibria and find the following affirmative answer.

Proposition 14 *A stone age equilibrium $z = \left(z^0, z^1, \ldots, z^n\right)$ is weakly Pareto efficient.*

Proof. Suppose the stone age equilibrium $z = \left(z^0, z^1, \ldots, z^n\right)$ is not weakly Pareto efficient. Although $z = \left(z^0, z^1, \ldots, z^n\right)$ is pairwise weakly Pareto efficient there exists, by assumption, an allocation $\hat{z} = (\hat{z}^0, \hat{z}^1, \ldots, \hat{z}^n)$ such that $M^i(\hat{z}^i) \succ_i^C M^i(z^i)$ for all agents $i = 1, \ldots, n$. Then, $M^i(\hat{z}^i) \succ_i^C M^i(z^i)$ implies $M^i(\hat{z}^i) \neq M^i(z^i)$. Since z is pairwise weakly Pareto efficient, no pair i and i', $i, i' \in \{1, \ldots, n\}$, can realize $(\tilde{z}^i, \tilde{z}^{i'}) = (M^i(\hat{z}^i), M^{i'}(\hat{z}^{i'}))$ in their pair. Note that $(\tilde{z}^i, \tilde{z}^{i'})$ is the most efficient way of allocating goods in order to achieve $(M^i(\hat{z}^i), M^{i'}(\hat{z}^{i'}))$. Therefore, it cannot hold that $\tilde{z}^i + \tilde{z}^{i'} \leq z^i + z^{i'}$ for any pair i and i'. Summing over all $i, i' \in \{1, \ldots, n\}$ such that $i' > i$ implies that

$$\text{Not} \sum_{i,i'=1,i'>i}^{n} \left(\tilde{z}^i + \tilde{z}^{i'}\right) \leq \sum_{i,i'=1,i'>i}^{n} \left(z^i + z^{i'}\right) \Longleftrightarrow \text{Not } \tilde{z}^0 \geq z^0,$$

where the equivalence is due to $\sum_{i,i'=1,i'>i}^{n} \left(z^i + z^{i'}\right) = (n-1) \sum_{i=1}^{n} z^i = (n-1)(\omega - z^0)$ and a similar equality for $\tilde{z}$. Then, for all goods k in z such that $z_k^0 = 0$, $\tilde{z}^0 \geq 0$ implies $\tilde{z}_k^0 \geq z_k^0$. So, not $\tilde{z}^0 \geq z^0$ requires at least one good $\kappa = 1, \ldots, m$ for which $0 \leq \tilde{z}_\kappa^0 < z_\kappa^0$ holds, i.e. good κ is satiated in z. For this good, $\sum_{i=1}^{n} \tilde{z}_\kappa^i = \omega - \tilde{z}_\kappa^0 > \omega - z_\kappa^0 = \sum_{i=1}^{n} z_\kappa^i$ implies that the consumption of the satiated good κ has to increase, a contradiction. So, z must be weakly Pareto efficient. ■

This results states that the pairwise weakly Pareto efficiency in Proposition 10 is enough for weakly Pareto efficiency of the entire economy. A stone age equilibrium may fail strongly Pareto efficiency, as the following example illustrates.

Example 15 *Consider a two-agent stone age economy with m goods, total resources ω, agent 1 is satiated at $\bar{z}^1 < \omega$, and agent 2 has monotonic preferences. For $c > 0$, let*

$$\Phi\left(z^2, s_{12}\right) = \left\{ y^{12} \in \mathbb{R}_+^2 \middle| \, y^{12} \leq c \cdot z^2 \right\}.$$

Then, $\Phi(z^2, s_{12}) \neq \{0\}$ unless $z^2 = 0$. By definition of the satiation point, $T^1(z^1) = \emptyset$ whenever $z^1 \geq \bar{z}^1$. So, for all feasible $z = (z^0, z^1, z^2)$ such that $z^1 \geq \bar{z}^1$, we have that $T^1(z^1) \cap \Phi(z^2, s_{12}) = \emptyset$. Any such z is also robust against voluntary trade. To see this: For $z^1 \geq \bar{z}^1$ and $z^1 \neq \bar{z}^1$, there does not exist any feasible $\hat{z}$ that can make agent 1 strictly better off. So, according to our definition, voluntary exchange is impossible in case agent 1 has excess holdings above his satiation point $\bar{z}^1$ even though giving up some or all of these excess *holdings would make agent* 2 *strictly better off. This means that all* z *such that* $z^1 \geq \bar{z}^1$ *and* $z^1 \neq \bar{z}^1$ *are stone age equilibria in which agent* 1 *withholds goods from agent* 2*. These stone age equilibria are weakly Pareto efficient, but not strongly Pareto efficient.*

3.4 Uniqueness

For monotonic preferences, the necessary and sufficient conditions such that the maximum of the lexicographic welfare function is the unique stone age equilibrium is that for every pair i and j we have $\Phi\left(z^j, s_{ij}\right) \neq \{0\}$ for all $\left(z^i, z^j\right)$ on the pairwise Pareto frontier whenever $z^j \neq 0$. For non-monotonic preferences, $\Phi\left(z^j, s_{ij}\right) \neq \{0\}$ for such $\left(z^i, z^j\right)$ is not sufficient and additionally requires that agent i also prefers at least one such coercive exchange in order to upset such allocation.

In general, stone age economies allow for large sets of stone age equilibria, as Example 13 shows. Some may criticize the stone age equilibrium concept for that, but we think it is rather natural. In the absence of coercive trade, hunter-gatherer economies with voluntary trade can have many equilibria depending upon who finds which quantities of the goods first. So, in terms of the traditional Edgeworth box, the initial endowments could be anything before exchange leads agents to a point on the contract curve. Also the lack of markets with uniform market prices, like in a Walrasian economy, provides more freedom in allocating goods across the economy. Nevertheless, coercive trade generally reduces the number of allocations that can be a stone age equilibrium, but as our results show, this can only be the case when the differences in relative strength are large enough such that a stronger player can take something from any weaker agent. In the extreme case, only lexicographic maximum, i.e., the jungle equilibrium, can survive. In case the difference in strength are relatively small such that agents cannot take from each other, voluntary exchange is the only thing they can rely on.

3.5 Gifts and the stone age equilibrium

The stone age equilibrium assumes voluntary exchange based upon quid pro quo that excludes gifts. In this subsection, we investigate allowing for gifts in which satiated agents voluntarily give away excess holdings of goods for nothing in return.

Voluntary exchange with gifts implies the following modification to Condition 2 of Definition 6:

2. Voluntary exchange: There does not exist a bilateral reallocation $(\tilde{z}^i, \tilde{z}^j)$ such that $\tilde{z}^i \succsim_i z^i$ and $\tilde{z}^j \succsim_i z^j$ with at least one strict preference satisfying $\tilde{z}^i + \tilde{z}^j \leq z^i + z^j$.

We call the modified equilibrium a 'stone age equilibrium with the possibility of gifts'. Modifying Proposition 10 implies the following strengthening of earlier results.

Proposition 16 *If $z = (z^0, z^1, \ldots, z^n)$ is a stone age equilibrium with the possibility of gifts, then*

1. *for each agent $i \in \{1, \ldots, n\}$ and good $k \in \{1, \ldots, m\}$: $M_k^i(z^i) < z_k^i$ implies all agents are satiated in good k.*
2. *for each good $k \in \{1, \ldots, m\}$ such that $z_k^0 > 0$: all agents are satiated in good k.*
3. *for each agent i and agent j, $i, i' \in \{1, \ldots, n\}$: $T^i(z^i) \cap \Phi(z^j, s_{ij}) = \emptyset$.*
4. *If for the pair i and i', $i, i' \in \{1, \ldots, n\}$: $(M^i(z^i), M^{i'}(z^{i'}))$ is strongly Pareto efficient in a two-agent exchange economy with total endowments $z^i + z^{i'}$ and preference relations $\succsim_i^C$ and $\succsim_{i'}^C$. Moreover, then necessarily $M^i(z^i + z^{i'} - M^{i'}(z^{i'})) = M^i(z^i)$ and $M^{i'}(z^i + z^{i'} - M^i(z^i)) = M^{i'}(z^{i'})$.*

Proof. Conditions 2 and 3 are the same as in Proposition 10. Next, we proof condition 4. For each pair i and i', $i, i' \in \{1, \ldots, n\}$: $(M^i(z^i), M^{i'}(z^{i'}))$ is strongly Pareto efficient in a two-agent exchange economy with total endowments $z^i + z^{i'}$ and preference relations

$\succsim_i^C$ and $\succsim_{i'}^C$. By definition, $M^i(z^i) \leq z^i$ and $M^{i'}(z^{i'}) \leq z^{i'}$. Possibly, there is some slack in excess holdings $z^i - M^i(z^i) \geq 0$ and $z^{i'} - M^{i'}(z^{i'}) \geq 0$. A feasible voluntary trade is to transfer one agent's entire slack to the other agent, which does not negatively affect the well-being of the giver and might improve the well-being of the receiver. If $(M^i(z^i), M^{i'}(z^{i'}))$ is pairwise strongly Pareto efficient, then by definition such transfer should not be Pareto improving. Consider the feasible reallocation $(\tilde{z}^i, \tilde{z}^{i'}) = (z^i + z^{i'} - M^{i'}(z^{i'}), M^{i'}(z^{i'}))$, where player i receives player i''s slack. Note that $M(\tilde{z}^{i'}) = \tilde{z}^{i'} \sim_{i'}^C M(z^{i'})$ and $\tilde{z}^i \geq z^i$. By weak monotonicity of $M(\cdot)$, we have $M(\tilde{z}^i) \geq M(z^i)$. Either $M(\tilde{z}^i) = M(z^i)$ implies $M(\tilde{z}^i) \sim_i^C M(z^i)$ and, therefore, $(z^i, z^{i'})$ is strongly Pareto efficient with respect to $\succsim_i^C$ and $\succsim_{i'}^C$. Or $M(\tilde{z}^i) \neq M(z^i)$ and strict convexity of $\succsim_i^C$ imply $M(\tilde{z}^i) \succ_i^C M(z^i)$ and, therefore, $(z^i, z^{i'})$ is not strongly Pareto efficient with respect to $\succsim_i^C$ and $\succsim_{i'}^C$. So, given $(z^i, z^{i'})$ is strongly Pareto efficient with respect to $\succsim_i^C$ and $\succsim_{i'}^C$, it is necessary that $M^i(\tilde{z}^i) = M^i(z^i)$. By interchanging i and i' we obtain $M(z^i + z^{i'} - M(z^i)) = M(z^{i'})$.

Finally, consider $M_k^i(z^i) < z_k^i$. then, agent i is satiated and has positive excess holdings in good k. There cannot be an agent $i' \neq i$ that is nonsatiated in good k, because by the previous arguments a gift from agent i to agent i' would be a pairwise strongly Pareto improvement. This shows condition 1. ■

The conditions state that receiving the most generous voluntary exchange, i.e. a gift, in which one agent gives up his entire slack of excess holdings to the other agent in return for nothing, should not make the receiving player better off in equilibrium. In case there is a slack of some good, say good k, it must be the case that both agents should be satiated in good k even though the receiving player may not have any excess holdings for this good. Since this condition should hold across all pairs of agents, all agents will be satiated in good k whenever $\sum_{i=1}^n M_k^i(z^i) < \omega_k$. To put it differently, as long as there is a single agent that is nonsatiated in good k, the stone age equilibrium with the possibility of gifts demands that all the other agents' excess holdings of good k are voluntarily given to the nonsatiated agent as a gift. For monotonic preferences, $M^i(z^i) = z^i$ and $M^{i'}(z^{i'}) = z^{i'}$ implies that there can be no slack and that the conditions trivially hold.

The power to withhold is an empty threat in stone age economies with the possibility of gifts. Strong Pareto efficient allocations allow for excess holding in satiated goods, but only in trivial cases where there is an abundance of goods to satiate all agents. The following example illustrates these issues, which we do not further discuss.

Example 17 *Consider stone age economy with two agents of equal strength, $s_{ij} = 1$, two goods, total resources $\omega = (3, 2)$, and identical preferences $\succsim_i^C$, $i = 1, 2$, such that $\hat{z}^i \succsim_i^C z^i$ if and only if $-\left(\hat{z}_1^i - 1\right)^2 + \hat{z}_2^i \geq -\left(z_1^i - 1\right)^2 + z_2^i$. Agent i is satiated in good 1 if $z_1^i \geq 1$ and has excess holdings $z_1^i - 1 \geq 0$. The set of strongly Pareto efficient holdings $z = (z^0, z^1, z^2)$ is given by $z^1 = (\alpha, \beta)$, $z^2 = (\gamma, 2 - \beta)$ and $z^0 = (3 - \alpha - \gamma, 0)$, where $\alpha, \gamma \geq 1$, $\alpha + \gamma \leq 3$ and $0 \leq \beta \leq 2$. Since good 1 is in excess of both agents satiation levels, any distribution over holdings is strongly Pareto efficient as long as both satiation levels are met. Let*

$$\Phi\left(z^2, s_{12}\right) = \left\{ (0, y) |\, 0 \leq y \leq \max\{\tfrac{1}{2}, z_2^2\} \right\}.$$

Then, any stone age equilibria $z = (z^0, z^1, z^2)$ is Pareto efficient and the no-coercive trade condition additional imposes $\beta \geq \frac{1}{2}$.

3.6 Concluding remarks

This paper offers a unified framework to study coercive and voluntary exchange. Our study is motivated by the fact that secure property rights, as assumed in the Walrasian economy, is rather the exception than the rule. Our model allows for coercive exchange driven by power relations that are absent in the Walrasian framework. In our model agents are characterized by their relative strength as well as by their preferences.

Our contribution is mainly conceptual. We introduce the notion of a stone age equilibrium in the context of an exchange economy and we abstract from production. In a stone age equilibrium no agent can take goods from any other agent and no pair of agents is willing to trade. Our model contains the Walrasian exchange economy and the jungle economy described by Piccione and Rubinstein (2007) as special cases. We obtain the Walrasian economy if the agents are equally strong such that no agent can take goods from others. The jungle economy is obtained as the other extreme when power relations are a strict ordering and a stronger agent can take everything away from any weaker agent. We find that the maximum of the lexicographical welfare function, i.e., the jungle equilibrium, is a robust equilibrium in all settings.

Novel is that we not only consider the relative power to take but also investigated the power to withhold resources to other agents. This power is nicely demonstrated in a stone age economy with a quid pro quo culture. It is also clear that withholding goods by satiated individuals is not (strongly) Pareto efficient in case other agents are nonsatiated in that good. Evolution may drift society towards gifts in good times and there does not seem to be any evolutionary pressure during bad times. There is however some reason for caution, withholding goods may serve some strategic goal, like a parent's withholding a of a candy from a child to induce good behavior. Strategic considerations of withholding and giving are not incorporated in our notion of a stone age equilibrium.

The discussion of quid pro quo and the possibility of gifts reflects the scientific fact that the typical hunter-gatherer society does not exists. As Bowles (2009) remarks: "whether ancestral humans were largely 'peaceful' or 'warlike' remains controversial". In any case, giving gifts is part of many cultures, see e.g. Ythier (2006). From our theoretical framework it is clear that the possibility of gifts allows for voluntary exchange that is pairwise strongly Pareto improving while any quid pro quo culture is only pairwise weakly Pareto improving. Therefore, economies based upon the gift culture are somewhat more efficient (and friendlier) than economies based upon quid pro quod. In future research, it seems more realistic to consider willingness to give gifts as attitudes of individual agents, similar to say risk attitudes, and consider societies with heterogeneity in these attitudes.

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