Extension of the probabilistic evacuation decision model

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Abstract: In 1995, due to danger of flooding, a massive evacuation in the province of Gelderland in the Netherlands took place. The decision about the evacuation was mainly a consequence of a presumption the dikes would not be able to withstand the forces of nature. The process of the evacuation went well, but life-threatening flooding did not occur. In the decision uncertainty in the load and uncertainty in the condition of the flood defences played a role. In general, such uncertainties are taken into consideration during operational flood management; however, not explicitly. There are several methods in decision theory, which can be used to include uncertainty in decision-making in an explicit way, for example the decision tree approach. This method can support decision-making when high costs and probabilities of undesirable events are involved. In this paper, we present an evacuation decision model, based on the decision tree theory and developed in FRIESER [4]. We present application of this model for three dike failure mechanisms, namely overflow, overflow/wave overtopping and uplifting/piping, what is an extension comparing to FRIESER [4]. The model requires specification of dike failure probabilities. Here we emphasize that operational decisions, like evacuation, require *operational* probabilities. These probabilities can be calculated using the concept of *fragility curves* and *short-term* water level density function. At the end, we shortly discuss usefulness the model in a real-life situation.

1 Introduction

The Netherlands is a country located in the delta of three European rivers, namely the Rhine, the Meuse and the Scheldt. Nearly 65% of the country is situated below sea level and/or is threatened by floods from the rivers (and tributaries). Natural barriers like dunes and high grounds, and man-made constructions like dikes and storm surge barriers protect the area from flooding. Despite the flood protection a flood danger remains; the flood (danger) in 1995 is a quite recent example.

At the beginning of 1995, heavy rainfalls in France and the Ardennes led to increase of water levels on the rivers Meuse, Rhine and Waal. In the Dutch province of Gelderland, the waters reached a level of serious concern. The regional management team decided to evacuate a part of the area. That was a large scale evacuation; nearly 250,000 people left their homes. The process of the evacuation went well; however, life-threatening flooding did not occur. For more information see BEZUYEN et al. [1], INT [6] and INT [7].

A primary goal of an evacuation is to save human lives. However, every evacuation, even an unnecessary one, has an great influence on the society, economy and animals. People are under a lot of stress; there are losses in income and production; there is a negative influence on livestock, horticulture and agriculture. Besides these, the costs of the operation and care for the evacuated people should be taken into account (see BEZUYEN et al. [1]). Because of these arguments every decision about an evacuation should be very carefully considered.

In 1995, the evacuation decision was mainly based on the opinion that the dikes would not be able to guarantee the safety in the region. In general, experience and experts' judgement had an influence on the evacuation decision. Because the flooding did not happen some question the evacuation (see FRIESER [4]). In the decision uncertainties in the coming water load and condition of the dikes played a role; these uncertainties were included into the decision-making process by means of experience and expertise (see BEZUYEN et al. [1]).

In FRIESER [4] a probabilistic evacuation decision model is developed. The model is based on the decision tree theory and aims to give a rational support to a decision-maker; in the model some of the uncertainties are explicitly (mathematically) included. According to the model, decision (evacuate yes or no) is based on a combination of costs and probabilities of a dike failure. The probabilities are *operational*, since the decision has to be taken in operational time. In FRIESER [4] a method is developed to calculate such probabilities for one dike failure mechanism, namely overflow. In this article we extend the model concept by implementing a wider approach to the calculation of the operational probabilities of a dike failure. We aim to compute the operational probabilities for two dike failure mechanisms, overflow/wave overtopping and uplifting/piping, using *fragility curves* and *short-term* water level density function. In the following sections we give a definition of the operational probabilities of a dike failure, present the concept of the probabilistic evacuation decision model and, by means of a case study, incorporate the extended approach for calculation of the probabilities into the decision model.

2 Dealing with uncertainties

2.1 Operational probability of a dike failure

Failure of a dike (a dike does not fulfil some of its functions, a lack of water retaining capacity) is governed by the strength, represented by geometry and material characteristics of the dike, and by the threats e.g. occurring hydraulic load. High water levels and waves usually constitute the most dominant danger to flood defences. Nevertheless, the failure can also be caused by a number of other physical, biological and human factors (see TAW [12]). There are several *failure mechanisms*, which can be distinguished in the context of a dike failure. Most of the mechanisms are presented in Fig. 1.

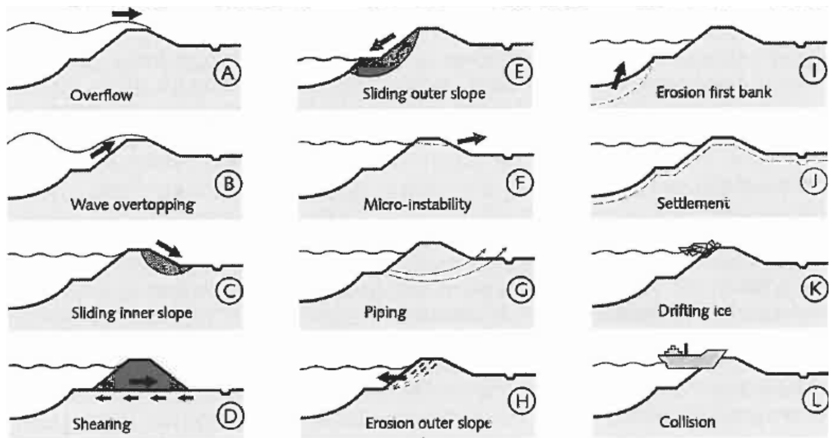

Fig. 1: Dike failure mechanisms (TAW [12]).

In this article, three dike failure mechanisms are considered more closely: overflow, overflow/wave overtopping and uplifting/piping. In case of overflow, the local water level is higher than the crest level of a dike causing erosion of the inner slope of the dike and entering of water in the area behind the dike. In case of overflow/wave overtopping also waves can lead to the failure (see TER HORST [5] and STEENBERGEN & VROUWENVELDER [11]). In case of the uplifting/piping failure mechanism, water seeps locally under a dike leading to the formation of channels (pipes), which weaken the dike. Pipes start to develop if the clay layer under the dike lifts up and cracks appear (see TER HORST [5]). A dike failure mechanism can be expressed by a model, which has an empirical or mathematical character. Usually, the model is formulated in terms of a limit state function Z, where strength of a dike is compared with the load imposed upon the dike. When $Z<0$ failure of the dike occurs. Examples of the limit state functions for overflow and overflow/wave overtopping are given in Tab. 1.

Tab. 1: Limit state functions for overflow, overflow/wave overtopping and uplifting/piping.

Failure mechanism	Limit state function	Parameters
Overflow Overflow/wave overtopping	$Z=h_d-h$ $Z=h_d-h$ or $Z=q_c-q$	h_d = the crest level of a dike, h = local water level, q_c = critical overtopping discharge, q = occurring overtopping discharge

The limit state functions depend on the local water level. Moreover, several other (random) variables and parameters are contained in Z. Keeping the above in mind, the general description of the probability of a dike (section) failure is:

$$P_f = P\{Z < 0\} = P\{R < S\} \tag{1}$$

where Z=R-S, P_f stands for the probability of a dike failure, Z stands for the limit state function, R stands for the strength of the dike and S stands for the load imposed upon the dike. R and S are assumed to be independent. Clearly, the probability of a dike failure is equal to the probability that the load imposed upon the dike exceeds the strength of the dike.

When the local water level is the dominant load, as is often the case with fluvial dikes (TAW [12]), it is useful to write the probability (1) as:

$$P_f = \int_{-\infty}^{\infty} P\{Z < 0 \mid H = h\} f_H(h) dh \tag{2}$$

where H stands for the local water level and f_H denotes the probability density function of H. The component $P\{Z < 0 \mid H = h\}$ represents the probability of a dike failure given a specific local water level. The function $h \to P\{Z < 0 \mid H = h\}$, in this context, is called a *fragility curve* (see BUIJS et al. [2] & [3] and VAN DER MEER et al. [8]). The component f_H is a long-term local water level density function. From now on, the term *local* will be omitted.

Keeping the above information in mind, we define the *operational* probability of a dike (section) failure, $P_{f|pred}$, as:

$$P_{f|pred} = \int_{-\infty}^{\infty} P\{Z < 0 \mid H = h\} g_H(h \mid h_p) dh \tag{3}$$

where g_H is a *short-term* water level density function and h_p is a forecasted water level. The operational probability of a dike failure is needed in operational decision-making (it incorporates present information concerning water levels).

The only difference between the equations (2) and (3) occurs in the formulation of the water level density function. The function f_H can be derived on the basis of distribution of annual river discharge maxima and relations between the discharge and water level (see TER HORST [5]), whereas the function g_H can be based on actual (operational) information concerning future water levels. The last function is more extensively considered in the following section.

2.2 Short-term water level density function

In our study, the short-term water level density function is assumed to be normal with mean μ, equal to the forecasted water level h_p and standard deviation $\sigma > 0$, which represents accuracy of the forecast. Having a dataset of observed and corresponding forecasted water levels, the standard deviation can be estimated as:

$$\hat{\sigma} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} {\varepsilon_i}^2} \qquad (4)$$

where ε is a historical forecast error (i.e. a difference between an observed water level and its forecast) and N denotes the size of the ensemble of the historical forecast errors. The right-hand side of (4) is the RMSE (Root Mean Squared Error), a measure often used to assess accuracy of a forecast model. It is important to emphasize that σ depends on a lead time of a forecast – indeed, the further in time a forecast is made, the less accurate it is. Therefore, the estimation should be made with respect to lead times.

It is used to say that the Rhine enters the Netherlands at Lobith. In FRIESER [4] the parameter σ is estimated for this location according to (4). The estimations are derived using a dataset of observed and corresponding forecasted water levels (high water periods in 1998-1999). In the analysis two water level forecast models are considered, namely model LOBITH and FloRIJN. The estimations are presented in Tab. 2.

Tab. 2: RMSE for two water level forecast models, location Lobith in the Netherlands, (round 30 observations per combination: model x lead time), FRIESER [4].

Forecasting model	RMSE 4 days ahead [m]	RMSE 3 days ahead [m]	RMSE 2 days ahead [m]	RMSE 1 day ahead [m]
LOBITH	0.49	0.30	0.31	0.07
FloRIJN	0.37	0.21	0.13	0.11

In general, the estimations decreases together with decrease of a lead time in case of both forecast models. Differences in the estimations follow from characters of the models. The model LOBITH is a multi-linear regression model (a statistical model), whereas a one-dimensional hydrodynamic model SOBEK constitutes the core of the model FloRIJN. For more information about the models see SPROKKEREEF [10].

As an example, consider forecasted water levels for the location Lobith: 14.24, 14.96, 15.28 and 15.44 m+NAP. These are 1, 2, 3 and 4 days ahead forecasts derived with model FloRIJN. On the basis of these values and estimations from Tab. 2, 5% and 95% quantiles of the normal distribution can be derived for the considered lead times and the forecast model. Fig. 2 presents the results.

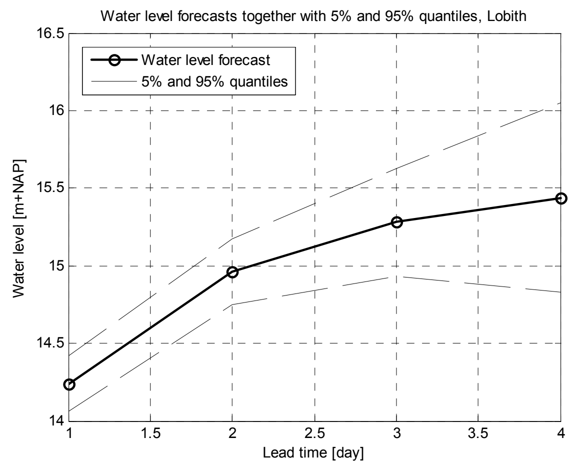

Fig. 2: Water level forecasts for four days ahead together with 5% and 95% quantiles, location Lobith, forecast model FloRIJN.

In the following section, the probabilistic evacuation decision model will be presented. The concept of operational probability of a dike (section) failure is used in the model.

3 Decision-tree for evacuation

Decision about evacuation of inhabitants of an area *endangered* by flooding is very difficult, mainly because the flood event is uncertain. The decision maker is concerned about consequences of an unnecessary evacuation on the one hand, and consequences (fatalities) if no evacuation decision is taken and the flood occurs on the other hand. An example is the evacuation in the province of Gelderland in 1995; despite expectations, flooding did not happen then. The decision about the evacuation was taken on the basis of deterministic information (like forecasted high water levels, see FRIESER [4]), experience and expertise; no robust technique, where uncertainties are explicitly incorporated, was used (see FRIESER [4]).

In FRIESER [4] a decision model for evacuation is developed. The aim of this model is to give a rational support to a decision maker, in case of a flooding danger. In the model uncertainties are explicitly (mathematically) included; the model uses the concept of a decision tree. This method can support decision-making process involving high costs and probabilities of undesirable events. Clearly, these factors are present in case of a decision about evacuation: costs of evacuation and flood damage and probability of a dike failure. The decision tree method applies a graphical approach to compare competing alternatives (decisions) and to assign values to those alternatives by combining probabilities and costs into expected values. Consequently, an alternative with the lowest expected value is indicated as the optimal. The decision model for evacuation is presented in Fig. 3.

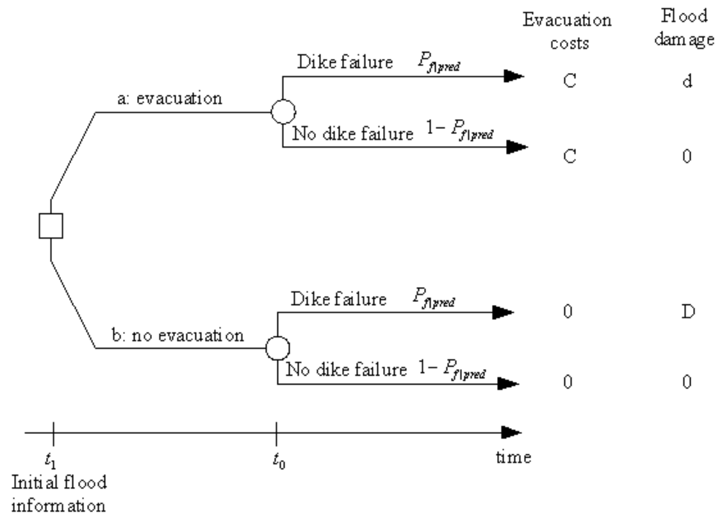

Fig. 3: Evacuation decision model.

The model describes four scenarios, which occur during evacuation decision-making. One of the scenarios is evacuation decision at time t_1 and a dike failure at time t_0 (the most upper branch in Fig. 3). Consequently, there is approximately $|t_1-t_0|$ time available for the evacuation. Costs of evacuation and flood damage are assigned to every scenario. The costs of evacuation is equal to C; if flooding occurs and evacuation did not take place then the flood damage is equal to D; if flooding occurs and evacuation took place then the flood damage equals d, where d<D. The last observation is connected with reduction of flood damage caused by the evacuation performed in the available time. Note that one of the model assumptions is equality of flooding with dike failure (this is not always the case, see FRIESER [4]). The model contains information about probability of a dike (section) failure, this is a probability that a dike fails at time t_0, given forecasted water level. The forecast is derived at time t_1 and attempts to estimate water level at time t_0. The probability can be derived for different dike failure mechanisms and can be expressed with the formula (3).

According to the model, the optimal decision is chosen on the basis of the minimum expected cost criterion:

$$Opt.decision = \arg\min_{\{a,b\}}\{EV(a), EV(b)\} \tag{5}$$

where EV stands for expected value of decision (combination of total costs and probability of a dike failure), a and b are possible decisions as defined in Fig. 3.

In the following section a case study is given, where application of the model is presented.

4 Case study

4.1 Introduction

Consider a fictive (although representative for the Netherlands) dike-ring area, situated along the river Rhine. It is assumed that 250,000 people inhabit the area; also several economic branches are present in the region. Suppose that a decision maker receives information about a danger of flooding in the area. The information is given in a form a forecasted water level for a location along the dike-ring. The forecast is 15.52 m+NAP on the third day from *now*. The crest level of a dike at the location is equal to 15.90 m+NAP; tangent of the slope angle of the dike is equal to 1/3.

The decision maker, concerned about condition of the dike and uncertainty in the load, considers evacuation of the area. It is assumed that the decision must be taken now using the given information. In order to support the decision, the decision maker uses the evacuation decision model. The model requires specification of costs and operational probability of a dike failure as input; these are considered in the following sections. These parameters depend on the lead time of the forecast (under the above assumptions there are approximately 3 days available for the evacuation).

4.2 Costs

In order to apply the evacuation decision model, costs of the evacuation (C) and flood damage (d or D, assuming that flood occurs) have to be estimated. It is assumed that the cost of evacuation emerges as the sum of initial evacuation cost and direct, and indirect economic damage. The last two arise as a consequence of evacuation. In this study, the initial evacuation cost is equal to an amount of compensations paid out to the evacuated people and expenses incurred by the police. The direct economic damage arises as a result of suspension of production and depends on the type of economic branch (e.g. minerals extraction, banks/insurance companies) and duration of the suspension. Indirect economic damage consists of losses to supply companies and customers, and losses caused by difficulties in traffic in the regions outside of the endangered area. Furthermore, it is assumed that the flood damage is equal to the sum of economic value of loss of human life and economic value of loss of replaceable moveable goods (e.g. television-set). The loss of human life is estimated using present value of the Net National Product per head of the country; obviously the loss decreases if an evacuation is carried out. Note that the flood damage does not contain cost of damage to e.g. buildings – since evacuation does not have any influence on such damage, it will not be considered here. The costs are estimated for the representative dike-ring area using data and models given in FRIESER [4]; the results are shown in Tab. 3.

Tab. 3: Costs of the evacuation and flood damage (in case of evacuation and no evacuation) for a representative dike-ring area; approximately 3 days available for the evacuation.

Costs	Value [mln]
Costs of evacuation (C)	107.5
Flood damage in case of no evacuation (D)	4430
Flood damage in case of evacuation (d)	85

4.3 Operational probability of a dike failure

The evacuation decision model requires specification of operational dike failure probability, $P_{f|pred}$. In FRIESER [4] this probability is estimated for one dike failure mechanism, namely overflow. Other mechanisms were not considered, due to study assumptions. In our research we extend the model concept by calculation of $P_{f|pred}$ for two dike failure mechanisms: overflow/wave overtopping and uplifting/piping. At the same time, we will give comparison with the overflow failure mechanism approach.

According to equation (3), the operational probability of dike failure can be computed as an integration of a fragility curve for a particular failure mechanism and short-term water level density function. In order to derive a fragility curve, a limit state function has to be specified. For the considered failure mechanisms, limit state functions are implemented according to STEENBERGEN & VROUWENVELDER [11] and TAW [12]. Every limit state function contains random variables e.g. crest height of a dike, significant wave height, etc.; usually experts determine types of probability distributions of the variables. Some of the most essential random variables, used in our study, are presented in Tab. 4. The mean values and standard deviations of the variables are realistic for a dike.

Tab. 4: Examples of random variables in the limit state functions.

Variable	Type	Mean	Standard deviation/coefficient of variation
Crest height	Normal	15.90 m+NAP	(s.d.) 0.1 m
Significant wave height	Gumbel	0.2 m	(s.d.) 0.13 m
Critical overtopping discharge	Log normal	10 l/s/m	(s.d.) 10 l/s/m
Length of seepage path	Log normal	100.4 m	(v.) 0.1
Thickness of a sand layer	Log normal	25 m	(v.) 0.1
Impervious covering layer	Log normal	2 m	(v.) 0.30
Inside water level	Normal	7 m+NAP	(s.d.) 0.1 m

Fragility curves for the considered location are presented in Fig. 4. Every fragility curve was derived using Monte Carlo simulations: for a given water level, variables in the limit state function(s) Z were sampled 30,000 times; since failure arises when Z is less than zero, the probability of a failure given the load was estimated as a number of times Z is less than zero divided by the number of the Monte Carlo runs (see BUIJS et al. [2]). The simulations were done under assumption of mutual independence of all random variables.

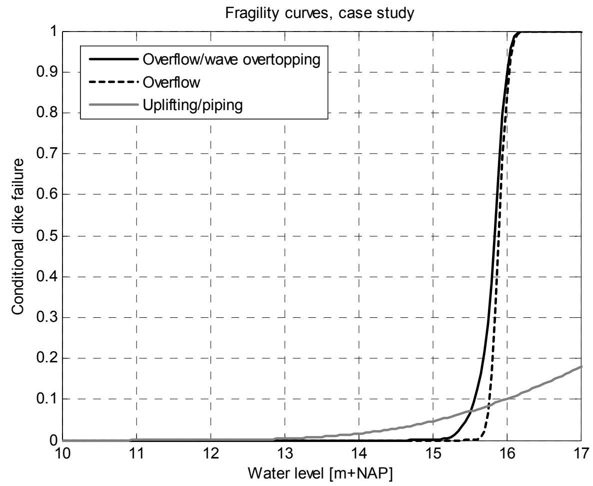

Fig. 4: Fragility curves for overflow/wave overtopping, overflow and uplifting/piping.

A few conclusions can be drawn from Fig. 4. First of all, it can be observed that uplifting/piping is a governing failure mechanism, for water levels lower than approximately 15.50 m+NAP. For higher water levels, the probability of dike failure due to overflow or overflow/wave overtopping is significantly higher than the probability of a dike failure due to uplifting/piping. Second of all, the probability of dike failure due to overflow or overflow/wave overtopping reaches critical values for water levels close to the crest level of the dike; this does not occur for uplifting/piping, since this mechanism does not depend on the height of a dike. Finally, the conditional probability of a dike failure due to overflow/wave overtopping is higher than due to (only) overflow – in case of overflow/wave overtopping, additionally, waves contribute to dike failure.

The next step in the calculation of the operational probability of a dike failure is derivation of the short-term water level density function. Recall that the decision maker is informed about the forecasted water level on the third day from now. As a result, the short-term water level density function is assumed to be normal with a mean equal to the forecasted water level 15.52 m+NAP and a standard deviation 0.30 m. The standard deviation is roughly chosen on the basis of Tab. 2.

For a given dike failure mechanism, integration of the corresponding fragility curve and short-term water level density function leads to the operational dike failure probability (see equation (3)). The probabilities, with respect to dike failure mechanisms, are presented in Tab. 5.

Tab. 5: Operational probability of dike failure for three failure mechanisms.

Failure mechanism	Operational probability of dike failure
Overflow/wave overflow	0.20
Overflow	0.12
Uplifting/piping	0.07

The operational probability of dike failure due to overflow/wave overtopping is the highest. Because of the form of the fragility curves, the probability of dike failure due to overflow/wave overtopping is higher than due to (only) overflow.

4.4 Optimal rational decision

In the evacuation decision model, the optimal decision is determined by the minimum expected cost criterion. Using the estimated costs (Tab. 3) and operational probabilities of dike failure (Tab. 5), the expected values of *a: evacuation* and *b: no evacuation* are derived, and shown in Tab. 6.

Tab. 6: Optimal decision according to the evacuation decision model for three dike failure mechanism; a: evacuation, b: no evacuation; 3 days available for the evacuation.

Failure mechanism	EV(a) [mln]	EV(b) [mln]	Decision
Overflow/wave overflow	**124.3**	874.5	a: evacuation
Overflow	**117.2**	507.7	a: evacuation
Uplifting/piping	**113.7**	321.2	a: evacuation

Since for every considered failure mechanism the expected value of evacuation is smaller than the expected value of no evacuation, the decision *a: evacuation* is indicated as optimal by the model. Thus, on the basis of this rational approach, the decision maker should evacuate the area.

5 Conclusions, remarks and discussion

In many situations, computation of operational probability of a dike failure cannot be only restricted to the overflow failure mechanism; a dike can fail due to other failure mechanisms. Indeed, in the case study the probability of a dike failure due to overflow/wave overtopping is higher than due to the other mechanisms. Application of the operational probabilities of dike failure, according to the formula (3), leads to improvement of the probabilistic evacuation decision model.

Other conclusions, remarks and discussion points following from our study are:

- In FRIESER [4] some variations of the probabilistic evacuation model (delay of evacuation is taken into account) are given. Application of such an approach can lead to other decision.

- In our study, the evacuation decision model was implemented for three dike failure mechanisms (three probabilities). The indicated optimal decision was the same in every situation; however, this is not always the case. In general, the probability of occurrence of single failure mechanisms and relations between them lead to the probability of the main event, i.e. a dike failure. A future work will consider implementation of a total dike failure probability in the evacuation decision model. Moreover, in the case study, we assume that the random variables are independent, what leads to *basic* Monte Carlo simulations. If dependence is of importance, other calculation routines (like e.g. FORM, see TAW [13]) are used.

- The presented model does not take into account possible changes in the decision caused by trend of water level forecast. More precisely, if the model (based on a point water level forecast) indicates evacuation and the trend of the water level forecast decreases, then *no evacuation* decision may be more reasonable. Consideration of the trend influence is a subject of future work.

- The short-term water level density function is assumed to be normal. In VAN SCHROJENSTEIN LANTMAN [9] the normality has been proven for a dataset of observed and corresponding forecasted water levels (forecast model FloMaas for the river Meuse). However, the assumption of normality is debatable. In fact, the type of the density function changes with actual circumstances. Here, an example of an ensemble forecasting approach (used for generation of probability forecast) can be given; usually the quantiles of the probability forecast do not correspond with the quantiles of a normal distribution.

- The historical forecast errors provide information about accuracy of a forecast model. Moreover, the errors can be used to estimate bias in the forecast model; e.g. the model can on the average forecasts 4 cm too low water levels 2 days ahead. The bias is not included in the considered definition of the short-term water level density function.

- The presented evacuation decision model constitutes a rational tool, which can assist in operational decision-making. An advantage of the model is that uncertainties (in dike failure) are mathematically included in it. It is important to say that the model can be applied as a support (source of additional information) in a real-life situation. However, the model cannot replace the usual decision-making, where often intuition is taken into account. In VAN SCHROJENSTEIN LANTMAN [9] a study is carried out, which shows that adjusting of a forecast model outcome by experts leads usually to improvements. This shows that expertise can indeed have a positive influence.

6 Acknowledgement

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