Estimating equivalent roughness lengths based on wake sizes

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ABSTRACT

Scaling expressions are derived for equivalent roughness lengths to describe average flow over cylindrical vegetation and two-dimensional dunes. The derivation is based on the assumption that the spatial extent of a flow separation zone behind a bluff body controls flow resistance. A comparison with experimental data available from literature shows that the scaling relations capture the dominant dependencies of the equivalent roughness length. Also, it is shown that the proposed methodology is consistent with the practice of using an equivalent grain roughness for flow over a rough bed. The derived scaling expressions are useful for engineering applications in large-scale flow situations where it is not feasible to make use of more detailed flow-modeling techniques.

INTRODUCTION

Hydraulically rough flows are affected by geometrical properties of the solid boundaries because the spatial extent of boundary irregularities exceeds the width of the laminar sublayer. Due to the dependency of the flow field on geometrical boundaries, many different roughness formulations have been proposed. Such formulations are seemingly unrelated and are restricted to particular conditions. Examples are the formulation of Van Rijn (1984), which applies to resistance caused by dunes, and the formulations proposed by Baptist et al. (2007) and Huthoff et al. (2007), which apply to flow resistance caused by cylindrical vegetation.

In the current work we make use of the conceptual framework proposed by Gioia and Bombardelli (2002) to link geometrical properties of flow obstructions to the overall flow resistance. Gioia and Bombardelli propose that the well-known Manning law for flow in rough channels is linked to the energy cascade concept in turbulent flows. The key assumptions in their derivation are that (i) turbulent energy generated at the largest scale of the flow field (the flow depth, or the hydraulic radius R) is transferred to smaller turbulence scales (energy cascade) and that (ii) the lower end of the energy cascade is characterized by a turbulent length scale r that is controlled by roughness elements on the channel bed. Further scaling assumptions regarding velocity fluctuations in the Reynolds stress result in an expression for the mean streamwise flow velocity:

$$U \sim \left(\frac{R}{r}\right)^{1/6} \sqrt{gRi} \qquad (1)$$

A comparison with this equation to the well-known Manning-Strickler equation shows that turbulence scale r is equivalent to Strickler's roughness height k_S.

In the following sections, scaling relations for r are derived, through which geometrical properties of flow obstructions are linked to an equivalent roughness height.

THE EQUIVALENT ROUGHNESS LENGTH RELATED TO WAKE SIZES

In hydraulically rough flow, the irregularities on the bed penetrate into the turbulent flow field and, hence, affect the turbulence characteristics. Here, we propose a methodology to estimate the flow resistance that is experienced by a free flowing layer over top of roughness elements, where the resistance is controlled by the size of the wakes that the roughness elements create. From numerous studies of flows around bluff bodies it has been shown that the turbulence intensity is strongly concentrated to the wake zones of the obstructing objects (e.g. Nakamura 1993, Williamson 1996). Given that increased turbulence intensity manifests itself on the scale of the flow domain as an increase in flow resistance, the size of the wake and the turbulence intensity inside the wake seem suitable quantities to describe the flow resistance caused by the obstructing objects.

Figure 1 gives a sketch of a flow field where the depth of flow is partly occupied by large-scale obstructing objects (cylinders or dunes). In the figure an artificial rough bed is introduced (dashed line), which is located at the top of the obstructing objects. An equivalent roughness height r is assigned to this artificial rough bed. It is hypothesized that this equivalent roughness height is related to the spatially-averaged turbulence intensity that is created in wakes of the roughness elements.

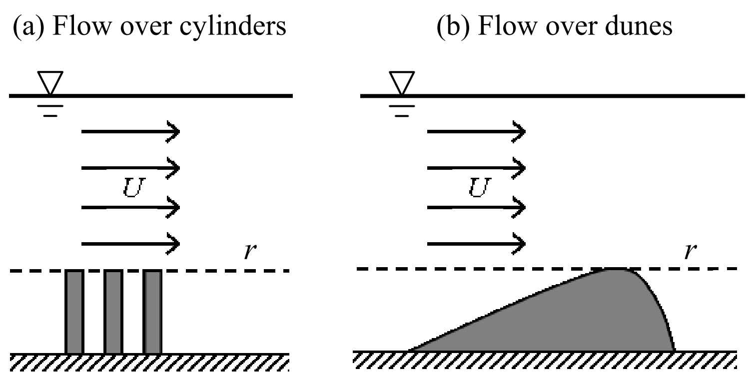

Figure 1: Flow over large-scale roughness elements: (a) cylinders and (b) dunes.

For the case of a single obstructing object, an integral turbulence length scale can be defined that is associated with generation of the turbulence in the wake; the momentum thickness θ (e.g. Tennekes and Lumley, 1972). This length scale is based on the momentum defect in the wake and is related to the drag coefficient of a bluff body C_D as:

$$\theta = \frac{1}{2} C_D D \tag{2}$$

Here, D is the transverse size or diameter of the obstructing object. To arrive at a relation for the flow resistance caused by a group of obstructing objects it is important to take into account *how many* wakes contribute to intensify the turbulence in the flow field. Therefore, a *wake filling factor* f is introduced:

$$f = \frac{V_W}{V}, \tag{3}$$

which gives the fraction of the flow field that, in between the obstructing objects, is 'filled' with wakes (V = total volume between obstructing objects, V_W = wake volume). Finally, the equivalent roughness length for flow over obstructing objects is assumed to be proportional to the volumetric average of the turbulence length scale in the wake, i.e.

$$r \sim f\theta . \tag{4}$$

In this expression for r it is assumed that only the turbulence mixing in the wakes of the obstructing objects contribute to the overall flow resistance, and that any flow mixing outside of the wake zones is negligible for the hydraulic resistance that is experienced in the free flowing layer above.

Equations (2), (3) and (4) are consistent with the well-known empirical result that the grain size is a suitable quantity to estimate the equivalent roughness length in rough bed channel flows. In such cases, the wakes caused by the grains are space filling (f =1), and thus from equation (2) and (4) it follows that $r \sim D$. Furthermore, Schlichting (1936) showed that for sparse bed grain coverage a nearly linear relationship is found between the equivalent roughness length and the solidity of grains (i.e. the projected frontal area of the roughness elements per unit bed area). This is in agreement with equation (4), assuming that the wake-filling factor is proportional to the solidity. At relatively high solidities (above 15%) roughness elements shelter each other, resulting in a lower equivalent sand roughness (e.g. Jiménez 2004). This observation provides further support for equations (2) – (4), because a maximum wake filling factor (f = 1) in combination with a modified drag coefficient C_D, allows incorporation of the observed wake sheltering effect on r.

Making use of equations (2) – (4), scaling relations can be derived for the equivalent roughness length r for flows over various types of roughness elements. In the following sections two cases of flow over large-scale obstructing objects are considered: flow over dunes and flow over cylindrical vegetation. For both cases the derived expressions for r are compared to data from literature.

EQUIVALENT ROUGHNESS LENGTH FOR CYLINDRICAL ELEMENTS

Cylindrical roughness elements are characterized by the following parameters: diameter D, height k and surface density m ($m = 1/s^2$, with s the separation between individual elements). If we consider a bed area s^2 then the flow volume up to the cylinder top is $V = ks^2$, which can be used in equation (3) to calculate the wake filling factor (for simplicity, the solidity of the cylinders is neglected). Next, the wake volume V_w within V needs to be estimated. One cylinder is contained within V, therefore, V_w corresponds to the wake volume produced by a single cylinder. We estimate the size of the wake by considering a steady wake in the horizontal plane that remains the same at all depths behind the cylinder. The width of the wake and the extent of the wake in streamwise direction both scale with the momentum thickness (e.g. Williamson 1996). Therefore, multiplying the scaling assumptions for the wake size in the horizontal plane with the height of the cylinder yields the volume of the wake $V_W \sim \theta^2 k$. Next, by substituting the expressions for V and Vw into equation (3), the scaling relation for the wake filling factor becomes

$$f \sim \frac{\theta^2}{s^2}. \tag{5}$$

Following equation (4), the scaling relation for the equivalent roughness length yields

$$r \sim \frac{\theta^3}{s^2}. \tag{6}$$

Equation (6) can be modified to include the effect of different streamwise and lateral spacings between cylinders (using $s_x s_y$ instead of s^2). However, here the analysis is restricted to homogeneously distributed cylinders.

Figure 2 shows the comparison between measured equivalent roughness heights for flow over cylindrical elements (in terms of Strickler's k_S) and calculated values for θ^3/s^2. In calculating the values for θ^3/s^2, the definition for the momentum thickness in equation (2) was used. For calculating the k_S values from the measurements, only the flow field above the cylinders was considered.

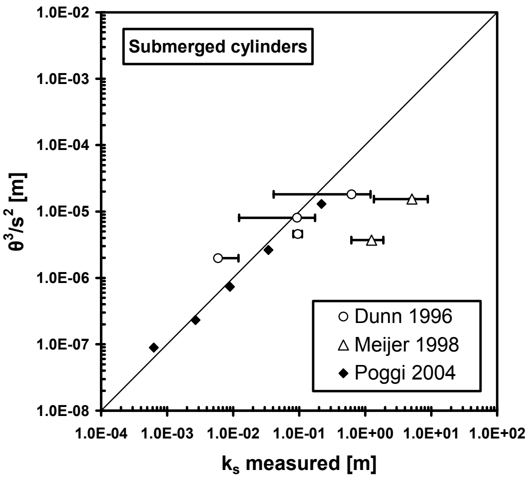

Figure 2: Equivalent roughness lengths of flows over cylinders, based on equation (11), compared to Strickler's roughness heights determined from experiments.

The data used in Figure 2 are summarized in Table 1. Because the data include a wide range of values for θ^3/s^2 and k_S, in Figure 2 a log-log representation is used. The diagonal line in Figure 2 has a slope of 1, which corresponds to a linear relationship between plotted quantities ($\theta^3/s^2 = 10^{-4}k_S$). In the figure it can be seen that data points from the same experiment set follow the linear relationship quite well, thus supporting the scaling relation $\theta^3/s^2 \sim k_S$. However, between experiment sets, there is a significant off-set of plotted values (a difference of about one order of magnitude in θ^3/s^2 or k_S). This off-set may be due to the way that drag coefficients were determined in the separate experiments, and to the difficulty in measuring the energy slope (which is required to calculate C_D and k_S). In the experiments by Dunn et al. (1996) and by Meijer and Van Velzen (1998) a bulk drag coefficient was used, while in Poggi et al. (2004) the drag coefficient is determined from shear stress measurements. This makes comparison between experiments difficult. Also, a surprising result from Poggi et al. (2004) is that a decrease in C_D, which is attributed to wake interference, becomes stronger for sparser cylinder arrangements.

Table 1: Laboratory experiments with flow over rigid cylindrical roughness elements.

Reference	h/k [-]	k [m]	D [m]	C_D [-]	m [m^{-2}]	s [m]	No. of exps.
Dunn et al. (1996)	1.6, 2.3	0.12	0.0064	1.13	42	0.154	2
	2.0, 2.6	0.12	0.0064	1.13	97	0.102	2
	1.4–2.9	0.12	0.0064	1.13	170	0.077	5
	1.8-3.3	0.12	0.0064	1.13	384	0.051	3
Meijer & van Velzen (1998)	2.2-5.5	0.45	0.008	0.98	64	0.125	8
	1.5-2.5	0.9	0.008	0.98	64	0.125	8
	1.3-1.6	1.5	0.008	0.98	64	0.125	8
	2.2-5.5	0.45	0.008	0.98	256	0.063	8
	1.7-2.8	0.9	0.008	0.98	256	0.063	8
	1.3-1.7	1.5	0.008	0.98	256	0.063	8
Poggi et al (2004)	5	0.12	0.004	0.55	67	0.122	1
	5	0.12	0.004	0.6	134	0.086	1
	5	0.12	0.004	0.7	268	0.061	1
	5	0.12	0.004	0.85	536	0.043	1
	5	0.12	0.004	1.15	1072	0.031	1

EQUIVALENT ROUGHNESS LENGTH FOR DUNES

For the case of flow over dunes, again, we estimate the wake filling factor f and the momentum thickness θ to arrive at a scaling relation for the roughness length r. Two geometrical parameters are used to describe the size of the dunes; the average height of the dunes Δ and the streamwise distance between two dunes λ. Again, for simplicity we neglect the solidity of the dunes. Considering a bed area between consecutive dunes that has a width B, the volume between dunes is estimated as $V = \lambda\Delta B$. In Venditti & Bennett (2000) integral length scales for the wake zones behind dunes are determined, which for the cross-stream and vertical velocity component are comparable to dune height. Therefore, by making use of the momentum thickness concept where detailed shape effects on the wake are absorbed into a drag coefficient, the wake volume scales as $V_W \sim \theta^2 B$ (where the momentum thickness of a dune is defined as $\theta = C_D\Delta$). As a result, the scaling relation for the wake filling factor becomes

$$f \sim \frac{\theta^2 B}{\lambda \Delta B} = C_D^2 \frac{\Delta}{\lambda}. \quad (7)$$

Table 2: Laboratory experiments with flow over dunes (artificial dunes had fixed shapes that resembled natural dunes).

Reference	Reference code	Δ [m]	λ [m]	D_{50} [-]	No. of exps.	Dune type
Tuijnder et al. (2009)	T 2009	0.005-0.082	0.23-1.51	0.008	33	Real
Blom et al. (2003)	B 2003	0.017-0.066	0.79-1.53	0.0013	4	Real
		0.017-0.049	0.9-1.38	0.0021	2	Real
		0.018-0.122	0.99-1.79	0.0008	2	Real
McLean et al. (1999)	McL 1999	0.04	0.41-0.81	0.0002	6	Artificial
Klaassen (1990)	K 1990	0.029-0.145	1.3-2.44	0.00066	6	Real
Ogink (1989)	O 1989	0.08	1.6	0.0017	3	Artificial
		0.04	1.6	0.00085	9	Artificial
Van Mierlo & de Ruiter (1988)	vMdR 1988	0.08	1.6	0.0016	2	Artificial
Driegen (1986)	D 1989	0.037-0.17	0.91-2.38	0.00078	40	Real

Next, the scaling relation for the equivalent roughness length ($r = f\theta$) yields

$$r \sim C_D^3 \frac{\Delta^2}{\lambda}. \qquad (8)$$

Table 2 gives an overview of various experiments that have been carried out with flows over dunes, which we use to evaluate equation (8). We assume that in the experiments from Table 2 the dune shapes were similar and thus that the drag coefficient C_D may be considered a fixed value. Figure 3 shows that there is reasonable agreement between calculated values for Δ^2/λ and Strickler roughness heights k_S determined in the experiments. The Strickler roughness heights in Figure 3 represent the total resistance to flow, and therefore include the effects of dune form drag and bed (grain) roughness. In the right graph in Figure 3 also results scaled to the mean grain size of the bed (D_{50}) are shown. It can be seen that a large part of the data points fall on a line having a slope close to 1, supporting the linear relation that is proposed for k_S and the equivalent roughness length r in equation (8). Whenever k_S is of the same order of magnitude as $D_{50,}$ then bed roughness starts to dominate the flow. In that case, the scaling relation in equation (8) is no longer valid.

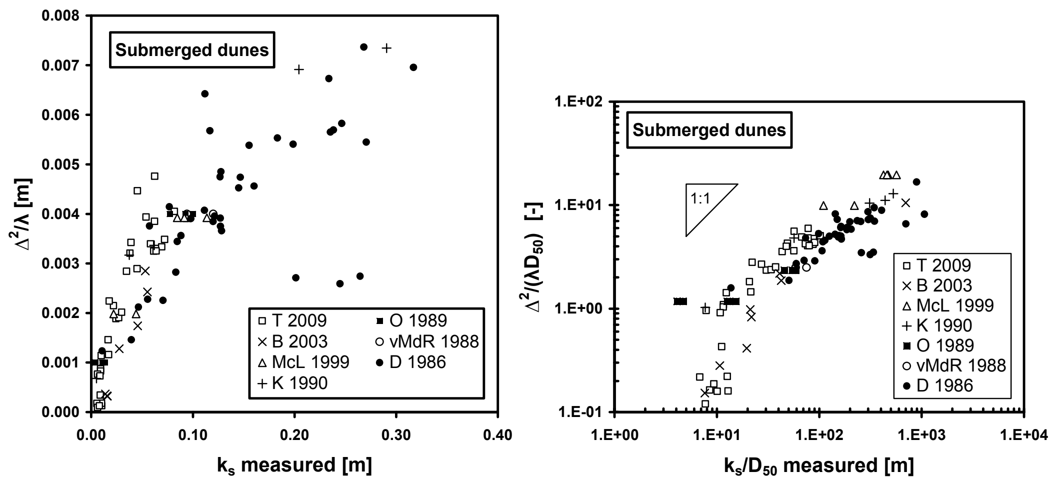

Figure 3: Equivalent roughness lengths of flows over dunes. In the graph on the right dimensionless quantities are used (scaled to the mean grain size D_{50}).

A possible explanation for slopes lower than 1 in the right graph of Figure 3 is the formation of dunes that have 3-dimensional shapes. In such cases, a transverse length scale should be incorporated in the expression for the equivalent roughness length. In principle it is possible to extend equation (8) with an additional length scale, but laboratory data for evaluation of such a dependency is lacking.

DISCUSSION & CONCLUSIONS

In the current paper, it is shown that simple assumptions regarding the role of wake turbulence in channel flows lead to unified view on hydraulic resistance caused by obstructing objects. Within this framework it appears that flows over rough walls, over dunes and over cylinders are closer related than what existing roughness formulations for these situations might suggest. Also, the methodology is consistent with well-known empirical results of flows over rough walls.

The proposed methodology states that the overall resistance experienced by a flow over obstructing objects is related to the volumetric average of wake zones. To be more specific, the dominant turbulence scale near the roughness elements is associated with wake size, and the equivalent roughness length is the volumetric average of the dominant turbulence scale. Regarding Gioia and Bombardelli's derivation of Manning's law, the results presented here imply that the effective velocity scale and the length scale that are associated with the roughness elements are not directly linked to wake turbulence, but that they are the spatially-averaged quantities. This conceptual picture may explain why the equivalent roughness length r does not necessarily mark the lower end of the inertial range in a measured turbulent energy spectrum. At different locations between the roughness elements the energy spectrum may look completely different, depending on whether one is measuring inside or outside of the wake zone (see, for example, Venditti 2007). To get a more representative view of the important bulk turbulence scales it seems more appropriate to spatially-average the energy spectra.

Finally, the proposed methodology, and the resulting roughness length scaling relations, are mathematically simple and provide insight into the dominating factors of flow resistance over bluff bodies. These properties make the proposed approach particularly suitable for engineering purposes where the overall effect caused roughness elements needs to be taken into account.

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