Comparison of Euler buckling, natural frequency and bending fractures

Peter Sterken*

Abstract

The wind-induced bending moments and stresses generated in the trunk of a 17,5m high *Cupressus sempervirens* were calculated in order to compare safeties regarding bending fractures and Euler buckling. Simultaneously, the natural frequency of the bare trunk was calculated. Afterwards, the height, and corresponding canopy size, were heightened in order to predict the limit from which traditional breakage calculations would not be able to predict the tree's safety anymore, since the slenderness would cause the stem to buckle (Euler buckling) instead of breaking due to pure bending stresses.

According to traditional breakage calculations, slender trees with a relatively small crown surface area might be safe, due to low bending stresses during gusts of approximately 32,5km/h. Nevertheless, the slenderness and modulus of elasticity (MOE) might cause the stem to fail, while the natural frequency approximates dangerously the frequency of violent gusts of wind. The results of the present study suggest that the slenderness and the MOE are critical factors when assessing the safety of slender trees.

Key words: Comparison · Wind load · Bending fractures · Euler buckling · Natural frequency

Introduction

Slender trees can fail by forming a large wave along the length of their stem. In reality, the tree "deviates itself" from the load and this type of failure seems to depend principally on its modulus of elasticity, height of the tree and the moment of inertia (Gordon, 1999). This phenomenon could be visualised by leaning on a thin walking stick which bends aside because of our weight. When we take away the pressure, the stick turns back to its original shape. But if we force it to bend too much – the stick escapes sideways - it breaks and we fall on the floor. Its slenderness and capacity to deform elastically determine how much of our weight the stick can take before it buckles.

Traditional calculations for assessing stem resistance to breakage are based upon bending in a cantilevered wood beam. These are not accommodated to cope with elastic stem flexure, i.e. elastic behaviour. Fracture safety is assumed to be the result of hypothetical wind-loads acting on inelastic stems and branches. Current methods based on the latter typically calculate the safety factor and the necessary residual wall thickness, regarding gusts at hurricane speed (see for example: SIA method, Wessolly and Erb, 1998).

The present paper proposes to incorporate the effect of the *elastic behaviour* of stems and branches, with the *wind induced bending moments*. This elastic behaviour is represented hypothetically here by Euler buckling and the natural bending frequency of stems. The here employed formulae cannot predict perfectly the tree's real safety, due to the enormous and natural diversity of (unpredictable) parameters and factors. Hence, the goal of the present study is to offer a theoretical exercise which might stimulate the process of integrating the MOE, slenderness and natural frequency, to traditional breakage calculations. Simultaneously, the protocol would equally be based on the "triangle of statics" (load, geometry and material properties).

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Materials and methods

- Subject tree

A 17,5m high *Cupressus sempervirens,* situated in Parc Sant Jordi, Spain, was selected for this study. The tree was enclosed in a dense vegetation of palms, cypresses and other Mediterranean tree-species. The height, diameters of the trunk and bark thickness were measured. A digital photograph was token, in order to draw its silhouette in the computer and to calculate the vertical crown surface area.

- *Wind speed profile*

The power-law model is used to predict the wind speed at a given height above ground level and is presented by the following formula (Berneiser & König, 1996):

$u^{(z)} = u^{(g)} * (h^{(z)} / h^{(g)})^{a}$

Where:

$u^{(z)}$ = wind speed "u" at a certain height "z" above ground level.

$u^{(g)}$ = maximum wind speed expected, not influenced by the roughness of the terrain.

$h^{(z)}$ = height above ground level at which a certain wind speed is reached (height of the analysis)

$h^{(g)}$ = height above ground level at which the maximum wind speed is reached.

a = surface friction coefficient

A turbulence factor is incorporated to count with the influence of strong incoming gusts (Wessolly & Erb, 1998).

- *Wind load analysis*

In accordance with the Eurocode 1 (AENOR, 1998), which recommend equations for predicting wind loads in structures, the wind load in the tree-crown is analysed. For trees, the following formula is a logical adaptation of those equations:

$F = ½*Cd*p*A* u^{(z)2}$

Where:

F = the force that a gust exerts in the crown

Cd = the aerodynamic coefficient describes the flexibility that the tree employs in order to diminish the force of the wind

p = density of the air, which depends on the pressure and humidity of the air, temperature and height above sea-level.

A = the exposed area of the crown to the wind.

$u^{(z)}$ = wind speed "u" at a certain height "z" above ground level.

- *Breaking safety: bending theory*

The calculation of the breaking safety of the tree is based on the bending theory of a beam and is the same as employed for the SIA method (Wessolly & Erb, 1998). The bending stress on the surface of a beam is calculated by the following equation:

$smax = M / W$

Where:

smax = the maximum bending stresses in the marginal fibres

M = the bending moment

W = the cross-section modulus

The cross-section modulus is represented by the following formula:

$W = d^3 *(p/32))$

Where:

W = the cross-section modulus

d = the diameter of the cross-section

The safety factor of the trunk is found from:

$S = s/smax$

Where:

S = the safety factor of the cross-section

s = the strength of the green wood

s_{max} = the maximum bending stress in the marginal fibres

- *Material properties*

For *Cupressus sempervirens*, values of 2 kN/cm² for longitudinal compression strength and 735 kN/ cm² for the modulus of elasticity (MOE) were assumed. To the knowledge of the author, no data have been published yet for this Mediterranean tree species. Therefore, the material properties of the wood of *Chamaecyparis lawsoniana* were employed (Wessolly and Erb, 1998).

- *Geometry*

The diameter and form of the trunk and the bark thickness were kept the same, just as wind speed parameters, material properties and others.The thickness of the bark is discounted from the diameter, to stay on the safe side, since its addition to the strength of the trunk is a parameter that is largely unknown.In the data-input of the mathematical model, only the height and corresponding vertical crown area varied.

Medium diameter of the trunk = 44,42cm
Medium bark thickness = 1,5cm
Cross-section modulus = 5144,13cm³

- *Necessary residual wall-thickness*

The residual wall thickness that a tree needs in order to withstand any given static wind speed can be calculated if assumed that the hollow trunk behaves like a perfectly formed hollow tube.
The equation employed here for calculating the minimum residual wall thickness, if completely concentrically and closed, is in accordance with the bending theory of the hollow beam and is the same as employed for the SIA method (Wessolly & Erb, 1998):

*t =0,5*dm*(1-(1-(100/SF))^(1/3))*
Where:
t = minimum residual wall thickness
dm = measured net diameter
SF = safety factor of the cross-section

- *Safety against Euler buckling*

The critical load for the collapse of a trunk by Euler buckling is calculated taking in account the modulus of elasticity of the green wood (MOE), the slenderness and height of the tree and the wind load in the crown. The formula found in Gordon (1999) for beams is employed here for trees:

Mcrit_euler = $p^2 * ((MOE * I) / (4 * h^2))$
Where:
Mcrit_euler = the critical moment that would lead to Euler buckling of the trunk or stem, expressed in kNm
MOE = the modulus of elasticity expressed in kN/ cm²
I = the geometrical moment of inertia
h = the height of the tree in m

- *Natural frequency*

In the employed model the bending-frequency of the bare trunk is represented by the following equation (AENOR, 1998):

$n = (el * d)/(h^2) * v(Ws / Wt)$
Where:
n = the bending-frequency of the trunk, expressed in Hz
el = the factor of frequency
d = the diameter of the trunk
h = the height of the palm or tree
Ws = the weight of the structural parts that contribute to the stiffness of the trunk
Wt = the total weight of the trunk

Results and discussion

The results of the wind load analysis, safety factors and residual wall thickness can be observed in diagram 1. The latter are compared with the results for Euler buckling and natural bending frequency.

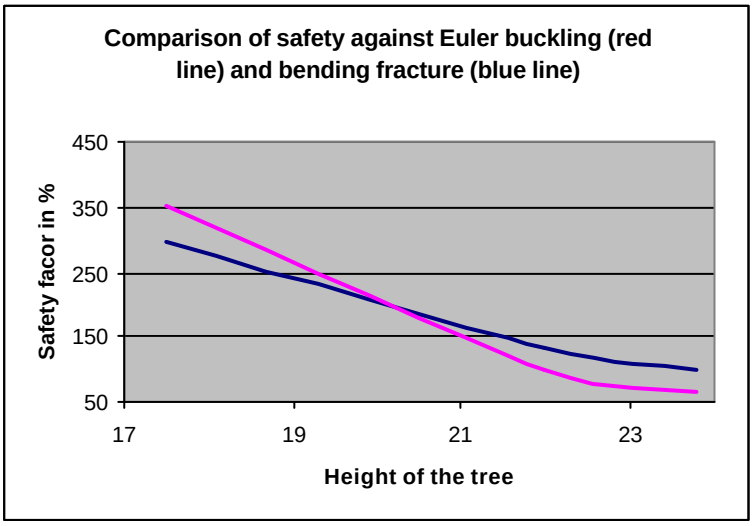
Diagram 1: results of the calculations

Tree height, m	bending moment kNm	stress s, kN/cm²	Safety bending, %	Required t in cm	Safety Euler, %	Natural frequency, Hz
17,5	34,53	0,67	**297,99**	2,57	351,15	5,8
22	77,97	1,52	131,95	7,61	**98,38**	3,7
23,78	102,86	2	100,02	18,95	**63,83**	3,2

Since its safety factor is relatively high, the original 17,5m high cypress would need, theoretically, a t/R ratio of 0,13 to withstand hurricanes of 117 km/h. Safety against Euler buckling, with a value of 351,15%, is higher than the latter. Therefore, the influence of its slenderness seems to be less important than pure bending fractures. In this case, the safety factor calculated according to the bending theory could be a good orientation for the tree its fracture safety. Note nevertheless that the low bending-frequency suggests that the influence of dynamics does needs attention: both the swinging of the tree and the dynamic nature of wind loads can excite resonant motions, generating dynamic responses that can be far larger than those that would be produced by equivalent static loads.

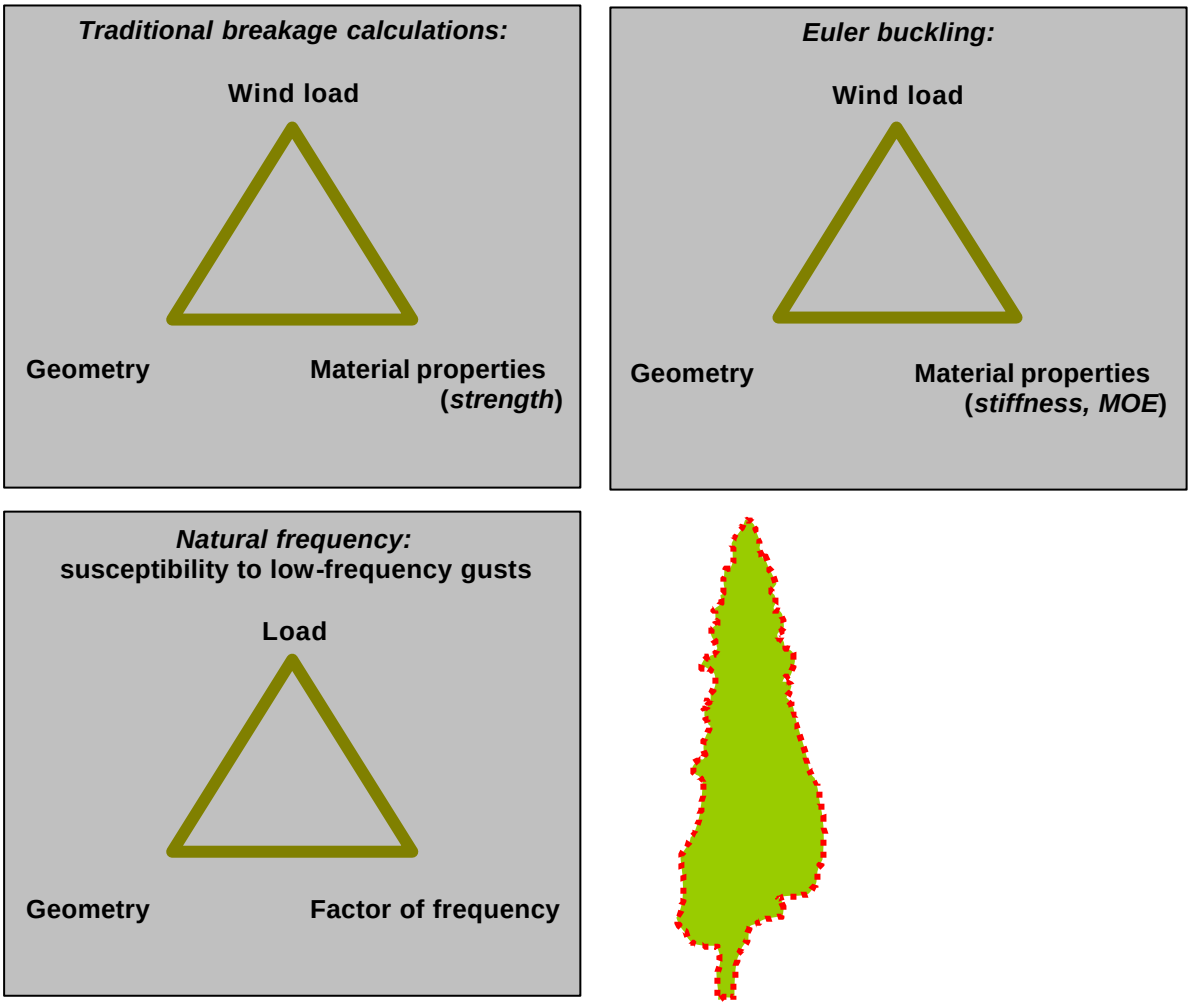
With a theoretical height of 22m, the increasing slenderness of the tree causes that Euler buckling seems to gain more importance. According to the traditional calculations, the tree would theoretically be safe (131,95%) and would need a residual wall thickness of 7,61cm to withstand a hurricane. This, due to a relatively small surface area (!!) and hence a low wind load. Nevertheless, safety against Euler buckling is slightly below the necessary 100% in order to withstand strong gusts of wind.
When the height is increased (23,78m), safety against bending levels the minimum necessary 100% to be theoretically safe. Nevertheless, the value for Euler buckling decreases such that the risk for buckling increases highly. The very low natural frequency also suggests the highly increased risk of failure. The closer the natural frequency gets to the frequency of gusts of wind (1 Hz for example), the lower its stability. Slenderness and MOE turn out to be the critical factors, more than pure wind-induced bending stresses.

Diagram 2: here is suggested that the safety against Euler buckling should be compared to the safety against bending fractures. Observe where the curves meet, which is the point from which slenderness and MOE gain a major importance.



The results suggest that, in very slender trunks, stems or branches, traditional calculations based on wind load analysis and the simple bending theory of a hollow beam might not be able to predict the safety, if the MOE and slenderness are not taken into account. The safety factor for Euler buckling can be employed together with the calculated natural frequency (in Hz), being both a method for assessing the sensitivity of the tree regarding unpredictable, and dangerous, dynamic swinging.

Diagram 3: basic components of the proposed model:



Conclusion

In slender and upright mains stems or branches, collapses similar to Euler buckling might gain a considerable importance. Not only the wind load or the compression strength of the wood can induce pure bending fractures (see traditional calculation of breakage), but also the MOE and the slenderness might influence largely the safety of a stem during meteorological events. Here is suspected that catastrophic dynamic behaviours and Euler buckling gain more importance than the traditional breakage calculations with very slender trees or stems.
Possibly, the SIA method (Wessolly & Erb, 1998) *only* employs uniformly distributed crown forms and relatively low and rigid tree-silhouettes, due to this reason.

According to the present study, the stem seems to be more prone to failure by Euler buckling due to its slenderness, than due to a static wind load and the simple bending theory of a stiff beam. This hypothesis might agree with how slender stems or branches fail in nature (see Sterken 2005 for a brief description of Euler buckling). Simultaneously, the assumption that the bending of a stem is directly proportional to the mean wind force acting on the crown centre and inversely proportional to the stiffness of the stem (Peltola et al. 2000), supports this hypothesis.

Additional notes:

- At the end, in real life tree-assessment there is no need for analysing this profoundly, since expert visual assessment and a good dose of common sense will generally achieve the most reliable diagnosis.
- The MOE of the trunk of this cypress was measured by the author in several cross-sections with Young's modulus sensors, measuring the longitudinal deformation of the exterior wood fibres, when applying a bending moment artificially (see: Peltola et al. 2000). Values as low as 414 kn/cm² at 1,03m height were recorded in this tree. Which means that the safety against Euler buckling will even be lower than calculated with the values assumed for *Chamaecyparis lawsoniana*.
- For the wind load analysis, an aerodynamic drag factor of 0,20 was chosen which is the value proposed by Wessolly and Erb (1998) for *Chamaecyparis lawsoniana*. According to these authors, this drag factor represents partially the tree its elastic behaviour during events of wind. Nevertheless, it should be noted that these values are purely hypothetical, as yet untested (Mattheck & Bethge, 2005). Hence, and although in the present model these aerodynamic drag factors are employed in order to follow a certain protocol, care should be taken, since the final result of the wind load analysis and tree-stability are, naturally, also hypothetical.

This paper is dedicated to Spain.

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COMPARISON

of Euler buckling, natural frequency, bending fractures and Brazier buckling.

Wind load analysis in accordance with Eurocode 1, part 2-4

Species= ***Cupressus sempervirens***

Wind load analysis

h tree=	17,50	m
Cw=	0,20	
A=	38,16	m*m
F=	3,72	kN
P=	9,29	m
M=	34,53	kNm

Geometry

d sound=	40,42	cm
Bark=	1,50	cm
t=	4,00	cm

Material properties

MOE=	735,00	kN/cm*cm

wind speed

a=	0,28	
tu=	1,40	
h(g)=	400,00	m

Euler

E=	735,00	kN/cm*cm
h=	17,50	m
r=	20,21	cm
Safety euler=	351,15	%
V euler=	51,46	m/s
	185,24	km/h

Natural frequency

n=	5,80	Hz

"tree-statics"

strength	2,00	kN/cm*cm
safety=	297,99	%
treq=	2,57	cm
t/R required=	0,13	

Brazier

d=	37,42	cm
t=	4,00	cm
E=	735,00	kN/cm*cm
Fbrazier=	454,12	%
V brazier=	58,52	m/s
	210,66	km/h

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Calculations Euler buckling= © Peter Sterken 19/09/2004
Calculations natural frequency= © Peter Sterken 06/03/2005
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COMPARISON

of Euler buckling, natural frequency, bending fractures and Brazier buckling.

Wind load analysis in accordance with Eurocode 1, part 2-4

Species= ***Cupressus sempervirens***

Wind load analysis

h tree=	22,00	m
Cw=	0,20	
A=	60,31	m*m
F=	6,68	kN
P=	11,67	m
M=	77,97	kNm

Geometry

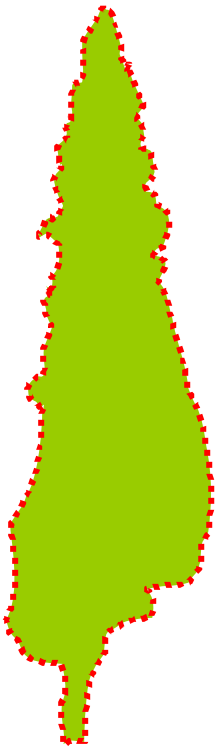
d sound=	40,42	cm
Bark=	1,50	cm
t=	4,00	cm

Material properties

MOE=	735,00	kN/cm*cm

wind speed

a=	0,28	
tu=	1,40	
h(g)=	400,00	m



Euler

E=	735,00	kN/cm*cm
h=	22,00	m
r=	20,21	cm
Safety euler=	98,38	%
V euler=	29,04	m/s
	104,54	km/h

Natural frequency

n=	3,70	Hz

"tree-statics"

strength	2,00	kN/cm*cm
safety=	131,95	%
treq=	7,61	cm
t/R required=	0,38	

Brazier

d=	37,42	cm
t=	4,00	cm
E=	735,00	kN/cm*cm
Fbrazier=	201,08	%
V brazier=	41,51	m/s
	149,45	km/h

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COMPARISON

of Euler buckling, natural frequency, bending fractures and Brazier buckling.

Wind load analysis in accordance with Eurocode 1, part 2-4

Species= ***Cupressus sempervirens***

Wind load analysis

h tree=	23,78	m
Cw=	0,20	
A=	70,47	m*m
F=	8,15	kN
P=	12,62	m
M=	102,86	kNm

Geometry

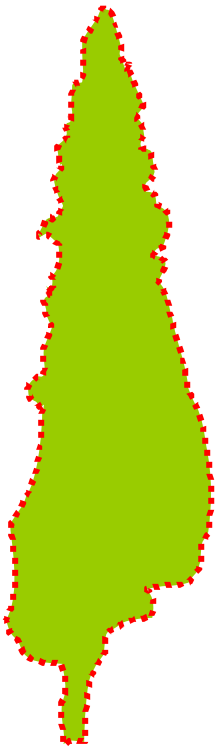
d sound=	40,42	cm
Bark=	1,50	cm
t=	4,00	cm

Material properties

MOE=	735,00	kN/cm*cm

wind speed

a=	0,28	
tu=	1,40	
h(g)=	400,00	m



Euler

E=	735,00	kN/cm*cm
h=	23,78	m
r=	20,21	cm
Safety euler=	63,83	%
V euler=	23,91	m/s
	86,06	km/h

Natural frequency

n=	3,20	Hz

"tree-statics"

strength	2,00	kN/cm*cm
safety=	100,02	%
treq=	18,95	cm
t/R required=	0,94	

Brazier

d=	37,42	cm
t=	4,00	cm
E=	735,00	kN/cm*cm
Fbrazier=	152,43	%
V brazier=	36,94	m/s
	132,99	km/h

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