Land supply and Ricardian rent

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Abstract

This paper investigates opportunities to improve the land market in GTAP. Different theoretical foundations for land supply, land substitution and land intensification are investigated and related with the current approaches for land use modeling in the LEI-version of GTAP, LEITAP.

Introduction

Biofuels are blamed for hunger, an increase in land use and a decrease in global biodiversity. Although the current high levels of food prices may have to do something with the increase in biofuel use, the main backgrounds are increases in animal feed demand as a consequence of the fast growth of China and India and temporary harvest problems. In order to analyze the long-term effects of biofuels, land use and dynamics of the long-term agricultural markets are essential. Banse et al (2008) used a GTAP world trade model with endogenous land supply and a constant elasticity of transformation (CET) nesting structure to distribute land supply over agricultural sectors in order to analyze the effects of an increase in the use of biofuels for land use and agricultural prices (Van Meijl et al, 2006). The land supply curves of Banse et al are derived from biophysical data, assuming that there is a direct relationship between land prices and the marginal productivity of land. Because additional land use has a lower productivity than the average land use, and GTAP is using average productivities, the average productivity of land use in GTAP is iterated with calculations by the land allocation model IMAGE (MNP, 2006) in order to correct for the decrease in average land productivity with increasing land use. The disadvantage of this procedure is that GTAP will interpret a decrease in land productivity as a decrease in relative cost of land services, and therefore will intensify agricultural production. For this effect there is no economic rationale, and this problem has to be solved.

Also the use of a CET land function has some problems, because it implies that the number of hectares changes when one land use is substituted for another land use (Baltzer and Kløverpris, 2008). Baltzer and Kløverpris solve this problem by correcting for this factor, implying that average productivity for all types of land remains the same. This solves the problem of the inconsistency between hectares and economic units of land, but is a-theoretic in character. Therefore, a deeper theoretical underpinning may be worthwhile.

This paper develops the theory of the land market more in depth in order to improve the implementation of the land market in GTAP.

Ricardian rent theory and the land supply function

Ricardian rent theory is based on differences in land productivity. Biophysical land allocation models have this information available. Van Meijl et al (2006) use this information to construct land supply curves. The IMAGE model (MNP, 2006) allocates biophysical land productivity per grid cell per crop. If you order all grid cells from high to low productivity, a marginal productivity curve can be constructed. If labor and capital cost per ha is the same on all land, then marginal cost (i.e. cost per unit of product on an extra hectare) is inversely related to land productivity. According to Ricardian rent theory product price is determined by marginal cost (where land rent is 0 on the marginal land), and therefore inversely related with marginal cost. For the non-marginal land the land rent increases when more land is used, because the sales price of the produced product increases. This determines the supply curve of land (figure 1).
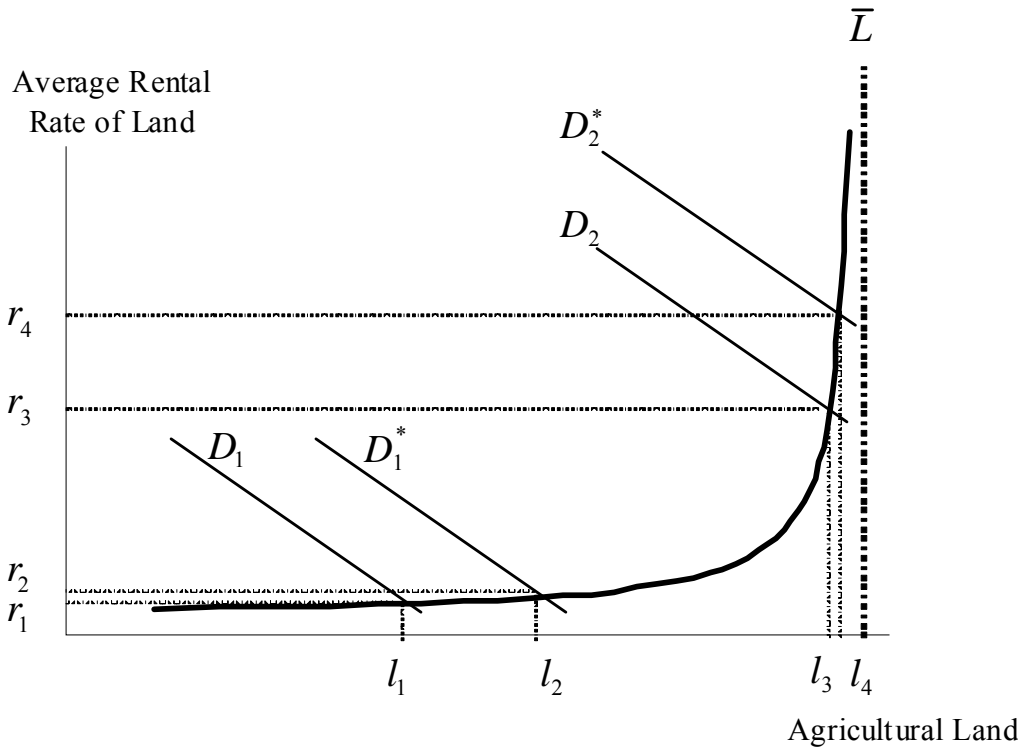
Figure 1 Land supply curve

Because GTAP uses average land productivities, the procedure above implies that the average land productivity should decrease when more land is used. This change in average land productivity is calculated in the IMAGE model, based on GTAP land use results. Then the average productivity of land is shocked in the GTAP model as a shock on land endowment productivity. This not only generates the required change in average productivity, but also creates an incentive to substitute away from land use with this lower productivity. So, the GTAP model generates intensification as a consequence of the use of extra marginal land. This intensification is additional to the intensification generated by the increase of average land rent. This additional effect on land intensification has no theoretical meaning at all, but as long as the effect of the increase in land price is also very rough, the inconsistency may not be very important from an empirical point of view.

According to the theory behind the derived supply curve, a more consistent procedure to adjust the land productivity changes would be to change output productivity with the adjustments that were generated by IMAGE. But why doing the IMAGE-LEITAP iteration procedure while most of the information is already in the land supply curve? By definition a change in land use implies that from an economic point the extra hectares are less useful. Therefore, the hectares could be adjusted for the change in marginal productivity. If the marginal hectare has a 10% lower productivity than the average hectare, the marginal hectare could count for 0.9 average hectares. Although this is not correct from a theoretical point of view, because the marginal hectare has by definition a rent of 0, in practice the extra amount paid as land rent may be seen as representing the difference in productivity. But this is not automatically correct. For this reason, we first try to develop a theory of land productivity based on Ricardian rent theory.

Correction for difference between aveerage and marginal land productivity

In figure … average and marginal productivity for land in Brazil is presented. When we divide average productivity by marginal productivity this shows how much extra land is required compared with the

original land. At current land use, this factor for Brazil is about 1.34. This would imply that the calculated percentage change in land use should be multiplied with 1.34 to generate the correct land use.
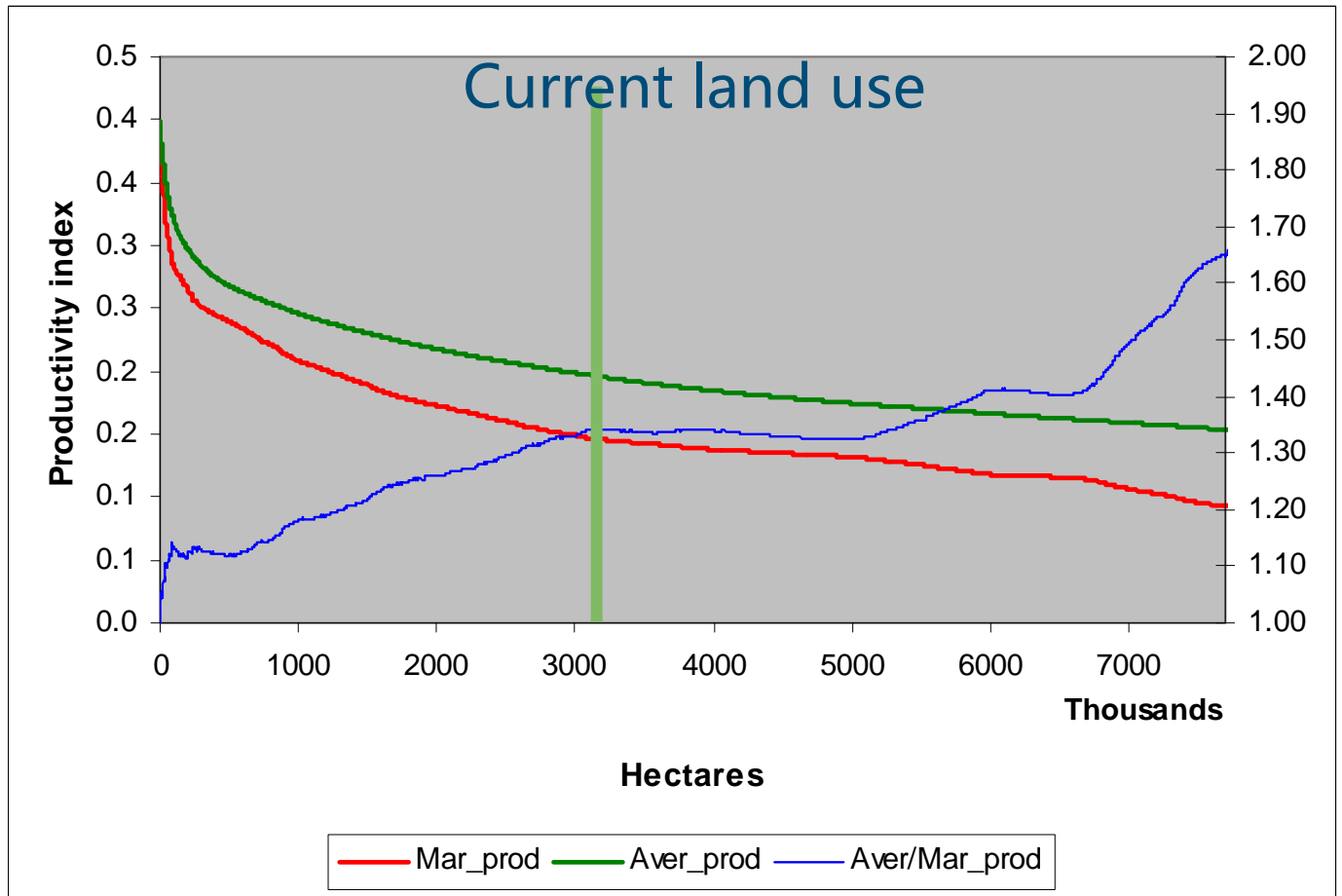
But this is not automatically correct. It assumes that all types of land have the same relation between marginal and average productivity. This should be checked. Furthermore, it assumes that the amount of other production factors per unit of product remains the same. This is consistent with the idea that marginal lands are farmed less intensively, but it is not automatically that the intensity is the inverse of land productivity. To investigate under what conditions this works, a further theoretical underpinning is required. Because the implicit theory behind the land supply curve is Ricardian theory, we first dig shortly in the basic idea of Ricardian theory, and then investigate
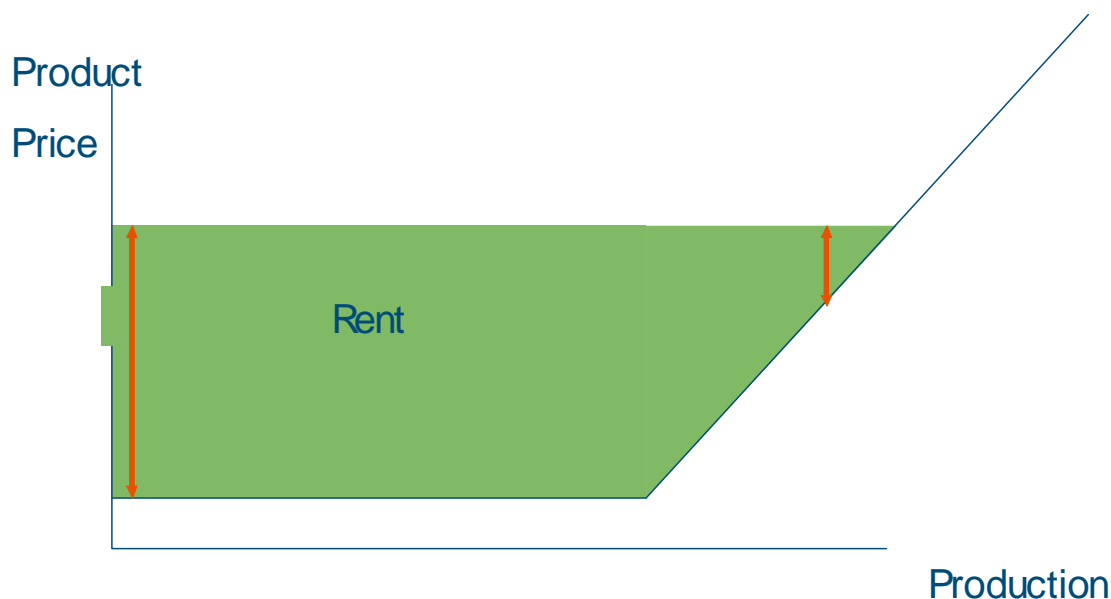
A short discussion of Ricardian rent theory

Figure 1. Land rent per unit of product.

Ricardian rent theory assumes differences in productivity between different types of land. When production in a country increases, this production can either be realized by extending production on new land, or intensifying production elsewhere. If it is realized by developing new land, then it will be used with minimum costs and a zero rent. The production per unit of product on the extra developed land, assumed to be land with the a low productivity, determines the price of the end product. So, the rent on marginal land is zero by definition, while the other rents per unit of product are the difference between price and optimal use of land. Because a higher prices increases the marginal revenue of non-marginal land, marginal cost can be higher, too. So, when more marginal lands are used, land productivity on non-marginal land increases. So, the market price of the end product of land depends on the productivity of marginal land, and this generates the supply curve for the end product. Land rent depends indirectly on demand for the end product.
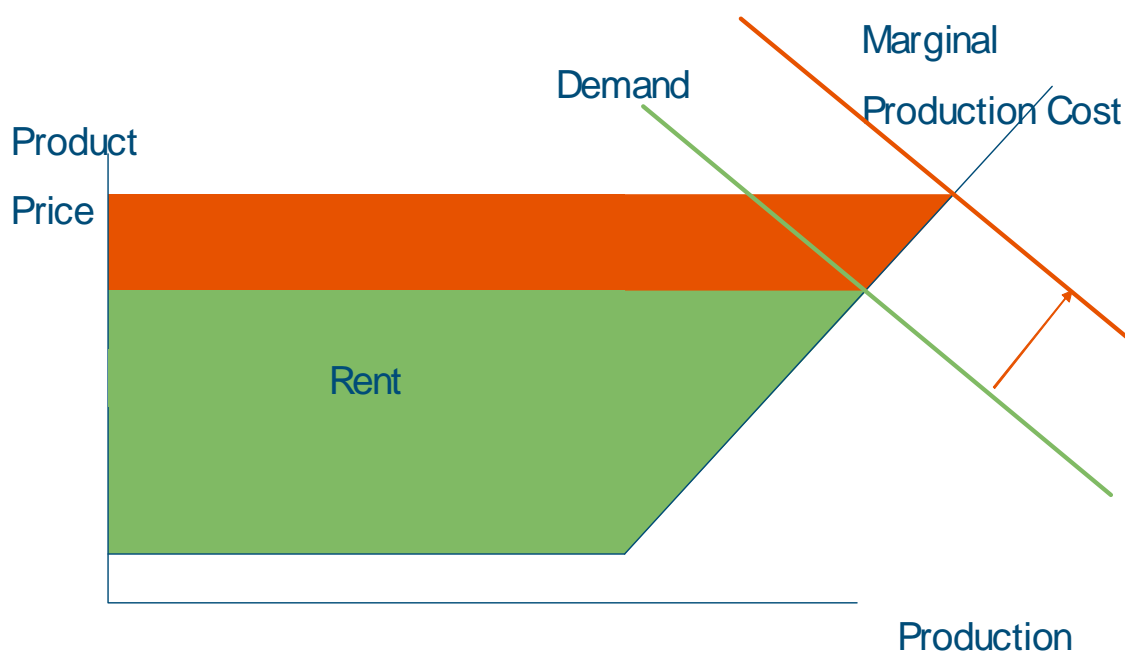
Figure 2. Increase in demand generates an increase in land rent per unit of product

Below we will develop the theory of rent, assuming first a Cobb Douglass production function, and the more general CES production function that is used in the LEITAP model.

The Cobb Douglass Production function

For simplicity of derivation we first use the Cobb Douglass production function for our derivations. This simplifies the exposition, while the main conclusions can be derived. We start with the production function:

$$Q = (\alpha_N N)^\beta (\alpha_L L)^{1-\beta}$$

Where Q = production, N = amount of variable non-land production factors, F = the amount of fixed non-land production factors, α_N non-land augmenting technology, α_L land augmenting technology, and β is a parameter. Rewriting this equation per hectare land generates:

$$\frac{Q}{L} = \left(\alpha_N \frac{N}{L}\right)^\beta (\alpha_L)^{1-\beta} \text{ and } \frac{N}{L} = \frac{1}{\alpha_N}\left(\frac{Q}{L}\right)^{\frac{1}{\beta}}(\alpha_L)^{\frac{\beta-1}{\beta}}$$

Assuming that each hectare of land has also a fixed cost F that uses non-land inputs, land rent per hectare R can be defined as

$$R = P\frac{Q}{L} - W\frac{N}{L} - WF$$

Where P is the price of the output, and W is the price of the non-land inputs. Assuming that land rent is maximized, with given rewards for non-land inputs and outputs, we get:

$$P\frac{\partial Q}{\partial N} - W = 0$$

$$P\beta\frac{Q}{N} - W = 0$$

$$\frac{Q}{L} = \frac{W}{\beta P}\frac{N}{L} \qquad \frac{Q}{L} = \alpha_N^{\frac{\beta}{1-\beta}}\beta^{\frac{\beta}{1-\beta}}\left(\frac{W}{P}\right)^{\frac{\beta}{\beta-1}}\alpha_L$$

$$\frac{N}{L} = \frac{Q}{L}\frac{\beta P}{W} \qquad \frac{N}{L} = \alpha_N^{\frac{\beta}{1-\beta}}\beta^{\frac{1}{1-\beta}}\left(\frac{W}{P}\right)^{\frac{1}{\beta-1}}\alpha_L$$

This implies that implicitly it is assumed that at constant input- and output prices output per hectare changes with the same percentage. If only α_L differs between different types of land, then measured percentage productivity differences indicate also percentage differences in α_L; otherwise the relationship is a little bit more complicated.

From these results we may derive the effect of price changes on land productivity:

$$\frac{\partial\left(\frac{Q}{L}\right)}{\partial P} = \frac{\beta}{1-\beta}\alpha_N^{\frac{\beta}{1-\beta}}\beta^{\frac{\beta}{1-\beta}}W^{\frac{\beta}{\beta-1}}\alpha_L P^{\frac{1}{\beta-1}}$$

$$\frac{P\partial\left(\frac{Q}{L}\right)}{\frac{Q}{L}\partial P} = \frac{\beta}{1-\beta}$$

and on the use of non-land inputs:

$$\frac{\partial\left(\frac{N}{L}\right)}{\partial P} = \frac{1}{1-\beta}\alpha_N^{\frac{\beta}{1-\beta}}\beta^{\frac{\beta}{1-\beta}}W^{\frac{1}{\beta-1}}\alpha_L P^{\frac{\beta}{1-\beta}}$$

$$\frac{P\partial\left(\frac{N}{L}\right)}{\frac{N}{L}\partial P} = \frac{1}{1-\beta}$$

For the marginal land holds R=0. But we have to be aware that the function always provides a positive rent for very small N because input productivity reaches infinity when N approaches 0. Therefore, we have to introduce fixed non-land inputs F, where we assume for simplicity that this has the same factor price W as non-land variable inputs N (i.e. W). These fixed non-land inputs were not relevant for the production decision on land as long as it was decided to put the land into use. But in the decision to put the land into use, it is relevant. We assume that differences in land productivity may be explained by a combination of α_L and α_N, and we denote these coefficients for the marginal lands as: $\overline{\alpha}_L$ and $\overline{\alpha}_N$.

So: $R = P\frac{Q}{L} - W\frac{N}{L} - W\frac{F}{L} = 0$

$$R = P\overline{\alpha}_N^{\frac{\beta}{1-\beta}}\beta^{\frac{\beta}{1-\beta}}\left(\frac{W}{P}\right)^{\frac{\beta}{\beta-1}}\overline{\alpha}_L - W\overline{\alpha}_N^{\frac{\beta}{1-\beta}}\beta^{\frac{1}{1-\beta}}\left(\frac{W}{P}\right)^{\frac{1}{\beta-1}}\overline{\alpha}_L - W\frac{F}{L} = 0$$

$$P = W\left(\frac{F}{L}\right)^{1-\beta}\left[(1-\beta)^{\beta-1}\beta^{-\beta}\right]\overline{\alpha}_N^{-\beta}\overline{\alpha}_L^{\beta-1}$$

It may be interesting to check what this implies for the price elasticity of decreasing productivity of marginal lands:

$$\frac{\partial P}{\partial\overline{\alpha}_L} = (\beta-1)W\left(\frac{F}{L}\right)^{1-\beta}\left[(1-\beta)^{\beta-1}\beta^{-\beta}\right]\overline{\alpha}_N^{-\beta}\overline{\alpha}_L^{\beta-2}$$

$$\frac{\overline{\alpha}_L\partial P}{P\partial\overline{\alpha}_L} = (\beta-1)$$

Combined with the elasticity for land productivity on non-marginal lands, we get:

$$\frac{\overline{\alpha}_L\partial\left(\frac{Q}{L}\right)}{\frac{Q}{L}\partial\overline{\alpha}_L} = \frac{P\partial\left(\frac{Q}{L}\right)}{\frac{Q}{L}\partial P}\frac{\overline{\alpha}_L\partial P}{P\partial\overline{\alpha}_L} = \frac{\beta}{1-\beta}(\beta-1) = -\beta$$

This is an extremely simple and elegant result. It holds if more marginal land uses per unit of product the same amount of non-land inputs. But this is not always a plausible assumption. We may interpret this is as follows. If land saving productivity on the marginal land is higher, then market price will be lower. Also

non-land saving technological change will reduce the output price. But the factor price for non-land and the size of the fixed costs raises output price. This is all very intuitive.

In the same we holds for changes in productivity of non-land inputs:

$$\frac{\partial P}{\partial \overline{\alpha}_N} = -\beta W \left(\frac{F}{L}\right)^{1-\beta} \left[(1-\beta)^{\beta-1} \beta^{-\beta}\right] \overline{\alpha}_N^{-\beta-1} \overline{\alpha}_L^{\beta-1}$$

$$\frac{\overline{\alpha}_N \partial P}{P \partial \overline{\alpha}_N} = -\beta$$

And

$$\frac{\overline{\alpha}_L \partial \left(\frac{Q}{L}\right)}{\frac{Q}{L} \partial \overline{\alpha}_L} = \frac{P \partial \left(\frac{Q}{L}\right)}{\frac{Q}{L} \partial P} \frac{\overline{\alpha}_L \partial P}{P \partial \overline{\alpha}_L} = \frac{\beta}{1-\beta}(-\beta) = \frac{\beta^2}{\beta - 1}$$

The next step is to relate the output price to the land rent of fertile land.

$$R = P\frac{Q}{L} - W\frac{N}{L} - W\frac{F}{L}$$

$$R = P\alpha_N^{\frac{\beta}{1-\beta}} \left(\frac{W}{P}\right)^{\frac{\beta}{\beta-1}} \alpha_L \left[\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right] - W\frac{F}{L}$$

$$R = W\frac{F}{L}\left[\frac{\alpha_L}{\overline{\alpha}_L}\left(\frac{\alpha_N}{\overline{\alpha}_N}\right)^{\frac{\beta}{1-\beta}} - 1\right]$$

If we differentiate this formula to the two productivities, then

$$dR = -W\frac{F}{L}\frac{\alpha_L}{\overline{\alpha}_L}\left(\frac{\alpha_N}{\overline{\alpha}_N}\right)^{\frac{\beta}{1-\beta}}\frac{1}{\alpha_L}d\overline{\alpha}_L - \frac{\beta}{1-\beta}W\frac{F}{L}\frac{\alpha_L}{\overline{\alpha}_L}\left(\frac{\alpha_N}{\overline{\alpha}_N}\right)^{\frac{\beta}{1-\beta}}\frac{1}{\alpha_N}d\overline{\alpha}_N$$

$$\frac{\partial R}{\partial \overline{\alpha}_L}\frac{\overline{\alpha}_L}{R} = \frac{-W\frac{F}{L}\frac{\alpha_L}{\overline{\alpha}_L}\left(\frac{\alpha_N}{\overline{\alpha}_N}\right)^{\frac{\beta}{1-\beta}}}{W\frac{F}{L}\left[\frac{\alpha_L}{\overline{\alpha}_L}\left(\frac{\alpha_N}{\overline{\alpha}_N}\right)^{\frac{\beta}{1-\beta}} - 1\right]} = -\frac{1}{1 - \frac{\alpha_L}{\overline{\alpha}_L}\left(\frac{\alpha_N}{\overline{\alpha}_N}\right)^{\frac{\beta}{\beta-1}}}$$

$$\frac{\partial R}{\partial \overline{\alpha}_N}\frac{\overline{\alpha}_N}{R} = -\frac{\beta}{1-\beta}\frac{-W\frac{F}{L}\frac{\alpha_L}{\overline{\alpha}_L}\left(\frac{\alpha_N}{\overline{\alpha}_N}\right)^{\frac{\beta}{1-\beta}}}{W\frac{F}{L}\left[\frac{\alpha_L}{\overline{\alpha}_L}\left(\frac{\alpha_N}{\overline{\alpha}_N}\right)^{\frac{\beta}{1-\beta}} - 1\right]} = -\frac{\beta}{1-\beta}\frac{1}{1 - \frac{\alpha_L}{\overline{\alpha}_L}\left(\frac{\alpha_N}{\overline{\alpha}_N}\right)^{\frac{\beta}{\beta-1}}}$$

This elasticity of rent on the change in marginal land productivity depends in the Cobb Douglass case on the distance in productivity and the shares of payments. The elasticity is minus infinity at the marginal land, which is the case by definition, because at the marginal land the rent equals 0.

What is the effect of using more marginal land on the productivity of the other land?

$$\frac{\partial\left(\frac{Q}{L}\right)}{\partial\overline{\alpha}_L}\frac{\overline{\alpha}_L}{\frac{Q}{L}} = \frac{\partial\left(\frac{Q}{L}\right)}{\partial P}\frac{P}{\frac{Q}{L}}\frac{\partial P}{\partial\overline{\alpha}_L}\frac{\overline{\alpha}_L}{P} = \frac{\beta}{1-\beta}(\beta-1) = \beta$$

$$\frac{\partial\left(\frac{Q}{L}\right)}{\partial\overline{\alpha}_N}\frac{\overline{\alpha}_N}{\frac{Q}{L}} = \frac{\partial\left(\frac{Q}{L}\right)}{\partial P}\frac{P}{\frac{Q}{L}}\frac{\partial P}{\partial\overline{\alpha}_N}\frac{\overline{\alpha}_N}{P} = \frac{\beta}{1-\beta}(-\beta) = \frac{-\beta^2}{\beta-1}$$

So, although the elasticities of land productivity of marginal lands on rent for non-marginal lands depends on the position of these land non-marginal lands on the land supply curve, this is not true for the effect of productivity of non-marginal lands on the average productivity of non-marginal lands.

What is the consequence of this exercise for the land supply curve? First, in the Cobb-Douglass case the elasticity of land productivity for the change in productivity in marginal land does not depend on the current land productivity of land. This implies that a general formula can be used for all land. But the formula with respect to the land rent shows that the elasticity of land rent is far from independent of the productivity of the land. So, the intermediate variable land rent that is used in the current LEITAP model is not directly defined on the input parameters. But if you know the elasticity of land intensity, the step towards defining the relevant average land rent to accomplish this is an easy one. But before jumping into conclusions, we have to check if the result also holds for the CES function used in the LEITAP model.

The CES Production function

In the LEITAP version of GTAP we are using a CES function between land and non-land inputs (with further nests for both inputs). Therefore, it is important to derive further results consistent with this general CES function.

$$Q = \left[\delta(\alpha_N N)^\gamma + (1-\delta)(\alpha_L L)^\gamma\right]^{\frac{1}{\gamma}} \tag{1}$$

$$\alpha_L, \alpha_N > 0; 0 < \delta < 1; \gamma < 1$$

Where the elasticity of substitution is defined as $\dfrac{1}{1-\gamma}$

Land rent:

$$R = PQ - WN - \text{WFL}$$

Land rent is maximized given available land leads to:

$$P\frac{\partial Q}{\partial N} = W$$

So,

$$P\frac{\partial Q}{\partial N} = P\delta\alpha_N{}^\gamma\left(\frac{Q}{N}\right)^{1-\gamma} = W$$

$$N = \left(\frac{P\delta\alpha_N^\gamma}{W}\right)^{\frac{1}{1-\gamma}} Q \qquad (2)$$

Now we substitute N in (1) by (2):

$$Q = \left[\delta\left(\alpha_N\left(\frac{P\delta\alpha_N{}^\gamma}{W}\right)^{\frac{1}{1-\gamma}}Q\right)^\gamma + (1-\delta)(\alpha_L L)^\gamma\right]^{\frac{1}{\gamma}}$$

$$\left(\frac{Q}{L}\right) = \frac{(1-\delta)^{\frac{1}{\gamma}}\alpha_L}{\left(1-\left(\delta^{\frac{1}{\gamma}}\alpha_N\frac{P}{W}\right)^{\frac{\gamma}{1-\gamma}}\right)^{\frac{1}{\gamma}}} \qquad (3)$$

Therefore from (3), output per hectare changes with the same percentage as land augmenting technology α_L, but has a much more complex relationship with non-land augmenting technology α_N.

In the same way we can derive the land intensity depending on prices and technology. From (2) and (3) follows

$$N = \left(\frac{P\delta\alpha_N{}^\gamma}{W}\right)^{\frac{1}{1-\gamma}}\frac{(1-\delta)^{\frac{1}{\gamma}}\alpha_L}{\left(1-\left(\delta^{\frac{1}{\gamma}}\alpha_N\frac{P}{W}\right)^{\frac{\gamma}{1-\gamma}}\right)^{\frac{1}{\gamma}}} L$$

so:

$$\left(\frac{N}{L}\right) = \frac{(1-\delta)^{\frac{1}{\gamma}} \alpha_L}{\left[\left(\frac{P\delta^{\frac{1}{\gamma}} \alpha_N}{W}\right)^{\frac{-\gamma}{1-\gamma}} - 1\right]^{\frac{1}{\gamma}}}$$ (4)

To calculate the elasticity of land productivity with respect to the change in output price, we rewrite (3):

$$\left(\frac{Q}{L}\right) = (1-\delta)^{\frac{1}{\gamma}} \alpha_L \left(1 - \delta^{\frac{1}{1-\gamma}}\left(\alpha_N \frac{P}{W}\right)^{\frac{\gamma}{1-\gamma}}\right)^{-\frac{1}{\gamma}}$$ (5)

Differentiating this function with respect to price gvies:

Substituting (5) in this equation gives:

$$\frac{\partial\left(\frac{Q}{L}\right)}{\partial P} = \frac{1}{1-\gamma}(1-\delta)^{\frac{1}{\gamma}} \alpha_L \delta^{\frac{1}{1-\gamma}} \frac{\left(\alpha_N \frac{P}{W}\right)^{\frac{\gamma}{1-\gamma}}}{P} \left(\left(\frac{Q}{L}\right)^{-\gamma}(1-\delta)\alpha_L^{\gamma}\right)^{\frac{-(1+\gamma)}{\gamma}}$$

$$\frac{\partial\left(\frac{Q}{L}\right)}{\partial P} = \frac{1}{1-\gamma}\delta^{\frac{1}{1-\gamma}} \frac{\left(\alpha_N \frac{P}{W}\right)^{\frac{\gamma}{1-\gamma}}}{P} \left(\frac{Q}{L}\right)^{1}\left(\frac{Q}{L}\right)^{\gamma}(1-\delta)^{-1}\alpha_L^{-\gamma}$$

Rewriting this by substituting $\left(\frac{Q}{L}\right)^{\gamma}$ using (3) gives the elasticity of land productivity with respect to land output price change:

$$\frac{P\partial\left(\frac{Q}{L}\right)}{\left(\frac{Q}{L}\right)\partial P} = \frac{1}{1-\gamma}\frac{1}{\left[\left(\delta^{\frac{1}{\gamma}}\alpha_N \frac{P}{W}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]}$$ (6)

If $\gamma = 0$ we have the Cobb-Douglas function case:

$$\frac{P\partial\left(\dfrac{Q}{L}\right)}{\left(\dfrac{Q}{L}\right)\partial P} = \frac{1}{\left(\delta^{-1}-1\right)} = \frac{\delta}{\left(1-\delta\right)}$$

So, the elasticity of land productivity with respect to land output price change does not depend on land saving technology α_L, but in contrast with the Cobb Douglass case does depend on non-land saving technology α_N.

The next step is derive the price for the productivity of marginal land. We start again with the rent function.

$$R = PQ - WN - WFL = 0$$

Substituting (2) gives:

$$P\frac{Q}{L}L - W\left(\frac{P\delta\overline{\alpha}_N{}^{\gamma}}{W}\right)^{\frac{1}{1-\gamma}}\frac{Q}{L}L - WF = 0$$

Substituting (3):

$$LP\left(1 - \delta^{\frac{1}{1-\gamma}}\left(\overline{\alpha}_N\frac{P}{W}\right)^{\frac{\gamma}{1-\gamma}}\right)\frac{(1-\delta)^{\frac{1}{\gamma}}\overline{\alpha}_L}{\left(1 - \delta^{\frac{1}{1-\gamma}}\left(\overline{\alpha}_N\frac{P}{W}\right)^{\frac{\gamma}{1-\gamma}}\right)^{\frac{1}{\gamma}}} = WF$$

$$P = W\left(\left(\frac{L}{F}\right)^{\frac{\gamma}{1-\gamma}}(1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}} + \delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}}\right)^{\frac{\gamma-1}{\gamma}} \qquad (9)$$

Now we can calculate the price elasticity with respect the productivity on marginal lands:

$$\frac{\partial P}{\partial \overline{\alpha}_L} = W\frac{\gamma-1}{\gamma}\left(\left(\frac{P}{W}\right)^{\frac{\gamma}{\gamma-1}}\right)^{-\frac{1}{\gamma}}\frac{\gamma}{1-\gamma}\left(\frac{L}{F}\right)^{\frac{\gamma}{1-\gamma}}(1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}\alpha_L^{-1}$$

$$\frac{\overline{\alpha}_L\partial P}{P\partial \overline{\alpha}_L} = -\left(\frac{P}{W}\right)^{\frac{\gamma}{1-\gamma}}\left((1-\delta)^{\frac{1}{\gamma}}\overline{\alpha}_L\frac{L}{F}\right)^{\frac{\gamma}{1-\gamma}} \qquad (10)$$

So, the elasticity is

$$\frac{\overline{\alpha}_L \partial P}{P \partial \overline{\alpha}_L} = -\left((1-\delta)^{\frac{1}{\gamma}} \overline{\alpha}_L \frac{PL}{WF} \right)^{\frac{\gamma}{1-\gamma}}$$

and then substituting $\left(\dfrac{P}{W} \right)^{\frac{\gamma}{1-\gamma}}$ in (10) using (8),

$$\frac{\overline{\alpha}_L \partial P}{P \partial \overline{\alpha}_L} = -\left((1-\delta)^{\frac{1}{\gamma}} \overline{\alpha}_L \frac{L}{F} \right)^{\frac{\gamma}{1-\gamma}} \left(\left(\frac{L}{F} \right)^{\frac{\gamma}{1-\gamma}} (1-\delta)^{\frac{1}{1-\gamma}} \overline{\alpha}_L^{\frac{\gamma}{1-\gamma}} + \delta^{\frac{1}{1-\gamma}} \overline{\alpha}_N^{\frac{\gamma}{1-\gamma}} \right)^{-1} \tag{12}$$

and this can be simplified into:

$$\frac{\overline{\alpha}_L \partial P}{P \partial \overline{\alpha}_L} = -\left(1 + \left(\left(\frac{1-\delta}{\delta} \right)^{\frac{1}{\gamma}} \frac{\overline{\alpha}_L}{\overline{\alpha}_N} \frac{L}{F} \right)^{\frac{\gamma}{\gamma-1}} \right)^{-1} \tag{14}$$

Applying $\gamma = 0$ to (12), we have the Cobb-Douglas function case:

$$\frac{\overline{\alpha}_L \partial P}{P \partial \overline{\alpha}_L} = \frac{-\left((1-\delta)^{\frac{1}{\gamma}} \overline{\alpha}_L \frac{L}{F} \right)^{\frac{\gamma}{1-\gamma}}}{\left(\left(\frac{L}{F} \right)^{\frac{\gamma}{1-\gamma}} (1-\delta)^{\frac{1}{1-\gamma}} \overline{\alpha}_L^{\frac{\gamma}{1-\gamma}} + \delta^{\frac{1}{1-\gamma}} \overline{\alpha}_N^{\frac{\gamma}{1-\gamma}} \right)} = \frac{-(1-\delta)}{((1-\delta)+\delta)} = (\delta - 1)$$

To calculate the elasticity of land productivity with respect of marginal land productivity, we combine (6) and (14):

$$\frac{\overline{\alpha}_L \partial \left(\frac{Q}{L} \right)}{\left(\frac{Q}{L} \right) \partial \overline{\alpha}_L} = \frac{P \partial \left(\frac{Q}{L} \right)}{\left(\frac{Q}{L} \right) \partial P} \frac{\overline{\alpha}_L \partial P}{P \partial \overline{\alpha}_L} = \frac{1}{1-\gamma} \frac{-\left(1 + \left(\left(\frac{1-\delta}{\delta} \right)^{\frac{1}{\gamma}} \frac{\overline{\alpha}_L}{\overline{\alpha}_N} \frac{L}{F} \right)^{\frac{\gamma}{\gamma-1}} \right)^{-1}}{\left(\left(\delta^{\frac{1}{\gamma}} \alpha_N \frac{P}{W} \right)^{\frac{\gamma}{\gamma-1}} - 1 \right)}$$

Substituting $\left(\dfrac{P}{W}\right)^{\frac{\gamma}{\gamma-1}}$ using (8)

$$\frac{\overline{\alpha}_L \partial\left(\dfrac{Q}{L}\right)}{\left(\dfrac{Q}{L}\right)\partial\overline{\alpha}_L} = -\frac{1}{(1-\gamma)}\left(\frac{\alpha_N}{\overline{\alpha}_N}\right)^{\frac{\gamma}{1-\gamma}}\frac{1}{\left(\dfrac{1-\delta}{\delta}\right)^{\frac{1}{1-\gamma}}\left(\dfrac{\overline{\alpha}_L}{\overline{\alpha}_N}\dfrac{L}{F}\right)^{\frac{\gamma}{1-\gamma}}+1} \tag{14}$$

This seems to be a very complex relationship, but [provides the opportunity to conclude that the elasticity of productivity with respect to land saving technology for marginal lands is indpendent of the position on the land supply curve. So, in that case a general factor could be used.

But the situation is more complex if

The next step is to relate the output price to the land rent of fertile land.

$$R = PQ - WN - WF$$

Similarly to (7):

$$R = LP\frac{(1-\delta)^{\frac{1}{\gamma}}\alpha_L}{\left(1-\delta^{\frac{1}{1-\gamma}}\left(\alpha_N\dfrac{P}{W}\right)^{\frac{\gamma}{1-\gamma}}\right)^{\frac{1-\gamma}{\gamma}}} - WF$$

Substituting $\left(\dfrac{P}{W}\right)^{\frac{\gamma}{1-\gamma}}$ using (8) and P using (9)

$$R = \frac{(1-\delta)^{\frac{1}{\gamma}}\alpha_L LW\left(\left(\dfrac{L}{F}\right)^{\frac{\gamma}{1-\gamma}}(1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}+\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}}\right)^{\frac{\gamma-1}{\gamma}}}{\left(1-\delta^{\frac{1}{1-\gamma}}\alpha_N^{\frac{\gamma}{1-\gamma}}\left(\left(\dfrac{L}{F}\right)^{\frac{\gamma}{1-\gamma}}(1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}+\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}}\right)^{-1}\right)^{\frac{1-\gamma}{\gamma}}} - WF$$

$$R = W\left[(1-\delta)^{\frac{1}{\gamma}}\alpha_L\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}} + \left(\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}} - \delta^{\frac{1}{1-\gamma}}\alpha_N^{\frac{\gamma}{1-\gamma}}\right)\left(\frac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1-\gamma}{-\gamma}} - 1\right]$$

To calculate the elasticity of rent on fertile land with respect to technology on marginal land, we get:

$$\frac{\partial R}{\partial \overline{\alpha}_L} = \frac{1-\gamma}{-\gamma}(1-\delta)^{\frac{1}{1-\gamma}}\frac{\gamma}{1-\gamma}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}-1}W(1-\delta)^{\frac{1}{\gamma}}\alpha_L\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}} + \left(\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}} - \delta^{\frac{1}{1-\gamma}}\alpha_N^{\frac{\gamma}{1-\gamma}}\right)\left(\frac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1-\gamma}{-\gamma}-1}$$

$$\frac{\partial R}{\partial \overline{\alpha}_L} = -(1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}-1}W(1-\delta)^{\frac{1}{\gamma}}\alpha_L\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}} + \left(\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}} - \delta^{\frac{1}{1-\gamma}}\alpha_N^{\frac{\gamma}{1-\gamma}}\right)\left(\frac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1}{-\gamma}}$$

$$\frac{\overline{\alpha}_L\partial R}{R\partial \overline{\alpha}_L} = \frac{-(1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}W(1-\delta)^{\frac{1}{\gamma}}\alpha_L\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}} + \left(\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}} - \delta^{\frac{1}{1-\gamma}}\alpha_N^{\frac{\gamma}{1-\gamma}}\right)\left(\frac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1}{-\gamma}}}{W\left[(1-\delta)^{\frac{1}{\gamma}}\alpha_L\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}} + \left(\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}} - \delta^{\frac{1}{1-\gamma}}\alpha_N^{\frac{\gamma}{1-\gamma}}\right)\left(\frac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1-\gamma}{-\gamma}} - 1\right]}$$

$$\frac{\overline{\alpha}_L\partial R}{R\partial \overline{\alpha}_L} = \frac{-(1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}} + \left(\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}} - \delta^{\frac{1}{1-\gamma}}\alpha_N^{\frac{\gamma}{1-\gamma}}\right)\left(\frac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1}{-\gamma}}}{\left(\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}} + \left(\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}} - \delta^{\frac{1}{1-\gamma}}\alpha_N^{\frac{\gamma}{1-\gamma}}\right)\left(\frac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1-\gamma}{-\gamma}} - (1-\delta)^{\frac{1}{\gamma}}\alpha_L^{-1}\right)}$$

If $\overline{\alpha}_N = \alpha_N$ then

$$\frac{\overline{\alpha}_L\partial R}{R\partial \overline{\alpha}_L} = \frac{-(1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}(1-\delta)^{\frac{1}{\gamma}}\alpha_L\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}\right)^{\frac{1}{-\gamma}}}{\left((1-\delta)^{\frac{1}{\gamma}}\alpha_L\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}\right)^{\frac{1-\gamma}{-\gamma}} - 1\right)}$$

$$\frac{\overline{\alpha}_L \partial R}{R \partial \overline{\alpha}_L} = \frac{\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}}{\left(\dfrac{\overline{\alpha}_L}{\alpha_L} - 1\right)}$$

If $\gamma = 0$ then we have Cobb-Douglas case:

$$\frac{\overline{\alpha}_L \partial R}{R \partial \overline{\alpha}_L} = \frac{1}{\left(\dfrac{\overline{\alpha}_L}{\alpha_L} - 1\right)}$$

Now we may calculate the elasticities for the non-land inputs. Starting from (4)

$$\left(\frac{N}{L}\right) = \frac{(1-\delta)^{\frac{1}{\gamma}}\alpha_L}{\left(\left(\frac{P\delta^{\frac{1}{\gamma}}\alpha_N}{W}\right)^{\frac{-\gamma}{1-\gamma}} - 1\right)^{\frac{1}{\gamma}}} = (1-\delta)^{\frac{1}{\gamma}}\alpha_L\left(\left(\frac{P\delta^{\frac{1}{\gamma}}\alpha_N}{W}\right)^{\frac{-\gamma}{1-\gamma}} - 1\right)^{\frac{-1}{\gamma}}$$

So,

$$\frac{\partial\left(\frac{N}{L}\right)}{\partial P} = \frac{-1}{\gamma}\frac{-\gamma}{1-\gamma}(1-\delta)^{\frac{1}{\gamma}}\alpha_L\frac{\left(\frac{P\delta^{\frac{1}{\gamma}}\alpha_N}{W}\right)^{\frac{-\gamma}{1-\gamma}}}{P}\left(\left(\frac{P\delta^{\frac{1}{\gamma}}\alpha_N}{W}\right)^{\frac{-\gamma}{1-\gamma}} - 1\right)^{\frac{-1}{\gamma}-1}$$

Since from (4):

$$\left(\left(\frac{P\delta^{\frac{1}{\gamma}}\alpha_N}{W}\right)^{\frac{-\gamma}{1-\gamma}} - 1\right) = \frac{(1-\delta)\ \alpha_L^{\gamma}}{\left(\frac{N}{L}\right)^{\gamma}}$$

then

$$\frac{\partial\left(\frac{N}{L}\right)}{\partial P} = \frac{\gamma}{1-\gamma}(1-\delta)^{\frac{1}{\gamma}}\alpha_L\frac{\left(\frac{P\delta^{\frac{1}{\gamma}}\alpha_N}{W}\right)^{\frac{-\gamma}{1-\gamma}}}{P}\left(\left(\frac{N}{L}\right)^{-\gamma}(1-\delta)\ \alpha_L^{\gamma}\right)^{\frac{-(1+\gamma)}{\gamma}}$$

$$\frac{\partial\left(\frac{N}{L}\right)}{\partial P} = \frac{\gamma}{1-\gamma} \frac{\left(\frac{P\delta^{\frac{1}{\gamma}}\alpha_N}{W}\right)^{\frac{-\gamma}{1-\gamma}}}{P} \left(\frac{N}{L}\right)\left(\frac{N}{L}\right)^{\gamma}(1-\delta)^{-1}\alpha_L^{-\gamma}$$

Using (4):

$$\frac{\partial\left(\frac{N}{L}\right)}{\partial P} = \frac{\gamma}{1-\gamma} \frac{\left(\frac{P\delta^{\frac{1}{\gamma}}\alpha_N}{W}\right)^{\frac{-\gamma}{1-\gamma}}}{P} \left(\frac{N}{L}\right)\left((1-\delta)^{\frac{1}{\gamma}}\alpha_L\left(\left(\frac{P\delta^{\frac{1}{\gamma}}\alpha_N}{W}\right)^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{-1}{\gamma}} - 1\right)^{\gamma}(1-\delta)^{-1}\alpha_L^{-\gamma}$$

$$\frac{P\partial\left(\frac{N}{L}\right)}{\left(\frac{N}{L}\right)\partial P} = \frac{\gamma}{1-\gamma} \frac{1}{\left(1 - \left(\frac{P\delta^{\frac{1}{\gamma}}\alpha_N}{W}\right)^{\frac{\gamma}{1-\gamma}}\right)}$$

If $\gamma = 0$ we have the Cobb-Douglas function case:

$$\frac{P\partial\left(\frac{N}{L}\right)}{\left(\frac{N}{L}\right)\partial P} = \frac{1}{(1-\delta)}$$

Now starting from (9):

$$\frac{\partial P}{\partial\overline{\alpha}_N} = W\frac{\gamma-1}{\gamma}\left(\left(\frac{P}{W}\right)^{\frac{\gamma}{\gamma-1}}\right)^{-\frac{1}{\gamma}}\frac{\gamma}{1-\gamma}\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}}\overline{\alpha}_N^{-1}$$

$$\frac{\partial P}{\partial\overline{\alpha}_N} = -P\left(\frac{P}{W}\right)^{-1}\left(\frac{P}{W}\right)^{\frac{1}{1-\gamma}}\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}}\overline{\alpha}_N^{-1}$$

$$\frac{\overline{\alpha}_N \partial P}{P \partial \overline{\alpha}_N} = -\left(\frac{P}{W}\right)^{\frac{\gamma}{1-\gamma}} \left((\delta^{\gamma} \overline{\alpha}_N)^{\frac{1}{1-\gamma}}\right)^{\frac{\gamma}{1-\gamma}}$$

So, the elasticity is

$$\frac{\overline{\alpha}_N \partial P}{P \partial \overline{\alpha}_N} = -\left((\delta^{\gamma} \overline{\alpha}_N)^{\frac{1}{1-\gamma}} \frac{P}{W}\right)^{\frac{\gamma}{1-\gamma}}$$

and then substituting $\left(\dfrac{P}{W}\right)^{\frac{\gamma}{1-\gamma}}$ in (10) using (8),

$$\frac{\overline{\alpha}_N \partial P}{P \partial \overline{\alpha}_N} = -(\delta^{\frac{1}{\gamma}} \overline{\alpha}_N)^{\frac{\gamma}{1-\gamma}} \left(\left(\frac{L}{F}\right)^{\frac{\gamma}{1-\gamma}} (1-\delta)^{\frac{1}{1-\gamma}} \overline{\alpha}_L^{\frac{\gamma}{1-\gamma}} + \delta^{\frac{1}{1-\gamma}} \overline{\alpha}_N^{\frac{\gamma}{1-\gamma}}\right)^{-1}$$

$$\frac{\overline{\alpha}_N \partial P}{P \partial \overline{\alpha}_N} = -\left(1 + \left(\left(\frac{1-\delta}{\delta}\right)^{\frac{1}{\gamma}} \frac{\overline{\alpha}_L}{\overline{\alpha}_N} \frac{L}{F}\right)^{\frac{\gamma}{1-\gamma}}\right)^{-1}$$

If $\gamma = 0$, then we have Cobb-Douglas case:

$$\frac{\overline{\alpha}_N \partial P}{P \partial \overline{\alpha}_N} = -\left(1 + \frac{1-\delta}{\delta}\right)^{-1} = -\left(\frac{1}{\delta}\right)^{-1} = -\delta$$

To calculate the elasticity of land productivity with respect to marginal land productivity, we combine (6) and (14):

$$\frac{\overline{\alpha}_N \partial \left(\frac{Q}{L}\right)}{\left(\frac{Q}{L}\right) \partial \overline{\alpha}_N} = \frac{P \partial \left(\frac{Q}{L}\right)}{\left(\frac{Q}{L}\right) \partial P} \frac{\overline{\alpha}_N \partial P}{P \partial \overline{\alpha}_N} = \frac{1}{1-\gamma} \frac{-\left(1 + \left(\left(\frac{1-\delta}{\delta}\right)^{\frac{1}{\gamma}} \frac{\overline{\alpha}_L}{\overline{\alpha}_N} \frac{L}{F}\right)^{\frac{\gamma}{1-\gamma}}\right)^{-1}}{\left(\left(\delta^{\frac{1}{\gamma}} \alpha_N \frac{P}{W}\right)^{\frac{\gamma}{\gamma-1}} - 1\right)}$$

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Substituting $\left(\dfrac{P}{W}\right)^{\frac{\gamma}{\gamma-1}}$ using (8)

$$\frac{\overline{\alpha}_N \partial\left(\dfrac{Q}{L}\right)}{\left(\dfrac{Q}{L}\right)\partial\overline{\alpha}_N} = \frac{-1}{1-\gamma}\frac{\left(1+\left(\left(\dfrac{1-\delta}{\delta}\right)^{\frac{1}{\gamma}}\dfrac{\overline{\alpha}_L}{\overline{\alpha}_N}\dfrac{L}{F}\right)^{\frac{\gamma}{1-\gamma}}\right)}{\left(\left(\dfrac{L}{F}\right)^{\frac{\gamma}{1-\gamma}}(1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}+\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}}-1\right)}$$

This is second step in the relationship to be derived. The conclusion must be that also if non-land saving technology changes with the marginality of land, all land productivities change with the same factor.

Finally, to calculate rent elasticity in respect to the marginal of non-land factor productivity, we start with R:

$$R = W\left((1-\delta)^{\frac{1}{\gamma}}\alpha_L\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}+\left(\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}}-\delta^{\frac{1}{1-\gamma}}\alpha_N^{\frac{\gamma}{1-\gamma}}\right)\left(\dfrac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1-\gamma}{-\gamma}}-1\right)$$

To calculate elasticities:

$$\frac{\partial R}{\partial\overline{\alpha}_N} = \frac{1-\gamma}{-\gamma}\frac{\gamma}{1-\gamma}\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}-1}\left(\dfrac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}W(1-\delta)^{\frac{1}{\gamma}}\alpha_L\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}+\left(\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}}-\delta^{\frac{1}{1-\gamma}}\alpha_N^{\frac{\gamma}{1-\gamma}}\right)\left(\dfrac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1-\gamma}{-\gamma}-1}$$

$$\frac{\overline{\alpha}_N\partial R}{R\partial\overline{\alpha}_N}=\frac{-\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}}(1-\delta)^{\frac{1}{\gamma}}\alpha_L\left(\dfrac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}+\left(\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}}-\delta^{\frac{1}{1-\gamma}}\alpha_N^{\frac{\gamma}{1-\gamma}}\right)\left(\dfrac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1}{-\gamma}}}{\left((1-\delta)^{\frac{1}{\gamma}}\alpha_L\left((1-\delta)^{\frac{1}{1-\gamma}}\overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}+\left(\delta^{\frac{1}{1-\gamma}}\overline{\alpha}_N^{\frac{\gamma}{1-\gamma}}-\delta^{\frac{1}{1-\gamma}}\alpha_N^{\frac{\gamma}{1-\gamma}}\right)\left(\dfrac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}\right)^{\frac{1-\gamma}{-\gamma}}-1\right)}$$

If $\overline{\alpha}_N = \alpha_N$ then

$$\frac{\overline{\alpha}_N \partial R}{R \partial \overline{\alpha}_N} = \frac{-\delta^{\frac{1}{1-\gamma}} \overline{\alpha}_N^{\frac{\gamma}{1-\gamma}} (1-\delta)^{\frac{1}{\gamma}} \alpha_L \left(\frac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}} \left((1-\delta)^{\frac{1}{1-\gamma}} \overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}\right)^{\frac{1}{-\gamma}}}{\left((1-\delta)^{\frac{1}{\gamma}} \alpha_L \left((1-\delta)^{\frac{1}{1-\gamma}} \overline{\alpha}_L^{\frac{\gamma}{1-\gamma}}\right)^{\frac{1-\gamma}{-\gamma}} - 1\right)}$$

$$\frac{\overline{\alpha}_N \partial R}{R \partial \overline{\alpha}_N} = \frac{-(1-\delta)^{\frac{-1}{1-\gamma}} \delta^{\frac{1}{1-\gamma}} \overline{\alpha}_N^{\frac{1}{1-\gamma}} \alpha_L^{\frac{\gamma}{1-\gamma}} \left(\frac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}}{\left(\alpha_L \overline{\alpha}_L^{-1} - 1\right)}$$

$$\frac{\overline{\alpha}_N \partial R}{R \partial \overline{\alpha}_N} = \frac{-\left((1-\delta)^{\frac{1}{\gamma}} \delta^{\frac{-1}{\gamma}} \overline{\alpha}_N^{-1} \alpha_L \frac{L}{F}\right)^{\frac{-\gamma}{1-\gamma}}}{\left(\alpha_L \overline{\alpha}_L^{-1} - 1\right)}$$

If $\gamma = 0$ then we have Cobb-Douglas case:

$$\frac{\overline{\alpha}_N \partial R}{R \partial \overline{\alpha}_N} = \frac{-(1-\delta)^{-1} \delta}{\left(\alpha_L \overline{\alpha}_L^{-1} - 1\right)} = \frac{(1-\delta)^{-1} \delta}{\left(1 - \frac{\alpha_L}{\overline{\alpha}_L^{-1}}\right)}$$

What is the consequence of this exercise for the land supply curve? First, rent changes differently between different types of land. But the final effect on relative land productivity is the same for all types of land. This implies that a simplified formula, as used in LEITAP can be used to calculate the effects. But it is probably not the simple land supply curve as used in LEITAP. This does not seem very logical.

In summary, the land supply cuver idea with marginal productivity correction is correct if there is only difference in the land saving technology parameter. If there is a difference in the non-land technology parameter or in the fixed costs, the elasticity of land supply depends on the type of land used.

The rent on land is a residual. Even if mathematically the land supply is correct, the mechanism behind it differs from normal land supply. So, it may not be a bad idea to change the calculations consistent with the theory of rent if Ricardian theory is really relevant in explaining those rents.

Transport cost and urban pressure

The land supply curve is more complex than only Ricardian rent theory. First, a lot of marginal land is not only marginal because it has physical limitations, but also because investment in infrastructure is required before the land can be exploited profitably. This can be represented in the model by higher fixed costs. Sometimes the distance from the market is too far, which can be presented by higher variable costs. So, in principle this can be included into the model.

Sometimes it is argued that agricultural land prices near urban regions are more determined by urban pressure than by agricultural productivity. This may not be completely correct. First, the Von Thünen principle holds that land prices may vary with differences in transport cost to sales markets. This is agricultural in character, and can be simply explained by the theory of rent. But what is the consequence for agregate agricultural production? It implies that with more urbanization land prices will be higher and

therefore the intensity of production will be higher. But these higher prices can also be payed because these are higher because economic productivity is higher because of the lower transport (and maybe also communication) cost.

Urban pressure also has a different effect: there is demand for land for other than agricultural uses. This implies that land in the neighbourhood of places where land may be transferred in urban land may generate speculation on price rises. This is especially (or only?) the case if some zoning policy is active, implying that when land gets a different formal status land prices may increase a lot. When farmers anticipate on this price increase they may pay more for their land. But this higher price does not imply a higher rent, because they pay the higher price because of speculative motifs.
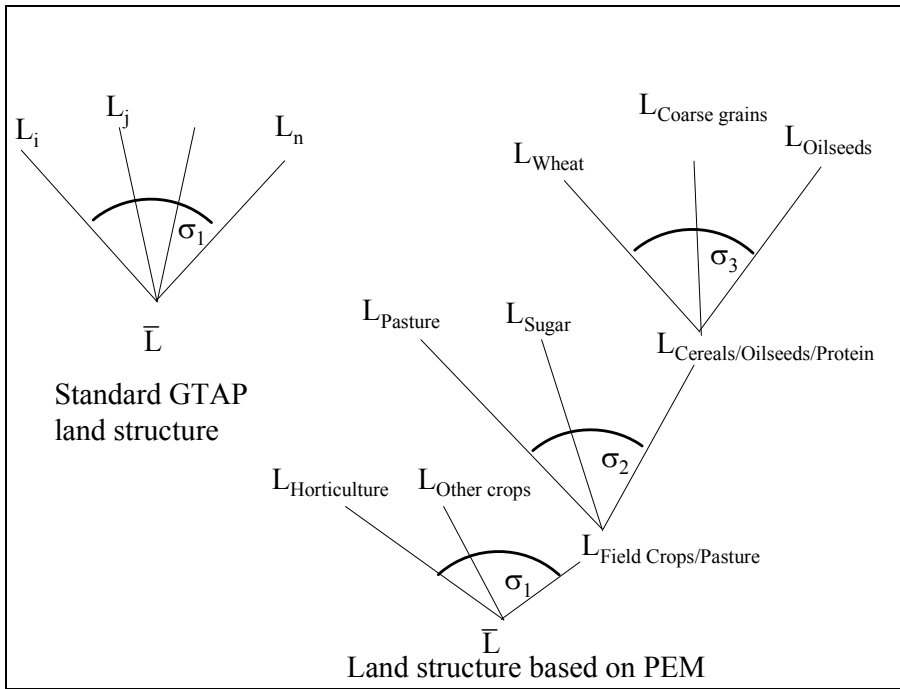
But the situation is more complicated than this. Near urban regions agricultural land use must decrease because of expansion of urban land use. But farmers don't stop their farm so easily. Therefore, they may stay even if they are farming at a loss if rewarded at market prices. Farm labour and farm capital stay in operation at below average market prices because of a relatively low factor mobility. As a consequence, it may turn out that the land price is too high for profitable agricultural production. But this is caused by a lack of non-land factor mobility, not because of urban pressure. This explanation implies that land prices and land rents have to be modeled dynamically, where farmers will quit gradually when land prices are too high, and therefore without new shocks (that occur in practice continuously) land prices would return to their zero profit value at normal labor and capital cost.

Although everything above can be put into a general equilibrium framework, and the Ricardian theory presented, it makes it much more complicated.

Theoretical considerations behind the CET land supply functions

The standard version of GTAP represents land allocation in a constant elasticity of transformation (CET) structure (left side of Figure 5) assuming that the various types of land use are imperfectly substitutable, but with equal substitutability among all land use types. Meijl et al (2006) extend this approach by account for the fact that the degree of substitutability differs between types of land (Huang *et al.*, 2004) using the more detailed OECD Policy Evaluation Model (PEM) structure (OECD, 2003) (right part of Figure 5). The extension of the standard GTAP model distinguishes different types of land in a nested three-level CET structure. The model covers several types of land use with different suitability levels for various crops (i.e., cereal grains, oilseeds, sugar cane/sugar beet, and other agricultural uses).

Figure 1: Land allocation tree in the extended version of GTAP

L_i L_j L_n

$L_{Coarse\ grains}$ $L_{Oilseeds}$

L_{Wheat}

σ_1

σ_3

$\overline{L}$

Standard GTAP
land structure

$L_{Pasture}$ L_{Sugar}

$L_{Cereals/Oilseeds/Protein}$

σ_2

$L_{Horticulture}$ $L_{Other\ crops}$

$L_{Field\ Crops/Pasture}$

σ_1

$\overline{L}$

Land structure based on PEM

The lower nest assumes a constant elasticity of transformation between 'vegetables, fruit, and nuts' (Horticulture), 'other crops' (e.g., rice, plant-based fibers), and the group of 'Field Crops and Pasture' (FCP). The transformation is governed by the elasticity of transformation, σ_1. The FCP group is itself a CET aggregate of cattle and raw milk (both aggregated under Pasture), 'Sugar,' and the group of 'Cereal, Oilseed, and Protein crops' (COP). Here, the elasticity of transformation is σ_2. Finally, the transformation of land within the upper nest, the COP group (i.e., wheat, coarse grains, and oilseeds), is modeled with an elasticity σ_3. In this way the degree of substitutability of types of land can be varied between the nests. Agronomic features are captured to some extent. In general it is assumed that $\sigma_3 > \sigma_2 > \sigma_1$, which implies that it is easier to change the allocation of land within the COP group and more difficult to move land out of COP production into, say, vegetables. The values of the elasticities are taken from PEM (OECD, 2003).

The CET-structure described above is an improvement compared with the general GTAP land function, but has the same theoretical disadvantages as this function. First, if the CET-function is used to transform these things

The rationale for the CET functional form may be interpreted in three ways (see for the first two Baltzer and Kloverpris, 2008: 20-1). First, different types of land have different productivities for different products. If the land is more or less allocated optimally to the different sectors, a change in allocation implies that the expanding has to use less productive lands, while the sector that reduces its land use may get rid of land with a lower than average productivity. As a consequence, a switch of production will generate an increase in the number of hectares needed.

Second, an increase in variety of production implies more opportunities for crop rotation, and therefore a higher land productivity. Although this may be the case between crop products, this can not be a defense for substitution between grassland and cropland.

Third, practice shows that farmers do not switch to different crop types immediately. A change in crop may require some investment or at least some adjustment in production processes. Especially for changes between grassland and cropland may require adjustments in the production process. Uncertainty and adjustment costs will delay this process, even if productivities may not be involved. To the extent that this dynamic process is relevant, a dynamic formulation of land adjustment would be required, implying that in the short run the effect of differences in rent between different sectors generate only small adjustments in land allocation, while after a long time the elasticity of transformation may be almost infinity.

The combination of the lines of reasoning above would imply a lower elasticity of transformation in the short run than in the long run. And it would imply an adjustment for the elasticity of substitution

In the current LEITAP implementation technological opportunities are not dependent on the share of land used for this product. Demand is determined by relative price, and it is assumed that relative price of a specific use of land increases with the share of land used by this product. This is not consistent with the first two arguments, and it could be conssitent with the adjustment cost argument in the short run. But in the long run it is not, because then price would go back to its original value. Baltzer and Kloverpris (2008) suggest a correction factor in hectares to adjust for this difference according to the CET function that is used. But this seems very ad hoc.

Based on the above line of argument, we could derive a more general theory of land substitution, based as much as possible on information about what happens in the real world. This will not be an easy exercise, but may improve the relevance of the model results a lot.

First, let us give an alternative formulation for the CET function. In the standard interpretation of a CET function the average productivity of land decreases for the expanding sector, and increases for the sector that reduces land use. Cloverpris et al (2008) suggest to put a correction factor into the model to correct for this phenomenon. But in practice the CET function may work differently in the model. An increase in land demand of one sector increases the price that has to be payed by this sector. Therefore, land use intensity increases. This may be explained by the dynamics of the land market. The land of your neighbours is not always for sales, so if you like to expand your production, you have to wait till you can buy extra land. In between you try to produce more through intensification. So, again, this is a dynamic mechanism, where the effect gradually fades away.
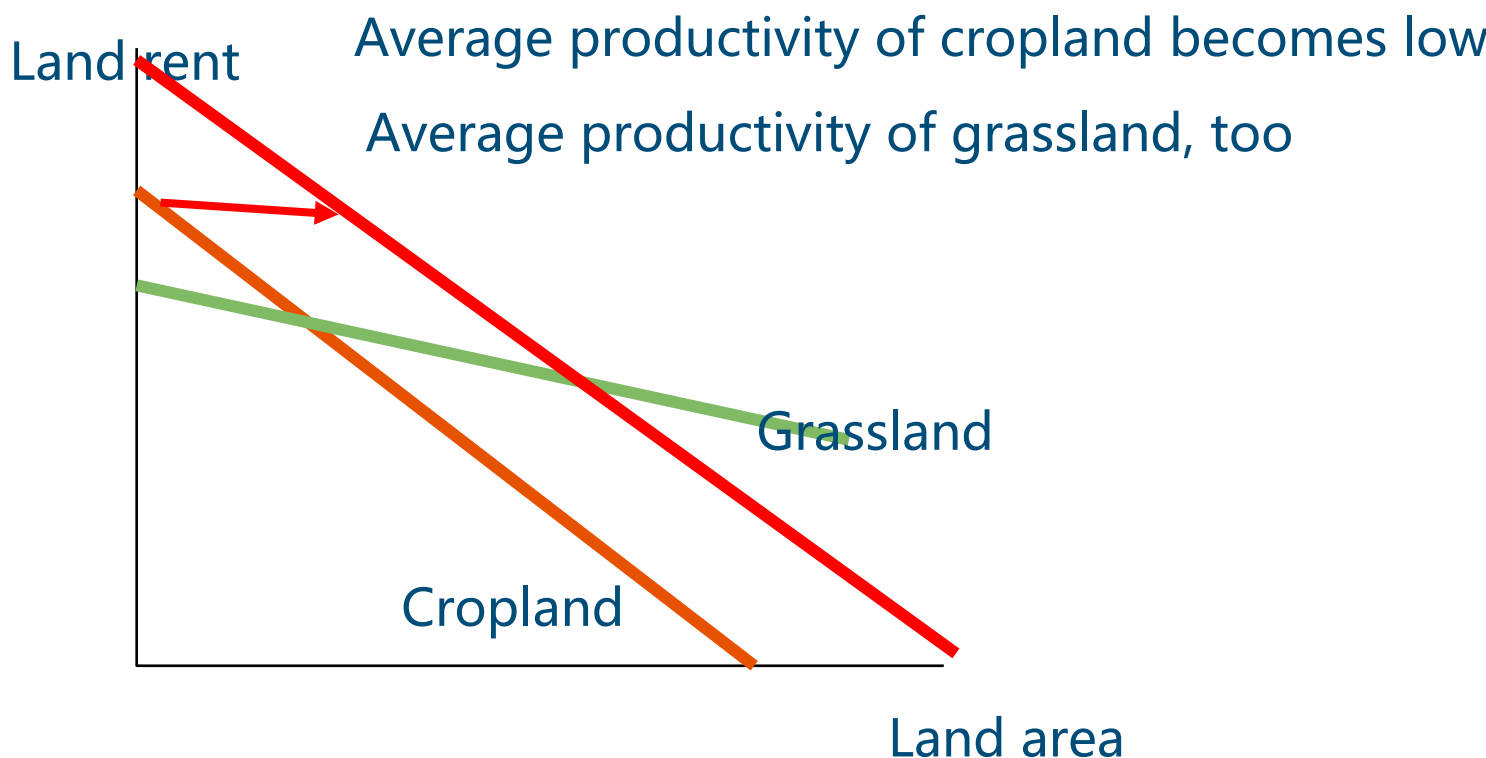
But the situation may be more coomplicated than this. Productivity of land differs, and there are indications that a lower productivity of land holds for most crops. For example (see figure …), the productivity of grassland may decrease less with a decrease in land quality than cropland productivity. Therefore, the most fertile land is used for crops. When crop prices increase and the crop sector expands, it will use the most

fertile grassland. So, for the crop sector average productivity will decrease, consistent with the CET function. But the grassland sector looses its most productive land, and therefore also in this sector average productivity will decrease, and this is in contrast with the CET function. The reduction in grassland use implies an increases in prices for grassland products, making more marginal land use profitable. So, when crop demand increases, the marginal land expansion is by the grassland, not the cropland. This has both consequence for the land supply function and for the land substitution mechanism in the model.

Conclusion

There are much more opportunities in GTAP to improve land supply than seems to be possible at first sight. It is possible to introduce heterogeneity of land in the model without making the model too complicated. It is also recommended to separate the land supply curve from the marginal productivity analysis and to tackle both problems in the model. The solution found can be defended theoretically, and is more consistent than the current approaches. But the approach also shows that empirical analyses about what happens in the real world is required to improve the further implementation of the ideas.

- The idea of the land supply curve derived from productivity differences on land is consistent with Ricardian theory under a CES production function.
- Correction of land productivities is not an easy task
- The effect of non-Ricardian factors requires some further research
- Correct land supply curves are essential for analyzing policies with large implications for land use, like biofuel policies.

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